TWO-DIMENSIONAL STRING THEORY AND D-BRANES

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Based on IK, hep-th/9108019;

IK, Maldacena, Seiberg, hep-th/0305159

Douglas, IK, Kutasov, Maldacena, Martinec, Seiberg, in preparation.

In the 80's and 90's, low-dimensional string theories based on $c \leq 1$ 2-d CFT's coupled to 2-d quantum gravity were extensively studied with both continuum (Liouville) and discretized (large N matrix model) approaches.

Discovery of double-scaling limits (Brezin, Kazakov; Douglas, Shenker; Gross, Migdal) led to their solution to all orders in string perturbation theory.

For reviews, see, e.g. Ginsparg, Moore; Kazakov; IK; Polchinski.

These dualities between large N matrix models and strings were precursors of the AdS/CFT duality and provided useful intuition towards its formulation.

Another general lesson (Shenker):

Non-perturbative effects are

$$\sim e^{-\mathrm{const}/g_s}.$$

In the $c=1$ matrix model they are due to single eigenvalue tunnelling.

Later it became clear that D-instantons produce effects of same order (Polchinski, Green).

Recently, connections between D-branes

and matrix eigenvalues in $c \leq 1$ models was made concrete.

Progress in both Liouville theory (especially, discovery of boundary states localized at infinite ϕ by A. Zamolodchikov and Al. Zamolodchikov) and in matrix models (McGreevy, Verlinde; Klebanov, Maldacena, Seiberg; Martinec; McGreevy, Verlinde, Teschner; Aleksandrov, Kazakov, Kutasov; ...)

Parallel developments in topological string theory (Aganagic, Klemm, Marino, Vafa).

REVIEW OF 2-D STRING THEORY

Start with a formal sum over world sheets embedded in ONE dim.

$$\sum_{\text{topol.}} \int [Dg_{\mu\nu}(\sigma)]\,[DX(\sigma)]\; e^{-S} \;;$$

$$S = \frac{1}{4\pi}\int d^2\sigma \sqrt{g}\left(\frac{1}{\alpha'} g^{\mu\nu}\partial_\mu X \partial_\nu X + \Phi R + \mu\right)$$

replace $\int [Dg_{\mu\nu}]$ by $\sum_{\text{discretizations}}$

pick conf. gauge $g_{\mu\nu} = e^{2\phi}\hat{g}_{\mu\nu}$

Large N hermitian matrix quantum mechanics (Kazakov, Migdal)

c=1 CFT + Liouville theory where dynamics of ϕ is induced (Polyakov)

Liouville field ϕ becomes the second dimension, although not translation inv.

In the DDK approach, the Liouville path integral becomes

$$\sum_h \tilde{g}_s^{2h-2} \int D\tau_i \int [Db][Dc][D\phi][DX] e^{-S_{gc}-S} ;$$

$$S = \frac{1}{4\pi}\int d^2\sigma \sqrt{\hat{g}}\left[(\partial X)^2 + (\partial\phi)^2 + 4\hat{R}\phi + \mu e^{2\phi}\right]$$

Linear dilaton is necessary to make $c_L = 25$. $g_s(\phi) \sim \tilde{g}_s e^{2\phi}$.

$\mu e^{2\phi}$ is the marginal closed string "tachyon" background corresponding to original worldsheet cosmological constant.

The "tachyon" is effectively massless in $D=2$ => perturbative stability.

Continuous spectrum of COM vertex ops:

$$T(k) = \int d^2\sigma \, e^{ikX + (2-|k|)\phi}$$

Oscillator excitations are physical only at integer momenta, eg. $\partial X \bar{\partial} X$

In some cases, path integral reduces
to free fields, e.g. sum over tori.

After integrating over the ϕ zero-mode,

$$Z_1 = V_L \int_{\mathcal{F}} d^2\tau \int [DX][D\tilde{\phi}][Db][Dc]\, e^{-S_{free}}$$

$$V_L = -\frac{1}{3}\ln\mu \,. \qquad q = e^{2\pi i\tau}$$

For a compact scalar, $X \sim X + 2\pi R$,

$$\frac{Z_1}{V_L} = \int_{\mathcal{F}} d^2\tau \left(\frac{|\eta(q)|^4}{2\tau_2}\right) \frac{1}{2\pi\sqrt{\tau_2}\,|\eta(q)|^2} \times$$

$$\frac{R}{\sqrt{\alpha'}} \frac{1}{\sqrt{\tau_2}\,|\eta(q)|^2} \sum_{n,m} e^{-\frac{\pi R^2 |n - m\tau|^2}{\alpha' \tau_2}}$$

All non-zero modes cancel! Left with

$$\sim \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \sum_{n,m} e^{-\frac{\pi R^2 |n-m\tau|^2}{\alpha'\tau_2}} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} +$$

$$+ 2\sum_{n=1}^{\infty} \int_0^{\infty} \frac{d\tau_2}{\tau_2^2} e^{-\frac{\pi R^2 n^2}{\alpha'\tau_2}} = \frac{\pi}{3}\left(1 + \frac{\alpha'}{R^2}\right)$$

$$Z_1 = -\frac{1}{24}\ln\mu \times \left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right)$$; Sakai, Tanii; Bershadsky, I.K.

This will be shown to give same answer as the discretized approach.

Quantum Mechanics of an $N \times N$ Hermitian Matrix:

$$Z = \int [D^{N^2}\Phi(x)]\, e^{-\beta S};$$

$$S = \int_{-\infty}^{\infty} dx\, \mathrm{Tr}\left[\frac{1}{2}\left(\frac{d\Phi}{dx}\right)^2 + \frac{1}{2\alpha'}\Phi^2 - \frac{1}{3}\Phi^3\right]$$

$\beta \equiv$ inverse Planck constant.

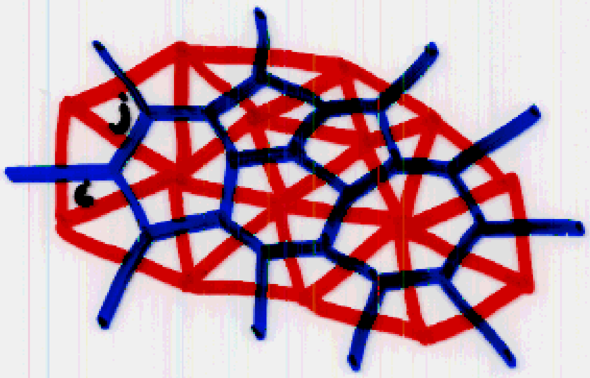
Consider a limit where $\beta \sim N \to \infty$

$$Z \sim \int [D^{N^2}\Phi(x)]\, e^{-N\int_{-\infty}^{\infty} dx\, \mathrm{Tr}\left[\frac{1}{2}\Phi'^2 + \frac{\Phi^2}{2\alpha'} - \frac{\kappa}{3}\Phi^3\right]}$$

where $\kappa^2 = \frac{N}{\beta}$.

The Feynman graph expansion produces triangulated random surfaces:

$V \equiv$ # of vertices of the blue lattice



$$-\ln Z = \sum_h N^{2h-2} \sum_{\text{Lattices}} K^V \times$$

$$\times \prod_{k=1}^{V} \int_{-\infty}^{\infty} dx_k \prod_{\langle ij \rangle} G(x_i, x_j)$$

The same sum appears in regularizing the sum over worldsheets embedded in 1d.

Here each edge contributes

$$G(x_i, x_j) = e^{-|x_i - x_j|/\sqrt{\alpha'}}$$

and gives same universality class as

$$G \sim e^{-(x_i - x_j)^2/\alpha'}.$$

To solve the matrix QM, represent

$$\Phi = \Omega^+ \Lambda \Omega \; ; \; \Omega^+ \Omega = I \; ;$$

$\Lambda \equiv$ diagonal matrix of eigenvalues

$$D\Phi = D\Omega \prod_{i=1}^{N} d\lambda_i \, \Delta^2(\lambda);$$

$$\Delta(\lambda) = \prod_{i<j} (\lambda_i - \lambda_j) \equiv$$ the Vandermonde determinant.

SU(N) singlet wave functions are independent of Ω and are symmetric in the eigenvalues, $\chi_{sym}(\lambda)$.

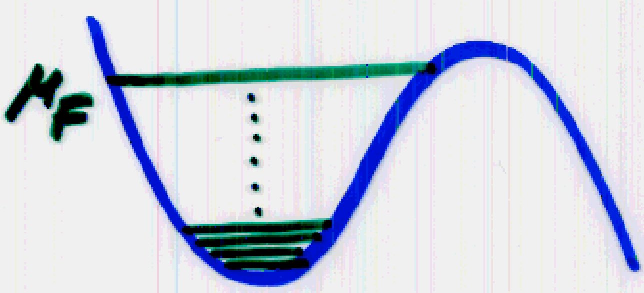
$$H\chi = E\chi \, ;$$

$$H = -\frac{1}{2\beta^2\Delta(\lambda)}\sum_{i=1}^{N}\frac{\partial^2}{\partial\lambda_i^2}\Delta(\lambda) + \sum_{j=1}^{N} v(\lambda_j)$$

Define $\Psi = \Delta(\lambda)\chi$, which is antisymmetric

$$\Big(\sum_{i=1}^{N} h_i\Big)\Psi = E\Psi \, ;$$

$$h_i = -\frac{1}{2\beta^2}\frac{\partial^2}{\partial\lambda_i^2} + \frac{\lambda_i^2}{2\alpha'} - \frac{\lambda_i^3}{3}$$



Find N free fermions whose ground state energy is $\sum_{i=1}^{N}\epsilon_i$.

Phase transition as $\mu = \mu_c - \mu_F \to 0$;

$\Delta = \kappa_c^2 - \kappa^2 \to 0$; $\mu_F = \epsilon_N$

Brezin, Itzykson, Parisi, Zuber.

In the double-scaling limit,

$\beta \sim N \to \infty$; $\mu \to 0$; $\beta\mu$ fixed.

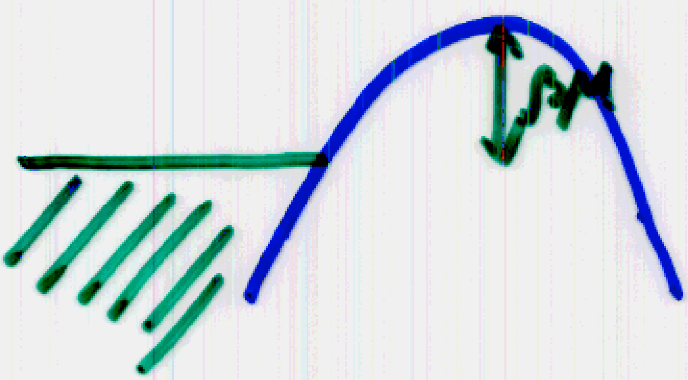
$g_s \sim \frac{1}{\beta\mu}$.

with $y = \sqrt{\beta}(\lambda - \lambda_c)$; $e = \mu_c - \epsilon$;

single-particle equ is

$$\left(-\frac{1}{2}\frac{d^2}{dy^2} - \frac{y^2}{2\alpha'}\right)\psi = \beta e \psi$$

Fermions fill one side of the upside-down harmonic oscillator



Tunnelling is suppressed by $e^{-\beta\mu\pi\sqrt{\alpha'}} \sim e^{-\mathrm{const}/g_s}$

A popular trick for finding perturbative series in $\frac{1}{\beta\mu}$ is to first fill both sides, and then divide by 2 (the two sides are indep. to all orders)

The density of states at the Fermi level satisfies

$$\frac{\partial \rho}{\partial(\beta\mu\sqrt{\alpha'})} = \frac{\sqrt{\alpha'}}{2\pi} \operatorname{Im} \int_0^\infty dT\, e^{-i\beta\mu\sqrt{\alpha'}\,T}\, \frac{T/2}{\sinh(T/2)}$$

This determines the ground state energy.

Compact Euclidean Time.

$$Z = \int [D^{N^2}\Phi(x)]\, e^{-\beta S},$$

$$S = \int_0^{2\pi R} dx\, \operatorname{Tr}\left[\frac{1}{2}\left(\frac{d\Phi}{dx}\right)^2 + \frac{1}{2\alpha'}\Phi^2 - \frac{1}{3}\Phi^3\right],$$

$$\Phi(x+2\pi R) = \Phi(x).$$

$$Z = \operatorname{Tr}\, e^{-\beta H \times (2\pi R)}.$$

This is matrix QM at finite temperature $T = \frac{1}{2\pi R}$.

The singlet contribution to the trace was found by Gross, IK.

The free energy is specified by

$$\frac{\partial F}{\partial \Delta} = \mu, \quad \frac{\partial \Delta}{\partial \mu} = \tilde{\rho} \; ;$$

$$\frac{\partial \tilde{\rho}}{\partial \mu} = \frac{\sqrt{\alpha'}}{2\pi\mu} \, \mathrm{Im} \int_0^\infty dt \, e^{-it} \, \frac{\frac{t}{2\beta\mu\sqrt{\alpha'}}}{\mathrm{sh}\,\frac{t}{2\beta\mu\sqrt{\alpha'}}} \; \frac{\frac{t}{2\beta\mu R}}{\mathrm{sh}\,\frac{t}{2\beta\mu R}} \; ;$$

The Legendre transform

$$\Gamma(\mu) = \Delta\mu - F(\Delta)$$

Satisfies

$$\frac{\partial^2 \Gamma}{\partial \mu^2} = \tilde{\rho} .$$

The Legendre transform naturally appears in matrix model with extra $g \int dx \, (\mathrm{Tr}\, \Phi^3)^2$ term.

It was argued to describe Liouville theory with operator $\mu e^{2\phi}$ (as opposed to $\Delta \phi e^{2\phi}$) Gubser, IK.

$$I = 2\pi R\,\beta^2\,\Gamma(\mu) = -\frac{1}{2}(\beta\mu)^2\frac{R}{\sqrt{\alpha'}}\ln\mu \underbrace{- \frac{1}{24}\ln\mu\left(\frac{R}{\sqrt{\alpha'}}+\frac{\sqrt{\alpha'}}{R}\right)}_{} + \ldots$$

agrees with the Liouville torus result.

The whole non-perturbative sum over surfaces I is T-dual:

$$R \to \frac{\alpha'}{R}\ ; \quad \beta\mu \to \beta\mu\,\frac{R}{\sqrt{\alpha'}}$$

is the expected symmetry.

Introduction of the matrix Φ seems to be just a trick to generate discretized random surface.

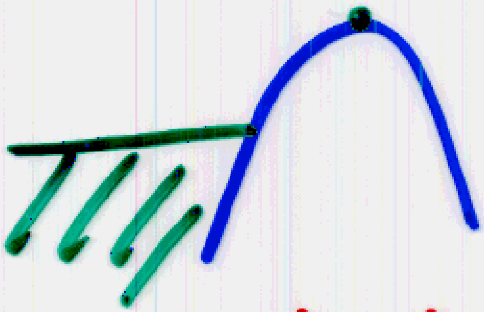
But recently it was suggested that Φ encodes open string degrees of freedom on unstable D-branes. McGreevy, Verlinde.

$\Phi(x)$ is the tachyon on unstable Dϕ-branes. The relevant boundary state is the product of the ZZ boundary state localized at $\phi \to \infty$ and the Neumann boundary state for X. IK, Maldacena, Seiberg

Semiclassically (for $c_L \to \infty$) the ZZ boundary state is described by the solution of the Liouville theory which is the Lobachevskiy plane (Poincaré disk).

The simplest ZZ state contains only the identity operator.

The only possible on-shell operators are e^{iEX} (open string tachyon of $m_T^2 = -\frac{1}{\alpha'}$) and ∂X (the gauge field)



Exciting a single eigen-value from the fermi surface to the top of the potential is dual to creating a single unstable D0-brane: $M = \beta\mu$.

The curvature of the potential (which was originally deduced from closed string physics), $U = -\frac{y^2}{2\alpha'}$, ⇐ e.g. IK review

agrees exactly with $m_T^2 = -\frac{1}{\alpha'}$.

In Minkowski signature ($X \to it$), the decay of unstable D-brane is described by the eigenvalue rolling to the left; at infinity it can be analyzed in terms of closed strings.

Open strings on N unstable "ZZ" D0-branes are described by the

GAUGED MATRIX QM

$$\int [D\Phi][DA]\, e^{-\int_0^{2\pi R} dx\, \mathrm{Tr}\left[\frac{1}{2}(D_x\Phi)^2 - \frac{\Phi^2}{4\alpha'}\right]}$$

$$D_x \Phi = \partial_x \Phi - [A, \Phi]$$

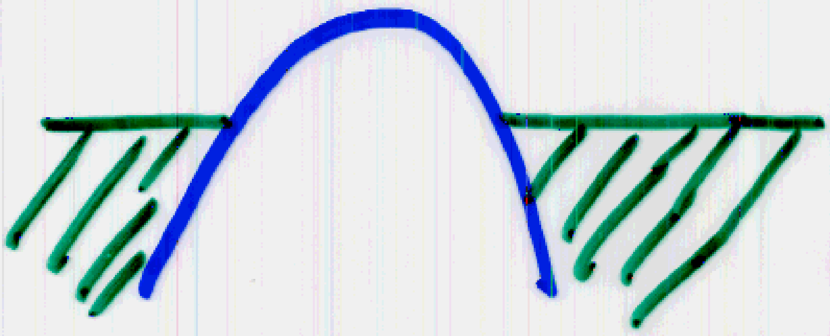
A is the Lagrange multiplier that restricts to singlet wave functions only, even at finite R.

The Kosterlitz-Thouless vortices (corresponding to non-singlets) are indeed excluded in conventional continuum definitions of string theory (otherwise T-duality is lost).

One may think of the gauged matrix model as open string dual

of 2-d bosonic closed string.
This duality requires $N \to \infty$:
an infinite number of unstable
D0-branes has to decay for the
fermi sea (closed string vacuum)
to form.

The bosonic string construction
is unstable non-perturbatively since
the Fermi sea is filled only on one
side.
A common device is to fill both
sides, which makes the matrix QM
stable. What is the meaning of this?
Is this a "doubled" bosonic string?



Actually, this is a Different 2-d string theory, a fermionic string!

(Douglas, Ik, Kudasov, Maldacena, Martinec, Seiberg; in preparation)

More precisely, this is the 2-d type 0B string theory.

Super-Liouville theory coupled to $\hat{c}=1$ matter, with NON-CHIRAL GSO projection $(-1)^{F+\tilde{F}}=1$

The continuous spectrum is a massless "tachyon" from (NS−, NS−) sector; AND

self-dual scalar from (R+, R+)

anti-selfdual scalar from (R−, R−)

So, the RR sector contributes another 2-d massless scalar.

To "derive" the matrix model dual to 2-d 0B strings, consider unstable Dϕ-branes.

Typically, in type 0 theories there are two different types of allowed D-branes, due to doubling of the RR sector compared to type II.

For example, in 2-d type 0A theory there are 2 gauge fields:

$$S = \int d^2x \sqrt{G} \left[\frac{1}{4} \left(F_{\mu\nu}^{(+)}\right)^2 (1+T+\dots) + \right.$$
$$\left. + \frac{1}{4} \left(F_{\mu\nu}^{(-)}\right)^2 (1-T+\dots) \right]$$

$\pm$ D0-branes couple to $A_\mu^{(\pm)}$.

Closed string tachyon $T \sim \mu e^{2\phi}$ breaks the symmetry between + and − D0-branes localized at large ϕ. Only one of the two types exists.

D-brane effective action is

$$\sim \int dt\, e^{-\Phi}(1 \pm T + \ldots)$$

Since $e^{-\Phi} \sim e^{-2\phi}$, the second term $= \pm \mu$ (as $\phi \to \infty$ this is the D-brane mass).

Only the positive mass brane is physical. This removes the doubling.

Unstable D0-branes of 0B string come from D0-$\overline{\text{D0}}$ pair of 0A via Sen's Z_2 projection.

Open string eff. action on N unstable 0B D0-branes is

$$\int [D^{N^2}\Phi(x)][D^{N^2}A(x)]\, e^{-\beta S} \; ;$$

$$S = \int dx\, \mathrm{Tr}\left[\frac{1}{2}(D_x\Phi)^2 + U(\Phi)\right].$$

$U(\Phi)$ is even, since the open string tachyon vertex op. is $\oint d\sigma\, k\cdot\psi\, e^{ik\cdot X}$.

$$U(\Phi) = -\frac{\Phi^2}{4\alpha'} + \alpha\,\Phi^4 + \ldots$$

With this double-well potential the Fermi sea is symmetric.

After double-scaling limit, free fermions move in potential $V = -\frac{y^2}{4\alpha'}$.

Related to the bosonic model by $\alpha' \to 2\alpha'$

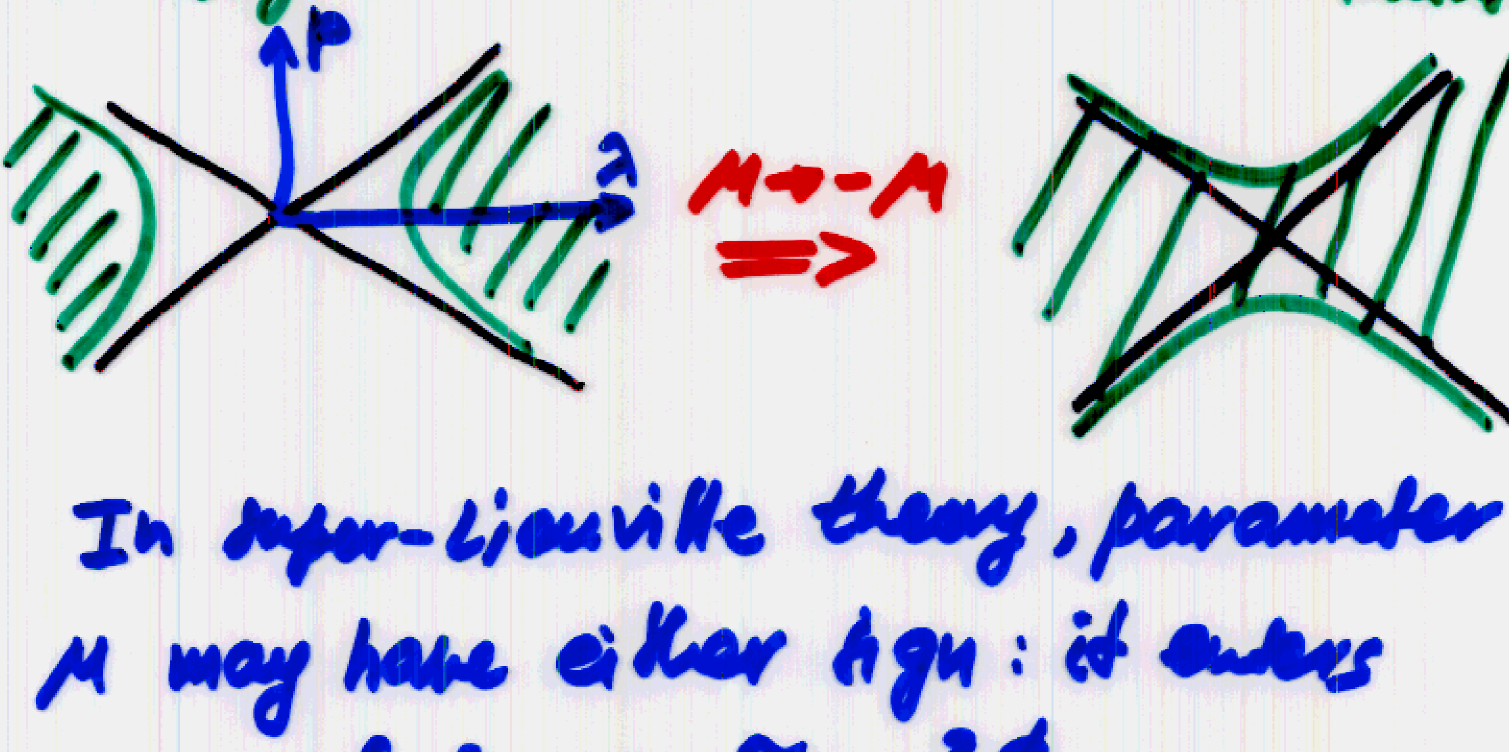
The NS-NS scalar is the symmetric perturbation of the Left and Right fermi seas:

$$T(q) = e^{i\delta_{NS}(q)} \frac{1}{\sqrt{2}}\left(T_L(q) + T_R(q)\right)$$

The R-R scalar is antisymmetric

$$S(q) = e^{i\delta_R(q)} \frac{1}{\sqrt{2}}\left(T_L(q) - T_R(q)\right)$$

This predicts remarkable relations between NS-NS and R-R correlators which, at tree level, agree with continuum results. Di Francesco, Kutasov

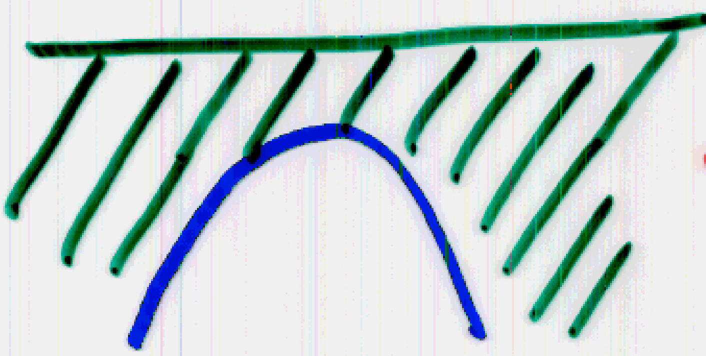


In super-Liouville theory, parameter μ may have either sign: it enters as $\mu \int d^2\sigma\, \psi_\phi \tilde{\psi}_\phi\, e^{2\phi}$.

$\mu \to -\mu$ is equivalent to $\psi \to -\psi$.

In the double-scaled QM, $\mu \to -\mu$ is equivalent to interchanging λ with conjugate momentum p, accompanied by particle-hole conjugation. IK

This is target space T-duality: in string theory $\psi \to -\psi$ reverses the sign of the (R+,R+) state, relative to (R−,R−).



Fermi sea for $p\mu < 0$

Another test of the matrix model for type 0B string is the genus one contribution:

$$Z_1 = -\frac{1}{12} \ln \mu \left(\frac{R}{\sqrt{2\alpha'}} + \frac{\sqrt{2\alpha'}}{R} \right)$$

This prediction is non-trivial from the continuum point of view.

Need to sum over the spin structures.

The 3 even spin structures $(-,-)$, $(-,+)$ and $(+,-)$ contribute

$$-\frac{1}{8\sqrt{2}}\ln\mu\left(\frac{R}{\sqrt{\alpha'}}+\frac{\sqrt{\alpha'}}{R}\right) \Leftarrow \text{even under } R\to \alpha'/R$$

and the odd one $(+,+)$ gives

$$-\frac{1}{24\sqrt{2}}\ln\mu\left(\frac{\sqrt{\alpha'}}{R}-\frac{R}{\sqrt{\alpha'}}\right) \Leftarrow \text{odd under } R\to \alpha'/R$$

The sum agrees with the symmetric fermi sea matrix QM in potential $-\frac{y^2}{4\alpha'}$.

It is also possible to formulate a large-N matrix QM dual to 2-d type 0A string theory.