ENTROPY LITE

OR

A ROLE FOR BEKENSTEIN'S BOUND

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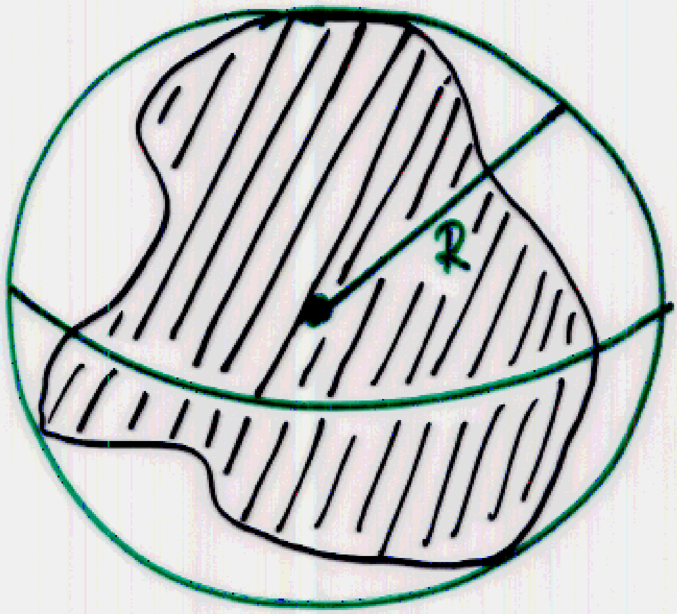
R.B., hep-th/0210295

R.B., to appear

For recent related work see also

A. Strominger & D. Thompson, hep-th/0303067

R.B., E. Flanagan & D. Marolf, hep-th/0305149 weakly gravitating system ($GM \ll R$)



Entropy S

Mass M

Radius R

$$S \leq \frac{2\pi M R}{\hbar} \ll \frac{\pi R^2}{G\hbar} = \frac{A}{4G\hbar}$$

Bekenstein 1974, 1981

holographic bound

't Hooft 1993

Susskind 1994

tight....

....lenient

Example :

massive particle (M)

size $R \sim \dfrac{\hbar}{M}$

holographic bound $\dfrac{A}{4G\hbar} \sim \dfrac{\ell_{Compton}^2}{\ell_{Pl}^2} \gg 1$

Bekenstein bound $\dfrac{2\pi M R}{\hbar} \sim \mathcal{O}(1)$

actual entropy $S \sim \mathcal{O}(1)$

QUESTIONS ABOUT THE BEKENSTEIN BOUND

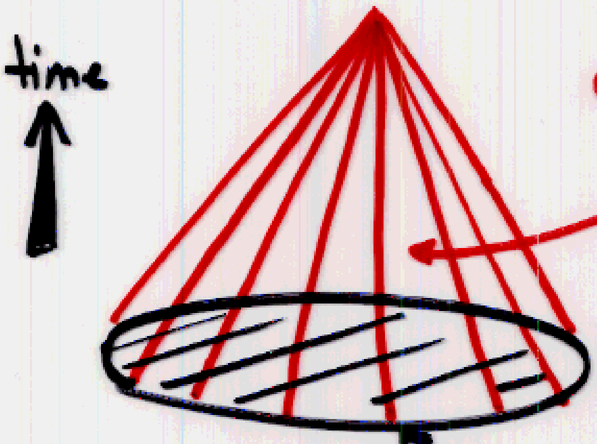
I. Is it true, and with what definition of S, M, and R?

II. Is it related to the holographic principle, and how?

III. What is the significance of the missing G?

DERIVATION OF BEKENSTEIN'S BOUND:

PRELIMINARIES



covariant entropy bound (RB 1999):

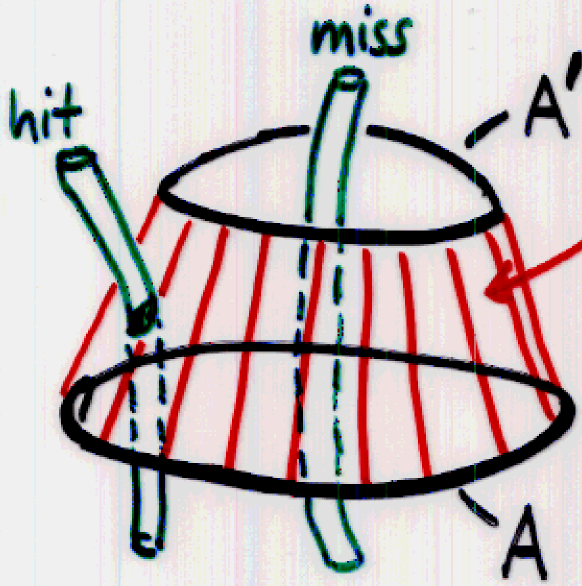
lightsheet L $(\theta \leq 0)$

$$S(L) \leq \frac{A}{4G\hbar}$$

surface, area A

- implies $S(V) \leq A/4G\hbar$ if gravity is weak
- implies GSL in black hole formation
- not strong enough to derive Bek.

generalization (Flanagan Marolf & Wald, 1999)



partial lightsheet L_p

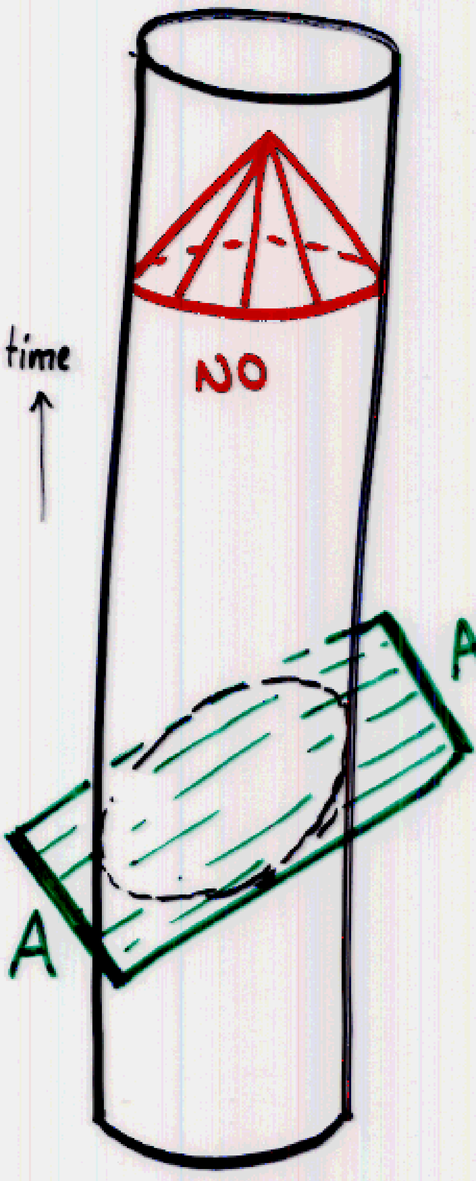
$$S(L_p) \leq \frac{A-A'}{4G\hbar}$$

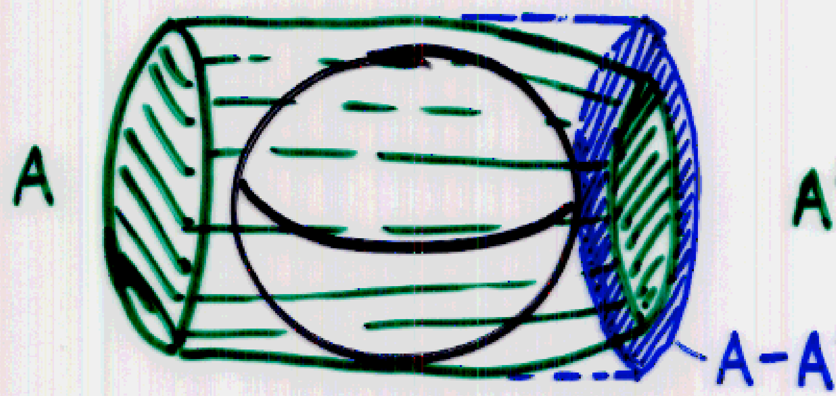
- implies GSL always
- implies Bek.!

DERIVATION OF BEKENSTEIN'S BOUND:

APPROACH

Choose initial surface astutely: $\theta_0 = 0$





angle $\sim \frac{GM}{R}$

annulus width $\sim GM$

annulus area $\sim GMR$

$$S \leq \frac{A-A'}{4G\hbar} \sim \frac{MR}{\hbar}$$

DERIVATION OF BEKENSTEIN'S BOUND:

PROOF

- Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = \underbrace{-\theta^2 - \sigma^2}_{\substack{\text{neglect} \\ (\theta_0 = 0,\ \text{weak gravity})}} - 8\pi G \underbrace{T_{ab} k^a k^b}_{\rho}$$

$$\rightarrow \theta \sim \int d\lambda \, G\rho$$

$$\rightarrow \Delta A \sim \int d^2x \int d\lambda \int d\lambda' \, G\rho$$

depends on detailed matter distribution ↯

- resolve by "X-raying" from both directions

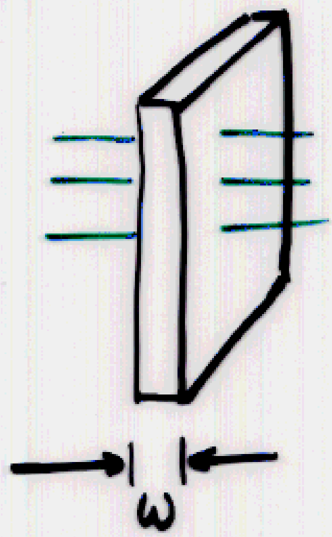
$$\rightarrow 2S \leq \frac{\Delta A_+ + \Delta A_-}{4G\hbar} \leq \frac{\left(\int d\lambda\right)\left(\int d^2x \, d\lambda' \rho\right) \cdot 2\pi}{\hbar}$$

$$\rightarrow S \leq \frac{\pi M \omega}{\hbar}$$

Derivation of Bekenstein Bound:

Result

- ω is the longest lightray width in the rest frame
- sphere $\rightarrow$ $\omega = 2R$
- generally $\omega \leq 2R$
- can be chosen the SMALLEST DIMENSION



$\rightarrow$ result stronger than Bek. bound

Testing the Bound

- Does Nature obey the Bekenstein bound?
- ... even with "width" ω instead of $2R$?
- Proof from simple phenomenological assumptions (not fundamental but common) R.B., Flanagan & Marolf 2003
  eliminates most potential counterexamples
- need domination by long wavelength modes, or violation of energy conditions

Specify M and ω.

Define $S \equiv \log\Big[\#$ of states with $E \leq M$, localized to region of width $\omega\Big]$

- construct challenges Unruh & Wald, Page, Unwin, ...

A) Species Problem

N scalars in spherical cavity

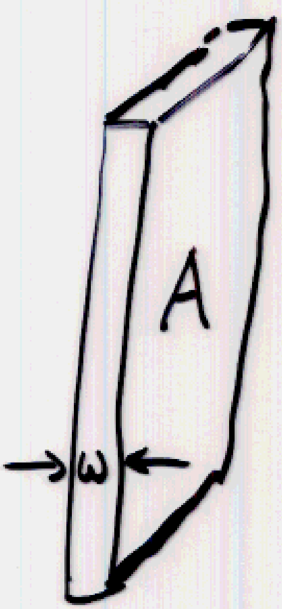
choose $M \sim \frac{\hbar}{w} \longrightarrow$ bound $\sim \mathcal{O}(1)$

$S = \log(1+N) \rightarrow \infty$ for $N \rightarrow \infty$ ↯

B) Casimir Problem

$$E(|0\rangle) < M < 0 \rightarrow \begin{cases} S \geq 0 \\ \text{bound} < 0 \end{cases}$$ ↯

C) Transverse Problem



$A \gg w^2, \quad M \sim \frac{\hbar}{w}$

$$\rightarrow \begin{cases} S \sim \log A \\ \text{bound} \sim \mathcal{O}(1) \end{cases}$$ ↯

(A) RESOLUTION (Bekenstein)

E must include all essential parts of complete, bounded system — also passive components such as confining matter

- add background field σ : $\ldots + \sum_{i=1}^{N} \lambda \phi_i^2 \sigma^2 + \ldots$
- construct bound state Fock space
- $E = E_\phi + \int (\nabla \sigma)^2$
- $(\nabla\sigma)^2 \gtrsim N \left(\frac{k_{max}}{\omega}\right)^2$

Casimir: $E_\sigma \gtrsim \frac{1}{\omega} \longrightarrow E(|0\rangle) > 0$ ✓

Species: $E_\sigma \gtrsim \frac{N}{\omega}$ while $S \sim \log N$ ✓

Transverse: $E_\sigma \gtrsim \frac{A}{\omega^3}$ while $S \sim \log A$ ✓

$\Rightarrow$ evidence that Bek. bound (even in $\pi M \omega$ form) is obeyed by all complete isolated systems.

Open questions:

- can't get no saturation?
- Is there a better definition of S, E, "isolated system"?
- What are the right definitions & why?

- should not be tied to quasistatic regime
- should involve null direction crucially
- should not fix anything but M and ω
- should not even fix M and ω separately ($\Delta A \sim M\omega$ is natural)
- hence, should not require that all states are at rest in the same frame

<u>Observation</u>: full result of derivation is

$$S \leq \pi \left(P_a k^a \right) \Delta\lambda$$

Light front frame

$\pi P_- \Delta\lambda$

$= \pi \mathcal{N}$ (number of partons)

rest frame

$\pi M \omega$

$\to$ Is $S = \log \#$ of states of DLCQ sector $\mathcal{N}$?

- not static
- involves null direction
- no extra information assumed
- fix only $\mathcal{N} = P_- \Delta\lambda$ & identify states under boosts

BUT: unclear how to resolve transverse problem

Absence of Newton's Constant

- consider holographic relation between information and geometry primary $\quad S \leq \frac{A - A'}{4 G \hbar} \quad (G, \hbar)$
- use classical GR $\quad G_{ab} = 8\pi G T_{ab} \quad (G)$
- derive Bekenstein bound $\quad S \leq \frac{\pi M \omega}{\hbar} \quad (\hbar)$
- which could be violated if one could have $\Delta x \Delta p \ll \hbar$
- discover that $\quad \Delta x \Delta p \gtrsim \hbar$