DUALITY-TWISTS, ORBIFOLDS & FLUXES

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TO APPEAR

STRINGS 2003

KYOTO

PLAN

1) GENERAL FORMALISM & MOTIVATION

2) LOW ENERGY POTENTIAL

3) ELLIPTIC TWISTS & ORBIFOLDS

4) PARABOLIC TWISTS & FLUXES

5) COMMENTS

MOTIVATION

1) GENERALIZATION OF TOROIDAL COMPACTIFICATION

2) INTRODUCES MASS PARAMETERS

- MODULI STABILIZATION

3) OFF-SHELL POTENTIAL ON THE MODULI SPACE THAT MAY NOT ADMIT FLAT SPACE OR AdS MINIMA BUT CAN HAVE NONTRIVIAL SOLNS e.g. DOMAIN WALLS

~ MASSIVE IIA THEORY

FORMALISM

GENERALIZATION OF SCHERK-SCHWARZ REDUCTION OF SUGRA

CONSIDER A THEORY IN (D+1) DIMENSIONS w/ SYMMETRY G

$$\psi \to g[\psi] \qquad g \in G$$

CIRCLE REDUCTION $\quad S^1 \times M_D$

$$\qquad\qquad y \qquad x^\mu$$

$$\psi(x, y) = g(y)\,[\psi(x^\mu)]$$

$$y \sim y + 2\pi R$$

y-DEPENDENT FIELDS BUT THE ACTION IN D-DIMENSIONS Y-INDEPT

$$g(y) = e^{\frac{My}{2\pi R}}$$

HAS A MONODROMY

$$\mathcal{M} = e^{M}$$

RESTRICTIONS

1) IN STRING THEORY ONLY $G(\mathbb{Z}) \in G$ THAT PRESERVES THE LATTICE OF INTEGRAL P-BRANE CHARGES IS A SYMMETRY

$$\mathcal{M} \in G(\mathbb{Z})$$

2) FOR ANY CONSTANT $h \in G(\mathbb{Z})$ $g(y)$ and $h\, g(y)\, h^{-1}$ GIVE

EQUIVALENT REDUCTIONS RELATED BY FIELD REDEFINITIONS

$$\Psi \longrightarrow h[\Psi]$$

$\Rightarrow$ INEQUIVALENT REDUCTIONS ARE CLASSIFIED BY CONJUGACY CLASSES OF DUALITY GROUPS $G(\mathbb{Z})$.

$$\mathcal{M} \sim h \mathcal{M} h^{-1}$$

IN STRING THEORY WE TYPICALLY ENCOUNTER $G(\mathbb{Z}) \sim$

$$SL(n;\mathbb{Z}),\ SO(p,q;\mathbb{Z}),\ E_{n,n}(\mathbb{Z})$$

STRATEGY

- FIND CONJUGACY CLASSES OF $G(\mathbb{Z})$. FOR A GIVEN TWIST $\mathcal{M}$. IN A CONJUGACY CLASS FIND M SUCH THAT $e^M = \mathcal{M}$

- THE LOW ENERGY IN D DIMENSIONS, e.g. THE SCALAR POTENTIAL ON THE MODULI SPACE IS COMPLETELY DETERMINED BY M.

SCALAR POTENTIAL

MODULI FIELDS PARAMETRIZE THE COSET G/K

K = MAXIMAL COMPACT SUBGROUP

VIELBEIN $\mathcal{V}(x) \in G$

$\mathcal{V}(x) \longrightarrow k(x)\, \mathcal{V}\, g$

k LOCAL, g GLOBAL

HERE $G = SL(n)$, $K = SO(n)$

$$L = -\frac{1}{2} \mathrm{Tr}\left[\mathcal{V}^{-1} D_m \mathcal{V}\, \mathcal{V}^{-1} D_m \mathcal{V}\right]$$

D_m = K-COVARIANT, LOCAL K REMOVES THE UNPHYSICAL D.O.F. OF $\mathcal{V}$

$$L = \frac{1}{2} \mathrm{Tr}\left[\partial_m \mathcal{H}^{-1} \partial^m \mathcal{H}\right]$$

$$\mathcal{H} = \mathcal{V}^t \mathcal{V} \qquad \text{K-INVARIANT}$$

$$\to g^t \mathcal{H} g$$

$$g(y) = e^{\frac{My}{2\pi R}}$$

$$\partial_y \mathcal{H} = M^t \mathcal{H} + \mathcal{H} M$$

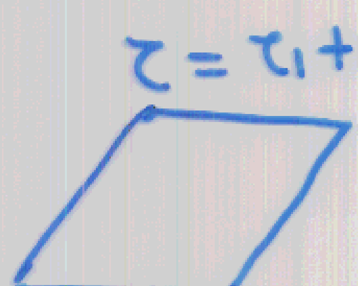
$$\partial_y \mathcal{H}^{-1} = -M \mathcal{H} - \mathcal{H} M^t$$

$$\boxed{V = e^{a\psi}\left(\mathrm{Tr}\, M^2 + M^t \mathcal{H} M \mathcal{H}^{-1}\right)}$$

$$G = SL(2) \qquad K = SO(2)$$

$$\tau = \tau_1 + i\tau_2$$

$$\mathcal{H} = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}$$



METRIC ON LATTICE OF CHARGES

$$G(\mathbb{Z}) = SL(2,\mathbb{Z})$$

ELLIPTIC $\longrightarrow$ ORBIFOLDS

$$M_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad M_3 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$$

$$M_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad M_6 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

M_n GENERATES SUBGROUPS OF FINITE ORDER

PARABOLIC $\longrightarrow$ FLUXES

$$M_{T_n} = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \qquad |\mathrm{Tr}\, M| = 2$$

HYPERBOLIC

$|\mathrm{Tr}\,\mathcal{M}| > 2$

$$\mathcal{M}_{H_n} = \begin{pmatrix} n & 1 \\ -1 & 0 \end{pmatrix} \to \text{FLUX + ORBIFOLD}$$

SPORADIC

$$\mathcal{M}(8) = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix} \qquad \mathcal{M}(10) = \begin{pmatrix} 1 & 4 \\ 2 & 9 \end{pmatrix}$$

$$\mathcal{M}(12) = \begin{pmatrix} 1 & 2 \\ 5 & 11 \end{pmatrix} \qquad \mathcal{M}(13) = \begin{pmatrix} 2 & 3 \\ 7, & 11 \end{pmatrix}$$

$$\mathcal{M}(14) = \begin{pmatrix} 1 & 2 \\ 6 & 13 \end{pmatrix} \quad \cdots\cdots$$

INTERPRETATION NOT CLEAR

ELLIPTIC TWISTS ~ ORBIFOLDS

$$\mathcal{M}_3 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \qquad M_3 = \frac{2\pi}{3\sqrt{3}} \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix}$$

$$V = e^{a\psi} \, \mathrm{Tr}(Y_3)^2$$

$$Y_3 = \frac{4\pi}{3\sqrt{3}\,\tau_2} \begin{bmatrix} \tau_2 - 2\tau_1\tau_2 & 1+\tau_1^2-\tau_2^2-\tau_1 \\ 1+\tau_1^2-\tau_2^2-\tau_1 & -\tau_2+2\tau_1\tau_2 \end{bmatrix}$$

$V \geq 0$ AND HAS A MINIMUM

AT $\tau = \tau_1 + i\tau_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

NOTE THAT THIS IS A FIXED PT OF $\mathcal{M}_3$ AND THE THEORY AT THE MINIMUM IS DESCRIBED BY AN ORBIFOLD. THIS GENERALIZES TO FINITE ORDER TWISTS OF ALL GROUPS

E1

USEFUL THEOREM:

FOR G NONCOMPACT, SEMISIMPLE EVERY FINITE ORDER SUBGROUP $H \subset G(\mathbb{Z}) \subset G$ IS CONJUGATE TO A SUBGROUP OF K

THERE EXISTS AN ELMENT $S \in G$ SUCH THAT $SHS^{-1} = K_1 \subset K$

COROLLARY:

$[S]$ IS A FIXED POINT OF H ON G/K

$g \sim kg$ IS THE EQUIVALENCE RELATION ON G/K

$$SH = K_1 S \sim S$$

$$\Rightarrow [S]\, H = [S]$$

$$G = SL(n) \qquad K = SO(n)$$

$$V(\Phi) = e^{a\psi}\left(\mathrm{Tr}\left(M^2 + M^t \mathcal{H} M \mathcal{H}^{-1}\right)\right)$$

$$\mathcal{H} = \mathcal{V}^t \mathcal{V}(\Phi)$$

$$\tilde{M} = \mathcal{V} M \mathcal{V}^{-1}$$

$$V(\Phi) = e^{a\psi}\, \mathrm{Tr}\left(\tilde{M} + \tilde{M}^t\right)^2 \geq 0$$

MINIMA OCCURS IF $\tilde{M} + \tilde{M}^t \Big|_{\phi = \phi_0} = 0$

THAT IS IF $\tilde{M}(\Phi_0)$ IS A ROTATION GENERATOR, OR $\tilde{\mathcal{M}}(\Phi_0) \in SO(n)$

$\Rightarrow$ IF THERE IS A POINT Φ_0 IN THE MODULI SPACE SUCH THAT

$$\mathcal{V}(\Phi_0)\, \mathcal{M}\, \mathcal{V}(\Phi_0)^{-1} = \tilde{\mathcal{M}}(\Phi_0) \in SO(n) = K$$

THE THM GUARANTEES THAT GIVEN $\mathcal{M}$ OF FINITE ORDER THERE EXISTS AN $S \in G$ SUCH THAT $S \mathcal{M} S^{-1} = \tilde{\mathcal{M}} \in K$

$\Rightarrow \nu(\Phi_0) = S$

MOREOVER Φ_0 IS A FIXED PT OF $\mathcal{M}$.

ENERGETICALLY, AT $\Phi = \Phi_0$ IN THE MODULI SPACE, IT COSTS NO GRADIENT ENERGY TO TWIST, AS YOU GO AROUND S^1

e.g. $S_3 = \sqrt{\frac{2}{\sqrt{3}}} \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \sqrt{3}/2 \end{pmatrix}$ $\quad \tau = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

PARABOLIC TWISTS

CONSIDER $SL(2,\mathbb{Z})$ ACTING ON THE τ-PARAMETER OF $\mathbb{T}^2_{x_1 x_2}$

$$\mathcal{M} = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \quad \text{GIVES METRIC}$$

$$dy^2 + \frac{1}{\tau_2} |dx_1 + \tau(y)\, dx_2|^2$$

$$\tau(y) = \tau_1 + i\tau_2 + ny$$

$$\begin{array}{c} \mathbb{T}^2_{x_1 x_2} \\ \downarrow \\ S^1_y \end{array}$$

$\mathbb{T}^2$ BUNDLE OVER S^1

$$\sim dy^2 + \frac{1}{\tau_2}(dx_1 + A)^2 + \tau_2 (dx_2)^2$$

$$A = (\tau_1 + ny)\, dx_2 \qquad F = n\, dy \wedge dx_2$$

S^1 BUNDLE OVER $\mathbb{T}^2$

$$\begin{array}{c} S^1_{x_1} \\ \downarrow \\ \mathbb{T}^2_{x_2 y} \end{array}$$

THIS TURNS ON n UNITS OF FLUX OF THE KK GAUGE FIELD. SIMILARLY ONE CAN TURN ON $H^{(3)}$ AND $F^{(3)}$ FLUX, BY DUALITY.

IN THIS SIMPLE CASE NO STABLE MINIMA OF THE POTENTIAL. BUT CAN HAVE DOMAIN WALL SOLNS

c/f IIB on S^1 w/ $SL(2,\mathbb{Z})$ PARABOLIC TWIST ~ MASSIVE TYPE IIA

HULL.

STABLE MINIMA ARE POSSIBLE AFTER INCLUDING NEGATIVE ENERGY OBJECTS LIKE ORIENTIFOLD PLANES.

~ FLUX COMPACTIFICATION

SL(4) TWISTS AROUND 2 CIRCLES

$$M_1 = \begin{pmatrix} 1 & n & 0 & 0 \\ & 1 & 0 & 0 \\ & & 1 & n \\ & & & 1 \end{pmatrix} \qquad S^1_{y_1}$$

$$M_2 = \begin{pmatrix} 1 & 0 & m & 0 \\ & 1 & 0 & m \\ & & 1 & 0 \\ & & & 1 \end{pmatrix} \qquad S^1_{y_2}$$

$$M_1 M_2 M_1^{-1} M_2^{-1} = \mathbb{1}$$

THIS IS DUAL TO $T^6/\mathbb{Z}_2$

ORIENTIFOLD W/ FLUXES

$$
\begin{aligned}
V \sim \Big(& |(1+\tau_1\tau_2)(1+\phi\bar{\tau}_3)|^2 \\
& + |(1+\tau_1\bar{\tau}_1)(1+\phi\tau_3)|^2 \Big) \\
& \qquad\qquad + 4 - 4
\end{aligned}
$$

$\tau_1\tau_3 = -1$

$\phi\tau_3 = -1$

ETC

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HYPERBOLIC TWISTS

SOME CAN BE VIEWED AS FLUX + ORBIFOLD

$$\begin{pmatrix} n & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

EQUIVALENT TO TURNING ON n UNITS OF FLUX ON A <u>SQUARE</u> TORUS, BECAUSE $\tau = i$ IS A FIXED PT OF $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

INTERPRETATION OF SPORADIC CLASSES NOT CLEAR.

ELLIPTIC TWISTS ARE EQUIVELENT TO TURNING ON MELVIN-TYPE FLUXES.

AT THE MINIMUM OF THE POTENTIAL, THE FLUXES VANISH.

$\Rightarrow$ NO CONTRDICTION WITH GKP RESULT ABOUT THE NECESSITY OF O-PLANES FOR STABLE MINIMA WITH FLUXES.