Blending Local Symmetries With Matrix Nonlocality

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INTRODUCTION

→ LOCAL SYMMETRIES determine much of the structure of low energy eff. actions in string theory

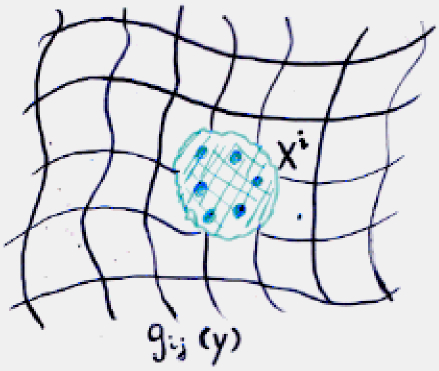
general covariance
gauge symmetries

$\xrightarrow{\text{bulk E.A.}}$ gauge theories + gravity

$\xrightarrow{\text{single brane E.A.}}$ $\int e^{-\phi}\sqrt{|g-B+F|} + \int C \wedge e^{F-B} + \ldots$

→ MATRIX description of multiple D-brane configurations makes incorporating local symmetries much harder

e.g. D0 branes in a metric



$\xi^i(y)$: inf. coordinate transform

$$\delta h_{ij}(y) = \partial_i \xi_j(y) + \partial_j \xi_i(y)$$

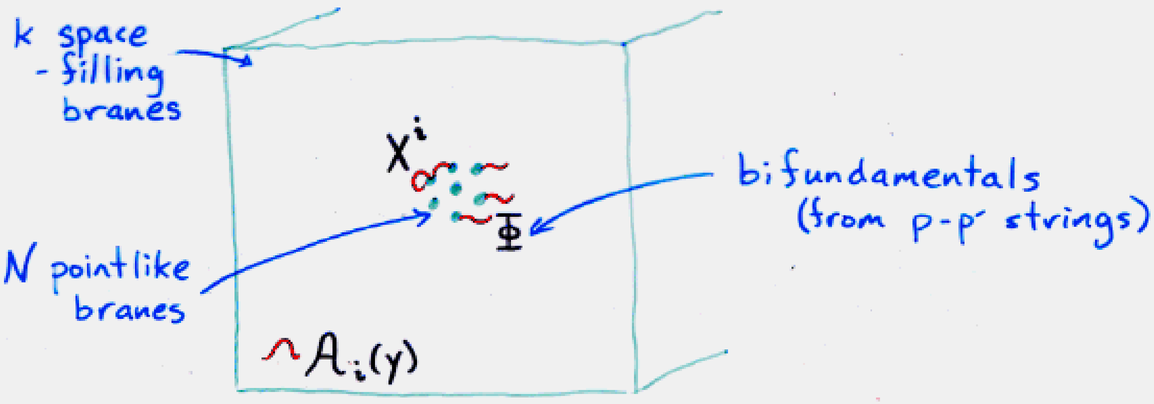
$$\delta x^i = \xi^i(x)$$

$$\delta X^i = ??? \qquad S_{INV} = ???$$

de Boer & Schalm : found consistent answer to 6th order in fields

→ complicated, many free parameters

THIS TALK: simpler open string analogue



Local gauge symmetry:

$$A_i(y) \to \mathcal{U}(y) A_i(y) \mathcal{U}^{-1}(y) + i\,\partial_i \mathcal{U}(y) \mathcal{U}^{-1}(y)$$

How does Φ transform?

$\Phi \to$ "$\mathcal{U}\Phi$" but $\mathcal{U}$ depends on y

Single pointlike brane: $\Phi \to \mathcal{U}(x)\Phi$

Many branes: $\Phi \to \mathcal{U}(X)\Phi$???

What does this mean?

Can we write a gauge invariant action?

Naive guess:

$$\Phi \to \sum \frac{1}{n!} \partial_{i_1} \cdots \partial_{i_n} U(0)\, \Phi\, X^{i_n} \cdots X^{i_1}$$

→ Gives right answer $\phi_m \to U(x_{mm})\phi_m$ for diagonal X^i

BUT: group multiplication property fails

$$U \circ (V \circ \Phi) \neq (UV) \circ \Phi$$

Can adjust $U \circ \Phi$ order-by-order to satisfy this:

$$U \circ \Phi = U(0)\Phi + \partial_i U(0) \Phi X^i + \tfrac{1}{2} \partial_i \partial_j U(0) \Phi X^j X^i + \ldots$$
$$+ \tfrac{i}{2} \partial_i U(0) A_j(0) \Phi [X^j, X^i]$$

depends explicitly on gauge field

INVARIANT ACTION?

→ Simple bilinears $\Phi^\dagger \Phi$, $\Phi^\dagger F_{ij} \Phi, \ldots$ not invariant

→ Can add terms order-by-order to build invariants

e.g.

$$S = \Phi^\dagger \Phi - i [X^i, \Phi^\dagger A_i \Phi] - \tfrac{1}{2} X^i X^j \Phi^\dagger A_i A_j \Phi + X^i \Phi^\dagger A_i A_j \Phi X^j$$
$$- \tfrac{1}{2} \Phi^\dagger A_j A_i \Phi X^i X^j - \tfrac{i}{2} X^i X^j \Phi^\dagger \partial_j A_i \Phi + \tfrac{i}{2} \Phi^\dagger \partial_i A_j \Phi X^i X^j + O(X^3)$$

MESSY!

A Covariant Object

Closer inspection reveals:

$$S_{INV} = \mathcal{F}^{\dagger}\mathcal{F}$$

$$\mathcal{F}(\Phi, X, A(y)) = \Phi + i A_i(0)\Phi X^i - \tfrac{1}{2}A_i(0)A_j(0)\,\Phi X^j X^i + \ldots$$
$$+ \tfrac{i}{2}\,\partial_j A_i(0)\,\Phi X^j X^i$$

Find $\mathcal{F} \to \mathcal{U}(0)\mathcal{F}$ $\therefore$ $\mathcal{F}$ is COVARIANT!

NOT UNIQUE: e.g.
$\mathcal{F}' = \mathcal{F} + F_{ij}(0)\mathcal{F}X^iX^j$

More generally, can define

$$\mathcal{F}_a = \mathcal{F}(\Phi, X-a, A(y+a)) \to \mathcal{U}(a)\mathcal{F}_a$$

Easy to write invariant actions:

$$\mathcal{F}_a^{\dagger}\mathcal{F}_a\,, \quad \mathcal{F}_a^{\dagger}F_{ij}(a)\,\mathcal{F}_a X^i X^j, \ldots$$

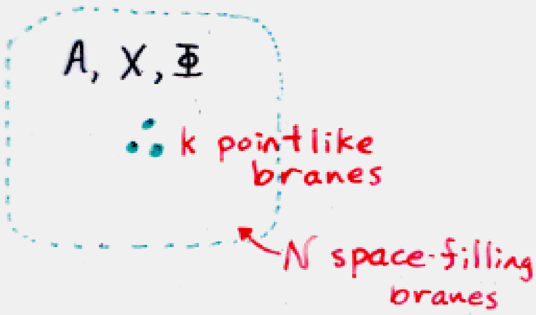
Abelian case: $\qquad \mathcal{F}_a = e^{i\int_a^X A}\cdot\Phi$

EXISTENCE OF $\mathcal{F}_a$ IN GENERAL SUGGESTS $\exists$

WILSON LINE FROM ORDINARY POINT TO MATRIX LOCATION

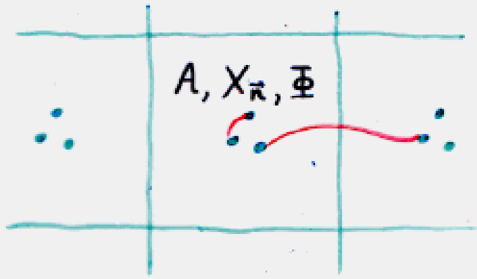
A MORE SYSTEMATIC APPROACH

Begin with constant A_i:

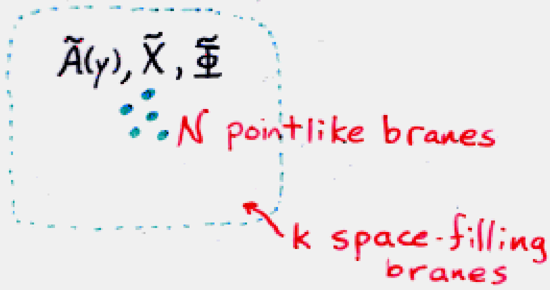


Compactify by adding images, mod out by translations

(Douglas+Moore, Taylor)



T-dualize all directions, decompactify



→ Action & transformation laws for original problem determined by those for constant A!

CONSTANT GAUGE FIELD

→ Still non-trivial: $U = e^{ik\cdot y}$ leaves A_i constant:

$$A \to A - k \qquad \mathcal{F}_a \to e^{ik\cdot a}\,\mathcal{F}_a \qquad \Phi \to \Phi_k \equiv (e^{ik\cdot y}) \circ \Phi$$

→ Compare translations:

$$X \to X - k \qquad \Phi_a \to e^{-ik\cdot a}\,\Phi_a \qquad \mathcal{F} \to \mathcal{F}_k$$

→ Suggest T-duality rules:

$$X \longleftrightarrow A \qquad \Phi_a \longleftrightarrow \mathcal{F}_a^\dagger$$

Can DEFINE $\mathcal{F}$ as image of $\Phi^\dagger$ under T-duality!

→ Need to define $\mathcal{F}$ in terms of Φ to complete symmetry transformation rules

DETERMINING $\mathcal{F}$

→ For constant A_i, have:

$$\mathcal{F} = \Phi + c_1 A_i \Phi X^i + c_{12} A_i A_j \Phi X^j X^i + \ldots$$
$$+ c_{21} A_j A_i \Phi X^j X^i$$

$$\equiv \mathcal{O} \circ \Phi$$

→ Consistency of transformation rules constrains $\mathcal{O}$:

I) T-duality: $\boxed{\mathcal{O}^\dagger = \mathcal{O}^{-1}}$

II) Symmetries:

- define $a_i \equiv$ inf. gauge tform. operator

$$\delta A_i = k_i \rightarrow \delta\Phi^\dagger = i k_i a_i \circ \Phi^\dagger$$

- find $\boxed{a_i = -i\,\partial_{X^i}\mathcal{O}\,\mathcal{O}^{-1}}$ $\partial_M f(M) \equiv \partial_y f(M+y)\big|_{y=0}$

- require $\boxed{\partial_{A_i} a_j = \delta_{ij}}$

SOLUTION:

X or A abelian: $\mathcal{O} = e^{iA\cdot X}$

General: Checked soln. to 5th order, appears unique up to field redefs:

$\Phi' = \mathcal{P}\circ\Phi$ $\partial_{X^i}\mathcal{P} = 0$

$$\mathcal{O}' = \mathcal{P}^\dagger \mathcal{O}\,\mathcal{P}^{-1}$$

$$a_i' = \mathcal{P}^\dagger a_i (\mathcal{P}^\dagger)^{-1} - i\,\partial_{X^i}\mathcal{P}^\dagger(\mathcal{P}^\dagger)^{-1}$$

INVARIANT ZERO-MODE ACTIONS

→ Basic gauge singlet: $\mathcal{F}_a^\dagger \mathcal{C}(A) \mathcal{F}_a$ ← commutator expression

$$\mathcal{F}_a^\dagger \mathcal{C}(A) \mathcal{F}_a \equiv f(a) \to f(a+b) \text{ under translations } X \to X - b$$

→ Impose translation invariance by integrating over space:

$$S^{\mathcal{F}}_{INV} = \int da \, \mathrm{Tr}\Big(\mathcal{F}_a^\dagger \mathcal{C}_1(A) \mathcal{F}_a \overbrace{\mathcal{C}_2(X)\, \delta(X-a)}^{\text{more general: } h(X-a)}\Big)$$

→ Matrix δ-funct. $\delta(X-a) \equiv \int dk \, e^{ik\cdot(X-a)}$ localizes action to individual brane locations for diagonal X

$$S^{\mathcal{F}}_{INV} \longrightarrow \sum_i \phi_i^\dagger \mathcal{C}_1(A) \phi_i \, \mathcal{C}_2(0)$$

→ Terms of this form give minimal basis of invariant bilinear actions

→ Alternate basis:

$$S^{\Phi}_{INV} = \int dk \, \mathrm{Tr}\big(\Phi_k \, \mathcal{C}_1(X) \, \Phi_k^\dagger \, \mathcal{C}_2(A) \, \delta(A-k)\big)$$

EXAMPLE

→ Consider A_i independent term:

$$S_0 = \mathrm{Tr}\left(\Phi^\dagger \Phi\, \mathcal{E}(X)\right)$$

→ Gauge invariant completion consistent with T-duality is:

$$S_{INV} = \int da\, \mathrm{Tr}_k\left(\mathcal{F}_a^\dagger \mathcal{F}_a\, \mathcal{E}(X)\, \delta(X-a)\right)$$
$$+ \int dk\, \mathrm{Tr}_N\left(\Phi_k \Phi_k^\dagger\, \mathcal{E}(A)\, \delta(A-k)\right)$$
$$+ \left\{\text{higher order invariants}\right\}$$

→ Expand in terms of X, A, Φ:

$$S_{INV} = \mathrm{Tr}\left(\Phi^\dagger \Phi\, \mathcal{E}(X) + i\, \Phi^\dagger A_i \Phi [X^i, \mathcal{E}(X)] + \ldots \right)$$
$$+ \Phi^\dagger \mathcal{E}(A) \Phi$$

next order: 7 terms, 2 arb. coefficients

→ Leading order coupling to gauge field determined uniquely, infinite series of higher order terms also fixed

BACK TO ORIGINAL PROBLEM

→ Start with zero-mode action + tform laws, apply quotienting/T-duality:

(I) S^{Φ}_{INV} gives:

$$\boxed{\hat{S}^{\mathcal{F}} = \int da \, \mathrm{Tr}\left(\mathcal{F}_a^{\dagger} f(F_{ij}(a), D_i F_{jk}(a), \ldots) \mathcal{F}_a \, \mathcal{C}(X) \, \delta(X-a) \right)}$$

$$\Phi^{\dagger} = \mathcal{O}(X-y, -iD(y)) \, \mathcal{F}_y^{\dagger}$$

$\mathcal{F}_a$ like parallel transport of Φ from X to a:

$$\mathcal{F}_a^{X\,\mathrm{abel}} = Pe^{i\int_a^X A} \Phi \qquad \mathcal{F}^{A=0} = \Phi$$

↑ straight line

(II) $S^{\mathcal{F}}_{INV}$ gives bilocal action:

$$\boxed{\hat{S}^{\Phi} = \int dx \int dy \, \mathrm{Tr}\left(\Phi^{\dagger}(y) \, W(y,x) \, \Phi(x) \, \mathcal{C}(X) \right)}$$

$W(y,x)$: straight open Wilson line with averaged insertions of gauge invariant ops.

$$\Phi(y) = \mathcal{O}^{\dagger}(-iD(y), X-a) \, \delta(y-a) \, \mathcal{F}_a$$

$\Phi(y)$: 2ND covariant object, like projection/pull-back of Φ from X to field on full space:

$$\Phi^{X\,\mathrm{abel.}}(y) = \delta(y-x)\Phi \qquad \Phi^{A=0}(y) = \delta(y-X)\Phi$$

Summary

→ Gauge invariant actions written naturally in terms of covariant objects $\Phi(y)$, $\mathcal{F}_y$ which promote Φ to (redundant) fields on full space

→ Gauge invariance demands infinite series of couplings to higher-brane gauge field for each A-independent term

→ OPEN PROBLEM: find full solution for Θ, prove unique up to field redefinitions

Can we write generally covariant action for D-branes by promoting $X^i \to X^i(y)$?