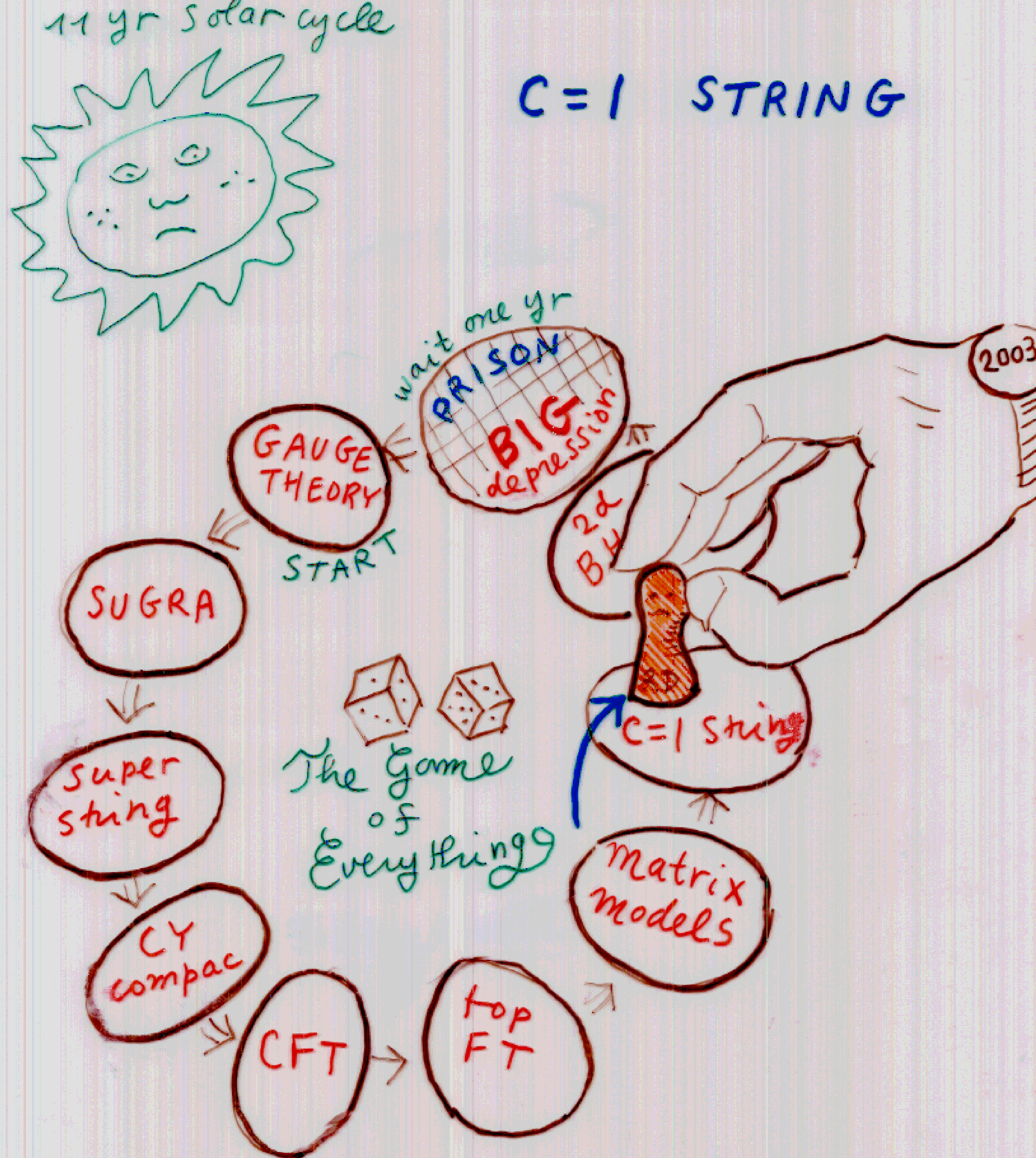
SUPERSYMMETRIC GAUGE THEORIES & MATRIX MODELS

w/ Cumrun Vafa

hep-th/0302011



Poincaré recurrences in hep-th space

"SUSY BOOTSTRAP"

$D = 4$
$\mathcal{N} = 1$ **SUSY**
F-terms

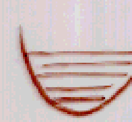
non-pert effects

SW curve
EFFECTIVE
GEOMETRY

pert
theory

hologr.

DYSON
gas of
eigenvalues



$D = 0$
"PLANAR"
Feynman
diagrams

$1/N$

large **N**
MATRIX
MODEL

[executive summary]

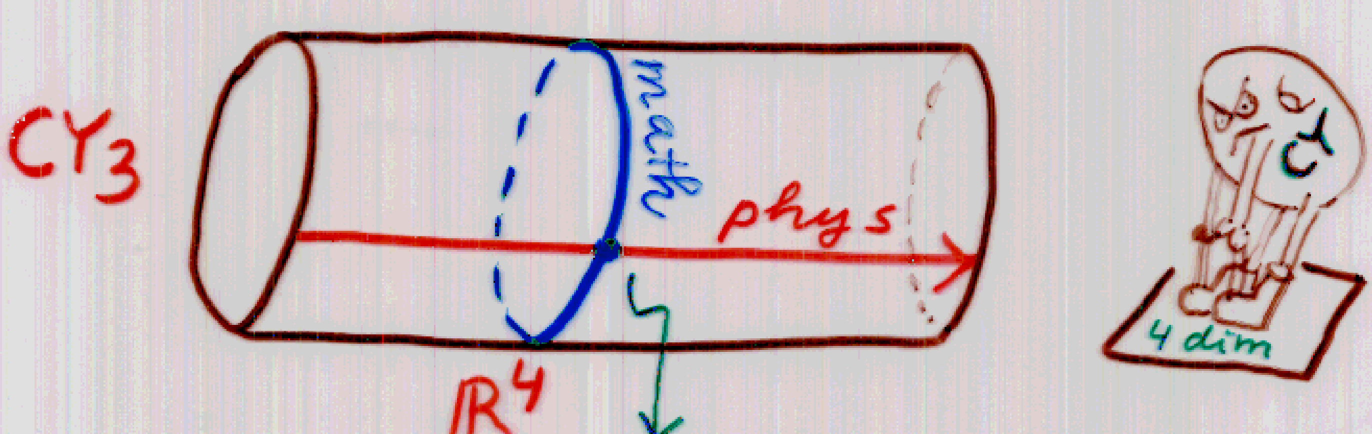
$\mathcal{N}=1$ GAUGE THEORY

F-terms $\int d^2\theta\, F(\phi)$

geom. engineer ↑ ↓ master field

CY_3 QUANTUM $(g_s^2=\hbar)$ GEOMETRY

(non-commutative ?)

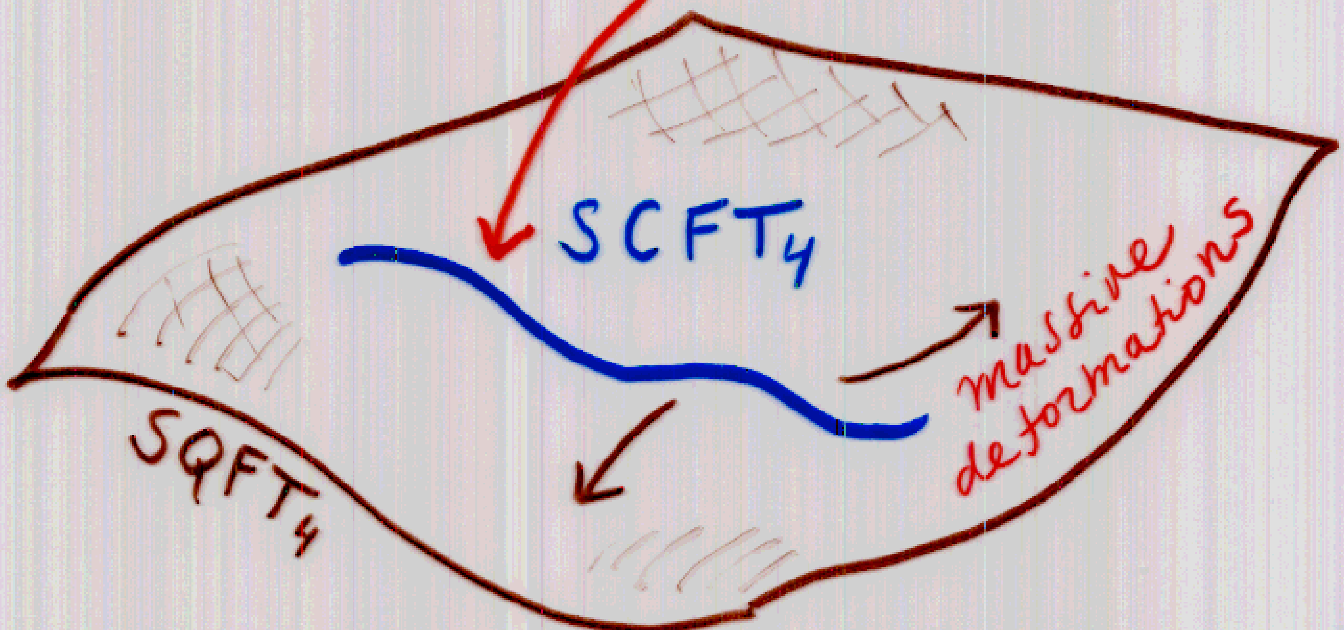


IR physics

massive theories

DO F-TERMS determine the world?

FANTASY: determine IR fixed points



True for SCFT$_2$ $\mathcal{N}=(2,2)$

simplest example $\mathcal{N}=2 \to \mathcal{N}=1$

[Cachazo-Intriligator-Vafa]

$U(N)$ + Adj Φ

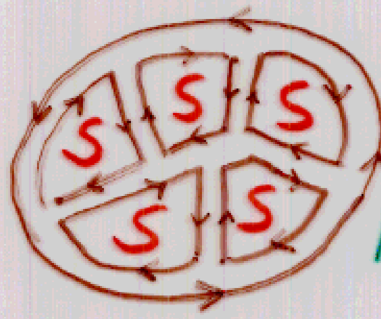
tree level superpotential

$$\mathrm{Tr}\, W(\Phi)$$

effective superpotential

$$S = \mathrm{Tr}\, \mathcal{W}_\alpha^2 \quad \text{(glueball)}$$

$$W_{eff} = \sum_{planar}$$



N

$$= N \frac{\partial \mathcal{F}_0}{\partial S}$$

$a_n \sim c^n$

ANALYTIC

$$\mathcal{F}_0 = \sum a_n S^n$$

matrix model free energy $\tilde{N} \to \infty$

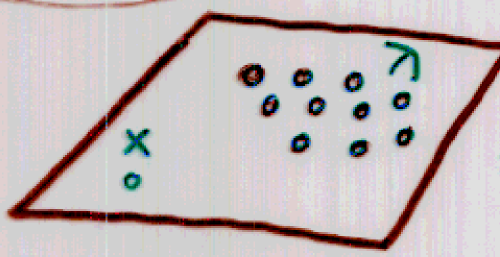
$$S = g_s \cdot \tilde{N}$$

MATRIX MODEL $\tilde{N} \times \tilde{N}$

$$e^{\mathcal{F}_0/g_s^2} \sim \frac{1}{\mathrm{vol}\, U(\tilde{N})} \int d\Phi \cdot e^{\frac{1}{g_s} \mathrm{Tr}\, W(\Phi)}$$

gas of eigenvalues

$$\int d\lambda \; \underbrace{\prod_{i<j} (\lambda_i - \lambda_j)^2 \, e^{\frac{1}{g_s} \sum W(\lambda_i)}}_{\exp S_{eff}(\lambda_i)}$$

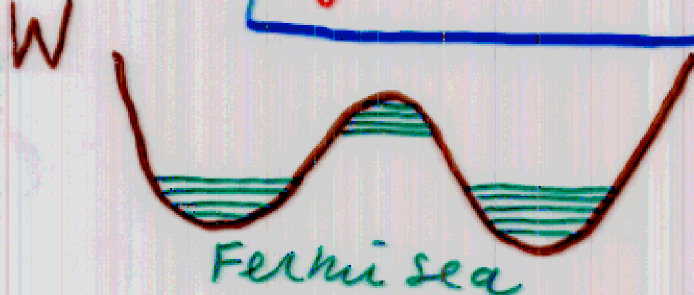


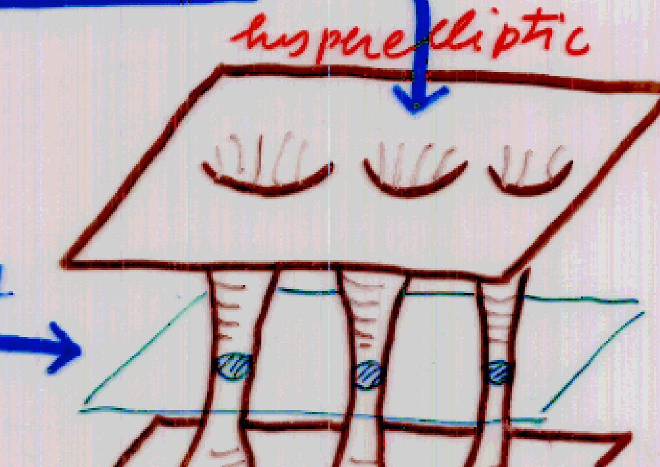
probe eigenvalue

$$y = \frac{\partial S_{eff}}{\partial x}(x, \lambda_1, \ldots, \lambda_{\tilde{N}})$$

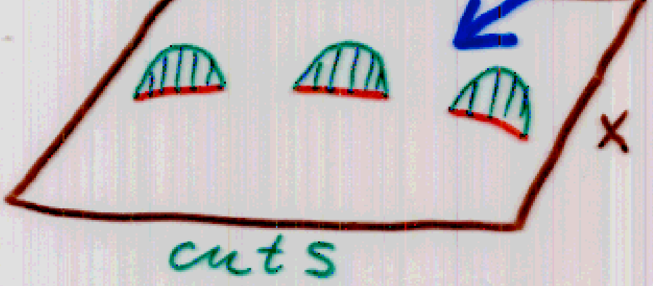
loop equation

$$\boxed{y^2 = W'(x)^2 + f_{qu}}$$



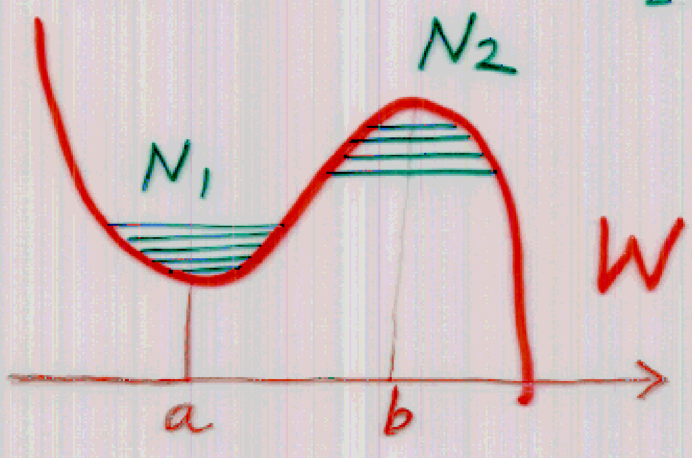


density



SYM BREAKING (multicuts)

[W. Gukov, Kazakov, Vafa



$$U(N) \longrightarrow U(N_1) \times U(N_2)$$

$$\Phi = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$$

ϕ_{12}, ϕ_{21} pure gauge

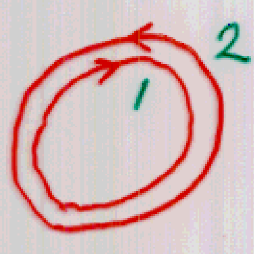
fix $\phi_{12} = \phi_{21} = 0$

GHOSTS $W = \mathrm{Tr}\, b\,[\phi, c]$

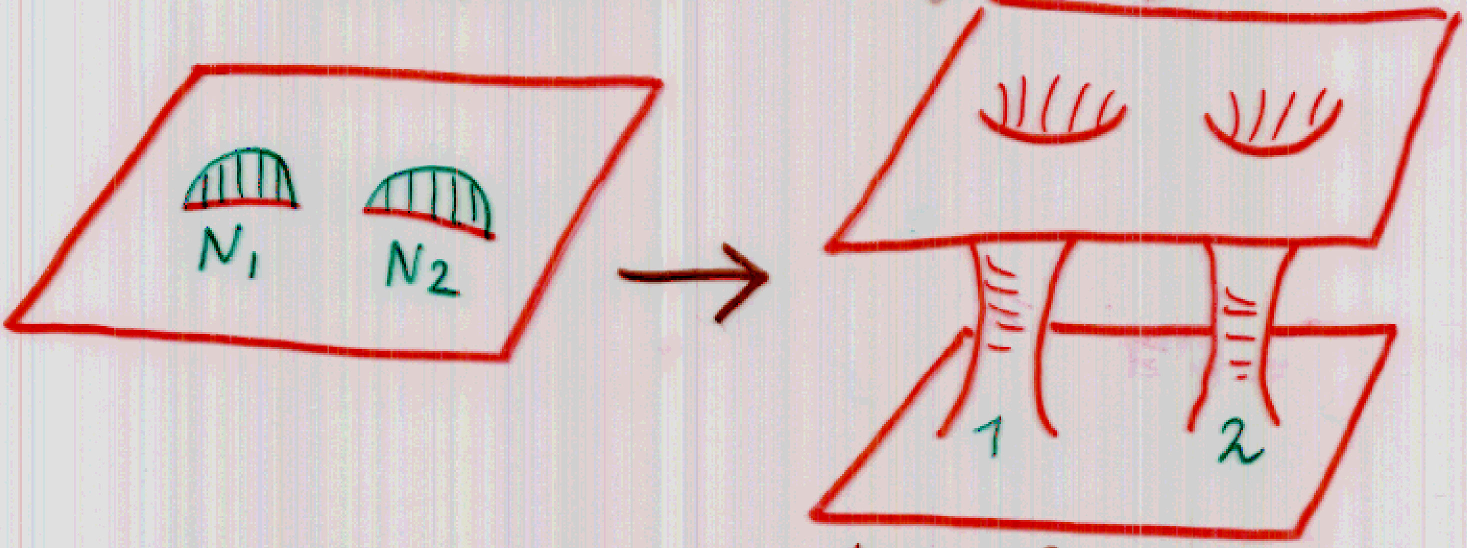
$$b = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix}, \quad c = \begin{pmatrix} 0 & c_{12} \\ c_{21} & 0 \end{pmatrix}$$

1-2 STRINGS fermions

ghost # conserved

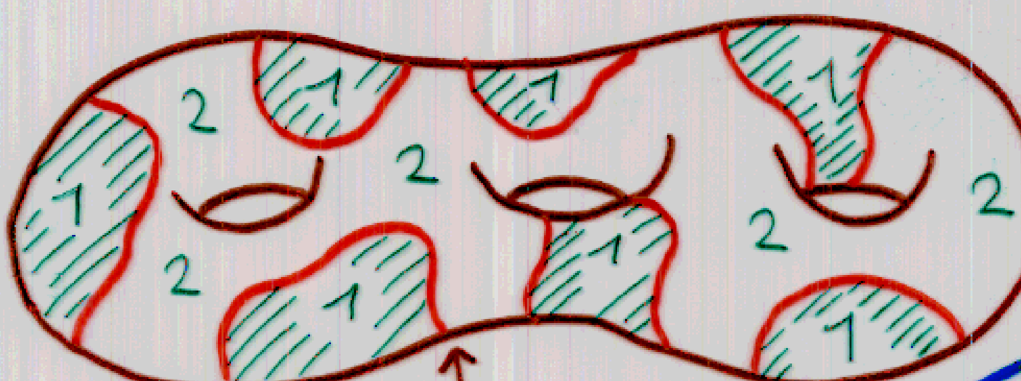


multicenter holography



two throats

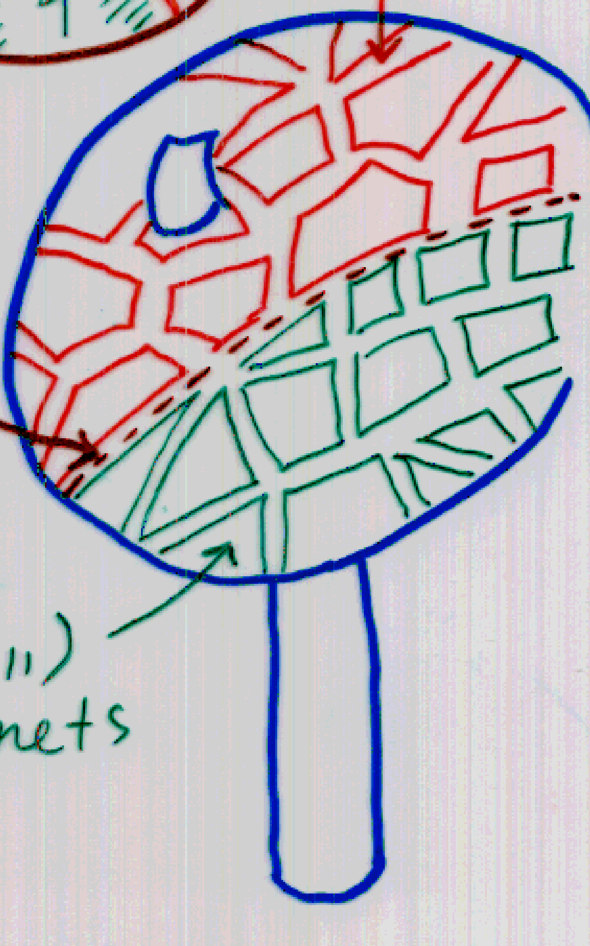
WORLD SHEET



$\tilde{W}(\phi_{22})$ fishnets

12-domain walls

patching worldsheet CFT's



$W(\phi_{11})$ fishnets

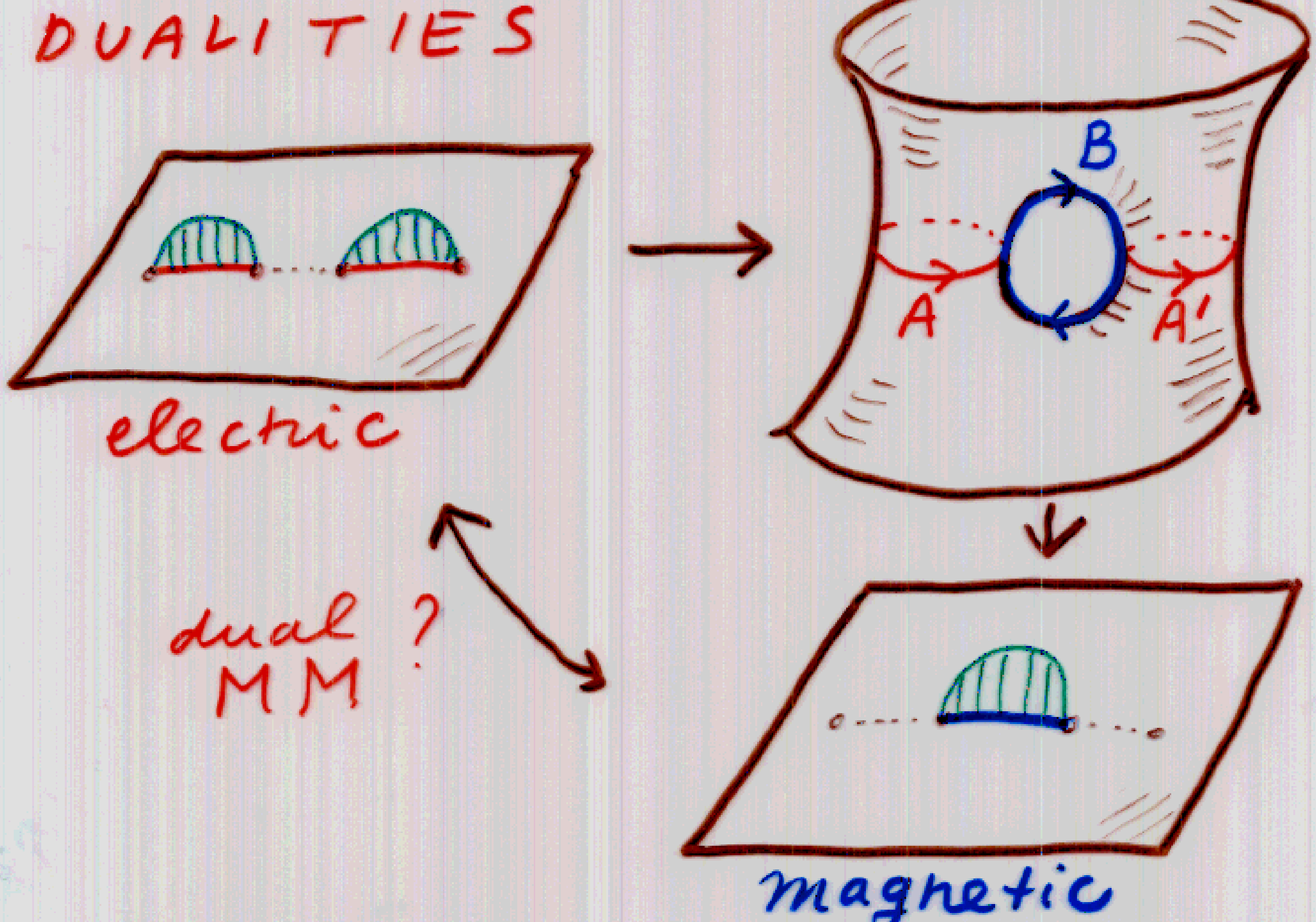
[cf Bachas, de Boer, D, Ooguri]

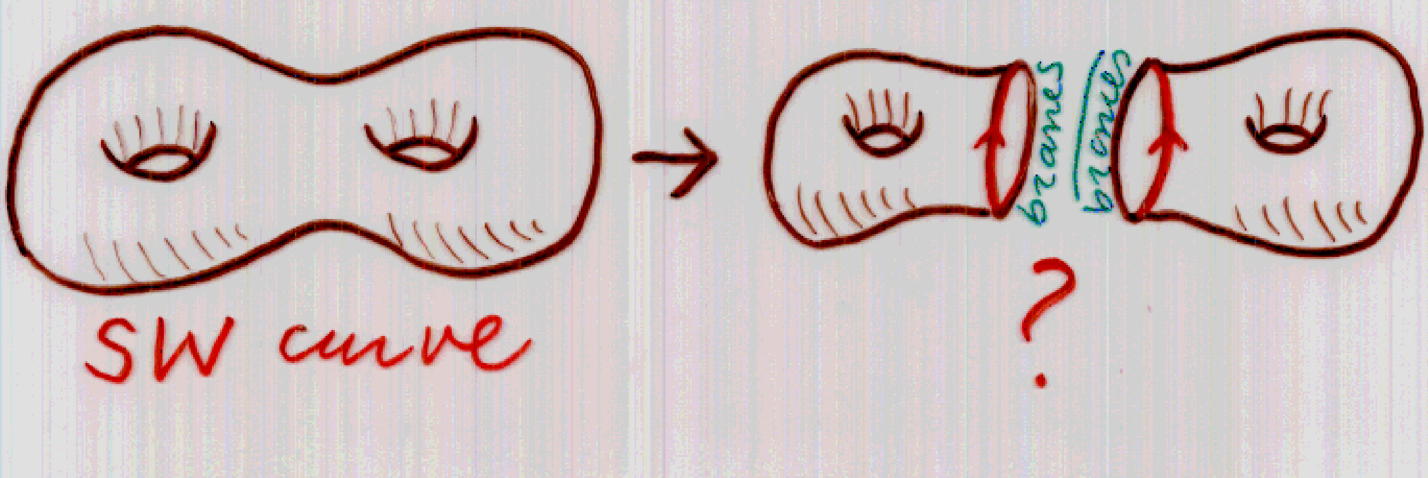
LOCAL field theory on effective geometry!

COLLECTIVE field

$$\varphi(x) = \mathrm{Tr}\log(x - \Phi)$$

DUALITIES



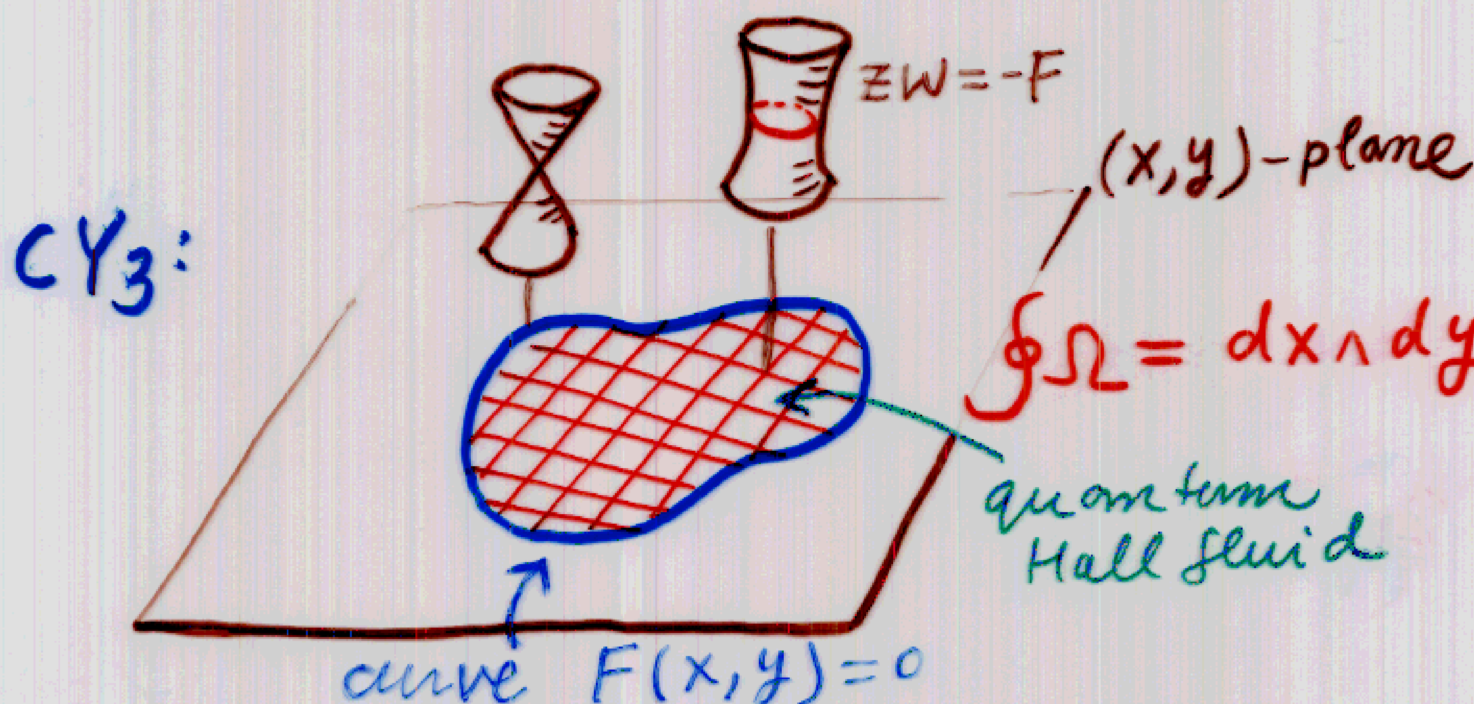


CY 3-FOLDS

$$F(x,y) + zw = 0 \qquad \text{in } \mathbb{C}^4$$

holomorphic 3-form

$$\Omega = \frac{dz\, dx\, dy}{z}$$



(x,y) plane : phase space

$$y = \frac{\partial S_{eff}}{\partial x} = p \qquad (\text{Ham-Jac})$$

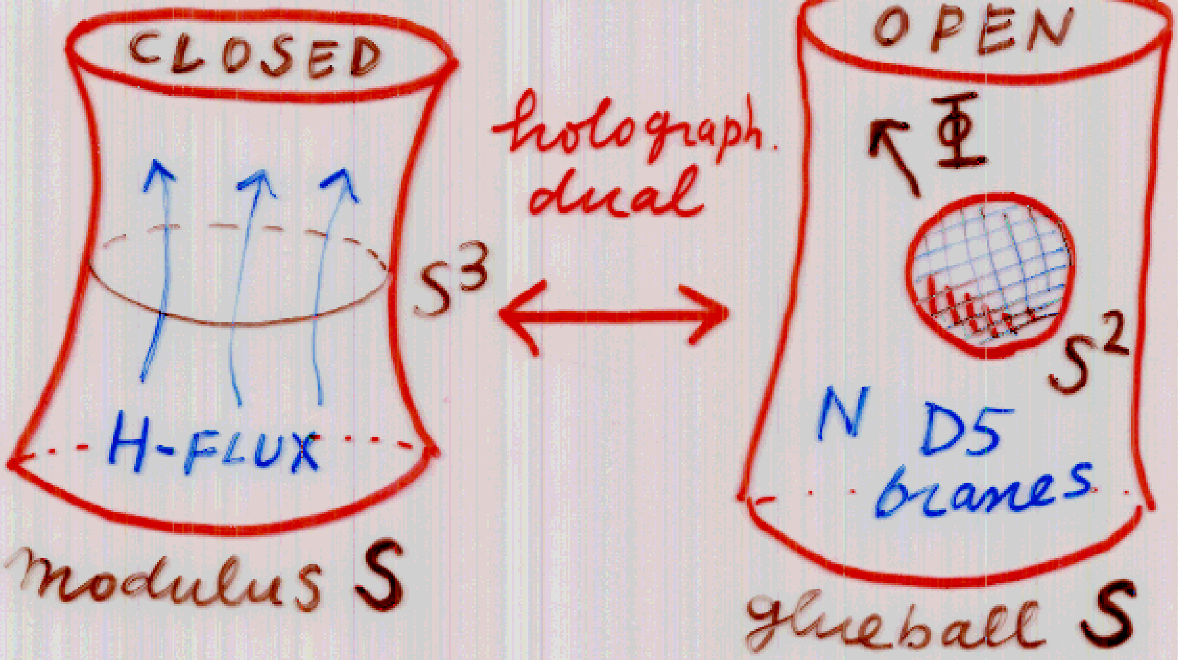
LEVEL SET

$$0 = F(x,y) \qquad \text{Hamiltonian}$$

$$\oint \Omega = \int_{F=0} y\, dx = \int dS_{eff}$$

WHERE IS THE CALABI-YAU ?

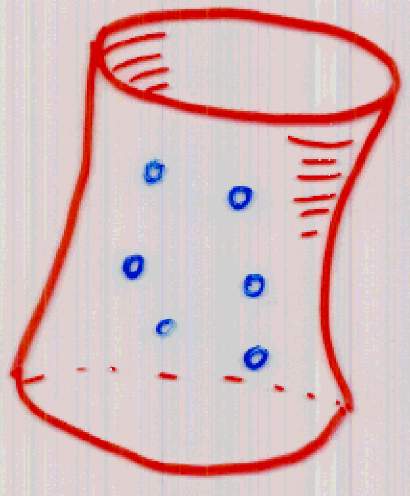
CONIFOLD



gaussian model $\quad W = m\Phi^2$

$$\int d\Phi \cdot e^{-\frac{1}{g_s}\mathrm{Tr}(m\Phi^2)}$$

add D3-branes , $\quad pt \in CY_3$

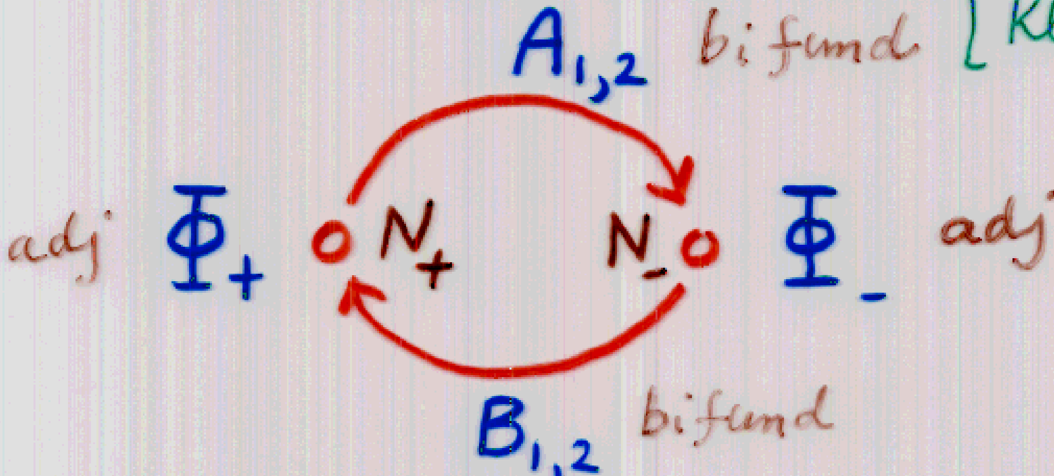


M D3-branes

[Klebanov, Strassler]

$\hat{A}_1$ QUIVER GAUGE THEORY

[Klebanov-Witten]



gauge group

$$\boxed{U(N_+) \times U(N_-)}$$

$$N_+ = N + M \qquad N_- = M$$

Superpotential

$$\boxed{W = \Phi_+^2 - \Phi_-^2 + B_i \Phi_+ A_i - A_i \Phi_- B_i}$$

$$\sim A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

RG cascade of SEIBERG dualities

$$(N_+, N_-) \rightarrow (2N_- - N_+, N_-) \dashrightarrow \begin{array}{c}\mathcal{N}=1\\ \text{SYM}\end{array}$$

[Klebanov-Strassler, Cachazo-Fiol-Intril.-Katz-Vafa]
+ . . . .

F-terms function $S = S_+ - S_-$

$\equiv U(N)$ theory

QUIVER MATRIX MODEL

$$\int d\Phi_\pm \, dA_i \, dB_i \, e^{W}$$

Integrate out A, B

reduce to eigenvalues

$$\Phi_\pm = \begin{pmatrix} \lambda_1^\pm & & 0 \\ & \ddots & \\ 0 & & \lambda^\pm_{\tilde{N}_+} \end{pmatrix}$$

general potential

$$W(\Phi_+) - W(\Phi_-)$$

eigenvalue gas

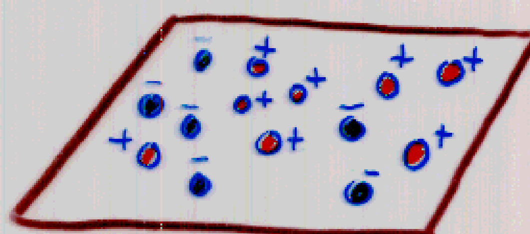
Vandermondes $\quad \Phi_\pm$

$$\int d\lambda_\pm \prod \frac{(\lambda_i^+ - \lambda_j^+)^2 (\lambda_i^- - \lambda_j^-)^2}{(\lambda_i^+ - \lambda_j^-)^2} e^{\sum_\pm W(\lambda_i^\pm)}$$

bifundamentals

charged fermions $\prod (\lambda_i - \lambda_j)^{q_i q_j}$

q_i = charge $\lambda_i^\pm = \pm 1$



net charge

$$\sum q_i = \tilde{N}_+ - \tilde{N}_- = \tilde{N}$$

effective action $\qquad$ Coulomb

$$S_{eff} = \sum q_i W(\lambda_i) + 2g_s q_i q_j \log(\lambda_i - \lambda_j)$$

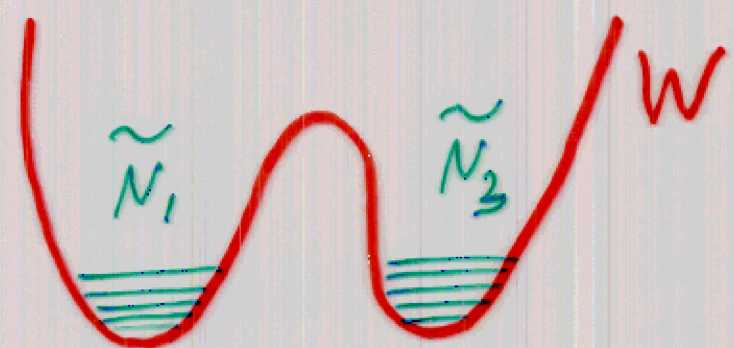
force $\quad y = \frac{\partial S_{eff}}{\partial x}$

$$y(x) = \pm \left[W'(x) + 2g_s \sum \frac{q_i}{x - \lambda_i} \right]$$

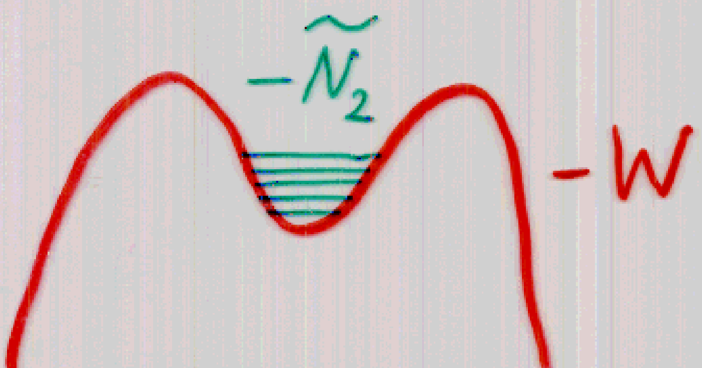
Spectral curve

$$y^2 = W'(x)^2 + f(x)$$

$$f_{qu}(x) = \frac{1}{\tilde{N}} \sum_i q_i \frac{W'(x) - W'(\lambda_i)}{x - \lambda_i}$$

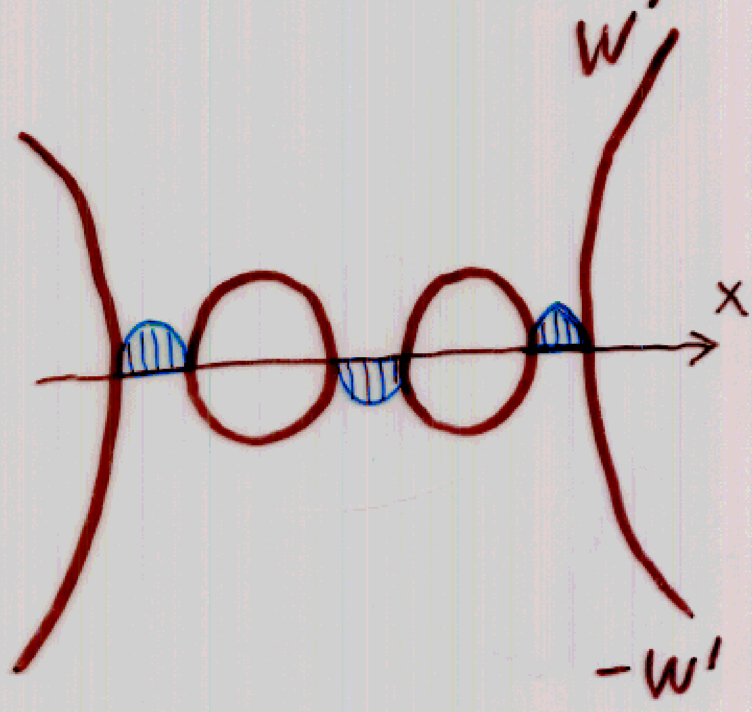


eigenvalues

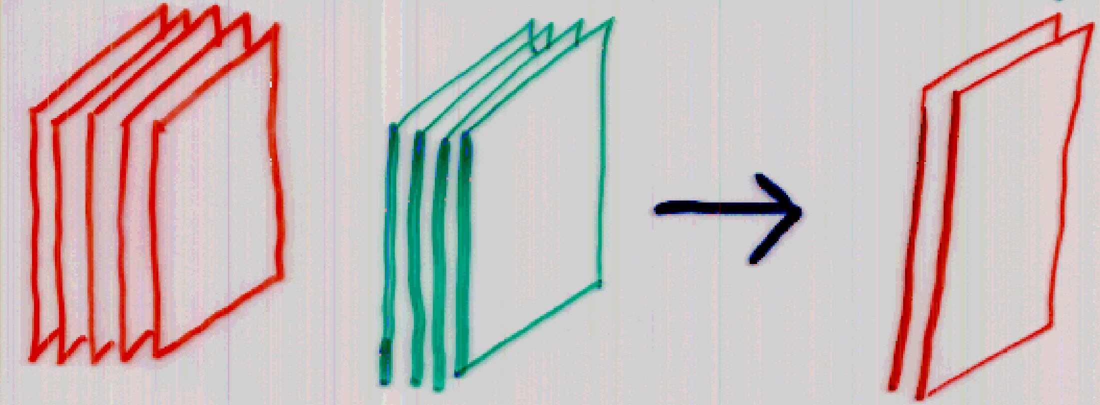


anti-eigenvalues (holes)

$$y^2 = W'(x)^2 + \ldots$$



TOPOLOGICAL BRANES-$\overline{\text{BRANES}}$ (Vafa)



N_+ N_- $N = N_+ - N_-$

SUPERGROUP $U(N_+|N_-)$

cf K-theory "stabilization"

$$N_+|N_- \simeq N_+ - N_-|0$$

Supermatrix

$$\Phi = \begin{pmatrix} \Phi_+ & * \\ * & \Phi_- \end{pmatrix}$$

odd components

gauge fixing

gauge fix to bosonic field

$$\Phi = \begin{pmatrix} \Phi_+ & 0 \\ 0 & \Phi_- \end{pmatrix}$$

$$U(N_+|N_-) \to U(N_+) \times U(N_-)$$

ghosts: chiral fields

$$\mathrm{STr}\left(b[\Phi, c]\right)$$

here <u>bosonic</u>

$$\text{ghost} = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}$$

superpotential

$$W_{\text{tree}} = \mathrm{str}\, W(\Phi) + \text{ghosts}$$

$$= \mathrm{Tr}\, W(\Phi_+) - \mathrm{Tr}\, W(\Phi_-)$$

$$+ \mathrm{Tr}\left(B_i \Phi_+ A_i - A_i \Phi_- B_i\right)$$

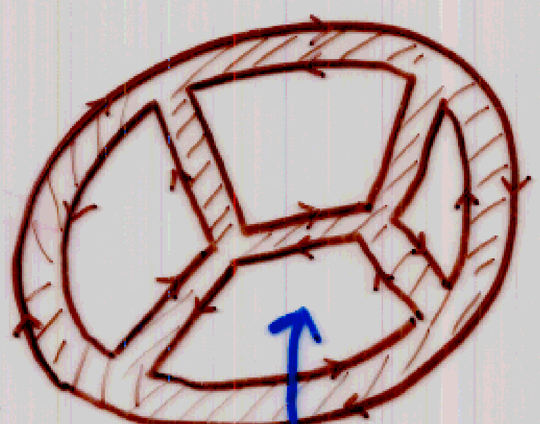
$\hat{A}_1$ quiver theory!

SUPERMATRIX MODEL

$$Z = \int_{\tilde{N}_+ | \tilde{N}_-} d\Phi \; e^{\mathrm{Str}\, W(\Phi)}$$

$\big\downarrow$ reduce

$$= \int_{\tilde{N}_+ - \tilde{N}_-} d\Phi \; e^{\mathrm{Tr}\, W(\Phi)}$$



$\mathrm{STr}(\mathbb{1}) = \tilde{N}_+ - \tilde{N}_-$

STABLE COMPUTATION

1. $S = S_+ - S_-$

$\mathrm{rk}\, S_\pm = N_\pm \to \infty$, $\underbrace{N_+ - N_- = N}_{\text{finite}}$

no relations $S^N \simeq 0$ in classical chiral ring

$$\mathcal{F}_0 = \sum a_n S^n$$

unambiguous

2. Stable chiral ring:

mesons $M_{ij} = A_i B_j \sim x_{ij}$

reconstruct conifold

$$\det x_{ij} = x_{11} x_{22} - x_{12} x_{21} = 0$$

TOPOLOGICAL STRINGS

$\Updownarrow$

$C \leq 1$ non-critical STRINGS

DOUBLE SCALING MATRIX MODELS

μ

[random surfaces]

KONTSEVICH MODELS

$1/N$

['t Hooft]

$c = 1$ String theory (X^0, φ)

(Eucl) time, Liouville

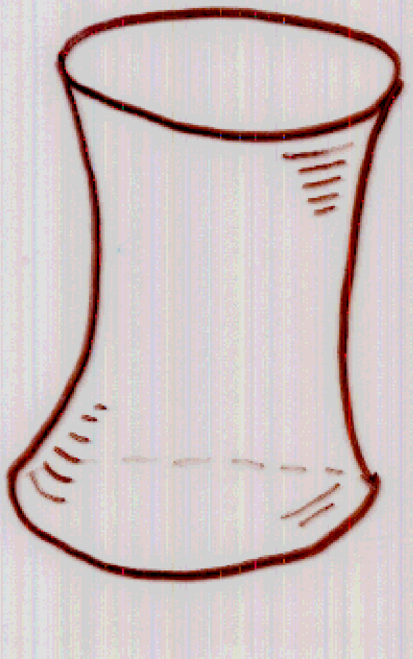
compactify $X^0 \simeq X^0 + 2\pi R$

Self-dual radius $R = 1$

$SU(2)_L \times SU(2)_R$

$\updownarrow$ [Goshal-Vafa, Mukhi-Vafa, ..]

topological string on deformed conifold



$x_{11} x_{22} - x_{12} x_{21} = \mu$

$xy - zw = \mu$

OBSERVABLES

tachyons e^{ikX}

winding modes $e^{im\tilde{X}}$

$SU(2)_L \times SU(2)_R$ symmetry

$\left(\frac{k}{2}, \frac{k}{2}\right)$ multiplets $\mathcal{O}_k$

GROUND RING [Witten]

generators x_{ij}

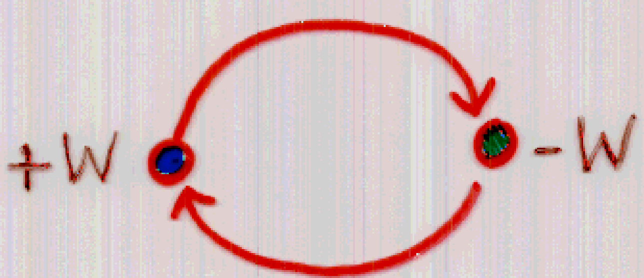
$$x_{11}x_{22} - x_{12}x_{21} = \mu$$

Quiver Matrix Model

$$\mathcal{O}_k = \mathrm{Tr}\left(A_{i_1}B_{j_1} \cdots A_{i_k}B_{j_k}\right)$$

DECONSTRUCTING $C=1$

$R=1$ $\quad \hat{A}_1$ quiver

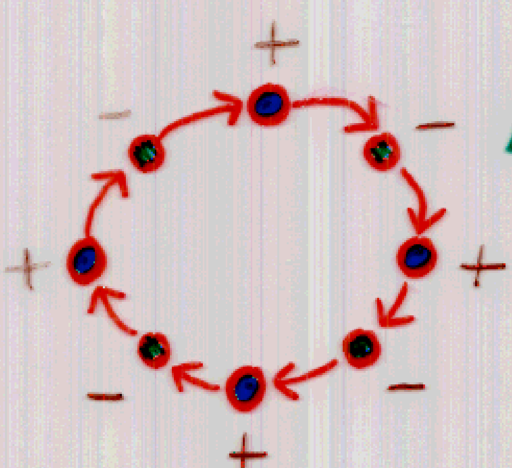


$\mathbb{Z}_k$ orbifold

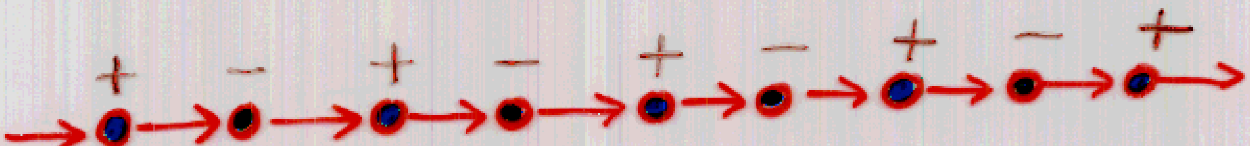
$x_{11} \to \omega\, x_{11}$

$x_{22} \to \omega^{-1} x_{22}$

$R=k$ $\quad \hat{A}_{2k-1}$ quiver

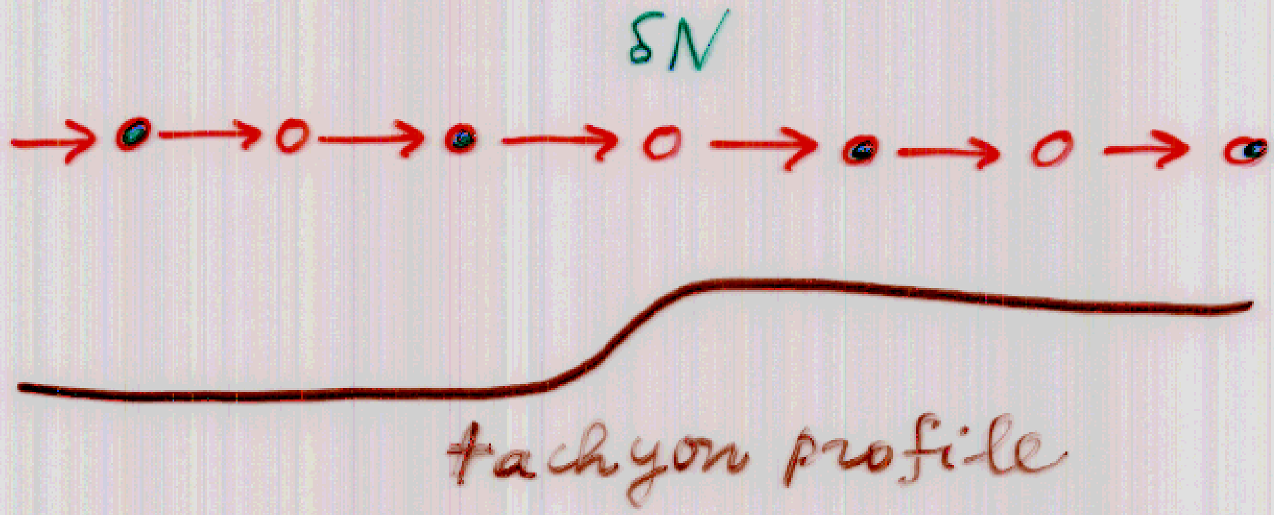


$R=\infty$ $\quad \hat{A}_\infty$ quiver

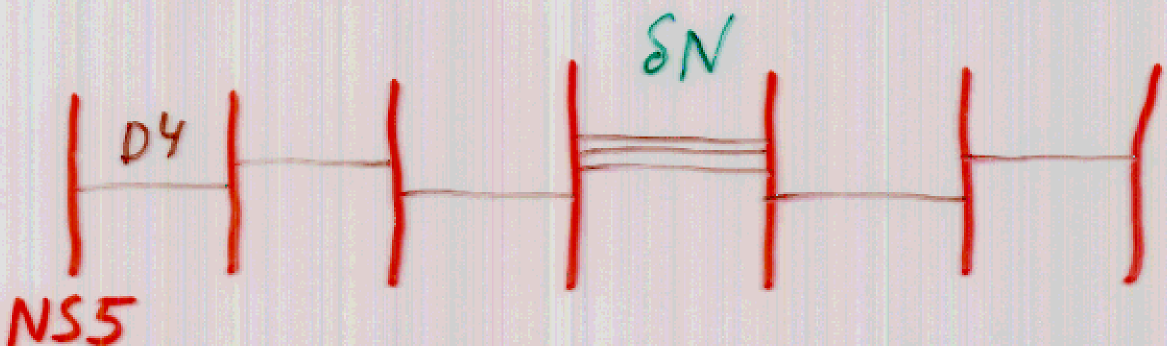


discretizing matrix QM
if all $\tilde{N}_i$ equal

$\tilde{N}_i$ not = added probe branes



Deconstructing (2,0) theory $SCFT_6$
(with mass perturbation)

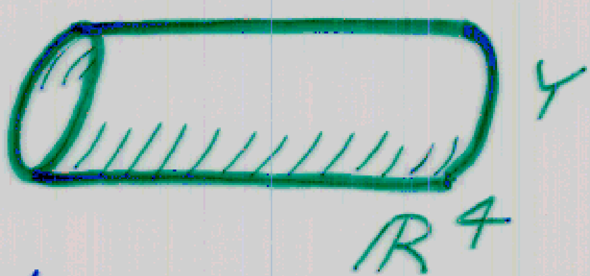


[Arkani-Hamed, Cohen, Kaplan,
Karch, Motl]

HIGHER DIMENSIONAL: $D>4$

$\mathcal{N}=1$ gauge theory $D=4+k$

$\mathbb{R}^4 \times Y^k$



use $D=4$, $\mathcal{N}=1$ superfields

$$\Phi(x^\mu, \theta^\alpha; y^i) \qquad y \in Y$$

ACTION: D & F terms

$$\int d^4x\, d^4\theta \int_Y d^ky\; K(\bar{\Phi}, \Phi)$$

$$+ \int d^4x\, d^2\theta \int_Y d^ky\; W(\Phi) + cc$$

generalized MM: $D=k$ bosonic gauge theory

$$\underset{\text{"matrix"}}{\Phi} \longrightarrow \underset{\text{field}}{\Phi(y)}$$

$$W(\Phi) \longrightarrow \int d^ky \cdot W(\Phi)$$

$$\begin{bmatrix} \mathcal{N}=1 \text{ gauge} \\ \text{theory} \end{bmatrix} \longrightarrow \begin{bmatrix} \infty \text{ \# massive} \\ \text{KK modes} \end{bmatrix}$$

$$e^{\mathcal{F}_0(S)} \sim \lim_{\tilde{N}\to\infty} \int \mathcal{D}\Phi \, e^{\int_Y W(\Phi)}$$

bosonic k-dim$^{\underline{al}}$

example: D=10 SYM

Y: CY_3 $(y^i, \bar{y}^{\bar{i}})$, $\Phi = A_{\bar{i}} \, d\bar{y}^{\bar{i}}$

$$W = \int_Y \Omega \wedge \mathrm{Tr}\left(A \bar{\partial} A + \frac{2}{3} A^3\right)$$

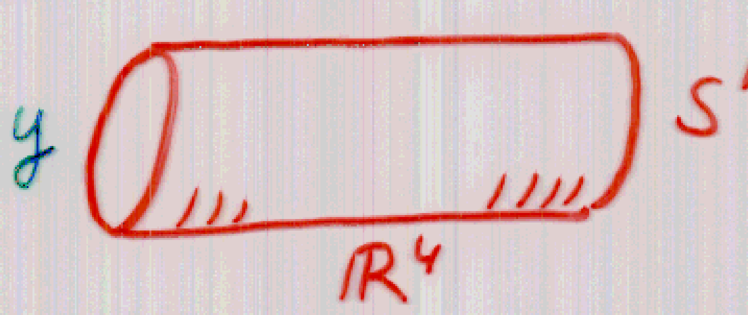
holomorphic CS

$$\frac{\partial W}{\delta A_{\bar{i}}} = \epsilon_{\bar{i}\bar{j}\bar{k}} F^{\bar{j}\bar{k}}$$

$$\left|\frac{\partial W}{\delta A_{\bar{i}}}\right|^2 = F_{jk} \bar{F}^{\bar{j}\bar{k}} \quad \text{YM action}$$

$\Rightarrow$ top open strings

$D=5$, $\mathcal{N}=2 \longrightarrow \mathcal{N}=1$



holonomy

$$U = \exp \int \phi + iA_5 \in U(N)$$

tree-level superpotential

$$\mathrm{Tr}\, W(U)$$

unitary matrix models

$$\int dU \cdot \exp \mathrm{Tr}\, W(U)$$

add HYPERS

$$P_i(y),\ Q^i(y)$$

$$\int dy \left\{ P_i \frac{DQ^i}{Dy} + H(P,Q) + W(U) \right\}$$

matrix QM [Hollowood]

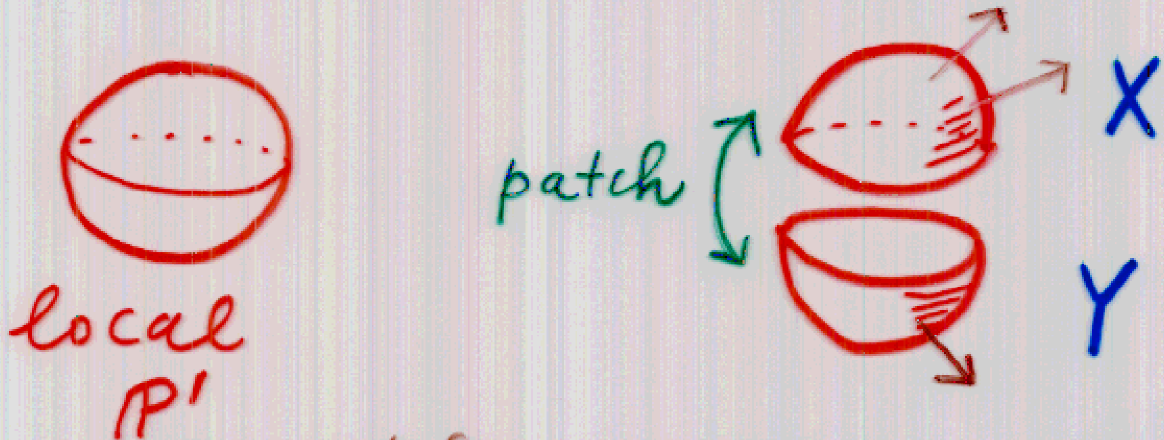
TACHYON SCATTERING
in C=1 at R=1

S-MATRIX

$$\langle 0 | e^{\sum t_n^{out} \alpha_n} \, \mathcal{S} \, e^{\sum t_n^{in} \alpha_{-n}} | 0 \rangle$$

Solution to 2-TODA hierarchy
[W. Moore, Plesser]

NORMAL MATRIX MODEL



2-matrix model

$$\int dX\, dY \cdot e^{\frac{1}{g_s}\operatorname{Tr}\left(XY + W(X) + \widetilde{W}(Y)\right)}$$

$$W = \sum t_n^{in} X^n$$

$$\widetilde{W} = \sum t_n^{out} Y^n$$

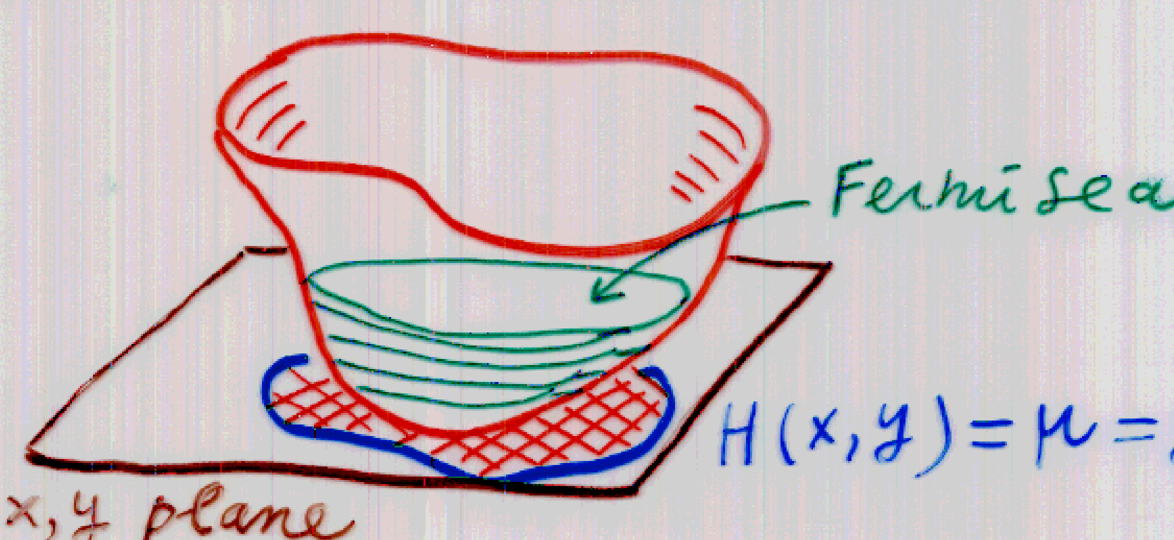
can assume

$$[X, Y] = 0$$

constant eigenvalue density

$$\rho(x,y) = dx \wedge dy$$

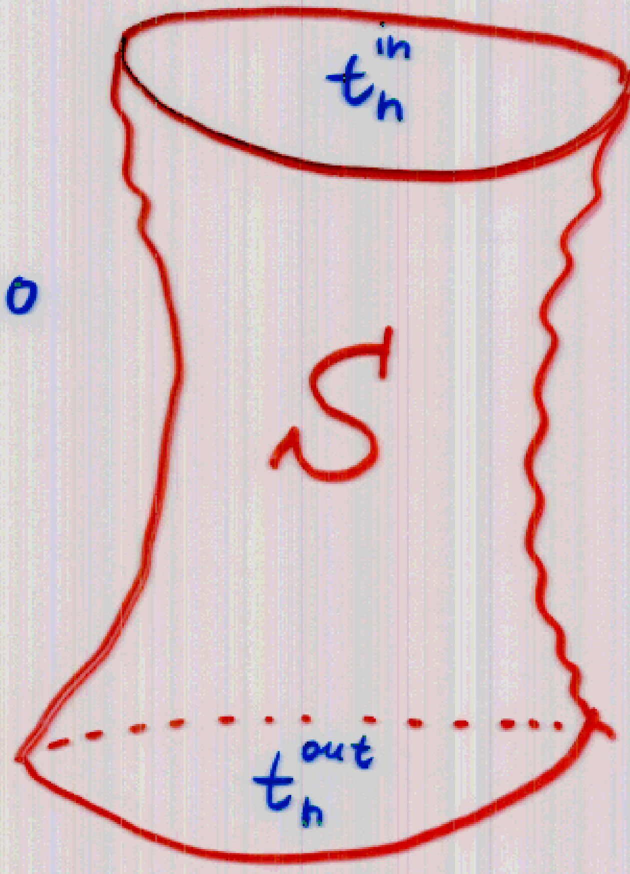
effective Hamiltonian $H(x,y)$



$$H(x,y) = \mu = g_s \widetilde{N}$$

SOLUTION $C=1$ at $R=1$ [D, Moore, Plesser]

genus 0



$$y = \frac{\mu}{x} + \partial_x \varphi^{in} = f(x)$$

DEFORM

$$xy = \mu$$

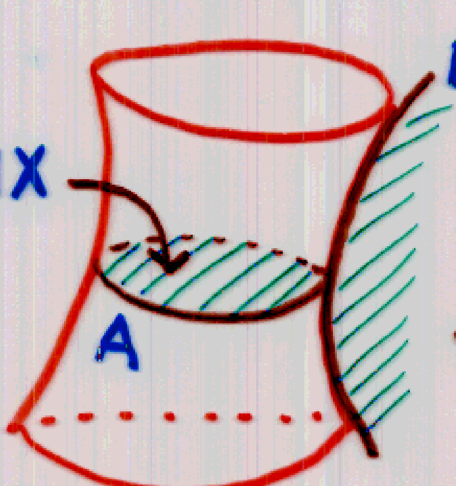
coord trans

$$x = \frac{\mu}{y} + \partial_y \varphi^{out} = f^{-1}(y)$$

$$\partial_x \varphi = \underbrace{\sum t_n x^{n-1}}_{\text{unnorm.}} + \underbrace{\frac{\partial \mathcal{F}_0}{\partial t_n} x^{-n-1}}_{\text{response}}$$

deform its own background

2-MATRIX



MATRIX QM

[Alexandrov-Kazakov-Kostov]

em-dual

all genus : FERMIONS / D-BRANES

W_∞ WARD IDENTITIES

$$W_{pq} = \oint dx \; x^p (\partial\varphi)^q + \dots$$
$$= \oint dx \; \psi^* x^p \partial_x^{q-1} \psi$$

$$\{x, y\} = 1 \longrightarrow y = -i \frac{\partial}{\partial x}$$

$$W_{pq}^{out} = S \, W_{pq}^{in} \, S^{-1} = W_{qp}^{in}$$

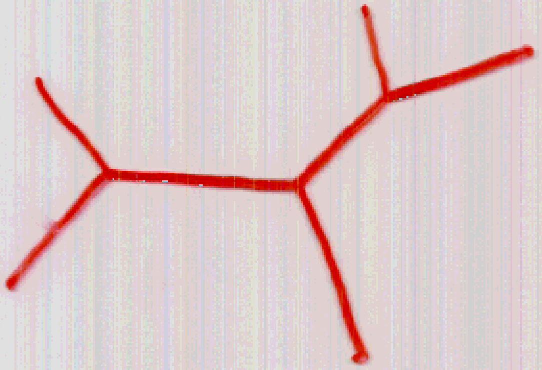
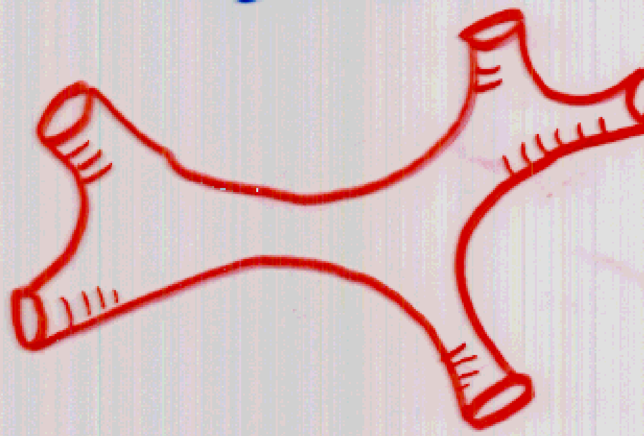
$$\psi_n^{out} = S \, \psi_n^{in} \, S^{-1} = \underbrace{e^{i\delta_n}}_{\text{phase}} \psi_n^{in}$$

S QM realization

$$(x, y) \xrightarrow[dx \wedge dy]{\text{canonical}} (y, -x)$$

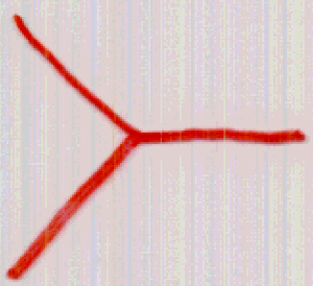
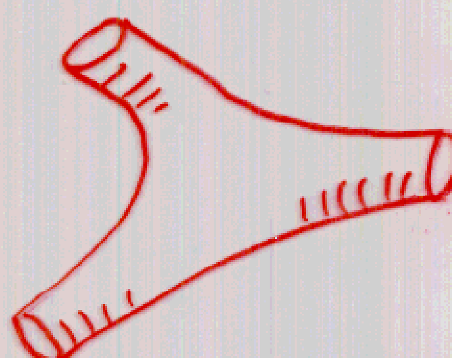
[See Aganagic's talk]

TORIC (Π^2) CY_3 $\xleftrightarrow{\text{mirror}}$ $F(u,v) + zw = 0$
u, v periodic

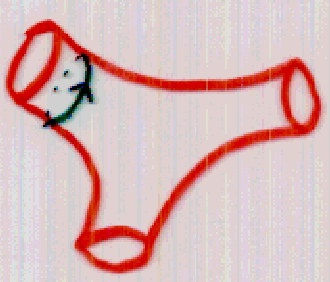
VERTEX

$F = e^{u} + e^{-v} + 1$

 $\longleftrightarrow$ 

VERTEX $\in \mathcal{H}^{\otimes 3}$ "3-TODA"

Ward identities for W_∞-alg

 +  +  $= 0$

FREE FERMIONS [w. Aganagic, Klemm, Marino, Vafa]

WHAT IS STRING THEORY?

"Fermi liquid in twistor space"

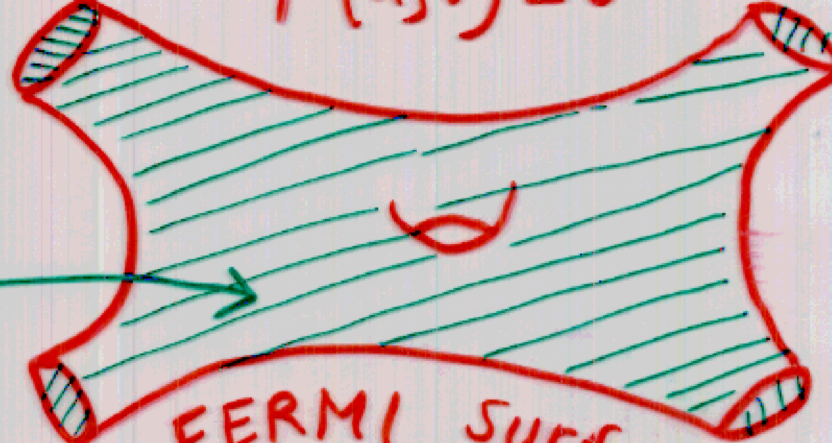
[Polchinski]

"TOPOLOGICAL Fermi liquid"

[Douglas]

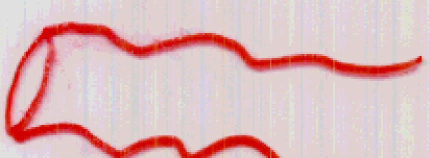
$F(u,v) = 0$

CY_3



FERMI Surface

Strongly interacting D-branes



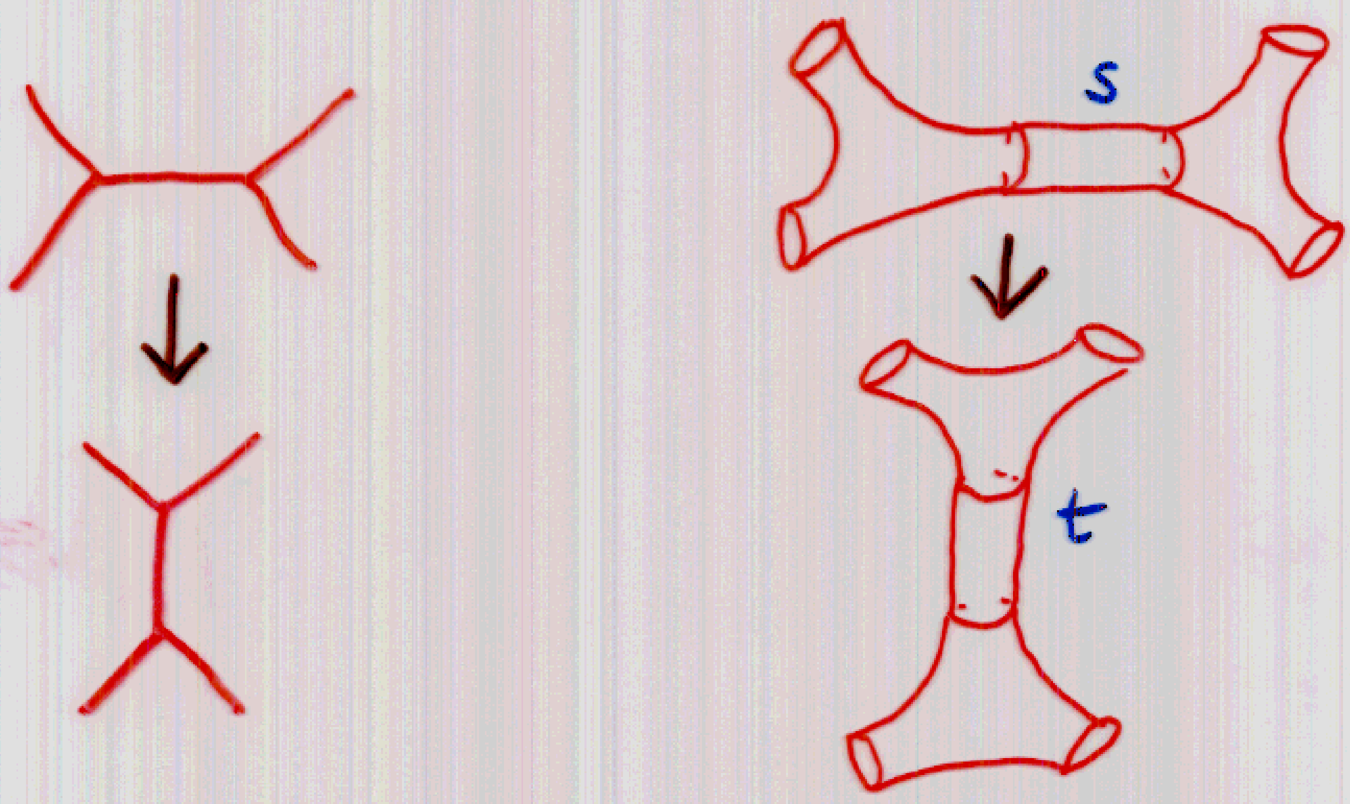
RIPPLES

CLOSED STRING COUPLINGS

KS collective field φ

QUASI-PARTICLES (free fermions) = probe D-branes

GEOMETRIC TRANSITIONS



DUALITY

modular geometry CY geometries?