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THE TOPOLOGICAL VERTEX

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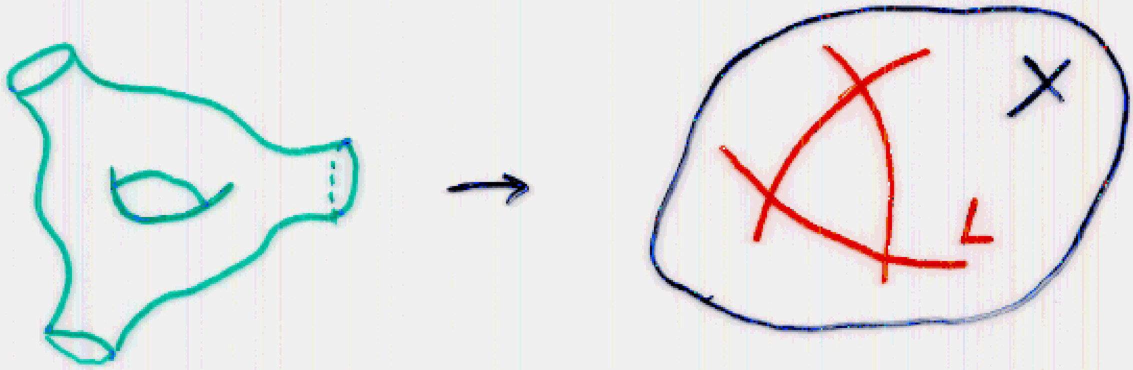
hep-th/0305132

TOPOLOGICAL A-MODEL AMPLITUDES ARE COUNTING HOLOMORPHIC MAPS



$$\Sigma_{g,h} \longrightarrow X \quad \text{- CY 3-FOLD}$$

$$\partial\Sigma_{g,h} \longrightarrow L \quad \text{- LAGRANGIAN}$$

THE CENTRAL QUESTION IS TO COMPUTE THE AMPLITUDES.

I WILL DESCRIBE THE EXACT ANSWER TO ALL GENERA

WHEN X IS A LOCAL CY FIBERED BY TORI.

THE ANSWER CAN BE GIVEN

IN TERMS OF A

LOCALLY DEFINED FREE CHIRAL SCALAR

ON A MIRROR RIEMANN SURFACE.

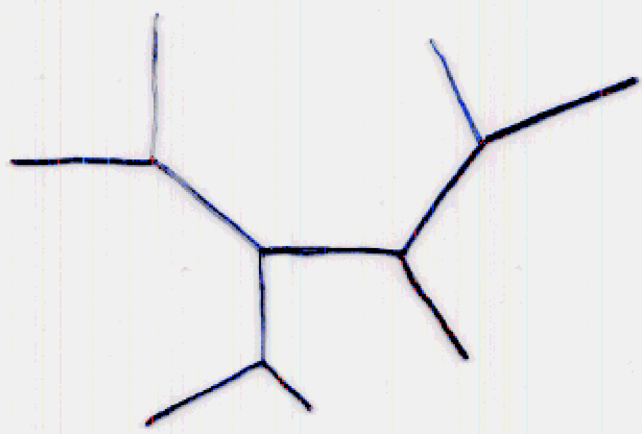
IT HAS IMPLICATIONS BEYOND THE

ORIGINAL QUESTION.

$T^3 \to X$
$\downarrow$
$B \simeq \mathbb{R}^3$

TO A LOCAL CY3 X FIBERED BY TORI

CORRESPONDS A GRAPH $\Gamma \in B$

Γ: 

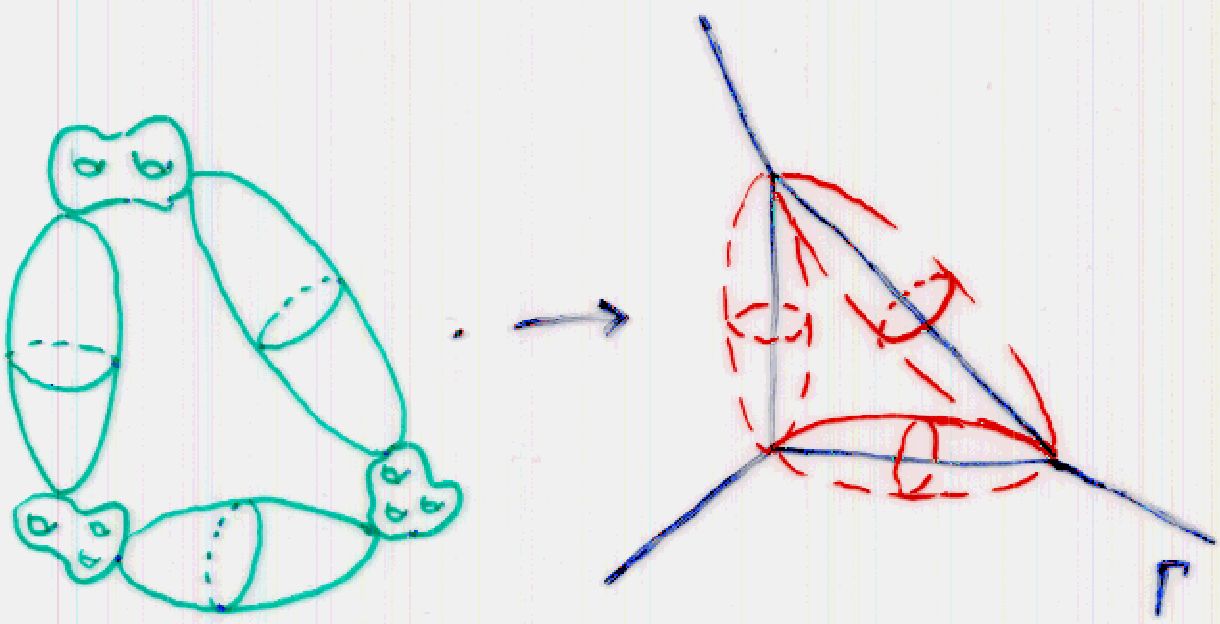
WHICH DESCRIBES WHERE THE T^3 DEGENERATES.

- EACH EDGE IN $\Gamma \Rightarrow \mathbb{P}^1$ IN X.

- ALL THE VERTICES ARE TRIVALENT.

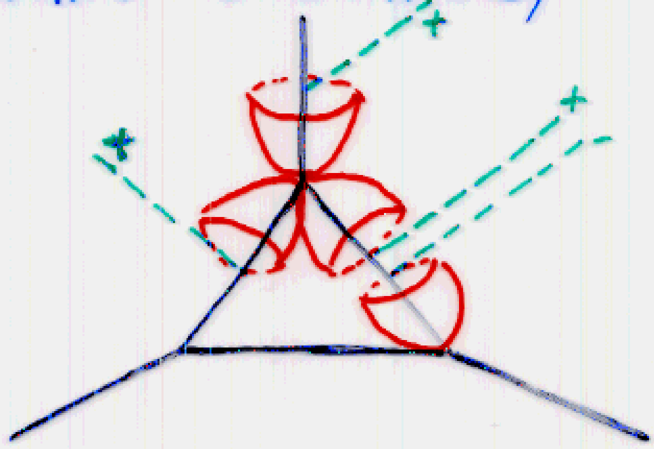
BY LOCALIZATION (KONTSEVICH, '94)

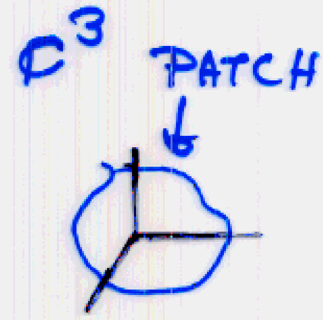
MAPS WHICH CONTRIBUTE TO TOPOLOGICAL A-MODEL AMPLITUDES ARE THE DEGENERATE MAPS THAT FALL ONTO THE EDGES OF THE GRAPH Γ.



THIS SUGGESTS A NATURAL REFORMULATION:

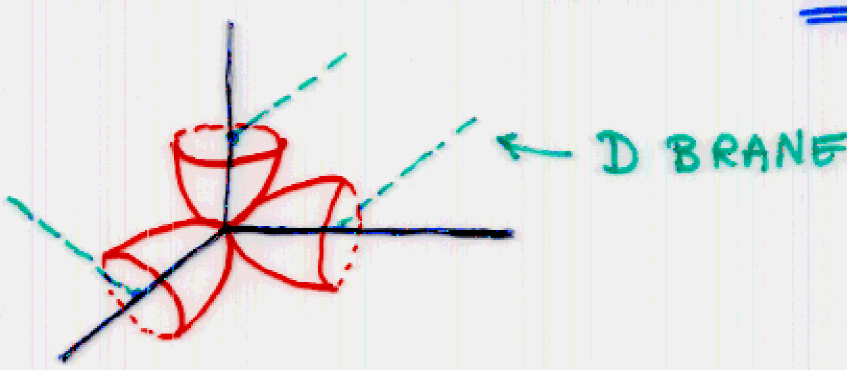
BY PLACING D-BRANES,



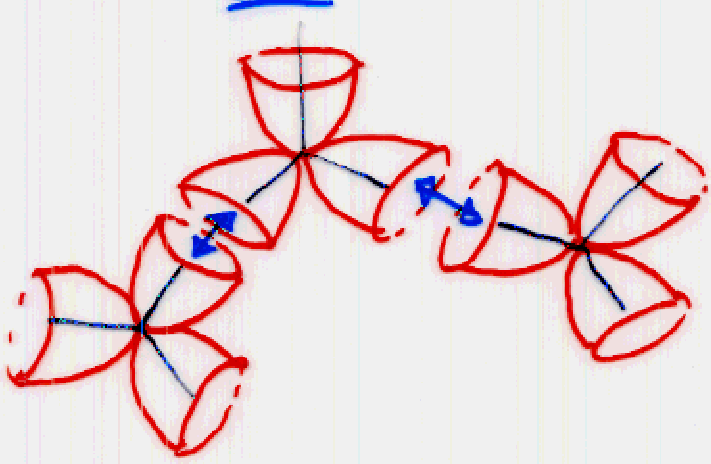


CUT Γ INTO 3-VALENT VERTICES.

IF WE KNOW THE AMPLITUDES ON <u>$\mathbb{C}^3$ WITH D-BRANES</u>

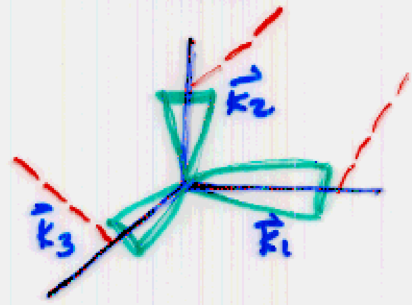


$\Rightarrow$ AMPLITUDES ON <u>ANY</u> TORIC CY3 BY <u>GLUING</u>:



WE NEED TO KNOW TOPOLOGICAL STRING AMPLITUDES

$$C_{\vec{k}_1,\vec{k}_2,\vec{k}_3}(g_s)$$

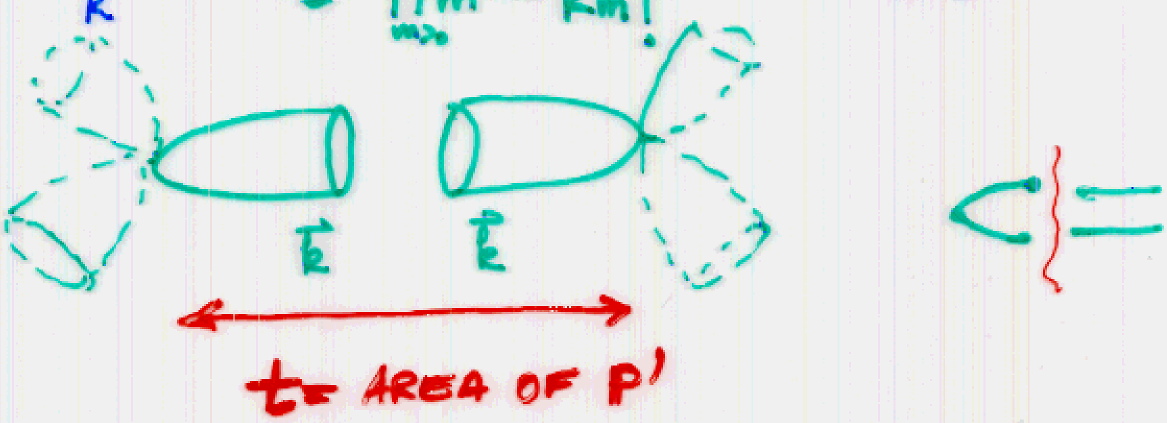


WHERE $\vec{k} = (k_1, \dots, k_m, \dots)$

k_m HOLES OF WINDING # m.

GLUING IS DONE BY MATCHING BOUNDARIES OF DISCONNECTED GRAPHS

$$\sum_{k} C_{\cdot,\cdot,\vec{k}} \frac{e^{-(\sum_m m k_m) t}}{\prod_{m>0} m^{k_m} k_m!} C_{\vec{k},\cdot,\cdot}$$



THE $\mathbb{C}^3$ AMPLITUDE WITH D-BRANES
CAN BE EVALUATED EXACTLY
TO ALL GENERA
BY USING A CERTAIN LARGE N
CHERN-SIMONS DUALITY (GOPAKUMAR-VAFA, '98)
AND TAKING $N \to \infty$ (g_s FIXED).

$$Z = \sum_{\vec{k}_i} C_{\vec{k}_1, \vec{k}_2, \vec{k}_3} \, \mathrm{Tr} V_1^{\vec{k}_1} \, \mathrm{Tr} V_2^{\vec{k}_2} \, \mathrm{Tr} V_3^{\vec{k}_3}$$

$$\mathrm{Tr} V^{\vec{k}} = \prod_m (\mathrm{Tr} V^m)^{k_m}$$

$$= \sum_R \chi_R (C(\vec{k})) \, \mathrm{Tr}_R V$$

$$\Rightarrow \quad C_{R_1 R_2 R_3} = \sum_{Q_1, Q_3} N_{Q_1, Q_3^t}^{R_1 \, R_3^t} \frac{W_{R_2^t Q_1} W_{R_2 Q_3^t}}{W_{R_2 0}} \, q^{\frac{k_{R_2} + k_{R_3}}{2}}$$

$\#$ OF TIMES $\quad Q_1 \otimes Q_3^t \rightarrow R_1 \otimes R_3^t$

$$W_{RQ} = \lim_{N \to \infty} \frac{S_{R\bar{Q}}}{S_{00}}$$

S MATRIX OF

$U(N)_k$ WZW MODEL

NOTE THAT THE VECTORS $\vec{k}$ ARE IN <u>1-1 CORRESPONDENCE</u> WITH <u>STATES</u> IN THE HILBERT SPACE $\mathcal{H}$ OF A <u>CHIRAL BOSON</u> ON S^1.

$$|\vec{k}\rangle = \prod_{m>0} (j_{-m})^{m} |0\rangle$$

AND THAT GLUING OPERATOR IS THE PROPAGATOR

$$|\vec{k}\rangle \frac{e^{-(\sum m k_m)t}}{\prod k_m! \, m^{k_m}} \langle \vec{k}|$$

$$= e^{-\sum_{m>0} \frac{t}{m} j_{-m}^{(1)} j_{m}^{(2)}} |0\rangle_{(1)} \otimes \langle 0|_{(2)}$$

t

<u>VERTEX:</u>

$$|C\rangle \in \mathcal{H}^{\otimes 3}$$

→ NATURAL EXPLANATION IS MIRROR SYMMETRY

MIRROR B-MODEL IS A THEORY OF VARIATIONS OF COMPLEX STRUCTURES ON MIRROR CY:

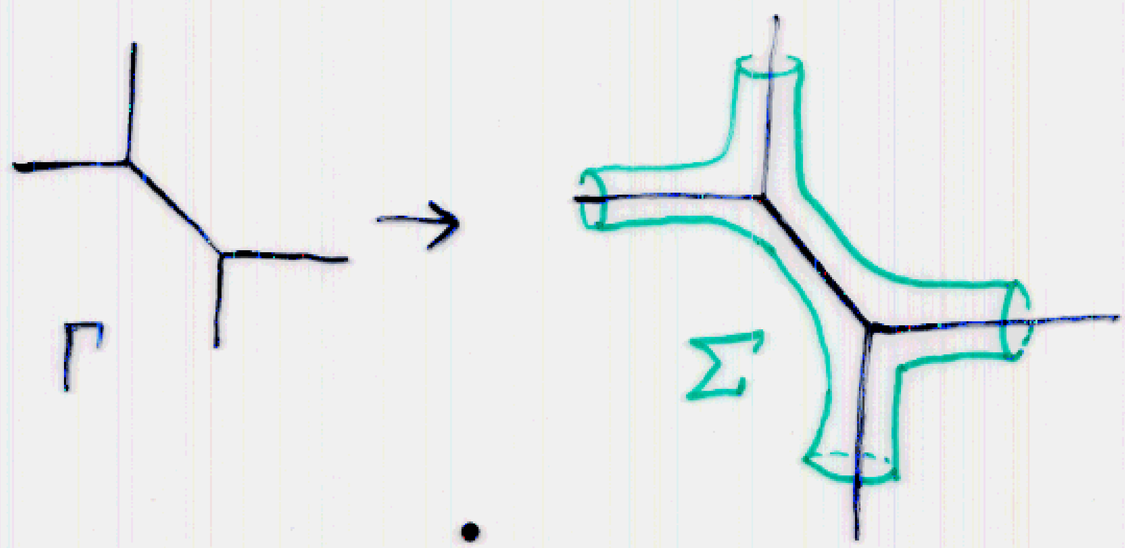
KODAIRA -SPENCER THEORY (B.C.O.V., '93)

$$xy = P(e^{-u}, e^{-v}) \in \mathbb{C}^2 \times (\mathbb{C}^*)^2$$

THESE ARE THE SAME AS VARIATIONS OF C.S. OF A RIEMANN SURFACE

$$\Sigma : \ P(e^{-u}, e^{-v}) = 0$$

HOLOMORPHIC 3-FORM ON X, $\Omega^{3,0}$ $\longrightarrow$ MEROMORPHIC 1-FORM λ ON Σ.



COMPLEX STRUCTURE DEFORMATIONS

DEFORM $\lambda \to \lambda + \delta\lambda$

TO ANOTHER MEROMORPHIC 1-FORM

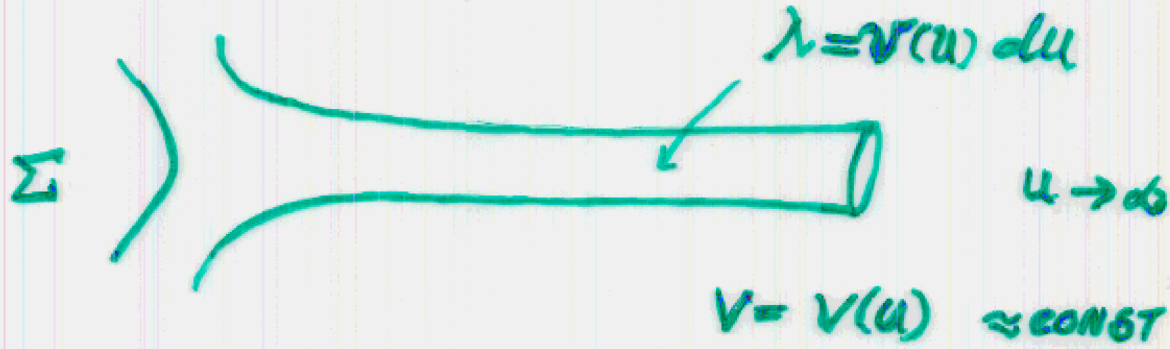
$$\bar{\partial}\,\delta\lambda = 0$$

$\delta\lambda$ CORRESPONDS TO A QUANTUM FIELD

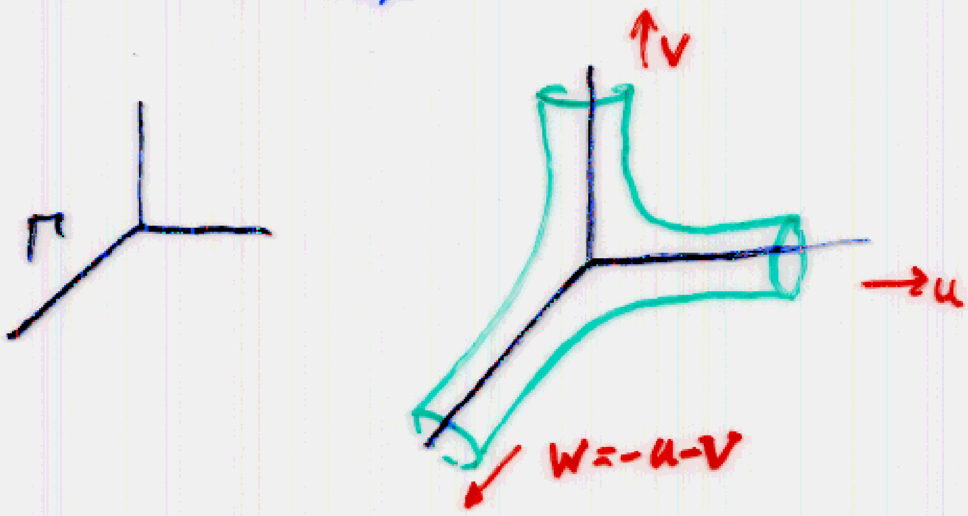
WHICH IS A CHIRAL SCALAR

$$\delta\lambda = \partial\varphi$$

AS λ IS DEFINED ONLY LOCALLY, SO IS φ.



E.G. THE VERTEX

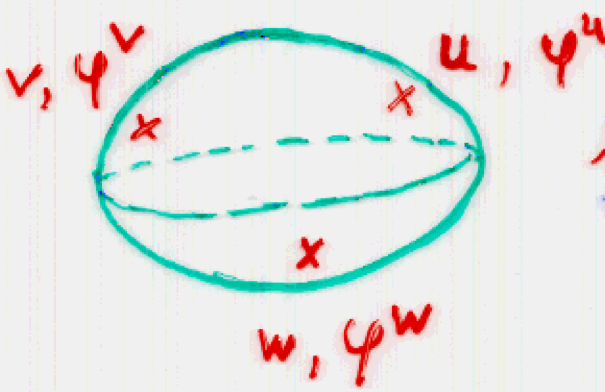


Σ IS A SPHERE WITH 3 PUNCTURES AND LOCAL COORDINATES PROVIDED BY MIRROR SYMMETRY:

$$e^v = -1 - e^u$$

$+\ \mathbb{Z}_3$ SYMMETRY $\quad u \to v \to w$

AND 3 CORRESPONDING SCALARS $\varphi^{u,v,w}$:



$$\underline{\lambda^u = v(u)\,du + \partial\varphi^u(u)}$$

$$\underline{|c\rangle \in \mathcal{H}^{\otimes 3}}$$

IN THE A-MODEL WE OBTAINED CLOSED STRING AMPLITUDES FROM OPEN STRINGS → BOSONISATION IN B-MODEL:

INSERTING A D-BRANE AT $u_* \in \Sigma$.

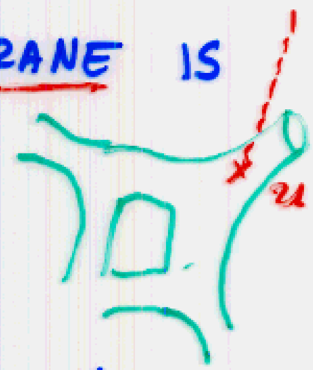
$$Z \to Z e^{\delta F}$$

$$\delta F = \int^{u_*} \lambda = \varphi_{cl}(u_*)$$

AGANAGIC, VAFA '00

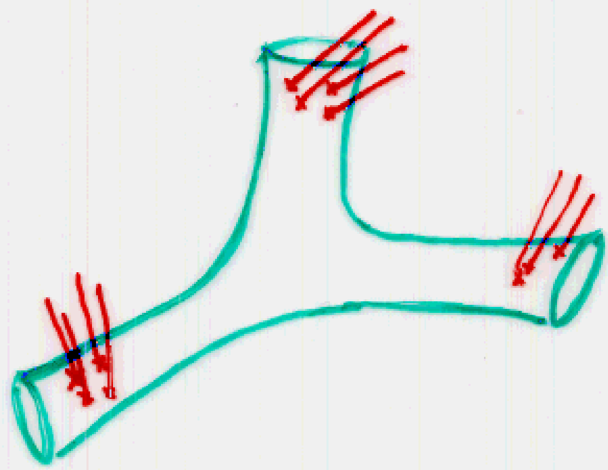
→ OPERATOR WHICH INSERTS A D-BRANE IS

$$e^{\varphi(u)} = \psi(u)$$



→ B-BRANE IS A FERMION!

INSERTING A LARGE # OF D-BRANES NEAR EACH OF 3-PUNCTURES

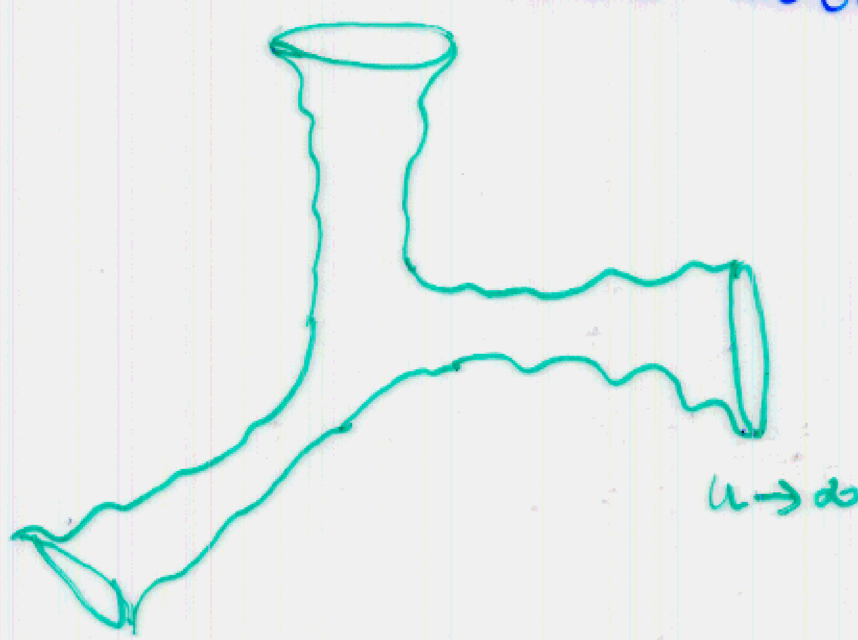


$\Leftrightarrow$ INSERTING COHERENT STATES $|t_i\rangle = e^{\sum \frac{\alpha_{-m}^{(i)} t_m^{(i)}}{m}} |0\rangle_{(i)}$

WHERE $t_m^{(i)} = \mathrm{Tr}\, V_{(i)}^m = \sum_a e^{m u_a^{(i)}}$

$V_i = P e^{\oint A_i}$
IN A-MODEL

WHICH EFFECTIVELY DEFORMS THE GEOMETRY:

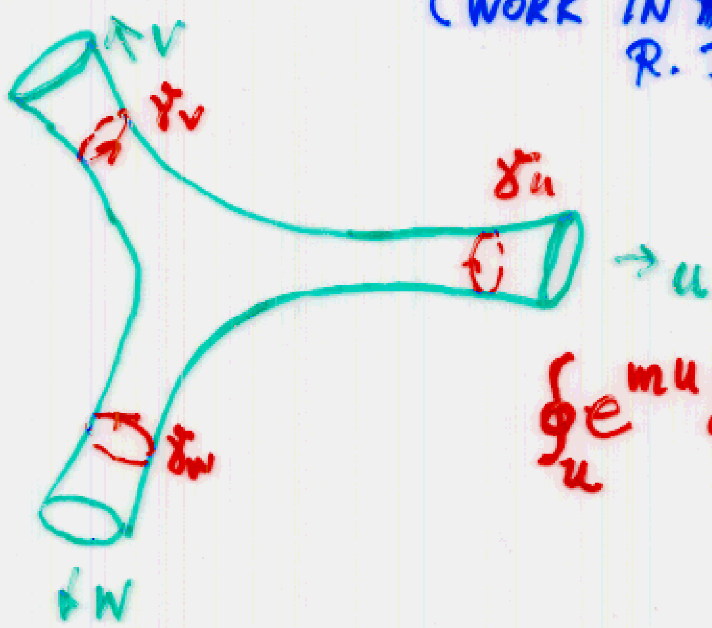


$$\partial\varphi^u = \sum_m t_m^{(u)} e^{mu} + \frac{\partial F}{\partial t_m^u} e^{-mu}$$

THE VERTEX $\langle t_1, t_2, t_3 | C \rangle$ IS HIGHLY CONSTRAINED; CORRESPONDING TO HAVING 3 FREE CHIRAL SCALARS DEFORMING THE COMPLEX STRUCTURE ON THE SAME RIEMANN SURFACE.

$\Rightarrow$ THERE ARE WARD IDENTITIES ARISING FROM THIS WHICH SUFFICE TO DETERMINE $|C\rangle$.

(WORK IN PREPARATION W/ R. DIJKGRAAF).



$$\oint_u e^{mu} \partial \varphi^u = \oint_u e^{mu} v(u)\, du$$

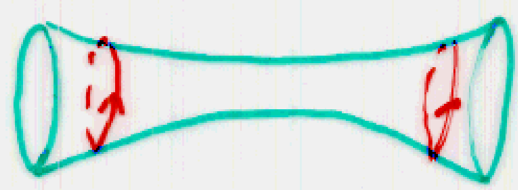
THIS IS IN THE SAME SPIRIT AS THE SOLUTION OF THE TACHYON SCATTERING IN $C=1$ STRING.

DIJKGRAAF, MOORE, PLESSER '92

THIS IS THE B-MODEL ON A RIEMANN SURFACE WITH 2 PUNCTURES

GOSHAL-VAFA

$\oint x^m \partial \varphi_x$



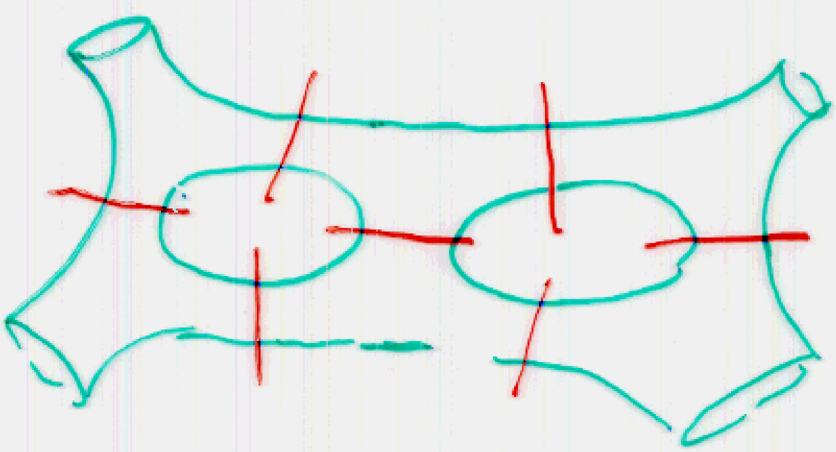
$xy = \mu$

AND W_∞ CONSTRAINTS ARE WARD IDENTITIES.

$\Rightarrow$ B-MODEL ON ANY RIEMANN SURFACE

(E.G. FROM MATRIX MODELS)

SHOULD BE SOLVABLE IN THIS WAY



Aganagic, Klemm, Vafa '01

OPEN STRING AMPLITUDES HAVE QUANTUM FRAMING AMBIGUITY

→ CHOICE OF INTEGER p

RECENTLY, M.C-C. LIU, K. LIU AND J. ZHOU PROVED THAT THE AMPLITUDES SATISFY A W_3 CONSTRAINT

$$\partial_p Z(p;t) = \sum_{m,n>0} mn\, t_{m+n} \partial_m \partial_n Z + t_m t_n (m+n) \partial_{m+n} Z$$

$$\partial_m = \frac{\partial}{\partial t_m},$$

$$t_m = \mathrm{Tr}\, V^m$$

THIS IS EASILY EXPLAINED PHYSICALLY BY CONSIDERING CHIRAL BOSON

FRAMING IS CHOICE OF COORDINATE IN u-PATCH

$$u \to u + pV = u + p\,\partial\varphi^u(u)$$

SINCE A CHANGE OF COORDINATE $u \to u + f(u)$ IS GENERATED BY

$$\oint f(u)T(u)$$

AND $T(u) = \frac{1}{2}(\partial\varphi(u))^2$

$\Rightarrow$ CUBIC GENERATOR

$$\sim \oint (\partial\varphi)^3$$