

CLASSIFYING

SUPERGRAVITY

SOLUTIONS

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# INTRODUCTION

Consider supersymmetric bosonic solutions of a supergravity theory:

Fluxes = 0:  $R_{\mu\nu} = 0$  } special holonomy  
 $\nabla_{\mu} \epsilon = 0$  }  $CY_n, G_2, \dots$

Fluxes  $\neq 0$ : ???

Motivation:

- Compactifications with flux
- Black holes - exotica?
- AdS/CFT

Key tool:

"G-structure" & intrinsic torsion

Results:

- Can classify susy solutions

- New solutions

previously: guess clever ansatz

Here: geometries can be given in a form quite different from usual ansatz

# G-structures on an $n$ -dimensional manifold

$\left\{ \begin{array}{l} F(M) \rightarrow \text{Frame bundle: principal } G(n) \text{ bundle} \\ G\text{-structure: principal } G \text{ sub-bundle of } F(M) \end{array} \right.$

$\Leftrightarrow$  no-where vanishing tensors

eg  $g_{ab} \leftrightarrow O(n)$

$g_{ab}, e_{a_1}, \dots, e_{a_n} \leftrightarrow SO(n)$

$J_a{}^b, J^2 = -1 \leftrightarrow G(M, \mathbb{R})$

$g_{ab}, J_a{}^b \leftrightarrow U(M)$

$g_{ab}, J_a{}^b, \chi^{(M,0)} \leftrightarrow SU(M)$

}  $n = 2m$

# Classification - Intrinsic Torsion

eg  $G \subset SO(n)$  structure

↳ tensors  $\eta$ , including  $g \rightarrow \nabla$

$$\nabla \eta \sim \bigoplus W:$$

More precisely

$\nabla \eta$  defines an element

$$T \in \Lambda^1 \otimes \mathfrak{g}^\perp = \bigoplus_i W_i:$$

$$\mathfrak{g} \oplus \mathfrak{g}^\perp = SO(n)$$

e.g. All  $W_i = 0 \Leftrightarrow \nabla \eta = 0$

$$\Leftrightarrow \nabla \text{ has holonomy } G$$

All  $W_i \neq 0 \rightarrow$  most general type of  $G$ -structure

The intrinsic torsion  $T$  is a measure of the deviation from special holonomy

## Example

4.

$SU(5)$  structures in  $D=10$

$$g_{ab}, J_a{}^b, \chi_{a_1 \dots a_5}^{(5,0)}$$

$$\begin{aligned} T &= \Lambda^1 \otimes g^\perp & g^\perp \oplus SU(5) &= SO(10) \\ &= (10 + \bar{10}) + (40 + \bar{40}) + (45 + \bar{45}) + (5 + \bar{5}) + (5' + \bar{5}') \\ &= \bigoplus_{i=1}^5 W_i \end{aligned}$$

All  $W_i$  can be expressed in terms of  $dJ, dX$ :

$$\begin{cases} dJ \rightarrow W^1, W^3, W^4 \\ dX \rightarrow W^1, W^2, W^5 \end{cases}$$

$$\text{eg } \begin{cases} (W_4)_a = J^{b_1 b_2} (dJ)_{b_1 b_2 a} \\ (W_5)_a = \chi^{b_1 \dots b_5} (dX)_{b_1 \dots b_5 a} \end{cases}$$

Examples of  $SU(5)$  structures:

$$\begin{cases} W_1 = W_2 = 0 & \rightarrow \text{complex} \\ W_1 = W_2 = W_3 = W_4 = 0 & \rightarrow \text{Kähler} \\ W_i = 0 & \rightarrow CY_5 \end{cases}$$

$\exists 2^5 = 32$  classes.

D=11 SUGRA

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}(F) \\ d * F + \frac{1}{2} F \wedge F = 0 \end{cases}$$

$$\hat{\nabla}_\mu \epsilon = \left( \nabla_\mu + \frac{1}{288} \left[ \Gamma_\mu^{\nu_1 \dots \nu_4} - 8 \delta_\mu^{\nu_1} \rho^{\nu_2 \nu_3 \nu_4} \right] F_{\nu_1 \dots \nu_4} \right) \epsilon = 0$$

F=0:  $\left. \begin{array}{l} R_{\mu\nu} = 0 \\ \nabla_\mu \epsilon = 0 \end{array} \right\} \rightarrow \text{special holonomy}$

F ≠ 0: Assume  $\exists$  1 Killing spinor

① Construct bi-linears

$$\begin{cases} K_\mu = \bar{\epsilon} \Gamma_\mu \epsilon \\ \Omega_{\mu\nu} = \bar{\epsilon} \rho_{\mu\nu} \epsilon \\ \Sigma_{\mu_1 \dots \mu_5} = \bar{\epsilon} \rho_{\mu_1 \dots \mu_5} \epsilon \end{cases}$$

② Algebraic Conditions

eg  $K^\mu \Omega_{\mu\nu} = 0$

i) Fierz identities

ii) Spinor defines a G-structure

where  $G \subset \text{Spin}(10,1)$  is the isotropy group of the spinor.

$\exists$  2 cases (Bryant)

Ⓐ Null Case  $K^2 = 0$

$(K, \Omega, \Sigma) \rightarrow Spin(7) \times \mathbb{R}^9$  structure  $D=11$

Ⓑ Timelike Case  $K^2 < 0$

$(K, \Omega, \Sigma) \rightarrow SU(5)$  structure in  $D=11$

Ⓒ Differential Conditions

$$\begin{cases} \nabla_\mu K_\nu = \dots \\ \nabla_\mu \Omega_{\nu_1 \nu_2} = \dots \\ \nabla_\mu \Sigma_{\nu_1 \nu_2 \dots \nu_5} = \dots \end{cases}$$

\* Constrain intrinsic torsion  $T$

\* Can solve for flux in terms of G-Structure

## Results for timelike case

K - Killing :  $K = \left(\frac{\partial}{\partial t}\right)$

$$ds^2 = -\Delta^2(x) [dt + w(x)]^2 + \Delta^{-1}(x) \left[ \underbrace{ds_{10}^2(x)} \right]$$

has an  $SU(5)$  structure,  $J$ ,  $x^{5,0}$   
only constraint is that

$$(W_5)_a = x^{b_1 \dots b_5} (dx)_{b_1 \dots b_5 a} \propto \partial_a (\log \Delta)$$

$$F = \left\{ \begin{array}{l} dJ, dx, dw, d\Delta \\ \text{terms} \end{array} \right\} + F_{75}$$

↑  
undetermined by susy

• NECESSARY & SUFFICIENT FOR SUSY

• SUSY  $\Rightarrow$  Equations of motion

IMPOSE  $d * F + \frac{1}{2} F \wedge F = 0$

• Solutions

eg Godel with  $\geq 16$  susies



TO DO

① Null case complete general classification

② Refine Classification

* 2 susies	→	T, T	2 $SU(5)$ structures
	→	T, N	$SU(5)$ , $Spin(4) \times \mathbb{R}^9$
	→	N, N	2 $Spin(4) \times \mathbb{R}^9$

\* 3 susies

⋮

* 32 susies	Done!	{ Figueroa-O'Farrill Papadopoulos
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V. HARD!

① Simpler SUGRA theories

* D=5 Minimal SUGRA	(D=11 on $T^5$ )
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* D=5 Minimal gauged SUGRA	(IIB on $S^5$ )
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Complete story

② Simpler classes of solutions

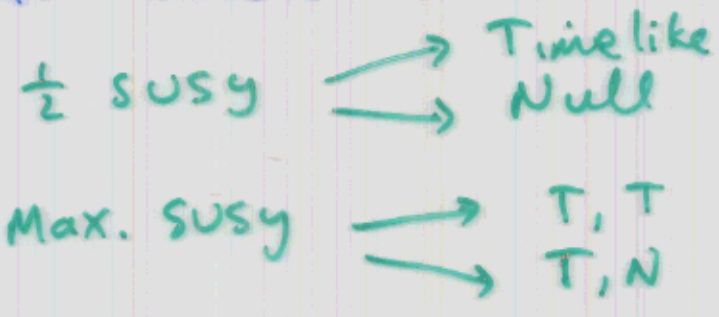
$\mathbb{R}^{1, 9-d} \times M_d$  in type I, II  
 $\phi, H \neq 0$

- wrapped NS 5-branes / Holography
- Generalised calibrations JPG, Kim, Patis, Weblum

MINIMAL D=5 SUGRA

$J_{uv}, F_{uv}$ , 8 supercharges

Complete classification



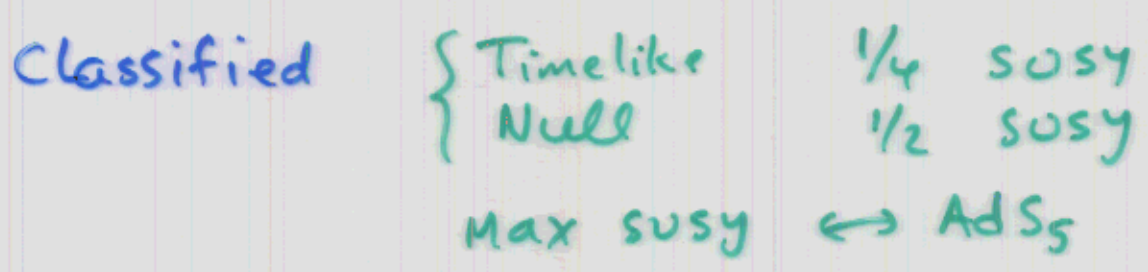
Solutions

- C.T.C.'s generic
- Gödel solution with Max susy (20 susies in D=11)

No black rings (Real)

MINIMAL GAUGED D=5 SUGRA

$J_{uv}, F_{uv}, \Lambda \neq 0$  8 supercharges



Timelike:  $dt$  is not static even though there are static susy solutions in this class  $\rightarrow$  New types of solutions

## D=5 Timelike cases

$$ds^2 = \Delta^2(x) [dt + \omega(x)]^2 - \Delta^{-1}(x) \underbrace{[ds_4^2(x)]}_B$$

$$F = \dots$$

### ungauged case:

- B is hyper Kähler
- Solve 2<sup>nd</sup> order p.d.e.'s

$$\left( \Delta d\omega = G^+ + G^- \right.$$

$$\left. \nabla^2 \Delta^{-1} = \frac{4}{9} (G^+)^2, \quad dG^+ = 0 \right)$$

### gauged case:

- B is Kähler & not hyper-Kähler
- $\Delta, \omega$  &  $F$  determined up to an antiholomorphic function.

## CONCLUSIONS

- Much progress can be made in classifying supersymmetric solutions of supergravity theories.
- Novel solutions
- Uniqueness theorems

Much more can be done.