Closed Strings and Unstable D-branes

AN ATTEMPT TO ASSIMILATE SOME OF THE RECENT RESULTS ON TACHYON CONDENSATION

Notations and conventions:

1. We shall set

$$\hbar = 1, \qquad c = 1, \qquad \alpha' = 1$$

2. g_s: closed string coupling constant

3. We shall be working in the weak coupling limit.

4. We shall always use the 'string metric' and dilaton as independent fields. (not Einstein metric)

5. Our objects of study will be unstable D-p-branes of type IIA, IIB and bosonic string theories and brane-antibrane systems involving BPS Dp-branes in type IIA and IIB string theories. (ALSO 0A & 0B) & TYPE I

6. Each of these systems has one (or more) tachyonic mode(s).

Known properties of the tree level tachyon effective action:

A.S.

1. The tachyon effective potential $V_{eff}(T)$ has a (local) minimum at some value $T = T_0$.

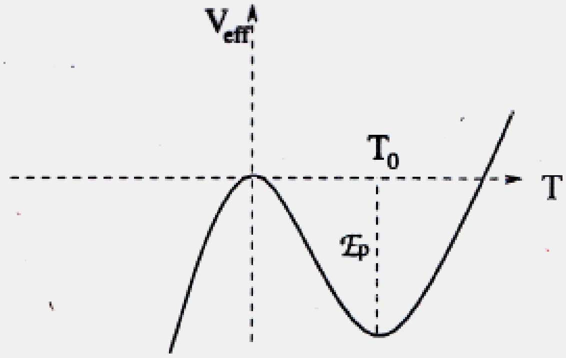
   At $T = T_0$ the total energy density vanishes identically.

$$V_{eff}(T_0) + \mathcal{E}_p = 0$$

$\mathcal{E}_p$ = tension of an unstable Dp brane in bosonic or type II string theory.

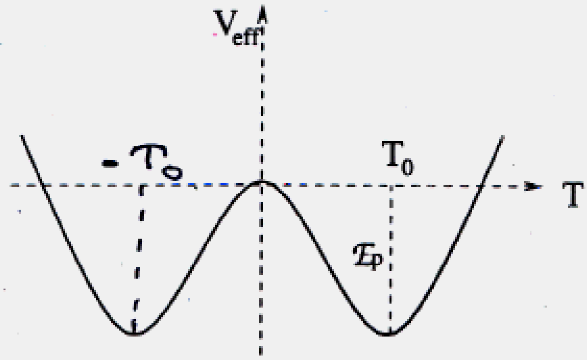
$\mathcal{E}_p$ = twice the tension of a BPS D-brane for Dp − $\bar{\text{D}}p$ system in type II string theory.

Shape of $V_{eff}(T)$ for a D-p-brane in bosonic string theory:



Shape of $V_{eff}(T)$ for non-BPS D-p-brane in type II string theory:

ALSO TYPE 0, TYPE I



For Dp-$\bar{\text{D}}p$ system we have to revolve this figure about the vertical axis to get the tachyon potential as a function of complex T.

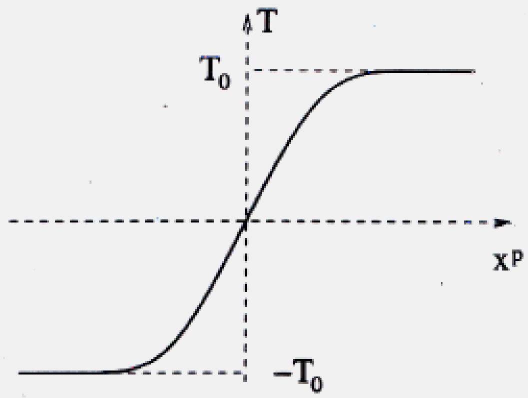
2. $T = T_0$ configuration describes the closed string vacuum without any D-brane.

   Thus around this minimum there are no physical open string excitations.

3. There are classical solutions of the equations of motion of T, representing lower dimensional D-branes.

   Example:

   On a non-BPS Dp-brane of type II string theory, a kink represents a D-$(p-1)$-brane.



Energy density is localized around a codimension 1 subspace ($x^p = 0$)

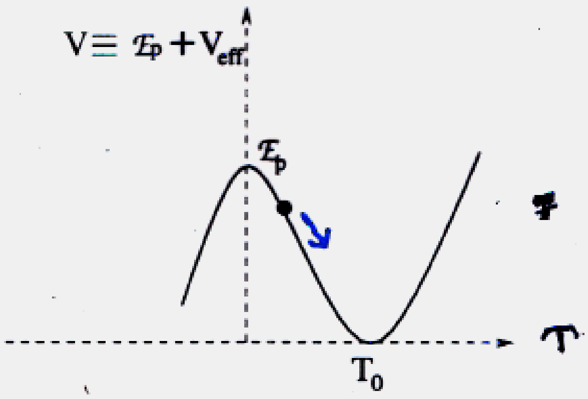
This way, all D-branes can be thought of as conventional soliton solutions of the tachyon effective field theory on sufficient number of space-filling branes.

Witten
Horava

Question: Can we also describe the fundamental closed strings and other known objects in string theory using the open string degrees of freedom on the unstable D-brane?

If so then the open string (field) theory on unstable D-branes could provide a new formulation of string theory.

Review of time dependent solutions:



A.S.
Gutperle, Strominger
Okuda, Sugimoto
Maloney, Strominger, Yin
Larsen, Naqvi, Terashima
Constable, Larsen
Schomerus

We can construct spatially homogeneous rolling tachyon solution on an unstable D-brane system characterized by one inequivalent parameter, – the energy of the system.

During the evolution the energy density T_{00} remains constant, but the other components of the stress tensor evolve as

$$T_{i0} = 0, \quad T_{ij}(x^0) = p(x^0)\delta_{ij}, \qquad 1 \le i,j \le p$$

$$T_{0\alpha} = T_{i\alpha} = T_{\alpha\beta} = 0, \qquad (p+1) \le \alpha, \beta \le 9 \text{ or } 25$$

$p(x^0) \equiv$ pressure $\qquad x^0 \equiv$ TIME

Result for non-BPS D-brane or D-$\bar{\text{D}}$ system in type II string theory: (ALSO TYPE 0)

Case 1: $T_{00} < \mathcal{E}_p$:

$$T_{00} = \mathcal{E}_p \cos^2(\pi\tilde{\lambda}), \qquad p(x^0) = -\mathcal{E}_p f(x^0)$$

$$f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0}\sin^2(\tilde{\lambda}\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0}\sin^2(\tilde{\lambda}\pi)} - 1$$

$\tilde{\lambda}$: the parameter characterizing the solution

The dilaton charge density associated with the solution (the charge to which the dilaton couples on the D-brane world-volume):

$$Q(x^0) = \mathcal{E}_p f(x^0)$$

Note: We have omitted $\delta(\vec{x}_\perp)$ localizing $T_{\mu\nu}$ and Q on the plane of the brane.

$$T_{00} = \mathcal{E}_p \cos^2(\pi\tilde{\lambda}), \qquad p(x^0) = -\mathcal{E}_p f(x^0)$$

$$Q(x^0) = \mathcal{E}_p f(x^0)$$

$$f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0}\sin^2(\tilde{\lambda}\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0}\sin^2(\tilde{\lambda}\pi)} - 1$$

1) As $x^0 \to \infty$, $f(x^0) \to 0$.

Hence, $p(x^0) \to 0$ and $Q(x^0) \to 0$

2) At $\tilde{\lambda} = \frac{1}{2}$, $T_{00} = 0$, $p(x^0) = 0$, $Q(x^0) = 0$.

$\to$ the tachyon is at the minimum of $V_{eff}(T)$.

Case 2: $T_{00} > \mathcal{E}_p$

$$T_{00} = \mathcal{E}_p \cosh^2(\pi\tilde{\lambda}), \quad p(x^0) = -\mathcal{E}_p f(x^0)$$

$$Q(x^0) = \mathcal{E}_p f(x^0)$$

$$f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0}\sinh^2(\tilde{\lambda}\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0}\sinh^2(\tilde{\lambda}\pi)} - 1$$

Again as $x^0 \to \infty$, $p(x^0) \to 0$ and $Q(x^0) \to 0$.

Very similar results also hold for the bosonic string theory.

So far in this analysis we have ignored the effect of coupling the open string degrees of freedom to the closed string degrees of freedom.

What is the effect of such a coupling?

For a subset of closed string states (associated with excitations of the oscillators associated with spatial coordinates) the rolling tachyon configuration provides a time dependent source $\propto f(x^0)$.

$\rightarrow$ some part of the energy will be radiated into closed strings.

Chen, Li, Lin
Lambert, Liu, Maldacena
Gaiotto, Itzhaki, Rastelli

For unstable D-p-branes wrapped on T^p, we have the following results:

Lambert, Liu, Maldacena: hep-th/0303139
Gaiotto, Itzhaki, Rastelli: hep-th/0304192

1. Total amount of energy radiated into closed strings is infinite.

2. The total amount of energy radiated into closed string modes with mass $\leq M$ is of order M.

3. Typical velocity of a closed string of mass M produced in this process is of order $1/\sqrt{M}$

This suggests that once backreaction is taken into account, the total amount of radiated energy will be equal to the initial energy $\sim \frac{1}{g_s}$.

Furthermore, most of the radiation will be into very massive closed string with mass of order $\frac{1}{g_s}$ and velocity of order $\sqrt{g_s}$.

From this analysis it would seem that the effect of closed string emission invalidates the tree level open string analysis.

However let us compare the properties of the emitted closed strings with those infered from the tree level open string analysis.

A. S. hep-th/0305011, 0306137
Karczmarek, Liu, Maldacena, Strominger, hep-th/0306132

1) Tree level open string analysis tells us that the final system has:

$$p/T_{00} = 0$$

On the other hand, closed string analysis tells us that the final closed strings are highly non-relativistic, with velocity $\sim \sqrt{g_s}$

For such a system

$$p/T_{00} \sim g_s \to 0 \qquad \text{as } g_s \to 0$$

Internal oscillations of the string do not affect this result.

2) Tree level open string analysis tells us that the final system has:

$$Q/T_{00} = 0$$

Q: Dilaton charge density

On the other hand we know that the closed string world-sheet does not couple to the zero momentum dilaton.

$$S_{world-sheet} = \int d^2z (G_{\mu\nu}(X) + B_{\mu\nu}(X)) \partial_z X^\mu \partial_{\bar{z}} X^\nu$$

Thus the final state closed strings carry zero total dilaton charge.

3) Finally, open string **tree level** analysis tells us that the various charge densities are localized on the plane of the original D-brane.

Closed string analysis tells us that the closed strings move with velocity of order $\sqrt{g_s}$ and hence, in the $g_s \to 0$ limit, takes infinite amount of time to carry the charges away from the plane of the brane.

→ charges are localized in the plane of the brane during any finite time interval.

However . . .

... if we take into account internal oscillation of the closed strings, then the various charges spread out away from the plane of the brane.

This seems to disagree with the open string result. $\propto \delta(\vec{x}_\perp)$

However, in string theory we have an intrinsic ambiguity in defining the local distribution of the various charge densities (including $T_{\mu\nu}(x)$).

e.g. given the Fourier transform of the dilaton field $\widetilde{\Phi}(k)$ which couples to the Fourier transform $\widetilde{Q}(k)$ of the dilaton charge density as

$$\int \widetilde{\Phi}(k)\widetilde{Q}(k),$$

we can make a redefinition

$$\widetilde{\Phi}(k) \to f(k^2)\widetilde{\Phi}(k) \quad \to \quad \widetilde{Q}(k) \to (f(k^2))^{-1}\widetilde{Q}(k)$$

$f(0) = 1$ for correct *on-shell* normalization.

This changes the local distribution of $Q(x)$ without changing the integrated value.

Conclusion: The results obtained from tree level open string analysis are consistent with the results obtained after taking into account the effect of closed string radiation.

Such agreements between open and closed string results also hold for more general cases, *e.g.* in the decay of unstable branes in the presence of electric field.

Mukhopadhyay & A.S. hep-th/0208142
A.S. hep-th/0306137
Rey, Sugimoto, hep-th/0301049

This leads to the

Conjecture: *Tree level open string theory provides a description of the rolling tachyon system which is dual to the description in terms of closed string emission.*

This is different from the usual open closed duality where one loop open string theory contains information about closed strings.

This also suggests that in the full quantum theory, the dynamics of an unstable D-brane can be described using only the open string degrees of freedom without ever talking about closed strings.

Furthermore, in the $g_s \to 0$ limit, the results are correctly described by tree level open string field theory, *i.e.* Ehrenfest theorem must hold.

If all states of string theory could be described as states in this open string field theory, then one could provide a new formulation of string theory purely in terms of the open string degrees of freedom living on unstable D-branes.

We shall now try to test these ideas in the context of $c = 1$ matrix model.

McGreevy & Verlinde, hep-th/0304224
Klebanov, Maldacena, Seiberg, hep-th/0305192
McGreevy, Teschner, Verlinde, hep-th/0305194
Also { Martinec, hep-th/0305148
Alexandrov, Kazakov, Kutasov, hep-th/0306177

C = 1 MATRIX MODEL: CONTINUUM THEORY

① WORLD-SHEET FIELDS:

$X^0 \oplus$ LIOUVILLE $\oplus$ GHOSTS

$c = 1$ $\quad$ $c = 25$ $\quad$ $c = -26$

② CLOSED STRING SPECTRUM

→ A MASSLESS "TACHYON"

③ ADMITS AN UNSTABLE D0-BRANE WITH A TACHYONIC MODE (ZZZ)

④ ADMITS TIME DEPENDENT SOLUTION DESCRIBING ROLLING OF THE TACHYON ON D0 BRANE

⑤ COUPLING TO CLOSE STRING

→ ALL THE ENERGY IS RADIATED AWAY INTO CLOSD STRINGS

? ⇝ INVALIDATES OPEN STRING ANALYSIS?

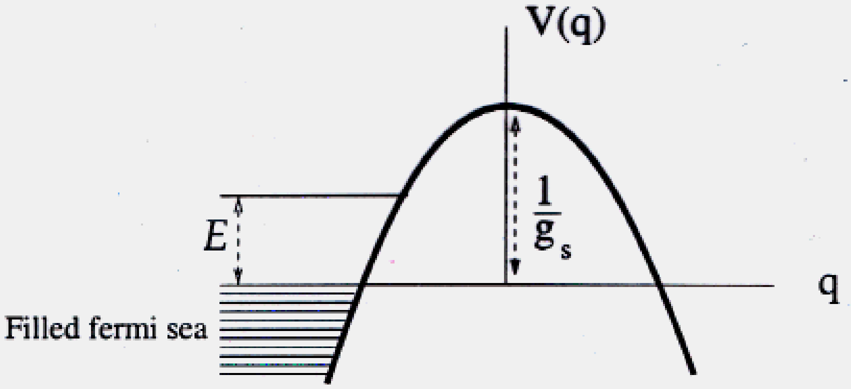
$c = 1$ matrix model:

Brezin, Itzykson, Parisi, Zuber; Gross, Miljkovic; Ginsparg, Zinn-Justin; Brezin, Kazakov, Zamolodchikov; Parisi; Gross, Klebanov; Das, Dhar, Sengupta, Wadia

Theory of free fermions moving in an inverted harmonic oscillator potential.

Hamiltonian

$$h = \frac{1}{2}p^2 - \frac{1}{2}q^2 + \frac{1}{g_s}$$



The rolling tachyon configuration on a D0-brane with energy $E = T_{00}$ corresponds to a single fermion excited from the fermi level to an energy level E.

McGreevy, Verlinde; Klebanov, Maldacena, Seiberg; McGreevy, Teschner, Verlinde

For $E = \frac{1}{g_s}$ we get the unstable D0-brane.

$g_s \to 0$ limit: We can descibe the system as a classical particle described by phase space coordinates (q, p).

Polchinski
Das, Dhar, Mandal, Wadia

In this limit the rolling tachyon configuration with energy E corresponds to a classical trajectory of energy E in the (p, q) space.

Thus the classical dynamics of q is the analog of the tree level tachyon dynamics on the unstable D0-brane.

$q \leftrightarrow$ classical open string 'field' on the D0-brane

This provides a perfectly good description of the system.

On the other hand, if we couple closed strings to this system in the continuum string description of the same theory, then all the energy is radiated away into closed strings.

Klebanov, Maldacena, Seiberg

???

[redacted] for large $|q|$ the system also allows a bosonic description.

Das, Jevicki
Sengupta, Wadia
Gross, Klebanov

A $(1+1)$ dimensional bosonic field obtained by bosonizing the second quantized fermi field $\Psi(q, x^0)$.

→ a closed string field

$g_s \to 0$ limit: classical closed string field theory in $(1+1)$ dimensions

We can describe the same rolling tachyon configuration as a classical closed string field configuration with the help of the usual bosonization rules.

This closed string field configuration agrees precisely with what one gets by analysing the closed string radiation 'emitted' by the rolling tachyon configuration.

Klebanov, Maldacena, Seiberg
McGreevy, Teschner, Verlinde

Thus we see that the closed string field theory in (1+1) dimensions and the open string field theory in (0+1) dimensions provide *equivalent but alternative* description of the same system.

If instead we had infered that closed string emission invalidates the open string results, we would be saying that the effect of 'emission of bosons' invalidates the fermionic description of the $c = 1$ matrix model!

This will clearly be the wrong viewpoint.

NOTE: CLOSED STRING DESCRIPTION IS VALID ONLY FOR $x^0 \to \pm\infty$ WHEN THE SOURCE TERMS $\propto f(x^0)$ VANISH

Does Ehrenfest theorem hold in the 'open string' (fermionic) description?

For energies of order $1/g_s$, we have typical initial condition:

$$q_{in} \sim \frac{1}{\sqrt{g_s}}, \qquad p_{in} \sim \frac{1}{\sqrt{g_s}}$$

$\hbar = 1 \rightarrow$ can choose $\Delta q \sim 1$, $\Delta p \sim 1$ initially.

Δq, Δp grow with x^0 as

$$\Delta q \sim e^{x^0}, \qquad \Delta p \sim e^{x^0}$$

Thus $\Delta q \lesssim q_{in}$, $\Delta p \lesssim p_{in}$ for

$$x^0 \lesssim \ln \frac{1}{\sqrt{g_s}}$$

Classical description breaks down after a time of order $\ln \frac{1}{\sqrt{g_s}}$ ($\rightarrow \infty$ as $g_s \rightarrow 0$)

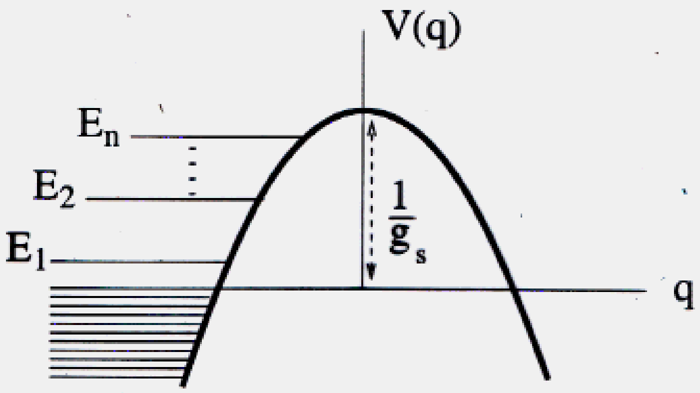
This could be related to the exponential growth of components of the boundary state involving time-like oscillators.

⇓

CLOSED STRING INTERACTIONS BECOME IMPORTANT FOR $x^0 \sim \ln 1/\sqrt{g_s}$

Okuda, Sugimoto hep-th/0208196

In the context of the $c = 1$ matrix model, we can now ask whether the degrees of freedom living on the D-brane are able to describe all the states of the system.

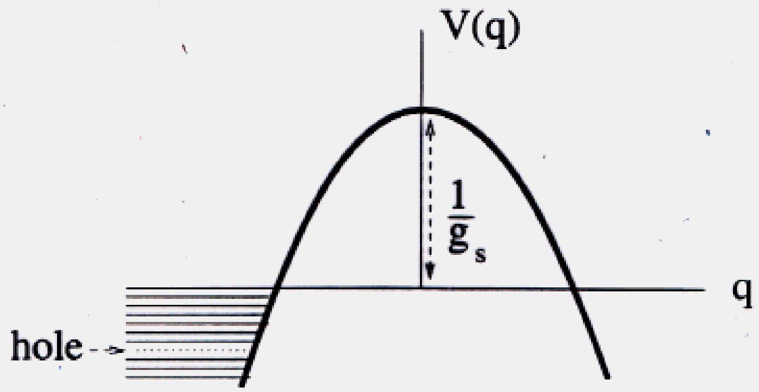


It is clear that any state which is represented as n fermions *raised* from the fermi level to various energy levels E_1, ... E_n above the fermi level can be described by a state of n D0-branes.

This is true even when the E_i's are small, *i.e.* for excitations close to the fermi level.

However, there is an important class of states missing from this list.

→ the holes created by lifting one or more fermions from below the fermi level to the fermi level.



These states do not seem to have a natural representation in terms of the degrees of freedom living on the D0-brane.

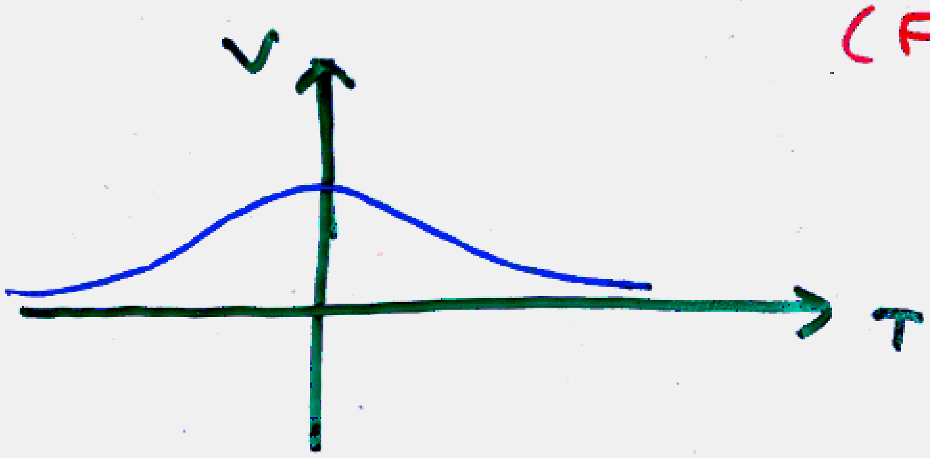
Thus it seems that we may need to add additional degrees of freedom to the open string field theory on unstable D0-branes to provide a complete description of the $c = 1$ matrix model.

HOW ARE (p, q) RELATED TO THE USUAL VARIABLES (π, T) DESCRIBING TACHYON EFFECTIVE DYNAMICS?

Garousi; Bergshoeff, de Roo, de Wit, Eyras, Panda; Gibbons, Hori, Yi; A.S.

$$\mathcal{H}(\pi, T) = \sqrt{\pi^2 + (V(T))^2}$$

(FOR OB)

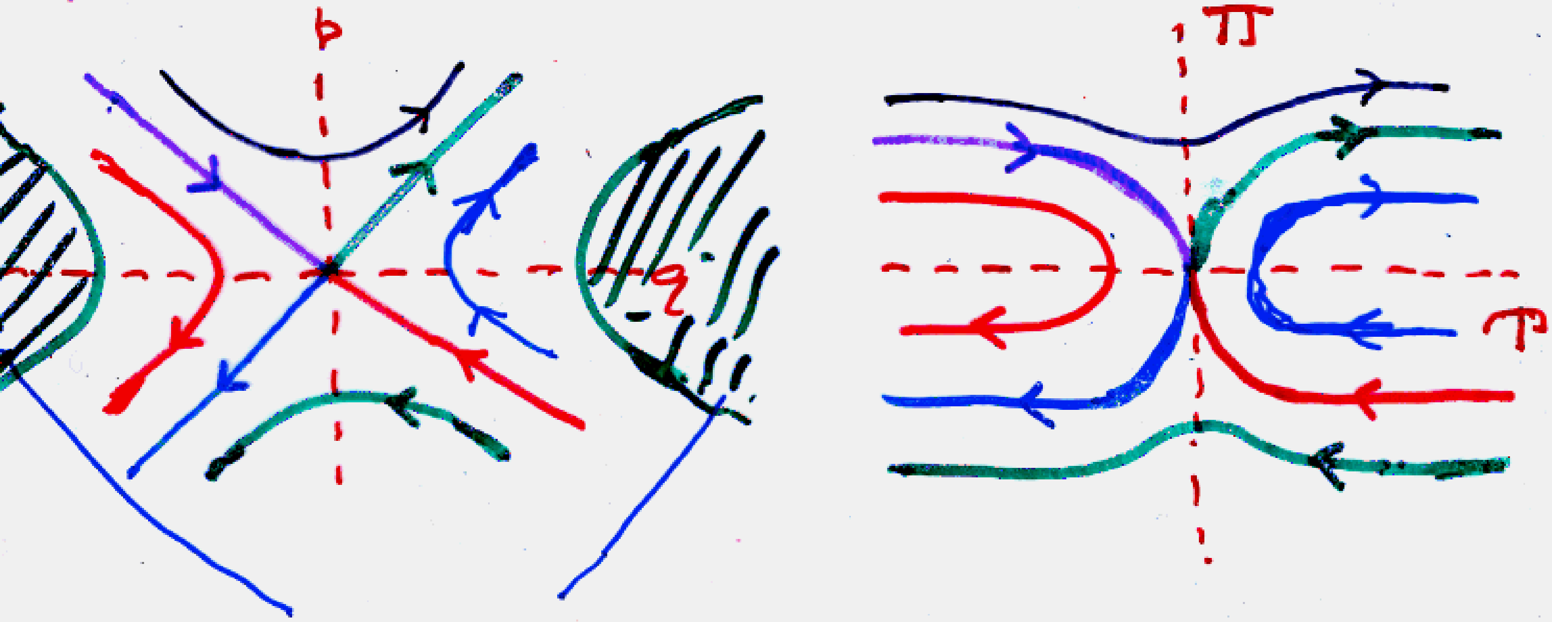


NOTE : $\mathcal{H}(\pi, T) \geq 0$

$\Rightarrow (\pi, T)$ MUST BE RELATED TO (p, q) BY A CANONICAL TRS. THAT REMOVES THE REGION BELOW THE FERMI SURFACE $(h(p, q) \leq 0)$

MAP FROM (p, q) PLANE TO (Π, T) PLANE FOR EVEN $V(T)$

(e.g. $V(T) = \frac{1}{g_s} \operatorname{sech} \frac{T}{2}$)



THE EXCLUDED REGIONS ARE MAPPED TO ∞ IN (Π, T) PLANE

$$h(p, q) = \frac{p^2}{2} - \frac{q^2}{2} + \frac{1}{g_s}$$

$$\mathcal{H}(\Pi, T) = \sqrt{\Pi^2 + V^2}$$

$V \to \frac{1}{g_s} \operatorname{sech} \frac{T}{2}$

$p = f(E, x^0)$ $\Leftrightarrow$ $\Pi = F(E, x^0)$

$q = g(E, x^0)$ $\quad$ $T = G(E, x^0)$

ELIMINATE E, x^0

$\Rightarrow$ $p = F(\Pi, T)$, $\quad q = G(\Pi, T)$

IN THE FULL QUANTUM THEORY THIS SIMPLY MEANS THAT THE ENERGY EIGENSTATES OF A D0-BRANE ARE IDENTICAL TO THOSE OF

$$\mathcal{H}(p, q) = \frac{1}{2} p^2 - \frac{1}{2} q^2 + \frac{1}{g_s}$$

EXCEPT THAT ALL THE NAGATIVE ENERGY STATES ARE REMOVED BY HAND.

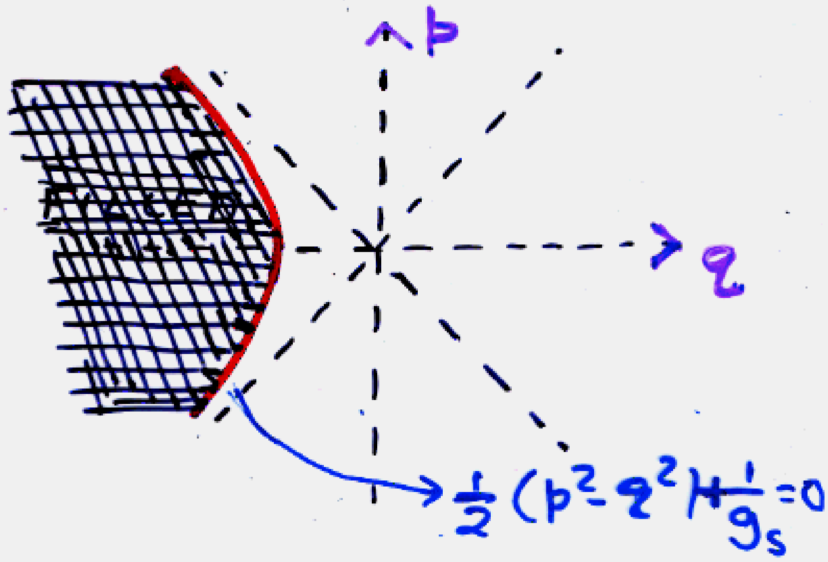
Question: Is there another class of (time dependent) boundary conformal field theories which describe the hole states in $c = 1$ matrix theory?

If so, the open string degrees of freedom associated with these boundary CFTs can be combined with those living on unstable D-branes in order to give a complete description of the $c = 1$ matrix model.

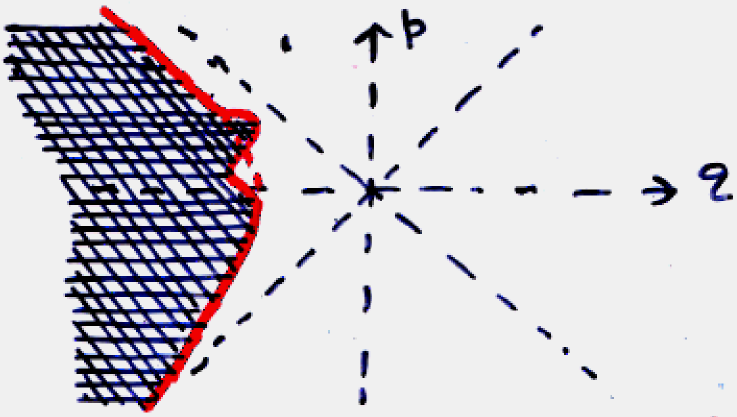
Identifying such open string degrees of freedom might also play a crucial role in reformulating critical string theories in terms of open string degrees of freedom on unstable D-branes.

A SEMI-CLASSICAL VIEW OF DIFFERENT STATES IN PHASE SPACE:

1) VACUUM

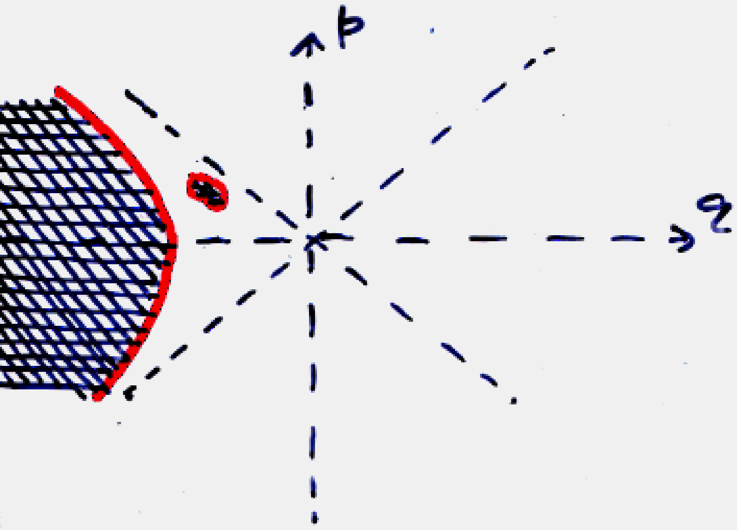


2) CLOSED STRINGS:

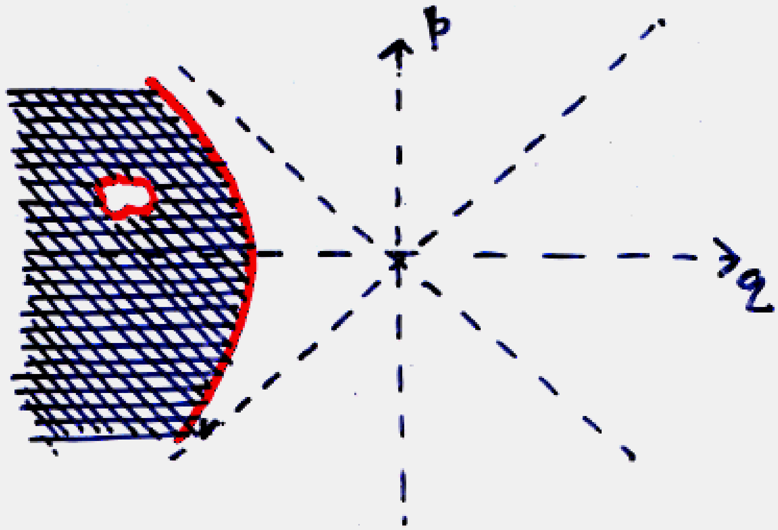


$= (3) + (4)$

3) D-BRANES



HOLES:
4) UNKNOWN BCFT



What can be the use of such a reformulation?

Any new formulation of string theory is likely to throw light on new aspects of string theory (as in the case of various dualities).

Example: The formulation in terms of open string degrees of freedom provides us with a field (the rolling tachyon) that monotonically increases with time at late time even when the system is in its ground state. $(q \sim e^{x^0})$ $T \sim x^0$

Thus it could provide a natural choice of the 'intrinsic time' in quantum cosmology.

"Cosmic time" associated with a space-time point

↔ value of the tachyon field T (or some function of T) at that space-time point.

ON A SPACE-FILLING BRANE

A.S. hep-th/0209122

Conclusion

Dynamics of open strings on unstable D-branes is quite rich.

Study of this dynamics could provide a reformulation of string theory, and hence throw light on new aspects of the theory.