D-Flation

Based on :

Sk, Kallosh, Linde, Trivedi hep-th/0301240

* work to appear soon with

Kallosh, Linde, Maldacena, McAllister, Trivedi

(hep-th/0307???)

[Plus works over ~2 yrs with :

Giddings, Liu, Pearson, Polchinski, Schulz,

Tripathy, Trivedi, H. Verlinde]

REFERENCES

(Very incomplete list of early work in subjects of my talk)

Flux comp:

Polchinski, Strominger

Becker[2]

Gukov, Vafa, Witten

Dasgupta, Rajesh, Sethi

Mayr

Curio, Klemm, Lüst, Theisen

...

Brane Inflation:

Dvali, Tye

Burgess, Majumdar, Nolte, Quevedo, Rajesh, Zhang

Shiu, Tye

Herdeiro, Hirano, Kallosh

García-Bellido, Rabadan, Zamora

Kyae, Shafi

...

Introduction

For past 18 years, there has been much work on string compactifications to 4d $\mathcal{N}=1$ SUSY :

- Heterotic strings on Calabi-Yau
- M theory on G_2 spaces
- Type II on CY orientifolds
  ($\subset$ F-theory on CY fourfolds)

The simplest constructions come with many chiral multiplet moduli. It is interesting to understand potentials which could arise for these for many reasons:

- Scalars with $m \sim$ TeV $\rightarrow$ cosmological issues
- Predictivity (particle physics parameters depend on moduli VEVs)

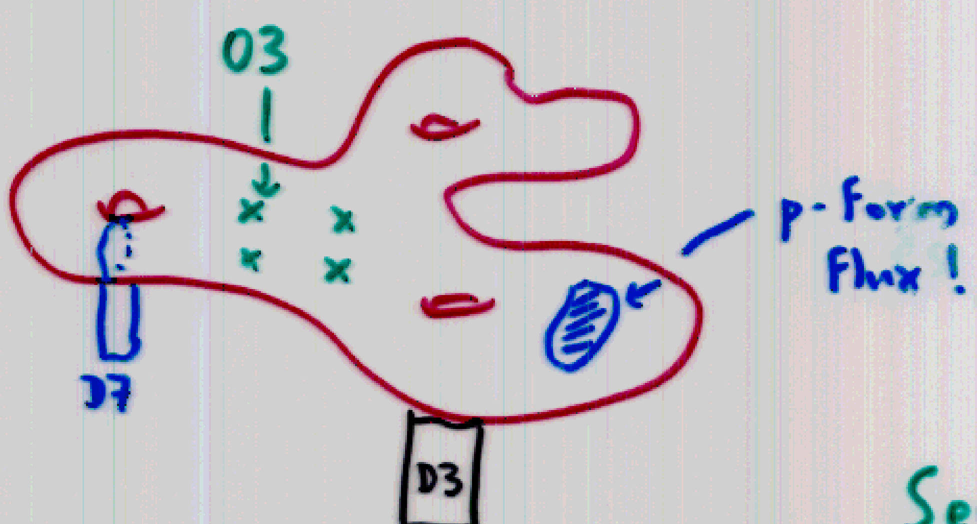
- Need to know $V(\phi_i)$ to "count" vacua (understand possibilities for 4d physics)
- Some of these moduli → serious problems for simple inflation proposals

I'll discuss these in this talk

"Pseudo-BPS" IIB compactifications

(c.f. Giddings, SK, Polchinski 0105097)

Consider IIB on a CY orientifold of D3/D7 type:



Sen

The orientifold M ← limit of F thy on X_4.

In addition to the branes, IIB has RR and NS 3-form fluxes that can a priori be nonzero:

$$\frac{1}{2\pi\alpha'}\int F_3, \quad \frac{1}{2\pi\alpha'}\int H_3 \quad \in 2\pi\mathbb{Z}$$

integrated over each $\Sigma_3 \in M$

The models are constrained by a tadpole condition:

$$\frac{\chi(X_4)}{24} = N_{D3} + \int_M H \wedge F$$

So generically, flux $\neq 0$ and $\exists$ some D3s.

Resulting class of IIB solutions:

Let:
$$ds_{10}^2 = e^{2A(y)}\eta_{\mu\nu}\,dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}(y)\,dy^m dy^n$$

$$F_5 = (1+*)\left[d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\right]$$

$$G_3 = F_3 - \tau H_3$$

$\tau = \tau(y)$

Then one finds that the solutions are:

$$\alpha = e^{4A}$$

$$*_6 G_3 = i\, G_3$$

+ eqns that fix $A(y)$, $\phi(y)$, $\tilde{g}_{mn}$

(→ F-theory sol'n if $\exists$ D7s
→ conformally CY_3 if no D7s)

For ease of language let's imagine we're in (conformally) CY_3 case. The eqn

$$*_6 G_3 = i\, G_3$$

is very interesting! Fixing $F, H \in H^3(M, \mathbb{Z})$, it generically fixes ϕ and complex structure of M completely.

4d Effective SUGRA description

For a 4d sugra, need to specify chiral fields and K, W, etc.

Kähler modulus ρ

Cplx str moduli z^α $\quad \alpha = 1, .., h^{2,1}(M)$

Dilaton τ

Brane positions ϕ

<u>Then</u>:

$$\mathcal{K}(\rho, \phi, \bar{\rho}, \bar{\phi}) = - 3 \ln [-i(\rho - \bar{\rho}) - k(\phi, \bar{\phi})]$$

kähler pot for CY metric!

$$\mathcal{K}(\tau, z^\alpha) = -\ln[-i(\tau - \bar{\tau})] - \ln[-i \int \Omega \wedge \bar{\Omega}]$$

$$W = \int G_3 \wedge \Omega$$

Gukov, Vafa, Witten

- Solutions indeed have $*_6 G = i G$
- ρ and ϕ moduli are <u>unfixed</u> while τ, z^α <u>fixed</u> (unfixed D3s $\leftrightarrow$ "pseudo-BPS")

The reason ρ, ϕ are unfixed:

$$V_{sugra} \approx e^{k}\left(g^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2\right)$$

But with this k, W:

$$g^{a\bar{b}} \partial_a k \, \partial_{\bar{b}} k = 3$$

a, b run over ρ, ϕ

$\rightarrow$ can write

$$V = e^{k} \sum_{\tau, z^{\alpha}} \left(g^{\alpha\bar{\beta}} D_\alpha W \overline{D_\beta W}\right) \geq 0$$

And then minima $\Rightarrow$ G ISD,

SUSY minima $\Rightarrow$ $\int G \wedge \Omega = 0$ $\Rightarrow$

G is type $(2,1)$

Imagine a model with no D3s. Resulting low energy theory quite dull:

$$W = W_0$$

$$K = -3 \ln[-i(\rho - \bar{\rho})]$$

However:

- α' corrections $\to \delta k$
- Can easily imagine (via duality to het. string, for instance) cases where $\exists$ a pure SYM sector from wrapped D7s, with $\theta + \frac{i}{g_{YM}^2} = \rho$

So

$$k \to k + \delta k$$

$$W \to W = W_0 + A e^{ia\rho}$$

Any such generic ρ dependence $\Rightarrow$ can expect to solve

$$D_\rho W = k_{,\rho} W + \frac{\partial W}{\partial \rho} = 0$$

$\to$ really got an isolated AdS vacuum!

Lifting to dS

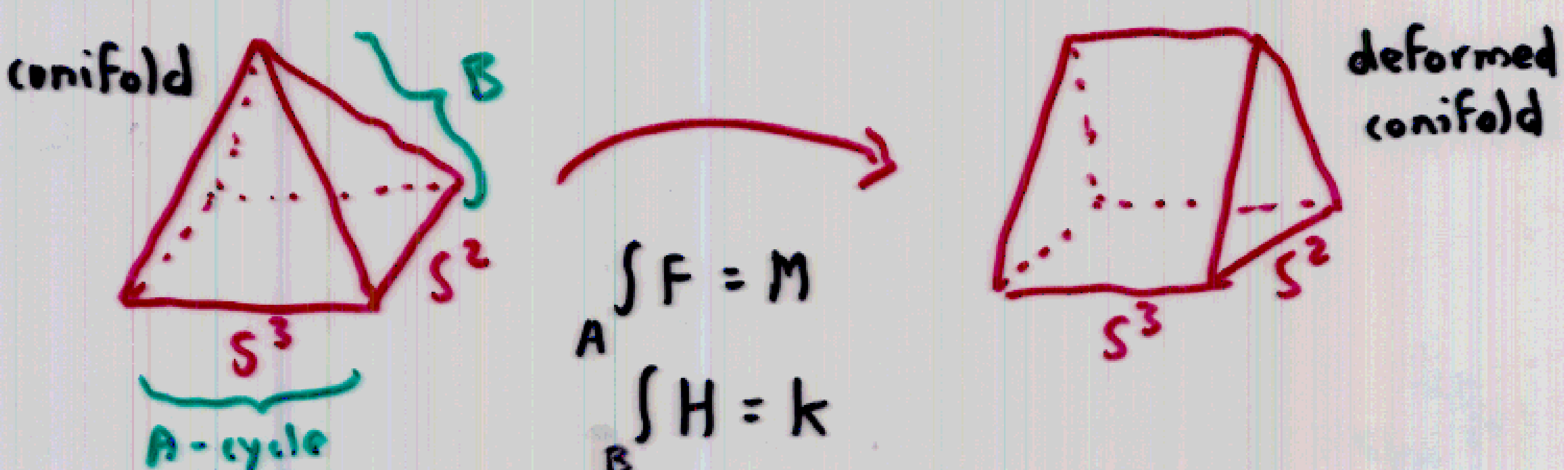
One might want to look for $\Lambda > 0$ instead.

Can we arrange this? Recall that these models are <u>warped</u>:

$$ds^2 = \underbrace{e^{2A(y)}}_{\text{warp factor}} \eta_{\mu\nu}\, dx^\mu dx^\nu + e^{-2A(y)}\, \tilde{g}_{mn}\, dy^m dy^n$$

- If $\exists$ regions of very small e^{2A} (as in RS toy models), can add a bit of energy by dropping a $\overline{D3}$ there!
- Can make such highly warped models: H. Verlinde, GKP

Embed Klebanov-Strassler sol'n into CY



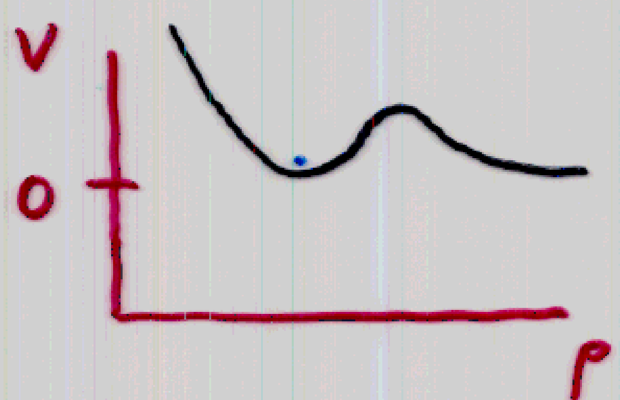
Result : $e^{2A}\Big|_{min} \approx e^{-4\pi k/3g_s M}$

So one can imagine adding to potential

$$\delta V \sim \frac{T_{\overline{D3}}\, e^{-8\pi k/3g_s M}}{(\operatorname{Im}\rho)^2} \Bigg\} \begin{array}{l}\overline{D3} \text{ at} \\ \text{end of} \\ \text{KS throat}\end{array}$$

Result for some tuned (but not <u>very</u> tuned)

choices of W_0, k, M, ...



Can get a dS sol'n with Λ sub-stringy

The more you can tune :

- W_0
- k, M → smaller you can imagine
- A, a Λ being, and better your control
- $g_s|_{min}$ gets

How many vacua are there?

Here we will be *very* rough (see Douglas' talk for more refined guesstimate).

- Tadpole condition

$$\int H \wedge F \le \frac{\chi}{24}$$

$\rightarrow$ replace with $\displaystyle\sum_{i=1}^{b^3} Q_i^2 \le \frac{\chi}{24}$

- Assume $O(1)$ fraction of fluxes $\rightarrow$ a vacuum.

$$\boxed{\#\text{ vacua} \sim \left(\frac{\chi}{24}\right)^{b^3/2}} \leftarrow \text{"The discretuum"}$$

This is *very* crude, but suggests *huge* #.

Real problem is # flux vacua per CY, NOT # of CYs (presently $\sim 10^4$ CYs known; this # is $\gg$ that).

In such a situation:

- Can hope large # models $\to$ ability to find dS vacua, even with small Λ (Bousso-Polchinski; Feng et al; Maloney, Silverstein, Strominger in $D>10$; KKLT)
- Can ask about eternal inflation as one filters through {vacua} (Susskind, Linde)
- Can try to really count / classify and see if we "should" exist (Douglas)
- Given dS constructions, can try to explain dS entropy (Fabinger, Silverstein)

It's clear existing dS proposals NOT fundamental or beautiful; as we learn to calculate W exactly, more + better models will exist.

Here, we will go in a different direction. Regardless of if/how we understand our vacuum, we know there was interesting dynamics in early Universe: e.g., inflation? We can hope to understand this via string thy.

Hasty Review of Inflation

Inflation explains:

- Homogeneity + isotropy of our Universe
- Absence of monopoles + other garbage
- Seeding of density perturbations

Simplest example:

Scalar field theory (+ gravity)

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \cdots$$

To have inflation, one needs:

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$ ← flat pot. → dS like expansion

$$\eta = M_p^2 \frac{V''}{V} \ll 1$$ ← Inflation should last long enough

One wants:

$$N_e = \frac{1}{M_p^2}\int \frac{V}{V'}\, d\phi \gtrsim 60$$

Finally, observed $\frac{\delta\rho}{\rho}$ →

$$\delta_H = \frac{1}{\sqrt{75\pi}}\frac{1}{M_p^3}\frac{V^{3/2}}{V'} \simeq 2\times 10^{-5}$$

(constrains scale of inflation).

So, we would like simple models that both fit naturally into string thy, and naturally → small ϵ, η.

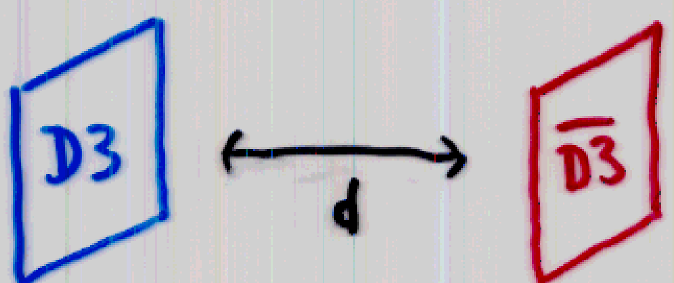
There is a huge and interesting literature which constructs D-brane inflation models in various approximations. I'll explain how compactification + stabilization of the extra dims generically → problems for the simple proposals.

(See Quevedo review)

D-flation

Simplest class of models:

Dvali / Tye



($\perp$ to M and filling $\mathbb{R}^{3,1}$).

$$V(r) = 2T_{D3}\left\{1 - \frac{1}{2\pi^3}\frac{T_{D3}}{M_{10}^8\, d^4}\right\}$$

→ (in terms of canonical ϕ)

$$V(\phi) = 2\, T_{D3}\left\{1 - \frac{1}{2\pi^3}\frac{T_{D3}^{\,3}}{M_{10}^8\, \phi^4}\right\}$$

Basic Idea:

Start branes far apart, let ϕ slow roll;

$D-\bar{D}$ tachyon $\rightarrow$ exit inflation!

Problem:

Let's calculate the slow roll parameters. Recall:

$$M_p^2 = M_{10}^8 L^6$$

(space of size L) $\rightarrow$

$$\eta = -\frac{10}{\pi^3}\left(\frac{L}{d}\right)^6 \approx -.3\left(\frac{L}{d}\right)^6$$

Small η $\rightarrow$ need $d > L$! Impossible.

Two comments:

1 - Basic strategies for avoiding this (anisotropic extra dim, fine tuning initial conditions, ...) cannot work.

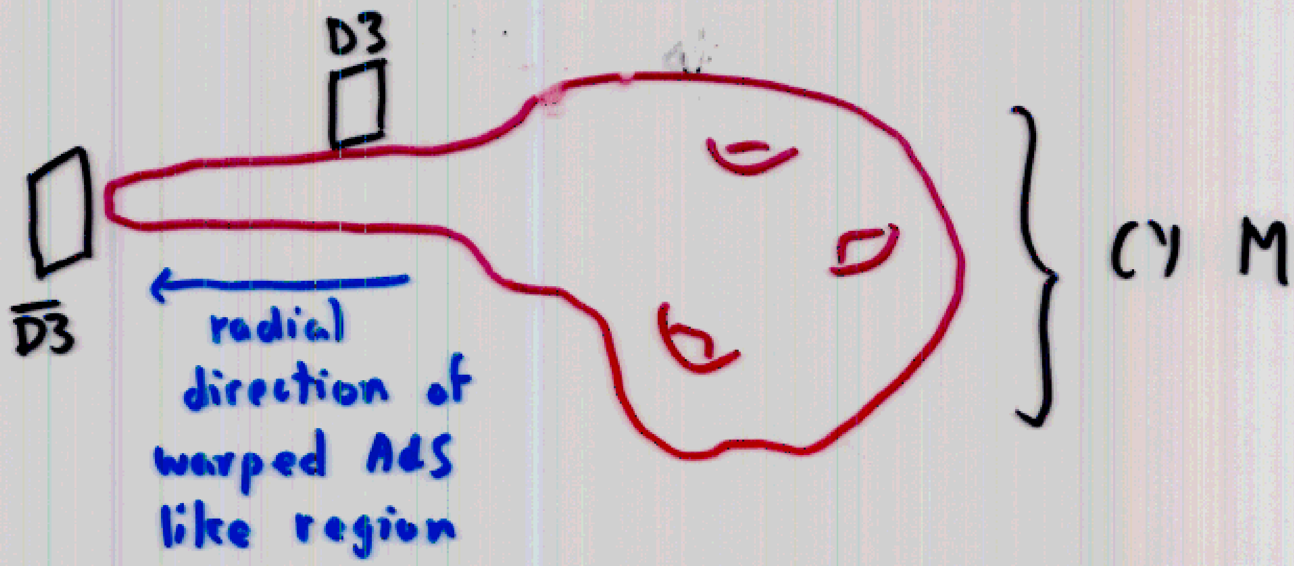
2- The "constant" in V which dominates during inflation is really a steep function of L:

$$V(L) \sim \frac{2\,T_{D3}}{L^{12}}$$

In absence of L stabilization, get fast roll to $L \to \infty$. But once we get around the problem that $\eta_{naive} \ll 1$ is impossible, we'll see generic methods of L fixing $\to$ unacceptably large η. This seems to be a problem for all D3 inflation models.

Evading the 0^{th} order problem: Use warping

Consider again the highly warped IIB models with e.g. a warped, deformed conifold region:



- We can basically compute the force by studying D3, $\overline{D3}$ in cut-off AdS_5:

$$ds^2 = h^{-1/2}(-dt^2 + d\vec{x}^2) + h^{1/2}(dr^2 + r^2 d\Omega^2)$$

$$h(r) = 4\pi g_s N \frac{l_s^4}{r^4}$$

$\overline{D3}$ sits at $r_0 = r_{min}$; D3 at $r_1 \leq r_{max}$

- $\overline{D3}$ "sees" perturbed h due to extra D3

$$h \to \tilde{h}(r) = 4\pi g_s l_s^4 \left[\frac{N}{r^4} + \frac{1}{|\vec{r}_1 - \vec{r}|^4} \right]$$

$$= h_0(r) + h_1(r)$$

So the warp factor at the $\overline{D3}$ location is:

$$e^{4A}(r_0) \sim \frac{1}{h_0(r_0)} \left\{ 1 - \frac{h_1(r_0)}{h_0(r_0)} \right\}$$

where I assumed large warping $(h_0(r_0) \gg 1)$.
Multiplying by T_{D3} $\rightarrow$ very flat pot'l as a function of r_1. This naive potential $\rightarrow$ excellent inflation with η, ϵ naturally exponentially small! Solves "$d \overset{??}{>} L$" problem.

More Problems:

This is too naive. Must worry about ρ stabilization to prevent decompactification. We'll do this now -- result is consistent with conformal coupling of D3 to gravity (which of course ruins inflation).

So back to 4d sugra with D3 (leave out $\overline{D3}$ for a moment) :

$$K = -3 \ln\left[-i(\rho - \bar{\rho}) - k(\phi, \bar{\phi})\right]$$

It follows that ρ, ϕ mix -- the axion in ρ is fibered over the ϕ moduli space.

If one defines

$$2r = \rho + \bar{\rho} - k(\phi, \bar{\phi})$$

then in fact :

- r is the real "volume modulus"

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + e^{2u(x)} \tilde{g}_{mn} dy^m dy^n$$

$$r \sim e^{4u} \quad (\sim L^4)$$

- But ρ, ϕ are the good chiral mults.

With this in mind, let's imagine how to inflate and fix the volume too.

Scenario I: Superpotential Stabilization

Very plausibly in various cases, NP effects

$\rightarrow$

$$W = W_0 + A e^{ia\rho} + \cdots$$
$$= W(\rho) \leftarrow$$ just assume a generic hol. function

Can compute energy during inflation by looking at (approximate) dS vacuum of

$$V_{TOT} = e^K \left(g^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2 \right)$$
$$+ \frac{D}{(2r)^2} \leftarrow$$ from brane tensions; this drives inflation

Suppose this V has min. at $r = r_0$, $\phi = 0$, with value $V|_{min} = V_0^{dS}$.

Algebra $\to$ quadratic term

$$\boxed{\delta V = \frac{2}{3}\left(V_0^{\,dS}\right)\varphi\bar{\varphi}}$$

For (canonically normalized) brane field.

This is actually what would arise from the curvature coupling of a conformally coupled scalar. [Same is true in AdS vacuum of the SUSY theory w/o $\overline{D3}$s].

Intuitive Explanation

The known sources of inflationary energy (internal fluxes + brane tensions) scale with r modulus like

$$V \sim r^{-\alpha}; \quad \alpha > 0 \text{ and } U(1)$$

But W fixes ρ (or comb. of ρ, ϕ other than r).

So as the brane moves, ϕ changes $\to$

$$\delta V \sim \alpha V \frac{\phi\bar{\phi}}{r}$$

You can quickly verify this $\Rightarrow$ $\eta = \mathcal{O}(1)$.

Scenario II: Kähler Stabilization

$$K = -3 \log (2r) + \cdots$$

Can hope the $\cdots$ together with

$$W = W_0$$

directly fix r (as the true volume, r controls α' expansion). Then inflation could proceed.

Hard to imagine an explicit model where this happens computably.

Summary / Conclusion

- $\exists$ many interesting brane inflation scenarios
- Real consideration of compactification $\rightarrow$ problems for most, generically. $\longleftrightarrow$ exceptions?

  E.g. D3 models where inflationary $V \sim r^{-\alpha}$ do not work, generically.
- If we're lucky and $H \sim 10^{14}$ GeV during inflation, maybe we need to be more creative (hard to believe the initial condition at stringy energies is: "start with the quintic and a 3-$\overline{3}$ pair...").