D-BRANES, ORIENTIFOLDS OF CALABI-YAU, AND $\mathcal{N}=1$ SUPERSYMMETRY

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STRINGS 2003, KYOTO

Based on:

- I. Brunner & K.H. hepth/0303135
- I. Brunner, K. Hosomichi, J. Walcher & K.H.
  work in progress & to appear

Also: Brunner-H 0208141

Roiban-Romelsberger-Walcher 0203272

Acharya-Aganagic-H-Vafa 0202208

H-Iqbal-Vafa 0005247

Brunner-Douglas-Lawrence-Romelsberger 9906200

⋮

Related Works:

Angelatonj Bianchi Pradisi Sagnotti Stanev 9607229

Blumenhagen Wisskirchen 9806131

Hikida 0201175

Huiszoon Schalm 0306091

Govindarajan Majumder 0306257

Misra 0304... Diaconescu Florea Misra 0305...

Related Talks in this Conference:

Mirjan Cvetic

Mike Douglas

⋮

4d $\mathcal{N}=1$ compactification of string theory

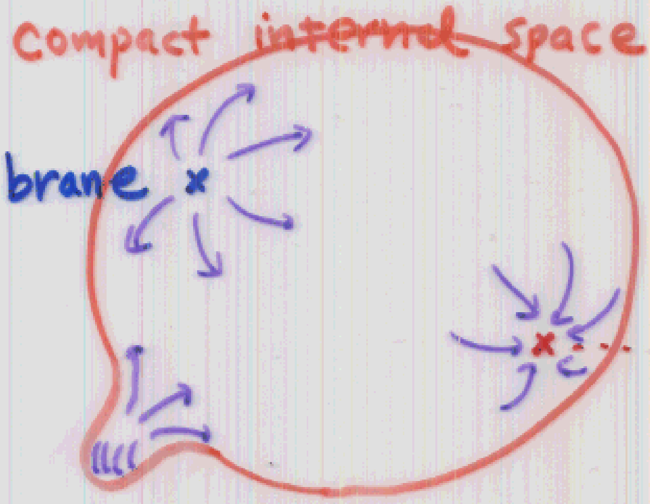
* interesting

** relevant for real world physics

Various ways :
- Heterotic string on CY^3 with gauge bundles
- M theory on G_2 holonomy manifolds
- F theory on CY^4

Many of them are dual to :

Type II string theory with space-filling D-branes
(or fluxes)



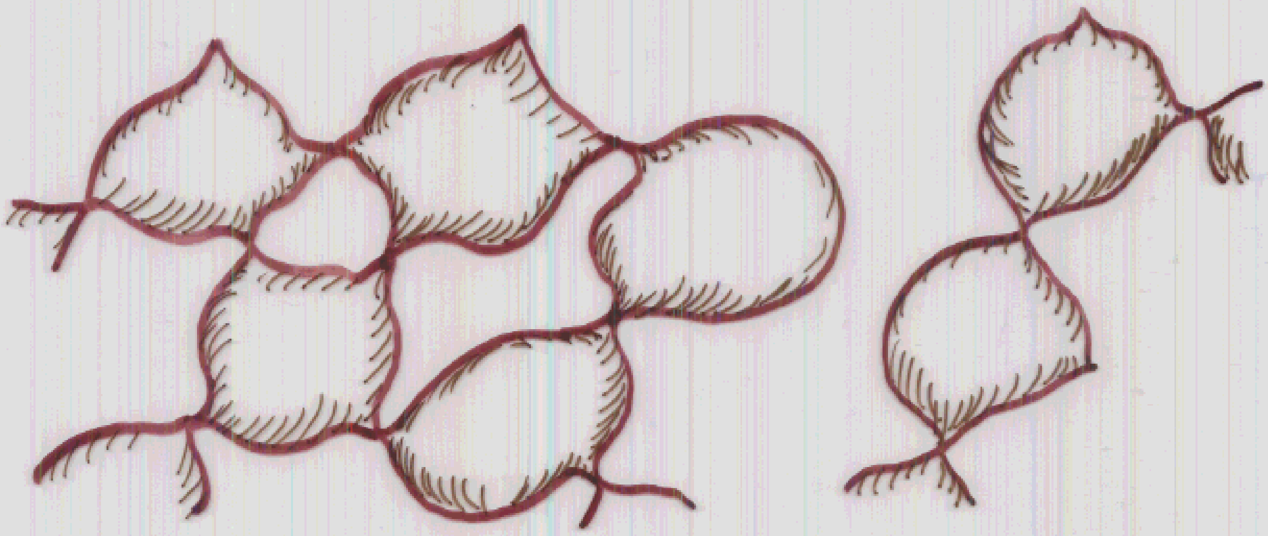
flux needs to be absorbed

$\Downarrow$

Orientifold is required

4d $\mathcal{N}=2$ compactifications (Type II on CY^3)

Moduli space looks like



What happens to this picture after Orientifold & addition of D-branes and fluxes ?

We expect a drastic change

① orientifold projection

② Open string fields

③ Potential

Orientifold is to gauge a worldsheet parity

$$P : X(t,\sigma) \to \tau X(t,-\sigma)$$

WS theory : (2,2) SUSY

σ-model on a Kähler mfd X $\begin{cases} \omega : \text{Kähler form} \\ J : \text{cplx str.} \end{cases}$

LG-model on X, superpotential W

Parity : preserving $\frac{1}{2}$ of (2,2) SUSY $\begin{cases} \text{A-type} \\ \text{B-type} \end{cases}$ like D-brane Ooguri Oz Yin

A-type (e.g. Type IIA Orientifolds on CY^3/Gepner)

$\tau : X \to X$ antiholomorphic ($\Leftrightarrow \tau^*\omega = -\omega$)

$\left(W(\tau x) = \overline{W(x)} + \text{const} \right)$

O-plane X^τ is a Lagrangian submfd (s.t. $\operatorname{Im} W|_{X^\tau} = \text{const}$)

B-type (e.g. Type IIB Orientifolds on CY^3/Gepner)

$\tau : X \to X$ holomorphic ($\Leftrightarrow \tau^*\omega = \omega$)

$\left(W(\tau x) = -W(x) + \text{const} \right)$

O-plane X^τ is a complex submfd (s.t. $W|_{X^\tau} = \text{const}$)

Reduction of Moduli Fields

A-type

complex structure

$$\tau^{*-1}\,\overline{\partial}_J\,\tau^* = \partial_J$$ antiholomorphic condition

(parameter in W

$$\tau^* W = \overline{W}$$ antihol. condition)

complexified Kähler class holomorphic condition

$$\tau^*(\omega - iB) = -\omega + iB + \underbrace{\pi i\, c_1(X)}_{\text{parity anomaly}} \mod 2\pi i$$

B-type

Complex structure

$$\tau^{*-1}\,\overline{\partial}_J\,\tau^* = \overline{\partial}_J$$ holomorphic condition

(parameter in W

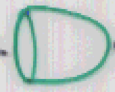
$$\tau^* W = -W$$ hol. condition)

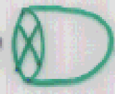
complexified Kähler class

$$\tau^*(\omega - iB) = \overline{\omega - iB} \mod 2\pi i$$ anti-holomorphic condition

Overlaps with RR ground states (NLσM)

RR ground states $|i\rangle_{RR} \leftrightarrow \omega_i \in H^{\cdot}(X)$

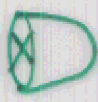
$\langle B_a | i \rangle_{RR}$ a  ← i (topologically twisted) disc 1-pt

$\langle C_p | i \rangle_{RR}$ p  ← i $\mathbb{RP}^2$ 1-pt

charge × tension of D-brane/O-plane

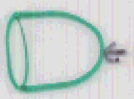
A-type

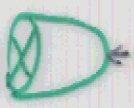
L  ← i $= \int_L \omega_i$

$\tau\Omega$  ← i $= \int_{X^\tau} \omega_i$

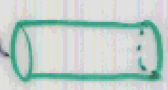
Period integrals

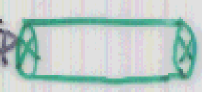
B-type

E  ← i $= \int_X e^{B+i\omega} \mathrm{ch}(\bar{E}) \sqrt{\mathrm{Td}(X)}\, \omega_i + \cdots$

$\tau\Omega$  ← i $= \int_{X^\tau} e^{i\omega} \sqrt{\dfrac{L(\frac{1}{4}TX^\tau)}{L(\frac{1}{4}NX^\tau)}}\, \omega_i + \cdots$

Witten index

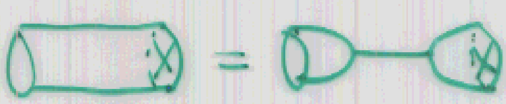
$\mathrm{Tr}_{a,b}(-1)^F$ a  b

$\mathrm{Tr}_{\mathcal{H}_{RR}} P(-1)^F$ $(-1)^P$  P

$\mathrm{Tr}_{a\,P(a)} P(-1)^F$ a  P

A: intersection # of D-branes/O-planes

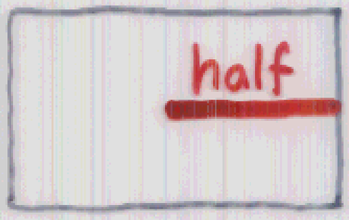
B: Euler/Hirzebruch/Lefschetz numbers

Factorization  etc.

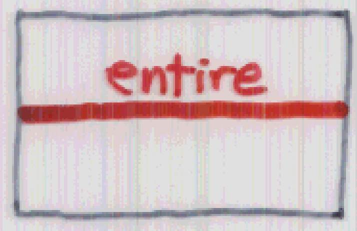
⟺ Riemann's bilinear identity

The case of LG model (A-type) H, Iqbal, Vafa; BH

D-brane or O-plane $\Big\}\ \gamma \xrightarrow{W} W(\gamma)$ horizontal line

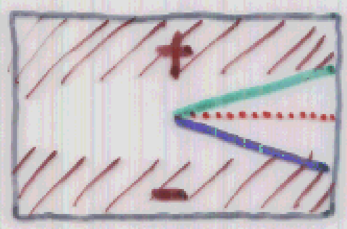


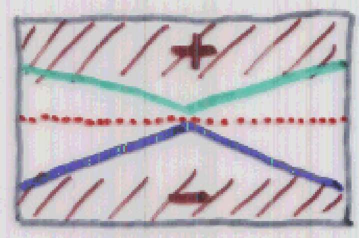
or



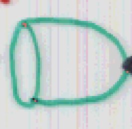
Deform the asymptotics

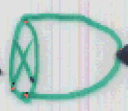
$\gamma^{\pm} \xrightarrow{W}$





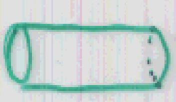
$$\boxed{\text{charge} = \gamma^{\pm} \in H_n(X^n, B_{\pm});\ \ B_{\pm} = \{\, \mathrm{Im}\, W \gtrless 0 \,\}}$$

γ  $i = \int_{\gamma^-} e^{-iW} \phi_i \Omega$

$\tau\Omega$  $i = \int_{O_\tau^-} e^{-iW} \phi_i \Omega$

γ : D-brane

O_τ : O-plane

γ_a  $\gamma_b = \#(\gamma_a^- \cap \gamma_b^+)$

$\tau\Omega$  $\tau\Omega = \#(O_\tau^- \cap O_\tau^+)$

γ  $\tau\Omega = \#(\gamma^- \cap O_\tau^+)$

Linear Sigma Model : $U(1)$ gauge theory

matters	P	X_1	X_2	X_3	X_4	X_5	
charges	-1	$1/h_1$	$1/h_2$	$1/h_3$	$1/h_4$	$1/h_5$	$\sum_{i=1}^{5} \frac{1}{h_i} = 1$

superpotential $W = P(X_1^{h_1} + X_2^{h_2} + X_3^{h_3} + X_4^{h_4} + X_5^{h_5})$

FI $\gg 0$: **NLσM** on a Calabi-Yau

$$X_1^{h_1} + \cdots + X_5^{h_5} = 0 \quad \text{in a } W\mathbb{CP}^4$$

FI $\ll 0$: $\langle P \rangle \neq 0$ **Landau Ginzburg Orbifold**

- $W = X_1^{h_1} + \cdots + X_5^{h_5}$
- $\Gamma =$ generated by $\gamma : X_i \to e^{\frac{2\pi i}{h_i}} X_i \quad \forall i$
  $\cong \mathbb{Z}_H \qquad H = \text{l.c.m.}\{h_1, \ldots, h_5\}$

= **Gepner Model**

$$M_{h_1} \times M_{h_2} \times M_{h_3} \times M_{h_4} \times M_{h_5} / \mathbb{Z}_H$$

M_h : IR fixed point of LG $W = X^h$

$= SU(2)_{k=h-2}/U(1)$ super coset

"$N=2$ minimal model"

A - Parity

$$X_i \to e^{\frac{2\pi i}{h_i} m_i} \overline{X}_{\sigma(i)}$$

$$P \to \overline{P}$$

σ: permutation of $\{1, \cdots, 5\}$

$h_{\sigma(i)} = h_i$

B - Parity

$$X_i \to e^{\frac{2\pi i}{h_i} m_i} X_{\sigma(i)}$$

$$P \to -P$$

$h_{\sigma(i)} = h_i$

$m_i + m_{\sigma(i)} = 0 \mod h_i$

Quintic

$$X_1^5+X_2^5+X_3^5+X_4^5+X_5^5=0 \quad \text{in } \mathbb{CP}^4$$

$$\begin{cases} \text{Kähler moduli} & 1_{\mathbb{C}} \\ \text{Complex structure moduli} & 101_{\mathbb{C}} \end{cases}$$

A

① $X_i \to \overline{X}_i$ $\quad (K=1_{\mathbb{C}} \quad C=101_{\mathbb{R}})$

O6-plane at $\mathbb{RP}^3$ (real quintic)

② $X_1 \leftrightarrow \overline{X}_2$, $X_i \to \overline{X}_i$ $i=3,4,5$ $\quad (K=1_{\mathbb{C}} \quad C=101_{\mathbb{R}})$

O6 at $\mathbb{RP}^3$

③ $X_1 \leftrightarrow \overline{X}_2$, $X_3 \leftrightarrow \overline{X}_4$, $X_5 \to \overline{X}_5$ $\quad (K=1_{\mathbb{C}} \quad C=101_{\mathbb{R}})$

O6 at $\mathbb{RP}^3$

B

④ $X_i \to X_i$ $\quad (K=1_{\mathbb{R}} \quad C=101_{\mathbb{C}})$

O9

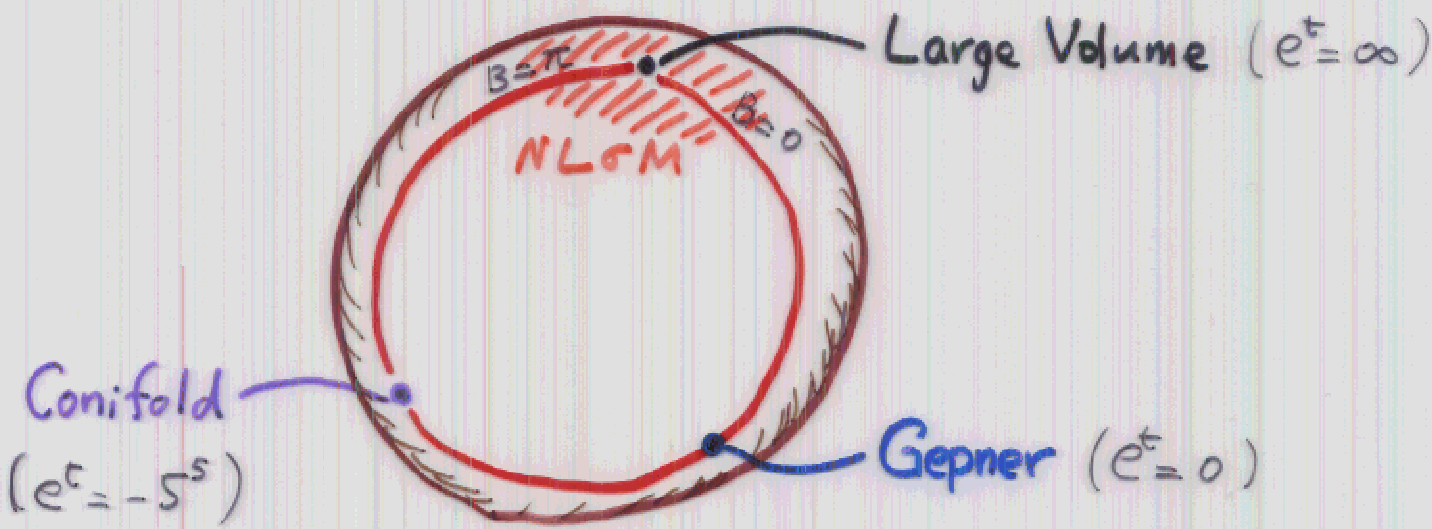
⑤ $X_1 \leftrightarrow X_2$, $X_i \to X_i$ $i=3,4,5.$ $\quad (K=1_{\mathbb{R}} \quad C=63_{\mathbb{C}})$

O3 at a pt & O7 along a hypersurface

⑥ $X_1 \leftrightarrow X_2$, $X_3 \leftrightarrow X_4$, $X_5 \to X_5$ $\quad (K=1_{\mathbb{R}} \quad C=53_{\mathbb{C}})$

O5 at a $\mathbb{CP}^1$ & O5 at a genus 6 curve

Kähler moduli of IIB Orientifolds ④⑤⑥ : e^t=real



- **Real** but combined with RR holonomy $\begin{cases} A_2 & \text{O9 or O5} \\ A_4^+ & \text{O7/O3} \end{cases}$ to form a **complex moduli field**

- Spacetime superpotential

- Passes through the **conifold point** (other description needed).

- At **Gepner point**, worldsheet is powerful.

No involutive A/B parities other than ①–⑥

Dressing by $X_i \to e^{\frac{2\pi i}{5} m_i} X_i$

- makes no difference

or • is impossible

$$A:\ X_i \longrightarrow \omega_i \overline{X_i}$$

$$\Updownarrow \qquad X_i' = \pm \omega_i^{-\frac{1}{2}} X_i \qquad (\pm \omega_i^{-\frac{1}{2}})^5 = 1$$

$$X_i' \to \overline{X}_i'$$

$$B:\ X_i \longrightarrow \omega_i X_i \qquad \text{not involutive}$$

The situation is different if some h_i even.

1. $\exists$ modification by $X_i \to e^{\frac{2\pi i}{h_i} n_i} X_i$
2. $\exists$ modification by quantum symmetry at Gepner pt.

To illustrate 1., we consider the example ...

A "Two parameter" Model

$$X_1^8 + X_2^8 + X_3^4 + X_4^4 + X_5^4 = 0 \quad \text{in } WCP^4_{11222}$$

$$K = 2c$$

$$X_i \to e^{\frac{2\pi i}{h_i} m_i} \overline{X_i}$$

RRW

fixed point : $X_i = e^{\frac{\pi i}{h_i} m_i} x_i \qquad x_i \in \mathbb{R}$

$$(-1)^{m_1} x_1^8 + (-1)^{m_2} x_2^8 + (-1)^{m_3} x_3^4 + (-1)^{m_4} x_4^4 + (-1)^{m_5} x_5^4 = 0$$

(i) $(-1)^{m_i} \equiv 1$: No solution

NO O-plane

(ii) $(-1)^{m_i} = (-1, 1, 1, 1, 1)$

O6 at S^3

(iii) $(-1)^{m_i} = (1, 1, 1, 1, -1)$

O6 at $S^3 \vee S^3$ (meeting at S^1) $\xrightarrow{\text{resolve}}$ a single $S^2 \times S^1$

(iv) $(-1)^{m_i} = (1, 1, 1, -1, -1)$

O6 at T^3

Description at Gepner Point

RCFT technique $\begin{cases} \text{boundary states} & \text{Cardy} \\ \text{crosscapstates} & \text{Pradisi-Sagnotti-Stanev} \end{cases}$

A-branes

$$|B^A\rangle = \frac{1}{\sqrt{H}} \sum_{l=1}^{H} \gamma^l |B^A\rangle^{\text{prod}}$$

product theory (BEFORE GSO)
$M_{h_1} \times \cdots \times M_{h_5}$

B-branes

$$|B^B_\omega\rangle = \frac{1}{\sqrt{H}} \sum_{l=1}^{H} \omega^l |B^B\rangle^{\text{prod}}_{\gamma^l} \qquad \omega^H = 1$$

boundary state in the γ^l-twisted circle.

+ transverse space + GSO : Recknagel Schomerus states

Crosscap States (Both A-parity & B-parity)

$$\boxed{|C_P\rangle = \frac{1}{\sqrt{H}} \sum_{l=1}^{H} |C_{\gamma^l P}\rangle^{\text{prod}}}$$

Dressing by quantum symmetry $g_\omega = \omega^l$ on γ^l-twisted states

$$|C_{g_\omega P}\rangle = \frac{1}{\sqrt{H}} \sum_{l=1}^{H} \omega^{-l} |C_{\gamma^l P}\rangle^{\text{prod}}$$

A-type : $(g_\omega P)^2 = (\pm 1)^F$ only if $\omega = \pm 1$ ... possible only if H even (some h_i even)

Full string theory

Polchinski – Cai

Callan-Lovelace-Nappi-Yost

"Tadpole state" $|T\rangle = \underbrace{|B\rangle}_{\substack{\parallel \\ |B\rangle_{NSNS} + |B\rangle_{RR}}} + \underbrace{|C\rangle}_{\substack{\parallel \\ |C\rangle_{NSNS} + |C\rangle_{RR}}}$

$$|B\rangle_{NSNS} = |B_+\rangle_{NSNS} + |B_-\rangle_{NSNS}$$

$$|B\rangle_{RR} = |B_+\rangle_{RR} + |B_-\rangle_{RR}$$

$$|C\rangle_{NSNS} = |C_{(-1)^{F_R}P}\rangle + |C_{(-1)^{F_L}P}\rangle$$

$$|C\rangle_{RR} = |C_P\rangle + |C_{(-1)^F P}\rangle \qquad P^2 = 1$$

$$\left|\begin{matrix}\text{each term}\\ \text{on RHS's}\end{matrix}\right\rangle = \underbrace{\left|\begin{matrix}\text{4d spacetime}\\ +\text{ghost}\\ +\text{superghost}\end{matrix}\right\rangle}_{\substack{\text{Standard}\\ \text{Coherent state}\\ \text{(universal)}}} \otimes \underbrace{|\,\text{internal}\,\rangle}_{\substack{\text{What I'm talking}\\ \text{about.}}}$$

Consistency Condition:

$${}_{RR}\langle i|C^{int}\rangle_{RR} + \frac{1}{4}\,{}_{RR}\langle i|B^{int}\rangle_{RR} = 0 \qquad \forall \text{(int) RR ground state } |i\rangle_{RR}$$

$\mathcal{N}=1$ supersymmetry:

$$\frac{{}_{RR}\langle 0|C^{int}\rangle_{RR}}{{}_{NSNS}\langle 0|C^{int}\rangle_{NSNS}} = \frac{{}_{RR}\langle 0|B^{int}\rangle_{RR}}{{}_{NSNS}\langle 0|B^{int}\rangle_{NSNS}} \qquad |0\rangle_{NSNS} \xleftrightarrow{\text{spec. flow}} |0\rangle_{RR}$$

A-type in more detail

All h_i odd $\quad H = \mathrm{l.c.m.}\{h_i\}$ odd, $\{\gamma^\ell\}_{\ell=1}^{H} = \{\gamma^{2\ell}\}_{\ell=1}^{H}$

$$|C_{\gamma^{2\ell}P}\rangle^{\mathrm{prod}} = \gamma^\ell |C_P\rangle^{\mathrm{prod}}$$

$$\therefore |C_P\rangle = \frac{1}{\sqrt{H}} \sum_{\ell=1}^{H} \gamma^\ell |C_P\rangle^{\mathrm{prod}}$$

Some h_i even $\quad H$ even, $\{\gamma^\ell\}_{\ell=1}^{H} = \{\gamma^{2\ell}\}_{\ell=1}^{H/2} \cup \{\gamma^{2\ell+1}\}_{\ell=1}^{H/2}$

$$|C_{P\pm}\rangle = \frac{1}{\sqrt{H}} \sum_{\ell=1}^{H/2} \left\{ \gamma^\ell |C_P\rangle^{\mathrm{prod}} \pm \gamma^\ell |C_{\gamma P}\rangle^{\mathrm{prod}} \right\}$$

$+ \leftrightarrow -$: dressing by $\mathbb{Z}_2$ quantum symmetry

- One of $\pm$ continues to Large Volume "Geometric"
- The other is locked at the Gepner point

— Kähler moduli removed "non-geometric"

"Geometric"/"non-geometric" ... distinguished by the **CHARGE**.

e.g. Suppose $P = \tau\Omega$ is "geometric".

If $[\text{O-plane}]^+ = [\gamma]^+ \in H_n(X^n, B_+)$ $\quad \gamma$ an A-brane

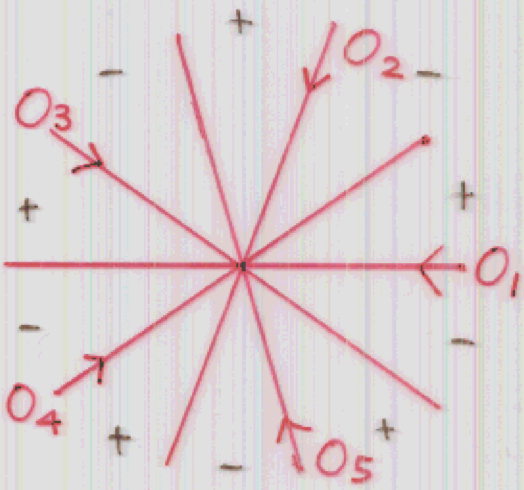
$$\tau : \gamma \longrightarrow -e^{i\theta_{LG} - i\theta_{CY}} \gamma$$

where $\quad \tau^* \Omega_{\substack{LG\\CY}} = e^{i\theta_{\substack{LG\\CY}}} \overline{\Omega_{\substack{LG\\CY}}}$

Examples

Quintic $\quad W = X_1^5 + X_2^5 + X_3^5 + X_4^5 + X_5^5 \,/\, \mathbb{Z}_5$

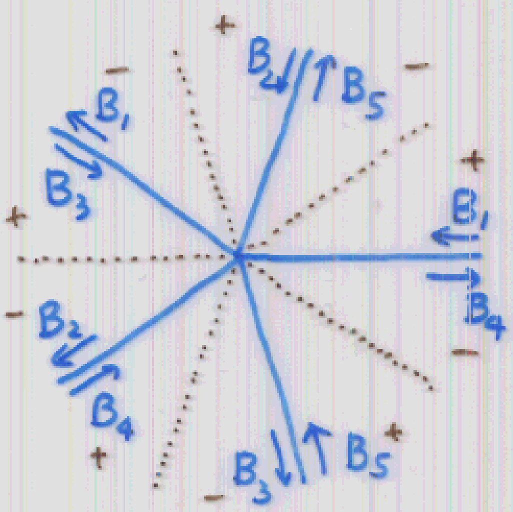
$$P_A : X_i \to \overline{X}_i \quad \forall i$$



$$O_{P_A} = \frac{1}{\sqrt{5}}\left(O_1^5 + O_2^5 + O_3^5 + O_4^5 + O_5^5\right)$$

$$[O_i]^+ = [B_i]^+$$

$$[O_{P_A}]^+ = [B]^+$$



$$B = \frac{1}{\sqrt{5}}\left(B_1^5 + B_2^5 + B_3^5 + B_4^5 + B_5^5\right)$$

$$= B_{j_0 n_0 s_0}$$

$$j_0 = \left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right)$$
$$n_0 = (2, 2, 2, 2, 2)$$
$$s_0 = (1, 1, 1, 1, 1)$$

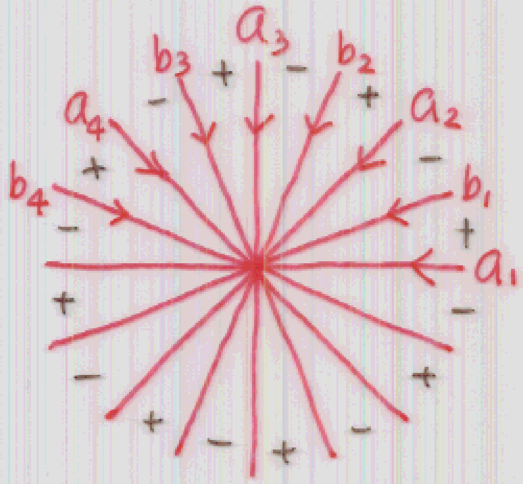
$\therefore$ D-brane on real quintic $(\mathbb{RP}^3)$ $= B_{j_0 n_0 s_0}$

Tension $\quad T_{O_{P_A}} = \langle 0 | C (-1)^{F_L} P_A \rangle = \dfrac{5}{\sqrt{5}} \left(\sqrt{\dfrac{2}{5 \sin(\frac{\pi}{5})}}\right)^5$

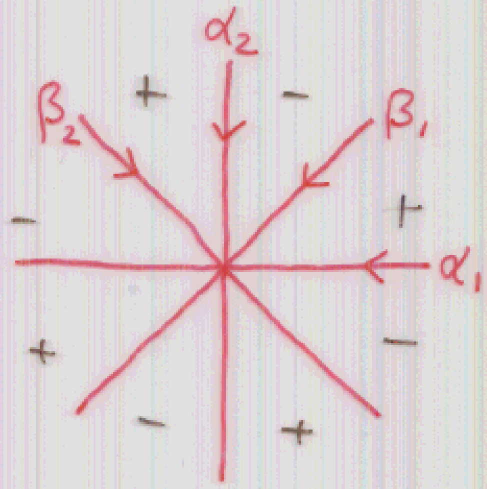
$$= \langle 0 | B_{j_0 n_0 s_0} \rangle_{NSNS}$$

SO-type O_{P_A} & four $B_{j_0 n_0 s_0}$: Consistent & supersymmetric

2 parameter Model $\quad W = X_1^8 + X_2^8 + X_3^4 + X_4^4 + X_5^4 \,/\, \mathbb{Z}_8$



$h=8$



$h=4$

(i) $\quad P_A : X_i \to \overline{X}_i \quad \forall i$

$$O_{P_A\pm} = \frac{1}{\sqrt{8}}\left(a_1^2\alpha_1^3 + a_2^2\alpha_2^3 + a_3^2(-\alpha_1)^3 + a_4^2(-\alpha_2)^4\right)$$
$$\pm \frac{1}{\sqrt{8}}\left(b_1^2\beta_1^3 + b_2^2\beta_2^3 + b_3^2(-\beta_1)^3 + b_4^2(-\beta_2)^3\right)$$

Homological relations : $\quad [a_i]^+ = [b_i]^+ \quad i=1,2,3,4$

$\qquad [\alpha_m]^+ = [\beta_m]^+ \quad m=1,2$

$$[O_{P_A+}]^+ = \frac{2}{\sqrt{8}}\left(a_1^2\alpha_1^3 + \cdots\right)^+ \neq 0$$
$$[O_{P_A-}]^+ = 0$$

Large volume : no O-plane

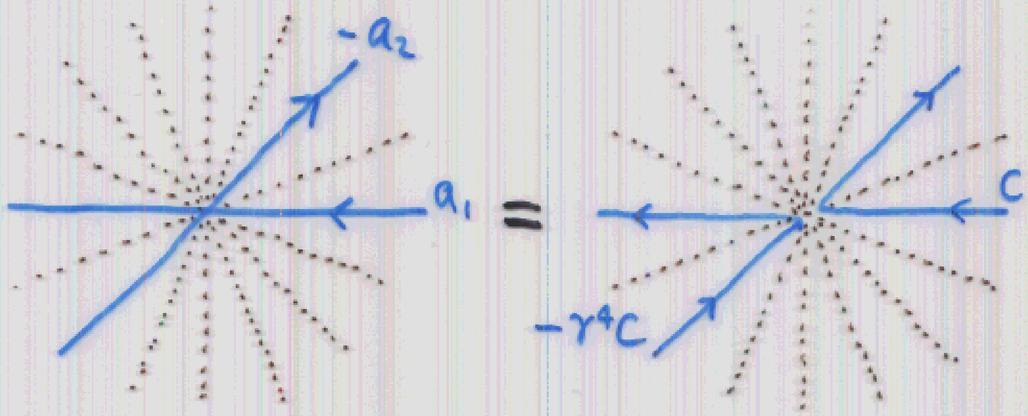
O_{P_A-} is the "Geometric" one.

(ii) $\gamma_{(1)}P_A : (X_1, X_2, X_3, X_4, X_5) \to (e^{\frac{2\pi i}{8}}\bar{X}_1, \bar{X}_2, \bar{X}_3, \bar{X}_4, \bar{X}_5)$

$$O_{\gamma_{(1)}P_A \pm} = \frac{1}{\sqrt{8}}\left(b_1 a_1 \alpha_1^3 + b_2 a_2 \alpha_2^3 + b_3 a_3 (-\alpha_1)^3 + b_4 a_4 (-\alpha_2)^3 \right)$$
$$\pm \frac{1}{\sqrt{8}}\left(a_2 b_1 \beta_1^3 + a_3 b_2 \beta_2^3 + a_4 b_3 (-\beta_1)^3 + (-a_1) b_4 (-\beta_2)^3 \right)$$

"Geometric"

$$\left[O_{\gamma_{(1)}P_A -}\right]^{\dagger} = \frac{1}{\sqrt{8}}\Big[(a_1 - a_2) a_1 \alpha_1^3 + (a_2 - a_3) a_2 \alpha_2^3 + (-a_3 + a_4) a_3 \alpha_1^3 + (-a_4 - a_1) a_4 \alpha_2^3 \Big]^{\dagger}$$



$$= \frac{1}{\sqrt{8}}\left[\sum_{l=1}^{8} \gamma^l \left(c\, a_1 \alpha_1^3 \right) \right]^{\dagger} = \left[B_{j_1 n_1 s_1} \right]^{\dagger}$$

$j_1 = (0, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ $n_1 = (1, 4, 2, 2, 2)$ $s_1 = (1, 1, 1, 1, 1)$

The fixed pt set of $\gamma_{(1)}P_A \cong S^3 \leftrightarrow B_{j_1 n_1 s_1}$

Tension $T_{O_{\gamma_{(1)}P_A -}} = \langle 0 | C_{(-1)^{F_L}\gamma_{(1)}P_A -} \rangle = \frac{4}{\sqrt{8}} \sqrt{\frac{2}{8 \sin\frac{\pi}{8}}}^{\,2} \sqrt{\frac{2}{4 \sin\frac{\pi}{4}}}^{\,3} \cdot 2\sin\left(\frac{\pi}{8}\right)$

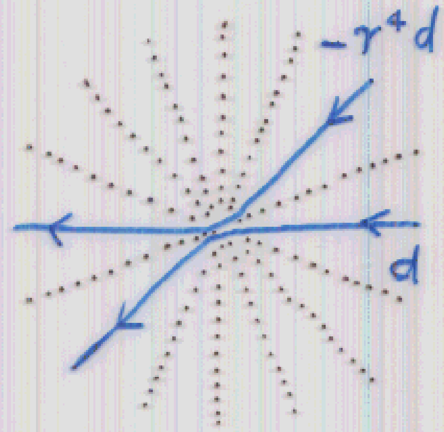
$$= \frac{1}{\sqrt{2}} = \langle 0 | B_{j_1 n_1 s_1} \rangle_{NSNS}$$

SO-type $O_{\gamma_{(1)}P_A -}$ & four $B_{j_1 n_1 s_1}$: Consistent & Supersymmetric

"Non-geometric"

$$\left[O_{\gamma_{(1)} P_A +}\right]^\dagger = \frac{1}{\sqrt{8}}\left[(a_1+a_2)\, a_1 \alpha_1^3 + (a_2+a_3)\, a_2 \alpha_2^3 - (a_3+a_4)\, a_3 \alpha_1^3 - (a_4-a_1)\, a_4 \alpha_2^3 \right]$$

$a_1 + a_2 =$



$$= \frac{1}{\sqrt{8}}\left[\sum_{l=1}^{8} \gamma^l \left(d\, a_1 \alpha_1^3 \right)\right]^\dagger = \left[B_{j_i' n_i' s_i} \right]^\dagger$$

$$j_i' = \left(2, \tfrac{3}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$n_i' = (5, 4, 2, 2, 2)$$

Tension $\quad T_{O_{\gamma_{(1)} P_A +}} = \langle 0 | C_{(-1)^{F_L} \gamma_{(1)} P_A +} \rangle = \frac{4}{\sqrt{8}} \sqrt{\frac{2}{8 \sin\frac{\pi}{8}}}^{\,2} \sqrt{\frac{2}{4 \sin\frac{\pi}{4}}}\, 2\cos\left(\frac{\pi}{8}\right)$

$$= \langle 0 | B_{j_i' n_i' s_i} \rangle_{NSNS}$$

SO-type $O_{\gamma_{(1)} P_A +}$ & four $B_{j_i' n_i' s_i}$: consistent & supersymmetric

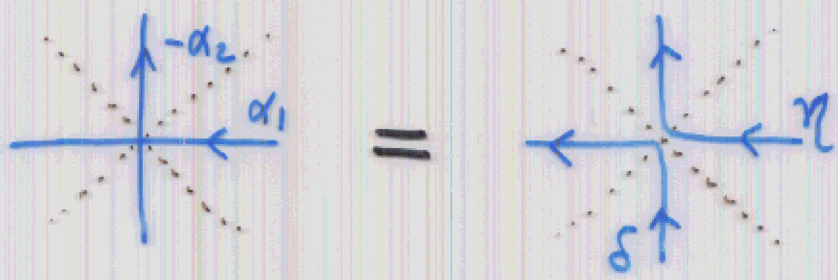
Moduli field of total size removed.

(iii) $\gamma_{(5)} P_A : (X_1 \cdots X_5) \to (\bar{X}_1 \cdots \bar{X}_4 , e^{\frac{2\pi i}{4}} \bar{X}_5)$

$$O_{\gamma_{(5)}P_A\pm} = \frac{1}{\sqrt{8}}\left(a_1^2\alpha_1^2\beta_1 + a_2^2\alpha_2^2\beta_2 + a_3^2(-\alpha_1)^2(-\beta_1) + a_4^2(-\alpha_2)^2(-\beta_2)\right)$$
$$\pm \frac{1}{\sqrt{8}}\left(b_1^2\beta_1^2\alpha_2 + b_2^2\beta_2^2(-\alpha_1) + b_3^2(-\beta_1)^2(-\alpha_2) + b_4^2(-\beta_2)^2\alpha_2 \right)$$

"Geometric"

$$\left[O_{\gamma_{(5)}P_A-}\right]^+ = \frac{1}{\sqrt{8}}\left[a_1^2\alpha_1^2(\alpha_1-\alpha_2) + a_2^2\alpha_2^2(\alpha_2+\alpha_1) + a_3^2\alpha_1^2(-\alpha_1+\alpha_2) + a_4^2\alpha_2^2(-\alpha_2-\alpha_1)\right]^+$$



$$= \left[\frac{1}{\sqrt{8}}\sum_{l=1}^{4}\gamma^l(a_1^2\alpha_1^2\eta) + \frac{1}{\sqrt{8}}\sum_{l=1}^{4}\gamma^l(a_1^2\alpha_1^2\delta)\right]^+$$

$$= \left[B_{j_2 n_2 s_1}\right]^+ + \left[B_{j_3 n_2 s_1}\right]^+$$

$j_2 = (\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, 0)$, $j_3 = (\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, 1)$, $n_2 = (4, 4, 2, 2, 1)$

$\gamma_{(5)}P_A$ - fixed point set $= S^3 \vee S^3 \leftrightarrow B_{j_2 n_2 s_1}$ & $B_{j_3 n_2 s_1}$

Tension $T_{O_{\gamma_{(5)}P_A-}} = \langle 0 | C_{(-1)^{F_L}\gamma_{(5)}P_A-}\rangle = \frac{4}{\sqrt{8}}\sqrt{\frac{2}{8\sin\frac{\pi}{8}}}^{\,2}\sqrt{\frac{2}{4\sin\frac{\pi}{4}}}^{\,3}\cdot 2\sin(\frac{\pi}{4})$

$$= \langle 0 | B_{j_2 n_2 s_1}\rangle_{NSNS} + \langle 0 | B_{j_3 n_2 s_1}\rangle_{NSNS}$$

SO type $O_{\gamma_{(5)}P_A-}$ ← four $B_{j_2 n_2 s_1}$ + four $B_{j_3 n_2 s_1}$: consistent ← supersymetric