CLASSIFYING SUPERGRAVITY SOLUTIONS

hepth 0205050 w. D. Martelli, S. Pakis, D. Waldram

0209114 w. J. Gutowski, C. Hull, S. Pakis, H. Reall

0212008 w. S. Pakis

0302158 w. D. Martelli, D. Waldram

0304064 w. J. Gutowski

INTRODUCTION

Consider supersymmetric bosonic solutions of a supergravity theory:

Fluxes $= 0$: $\left.\begin{array}{l} R_{\mu\nu} = 0 \\ D_\mu \epsilon = 0 \end{array}\right\}$ special holonomy CY_n, G_2, ...

Fluxes $\neq 0$: ???

Motivation:

- Compactifications with flux
- Black holes - exotica?
- AdS/CFT

Key tool:

"G-structure" & intrinsic torsion

Results:

- Can classify susy solutions
- New solutions

previously: guess clever ansatz

Here: geometries can be given in a form quite different from usual ansatz

G-structures on an n-dimensional manifold

$F(M)$ + Frame bundle: principal $Gl(n)$ bundle

G-Structure: principal G sub-bundle of $F(M)$

$\Leftrightarrow$ no-where vanishing tensors

eg

$$g_{ab} \longleftrightarrow O(n)$$

$$g_{ab}, \epsilon_{a_1 \dots a_n} \longleftrightarrow SO(n)$$

$$\left.\begin{aligned} J_a{}^b, \ J^2 = -1 &\longleftrightarrow Gl(m, \mathbb{C}) \\ g_{ab}, \ J_a{}^b &\longleftrightarrow U(m) \\ g_{ab}, \ J_a{}^b, \ \chi^{(m,0)} &\longleftrightarrow SU(m) \end{aligned}\right\} n = 2m$$

Classification - Intrinsic Torsion

eg $G \subset SO(n)$ structure

$\hookrightarrow$ tensors η, including $g \to \nabla$

$\nabla\eta \sim \bigoplus W_i$

More precisely

$\nabla\eta$ defines an element

$$T \in \Lambda^1 \otimes \mathfrak{g}^\perp = \bigoplus_i W_i$$

$$\mathfrak{g} \oplus \mathfrak{g}^\perp = \mathfrak{so}(n)$$

e.g. All $W_i = 0 \iff \nabla\eta = 0$

$\iff \nabla$ has holonomy G

All $W_i \neq 0 \to$ most general type of G-structure

$\boxed{\text{The intrinsic torsion } T \text{ is a measure of the deviation from special holonomy}}$

Example

SU(5) structures in D=10

g_{ab}, $J_a{}^b$, $\chi^{(5,0)}_{a_1 \dots a_5}$

$T = \Lambda^1 \otimes g^\perp$ $\qquad g^\perp \oplus su(5) = so(10)$

$= (10+\bar{10}) + (40+\bar{40}) + (45+\bar{45}) + (5+\bar{5}) + (5'+\bar{5}')$

$= \bigoplus_{i=1}^{5} W_i$

All W_i can be expressed in terms of $dJ, d\chi$:

$$\begin{cases} dJ \longrightarrow W^1, W^3, W^4 \\ d\chi \longrightarrow W^1, W^2, W^5 \end{cases}$$

eg $$\begin{cases} (W_4)_a = J^{b_1 b_2} (dJ)_{b_1 b_2 a} \\ (W_5)_a = \chi^{b_1 \dots b_5} (d\chi)_{b_1 \dots b_5 a} \end{cases}$$

Examples of su(5) structures:

$$\begin{cases} W_1 = W_2 = 0 \longrightarrow \text{complex} \\ W_1 = W_2 = W_3 = W_4 = 0 \longrightarrow \text{Kähler} \\ W_i = 0 \longrightarrow CY_5 \end{cases}$$

$\exists\ 2^5 = 32$ classes.

D=11 SUGRA

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}(F) \\ d * F + \frac{1}{2} F \wedge F = 0 \end{cases}$$

$$\hat{\nabla}_\mu \epsilon = \left(\nabla_\mu + \frac{1}{288} \left[\Gamma_\mu{}^{\nu_1 \dots \nu_4} - 8 \delta_\mu^{\nu_1} \Gamma^{\nu_2 \nu_3 \nu_4} \right] F_{\nu_1 \dots \nu_4} \right) \epsilon = 0$$

$F=0$: $\left.\begin{array}{l} R_{\mu\nu} = 0 \\ \nabla_\mu \epsilon = 0 \end{array}\right\} \longrightarrow$ special holonomy

$F \neq 0$: Assume $\exists$ 1 Killing spinor

① Construct bi-linears

$$\begin{cases} K_\mu = \bar{\epsilon} \Gamma_\mu \epsilon \\ \Omega_{\mu\nu} = \bar{\epsilon} \Gamma_{\mu\nu} \epsilon \\ \Sigma_{\mu_1 \dots \mu_5} = \bar{\epsilon} \Gamma_{\mu_1 \dots \mu_5} \epsilon \end{cases}$$

② Algebraic Conditions

eg $K^\mu \Omega_{\mu\nu} = 0$

i) Fièrz identities

ii) Spinor defines a G-structure where $G \subset \mathrm{Spin}(10,1)$ is the isotropy group of the spinor.

$\exists$ 2 cases (Bryant)

Ⓐ Null Case $K^2 = 0$

$(K, \Omega, \Sigma) \to Spin(7) \ltimes \mathbb{R}^9$ structure $D=11$

Ⓑ Timelike Case $K^2 < 0$

$(K, \Omega, \Sigma_1) \longrightarrow SU(5)$ structure in $D=11$

③ Differential Conditions

$$\begin{cases} D_\mu K_\nu = \cdots \\ D_\mu \Omega_{\nu_1 \nu_2} = \cdots \\ D_\mu \Sigma_{\nu_1 \nu_2 \ldots \nu_5} = \cdots \end{cases}$$

* Constrain intrinsic torsion T
* Can solve for flux in terms of G-structure

Results for timelike case

K - Killing : $K = \left(\frac{\partial}{\partial t}\right)$

$$ds^2 = -\Delta^2(x)\left[dt + \omega(x)\right]^2 + \Delta^{-1}(x)\left[\underbrace{ds^2_{10}(\underline{x})}\right]$$

has an SU(5) structure, J, $\chi^{5,0}$
<u>only</u> constraint is that

$$(W_5)_a = \chi^{b_1 \ldots b_5}(d\chi)_{b_1 \ldots b_5 a} \propto \partial_a(\log \Delta)$$

$$F = \left\{ \begin{array}{c} dJ,\ d\chi,\ d\omega,\ d\Delta \\ \text{terms} \end{array} \right\} + F_{\underline{75}}$$

undetermined by susy

- NECESSARY & SUFFICIENT FOR SUSY
- SUSY $\not\Rightarrow$ Equations of motion

  IMPOSE $\quad d*F + \frac{1}{2}F \wedge F = 0$

- Solutions

  eg Godel with ≥ 16 susies

TO DO

① Null case complete general classification

② Refine Classification

* 2 susies → T, T 2 $SU(5)$ structures
→ T, N $SU(5)$, $Spin(7) \ltimes \mathbb{R}^9$
→ N, N 2 $Spin(7) \ltimes \mathbb{R}^9$

* 3 susies

⋮

* 32 susies Done! {Figueroa-O'Farrill, Papadopoulos}

V. HARD!

① Simpler SUGRA theories

* D=5 minimal SUGRA (D=11 on T^5)
* D=5 minimal gauged SUGRA (IIB on S^5)

Complete story

② Simpler classes of solutions

$\mathbb{R}^{1,9-d} \times M_d$ in type I, II

$\phi, H \neq 0$

- wrapped NS 5-branes / Holography
- Generalised calibrations JPG, Kim, Pakis, Waldram

MINIMAL D=5 SUGRA

$g_{\mu\nu}$, $F_{\mu\nu}$, 8 supercharges

Complete classification

$\frac{1}{2}$ susy $\rightarrow$ Timelike
$\rightarrow$ Null

Max. susy $\rightarrow$ T, T
$\rightarrow$ T, N

Solutions
- C.T.C.'s generic
- Godel solution with Max susy
  (20 susies in D=11)

No black rings (Reall)

MINIMAL GAUGED D=5 SUGRA

$g_{\mu\nu}$, $F_{\mu\nu}$, $\Lambda \neq 0$ 8 supercharges

Classified $\begin{cases} \text{Timelike} & 1/4 \text{ susy} \\ \text{Null} & 1/2 \text{ susy} \end{cases}$

Max susy $\leftrightarrow$ AdS_5

Timelike: ∂_t is <u>not</u> static even though there are static susy solutions in this class $\longrightarrow$ New types of solutions

D=5 Timelike cases

$$ds^2 = \Delta^2(x)\left[dt + \omega(x)\right]^2 - \Delta^{-1}(x)\Big[\underbrace{ds_4^2(x)}_{B}\Big]$$

$$F = \ldots$$

ungauged case:

- B is hyper Kähler
- Solve 2^{nd} order p.d.e.'s

$$\left(\begin{array}{l} \Delta\, d\omega = G^+ + G^- \\ \nabla^2 \Delta^{-1} = \frac{4}{9}(G^+)^2 \quad , \quad dG^+ = 0 \end{array} \right)$$

gauged case:

- B is Kähler & <u>not</u> hyper-Kähler
- Δ, ω & F determined up to an antiholomorphic function.

CONCLUSIONS

- Much progress can be made in classifying supersymmetric solutions of supergravity theories.
- Novel solutions
- Uniqueness theorems

Much more can be done.