# Strings in a D-brane Pulse

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Strings 03, Kyoto

based on:

C.B. + C. Hull, th/0210269

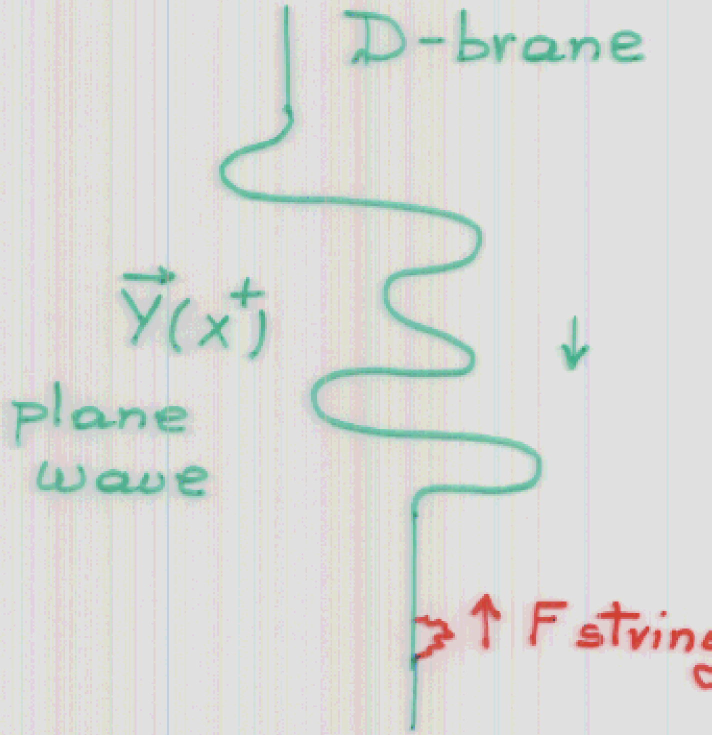
C.B., th/0212217

C.B. + M. Gaberdiel, th/0307xxx

work in progress:

with N. Couchoud

Problem I will consider, and solve in part :




Outline :

* Why interesting – put in context
* Why 'simple'; QM S-matrix exact
* Time-dependence & boundary states (subtleties of Wick rotn)
* prospects + 'hopes'

cf. also next talk (B. Pioline) for some related material

My own interest triggered by question: what is fate of spacelike or null singularities in string theory?

A setting in which question is sharply posed is space-time orbifolds:

$$\mathbb{R}^{1,9}/\{g\}$$

in Poincaré group

- $g$ = rotation 'understood'
- $g$ = boost (+ transln) hard to control
- $g$ = null boost (+ transln. by $b$) promising

Horowitz + Steif '90
Figueroa-O'Farrill + Simon '01

Liu + Moore + Seiberg
Lawrence
Fabinger + McGreevy
" + Hellerman
Robbins + Sethi '02
...

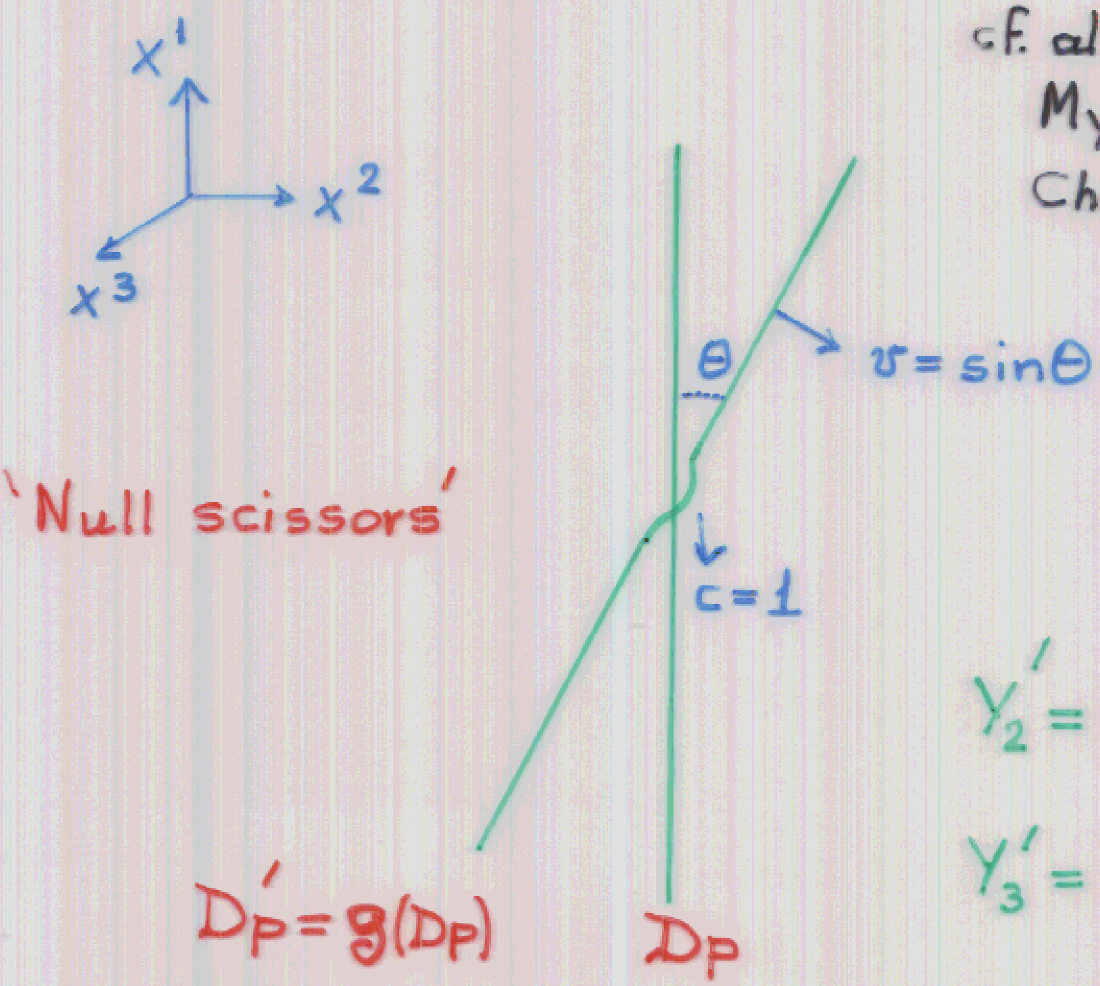
Promising because 'vacuum' stable

Susy only broken by matter falling through the singularity, so back-reaction could be finite + controlled.

Analogous strategy has proved very fruitful in study of stringy Black Holes.

With Chris Hull proposed to consider open-string analog of this problem:

cf. also
Myers + Winters
Chen + Chen + Lin
...




$$Y_2' = \tan\theta \cdot x^+$$

$$Y_3' = b$$

This is a sector of type-I generaliznts of the parabolic orbifold.

By focusing on the open strings one can, furthermore, disentangle 3 potential sources of divergent behavior:

* Light 'twisted' = stretched strings near singularity
* Graviton exchange
  disk versus annulus
* No asymptotic twisted states
  regularize by 'cutting-off' expansion: $y_3' = a \tanh \frac{x^+}{T}$


and try to focus on the first one.

A plane-fronted D-brane wave has $\infty$-dim. 'moduli space', corresponding to arbitrary profile

$$\vec{Y}(x^+) \quad \overset{T}{\longleftrightarrow} \quad \vec{A}(x^+)$$

with given total energy:

$$P^-_{\text{total}} = T_p \int \frac{dx'}{\sqrt{2}} \partial_+ \vec{Y} \cdot \partial_+ \vec{Y} \qquad \text{fixed.}$$

↳ This was appreciated in early days of dualities, and more recently

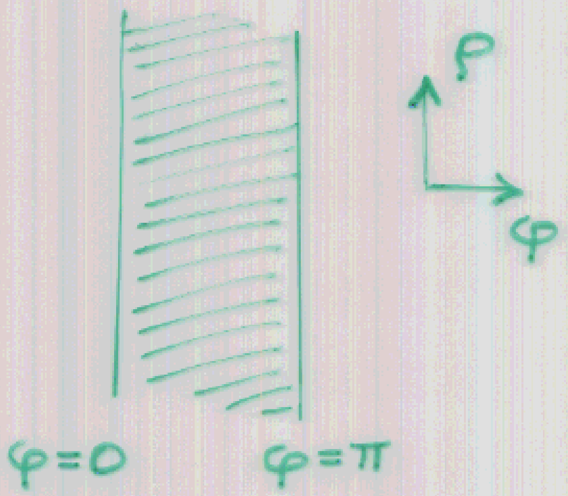
Callan, Maldacena, Peet '95
Dabholkar, Gauntlett, Harvey, Waldram '95
Mateos, Ng, Townsend '01, '02
Lunin, Mathur '01
Cho, Oh '01
Skenderis, Taylor . . . .

Will consider open strings in an arbitrary such background. Problem of more general interest:

* simple & 'malleable' time dependence
* interaction of strings with coherent radiation

Why problem is easy:

$$\delta \tilde{S} = \int_{-\infty}^{\infty} \frac{d\rho}{2\pi\alpha'} \left\{ \vec{Y}(X^+) \cdot \partial_\varphi \vec{X} \Big|_{\varphi=0} - \Big|_{\varphi=\pi} \right\}$$




assume both endpoints on same D-brane (generlz. is easy)

In light-cone gauge

$$X^+ = \underbrace{2\alpha'}_{=1} p^+ \rho$$

∴ profile function enters as source term, and there is

LINEAR RESPONSE

This should be compared to gravitnl waves, which in Brinkmann coords read

$$ds^2 = -2dx^+dx^- + d\vec{x}\cdot d\vec{x} + H(x^+,\vec{x})(dx^+)^2$$

with $R_{i+j+} = \partial_i \partial_j H$

worldsheet potential non-linear in $\vec{x}$

Amati + Klimcik '88
Metsaev '01
Papadopoulos + Russo + Tseytlin '02

In interaction representation:

$$S = \mathcal{T} \exp\left(i\int_{-\infty}^{\infty} d\rho\, H_I(\rho)\right)$$

$$= e^{i\delta_1 - \delta_2} :\exp\left(i\int_{-\infty}^{\infty} d\rho\, H_I(\rho)\right):$$

where: $H_I = \frac{\vec{Y}(p^+\rho)}{2\pi\alpha'}\cdot\left\{\partial_\varphi \vec{X}_{in}(0,\rho) - \partial_\varphi \vec{X}_{in}(\pi,\rho)\right\}$

& $\vec{X}_{in}(\varphi,\rho)$ is the non-interacting 'in-field' with standard →

Dirichlet b.cns:

$$\vec{X}_{in}(\varphi,\rho) = \vec{x} + \sum_{n\neq 0} \frac{i}{n}\vec{a}_n e^{-in\rho} \sin(n\varphi)$$

NB1 Expression valid for superstring since in NSR light-cone gauge the Fermionic completion $\propto \int \vec{\psi}\cdot\partial\vec{Y}\,\psi^+ = 0$

NB2 Under T-duality $\frac{\vec{Y}}{2\pi\alpha'} \to \vec{A}$, $\partial_\varphi \vec{X} \to \partial_\rho \vec{X}$ & some care with zero modes.

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The real & imaginary phase-shifts read:

$$\delta_1 + i\delta_2 = i\int \frac{d\rho\, d\rho'}{\pi^2}\, \vec{Y}(p^+\rho)\cdot\vec{Y}(p^+\rho') \times$$

$$\times\, \partial_\rho \partial_{\rho'} \left\{ D_F^{str}(\rho-\rho',0) - D_F^{str}(\rho-\rho',\pi) \right\}$$

with

$$D_F^{str} = -\theta(\rho-\rho')\log\left(1 \mp e^{i(\rho'-\rho)}\right) + (\rho \leftrightarrow \rho')$$

This is the FEYNMAN propagator with DD b.cns on a LORENTZIAN worldsheet.

In terms of the Fourrier components of the wave,

$$Y(x^+) = \int_{-\infty}^{\infty} \frac{dk}{k}\, y(k)\, e^{ikx^+},$$

one has:

$$S = e^{i\delta_1}\, e^{-\delta_2} : \exp\Big( \sum_{\substack{n \text{ odd} \\ >0}} \frac{4i}{n}\, \vec{a}_n \cdot \vec{y}\Big(\frac{n}{2\alpha' p^+}\Big)\Big)$$

time delay in classical limit

normalizn t

coherent outgoing state if in 'vacuum'

where:

$$\delta_2 = \sum_{\substack{n>0 \\ \text{odd}}} \frac{8}{n} \Big| \vec{y}\Big(\frac{n}{2\alpha' p^+}\Big)\Big|^2$$

What does this formula say ?

↳ the string is excited when the available CM energy is:

$$k_0 p^+ = \frac{n}{2\alpha'}$$ , as should be expected

$k_0$ : freq. of pulse

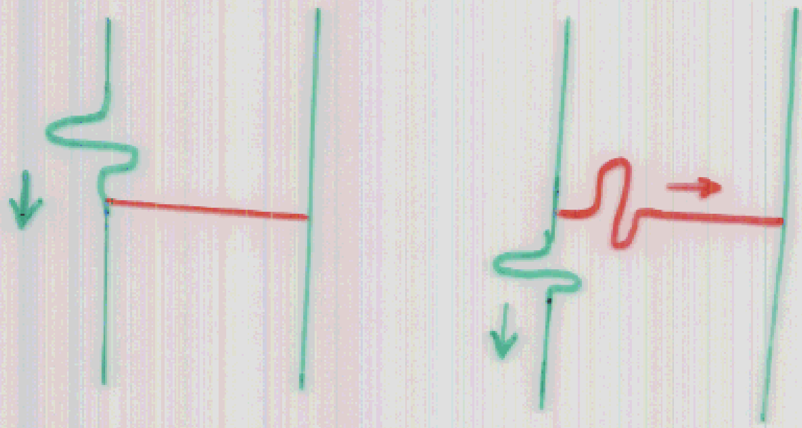
↳ in the limit $p^+ \to 0$, the string surfs adiabatically on the pulse. It can be elongated (for 2 D-branes) without being excited.

NB $\exists$ multipole expansion ($\leftarrow$ Euler-McLaurin formula) which could be used to put new constraints on non-abelian BI action.

↳ in the limit $p^+ \to \infty$ the string is 'effectively' very long ($L \sim \alpha' p^+$)

↪

the incident wave then sends a CLONE PULSE down the F-string:



As $k_0 \sim \frac{1}{\delta x^+} \to 0$, $\exists$ destructive interference between reflected waves & we recover the adiabatic limit discussed above.

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So this is an interesting BCFT, describing a time-dependent background, and having a non-trivial two-particle S-matrix. This is is non-singular for any finite-E pulse.

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To address the original questions, we would like to allow string emission ('back reaction') by turning on $g_s \neq 0$. Would like to know if process is finite (collinear divergences?) and/or whether potential divergences can be resummed.

Since we have not finished the calculations, I will not discuss this in the present talk.

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Will however comment on different question: how does channel duality work, or how does closed-string boundary state capture non-trivial time history of pulse?

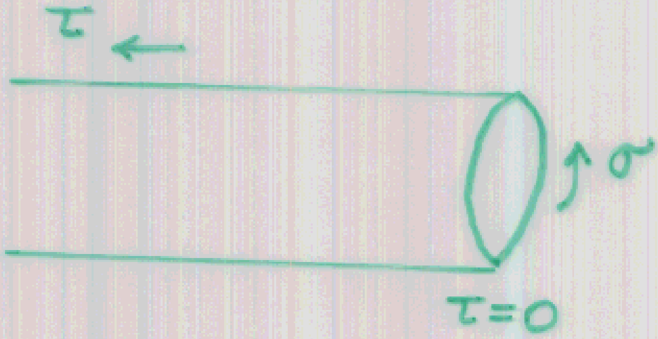
The bnry state for undulating D-branes has been written down recently by Hikida, Takayanagi * 2 (see also J. Blum)

For a BI wave it reads:

$$|B\rangle\rangle = P\exp\left(-i\int_0^\pi d\sigma\, \vec{A}(x^+)\cdot\partial_\sigma \vec{X}\Big|_{\tau=0}\right)|D_p\rangle\rangle$$

where:

$$X^\mu(\tau,\sigma) = x_c^\mu + p_c^\mu\tau + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}e^{-2in\tau}\cdot\left(\tilde{a}_n^\mu e^{2in\sigma} + a_n^\mu e^{-2in\sigma}\right)$$




Can be shown to impose correct b.cns $\left(\partial_\tau X^\mu - F^{\mu\nu}(x^+)\partial_\sigma X_\nu\right)|B\rangle\rangle = 0$

The Wilson-loop operator looks like the open-string S-matrix, but there are some significant diffs:

* $\sigma$-direction is spacelike
  $\Rightarrow$ path-ordering is trivial

* if we stay covariant, how does $p^+$ dependence get in?

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The way it all works in end is instructive. Let us illustrate it with tachyon $\rightarrow$ tachyon amplitude:

$$S_{0\to 0} = \mathcal{N} \langle 0| :e^{ipX(0,0)}: :e^{-ipX(\frac{\pi}{2},0)}: |B\rangle\rangle$$

normaliznt

normal ordering wrt open-string 'vacuum' (unperturbed ground state)

$$p^\mu p_\mu = \frac{1}{\alpha'}$$

Normal ordering for flat brane:

$$:e^{ipX}: = e^{ipx_c} e^{2ipX_<} e^{ip(X_> - X_<)}$$

where $X_{\gtrless} = \begin{cases}\text{positive}\\ \text{negative}\end{cases}$ part of $X^\mu$
frequency

(removes self-contractions because
$(X_> - X_<)|Dp\rangle\rangle = 0$ )

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Now using:

$$e^{C} g(B) e^{-C} = g(B + [C, B])$$

valid if $[C, B]$
commutes with $B, C$

and:

$$[X_>(\sigma), X_<(\sigma')] = -\frac{1}{4} \log(4 \sin^2(\sigma - \sigma'))$$

$$= D_F^{cyl}(\sigma, \sigma') + \text{constant}$$

can commute $X_<^\mu$ to left & $X_>^\mu - X_<^\mu$
to right till they hit $\langle 0|$ or $|Dp\rangle\rangle$.

Non-trivial part comes when $e^{ip^+(X^-_> - X^-_<)}$ commutes through Wilson-loop operator:

$$S_{0\to 0} = \frac{\mathcal{N}}{4} \langle 0| e^{-i\int d\sigma\, \partial_\sigma \vec{X}\cdot\vec{A}(x^+ - ip^+ \log\tan\sigma)} |D_p\rangle\rangle$$

imaginary argument shift

Now $X^+$ commutes with everything $\Rightarrow$ can replace by $x_c^+$.

Finally break $\partial_\sigma \vec{X} = 2\partial_\sigma \vec{X}_< + (\partial_\sigma \vec{X}_> - \partial_\sigma \vec{X}_<)$ & 'normal-order' to produce a phase:

$$S_{0\to 0} = \frac{\mathcal{N}}{4} \langle 0| \exp\Big\{ -\int d\sigma\, d\sigma'\, \partial_\sigma \partial_{\sigma'} D_F^{cyl}(\sigma,\sigma') \,*$$

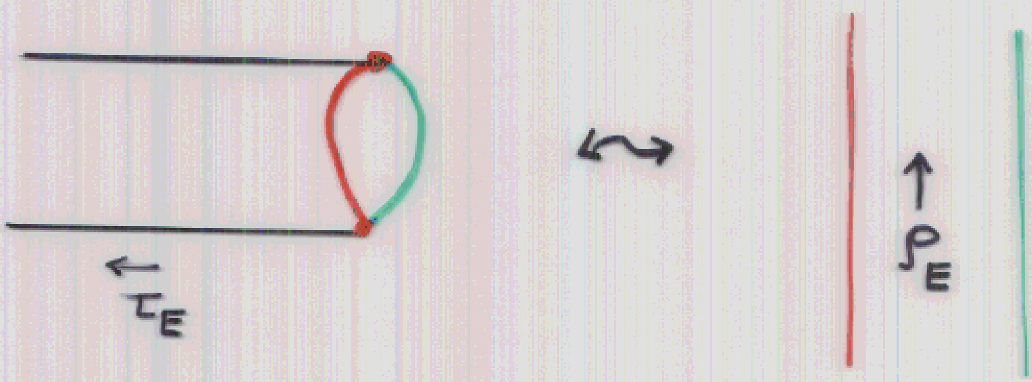
$$* \, \vec{A}(x_c^+ - ip^+ \rho_E(\sigma)) \cdot \vec{A}(x_c^+ - ip^+ \rho_E(\sigma')) \Big\} |D_p\rangle\rangle$$

with $\rho_E(\sigma) = \log\tan\sigma$

Now

$$\rho_E + i\varphi = \log \tan(\sigma + i\tau_E)$$

is the conformal mapping from the Euclidean half-cylinder to the Euclidean strip.




To get the correct S-matrix we must Wick rotate the worldsheet time of the strip.

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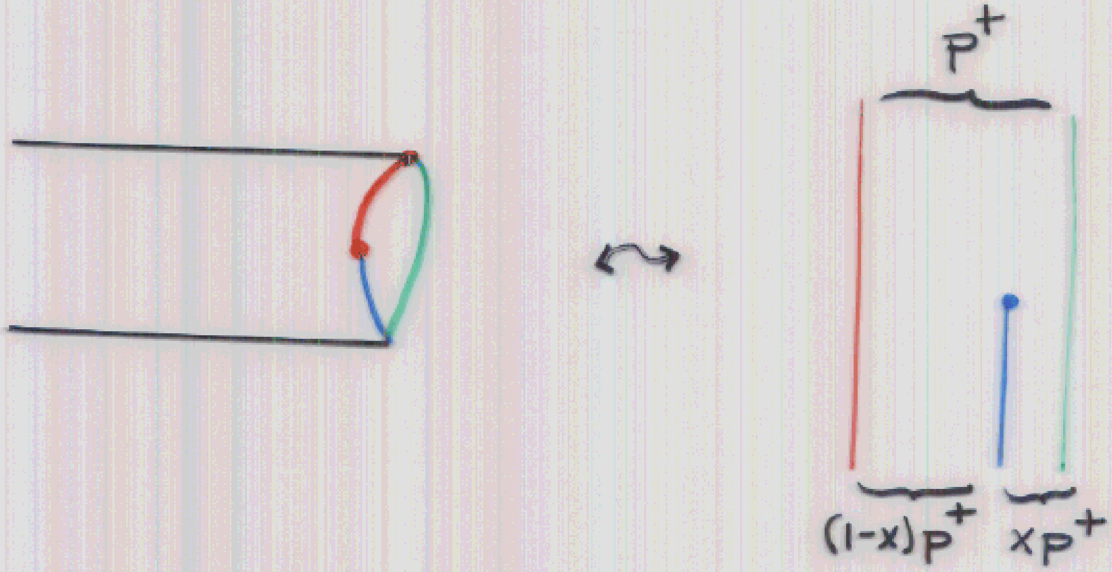
Repeating the argument for N-point amplitude, one finds $\rho_E(\sigma)$ = bnry restriction of conf. mapping between ½ cylinder with N marked points & light-cone gauge Euclidean worldsheet




CONCLUSION: Analytic continuation uniquely determined by causal structure of Lorentzian light-cone gauge worldsheet.

NB If no insertions

$\langle 0 | \mathbb{B} \rangle\rangle$, $\langle\langle \mathbb{B}' \| \mathbb{B} \rangle\rangle$ profile-independent

consistent with fact that open-string-field theory action $= Z_{disk}$ on-shell

Witten '93
Shatashvili

# Summary + prospects:

↳ Plane D-brane waves are soluble & malleable time-dependent backgrds. They differ significantly from conventional BCFTs (non-diagonal 2pnt fncs, ....) & should prove useful in understanding time-dependence & non-timelike singularities.

↳ Back-reaction as matter falls through resembles approach to thermal equilibrium of stringy black holes (except excnts interact only once). Can this be made precise ?

↳ Pulse/antipulse backgrnds can be used to regularize + control many unstable (non susy) backgrnds, such as brane collisions. Useful approach ?

# What about 'real world' ?

3 kinds of strings

fundml
QCD
cosmic

2 kinds of coherent radiatn. fields

electromagnt
gravitnl

↳ In laser beam: $p^+ \gtrsim \frac{1}{\alpha' \omega} \sim 10^8$ GeV for QCD

Flux in cosmic rays $\sim \frac{1 \text{ event}}{m^2 \times \text{year}}$

ridiculously small. But if hard X-ray or γ-ray lasers were available, possible to excite highly cosmic QCD strings on leading Regge trajectory ??

↳ Finally, although our model doesn't strictly-speaking apply, amusing to entertain idea that cosmic strings could transport like 'fibers' localized gravitnl bursts.