Classical brane dynamics in Noncritical String Theory

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Strings 2003

Motivation

- Understanding the role of the D-strings in the duality
  2D string theory $\longleftrightarrow$ matrix models
  ($\rightarrow$ Talks by I. Klebanov, V. Kazakov, H. Verlinde, J. Maldacena, R. Dijkgraaf)
- Toy model for time-dependent phenomena in string theory

D-strings in 2d string Theory

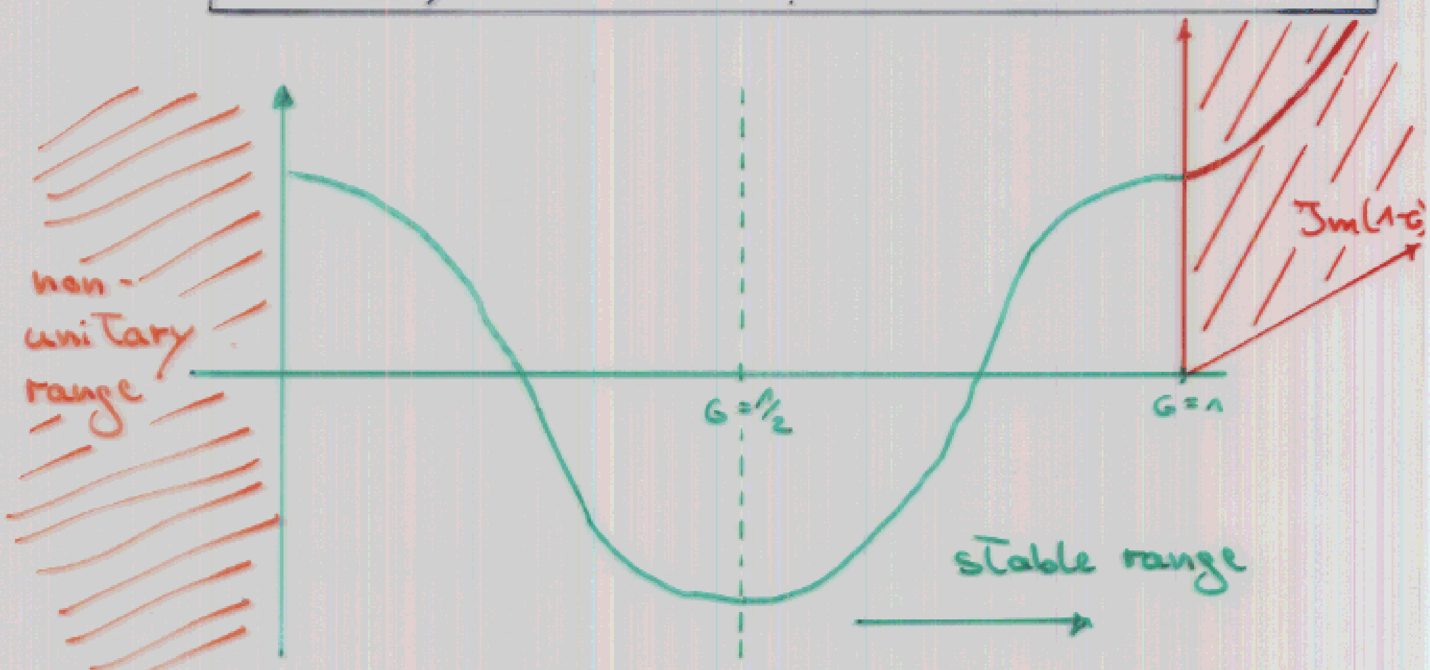
$$S_B = -\mu_B \int_{\partial\Sigma} dx \, \left(\varphi + \tfrac{1}{2}\log \pi\mu\right) e^{\varphi}$$

Boundary state:

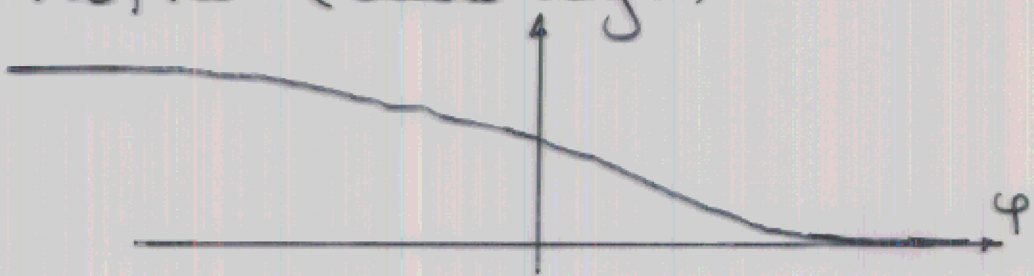
$$\langle B_\sigma | = \int_0^\infty dP \; i\, e^{i\delta(P)} \, \frac{\cosh 4\pi(1-\sigma)P}{\sinh 2\pi P} \, \langle\langle P |$$

- $e^{i\delta(P)} = (\pi\mu)^{-iP} \dfrac{\Gamma(1+2iP)}{\Gamma(1-2iP)}$
- $\cos 2\pi(1-\sigma) = \dfrac{\pi\mu_B}{\sqrt{\pi\mu}}$

$\boxed{\text{Theory is (entire)? analytic w.r.t. } \sigma}$



Profile (stable range)



Slow processes

$$\text{Ansatz}: \quad \frac{1}{\kappa} S_B = \int_{\partial\Sigma} dx \big(e^{\nu X^0} \Psi^{\sigma}_{1-\nu}\big)(x).$$

where

- $\Psi^{\sigma}_{1-\nu}(x)$: Primary boundary field in Liouville theory,
- $0 < \nu \ll 1 \quad \Rightarrow \quad 0 < 1 - \Delta_\nu = \nu^2 \ll 1.$

Two standard approaches:

- **Euclidean:** Boundary RG flows
  - forget about X^0,
  - study pert. th. of relevant boundary field $\Psi^{\sigma}_{1-\nu}(x)$.
- **Minkowskian:** Modify ansatz:

$$\frac{1}{\kappa} S_B = \int_{\partial\Sigma} dx \, G\big(\nu X^0(x)\big) \Psi^{\sigma}_{1-\nu}(x),$$

$$G(t) = e^t + \mathcal{O}(\kappa).$$

Boundary RG flows I

$$\langle \mathcal{O} \rangle_{\sigma,\lambda} = \langle \mathcal{O}\, \mathrm{P} e^{-S_{\mathrm{pert}}} \rangle_{\sigma},$$

$$S_{\mathrm{pert}} = -\lambda \int_{\partial\Sigma} dx \Psi^{\sigma}_{\alpha}(x)$$

In order to describe resulting scale dependence,

- introduce cut-offs a, L, (scale-parameter: $e^{l} = a/L$),
- make λ scale-dependent such that resulting eff. action is cut-off independent.

Consider $\mathcal{O}(\lambda^2)$:

$$\frac{1}{2}\int_{\partial\Sigma} d\varphi_2 d\varphi_1\, \Phi^{\sigma}_{\alpha}(\varphi_2)\Phi^{\sigma}_{\alpha}(\varphi_1)\Theta\big(|\varphi_2 - \varphi_1| - e^{l}\big).$$

Scale-variation $\delta_\epsilon : a \to a(1+\epsilon)$:

$$-\frac{\epsilon}{2}\int_{\partial\Sigma} d\varphi_2 d\varphi_1\, \Phi^{\sigma}_{\alpha}(\varphi_2)\Phi^{\sigma}_{\alpha}(\varphi_1)\delta\big(|\varphi_2 - \varphi_1| - e^{l}\big).$$

For $e^{l} \ll 1$ use OPE:

$$-\,\epsilon \sum\!\!\!\!\!\!\int d\beta\, E^{\sigma}_{\alpha\beta}\left(\frac{a}{L}\right)^{\Delta_{2\sigma} - 2\Delta(\alpha)} \int_{\partial\Sigma} d\varphi_1\, \Phi^{\sigma}_{\beta}(\varphi_1).$$

OPE controls perturbative scale dependence!

OPE of Liouville boundary fields

... is encoded (J.T. 0307???) in the analytic structure of the boundary three point functions (B. Ponsot, J.T., 0110244) ...

Two relevant examples ($1 < 2\alpha < 2$):

A $2 > 2\sigma > 1$:

$$\Phi_\alpha^\sigma(\varphi_2)\Phi_\alpha^\sigma(\varphi_1) \;=\; \text{irrelevant fields.}$$

B $1 > 2\sigma > 0$:

$$\begin{aligned}\Phi_\alpha^\sigma(\varphi_2)\Phi_\alpha^\sigma(\varphi_1) \;&=\; E_{\alpha\alpha}^\sigma \, |\varphi_2 - \varphi_1|^{\Delta_{2\sigma}-2\Delta(\alpha)} \, \Phi_{2\sigma}^\sigma(\varphi_1) \\ &\qquad + \;\text{irrelevant fields.}\end{aligned}$$

Easy to understand:

$$\mathcal{H}_\sigma \simeq \int_0^\infty dP \, \mathcal{V}_P \oplus \mathcal{D}_\sigma,$$

$$\mathcal{D}_\sigma = \begin{cases} \emptyset & \text{if } 2 > 2\sigma > 1 \\ \mathcal{V}_{i(1-2\sigma)} & \text{if } 1 > 2\sigma > 0. \end{cases}$$

Boundary RG flows II

Consequences:

A $\underline{2 > 2\sigma > 1}$:

Only trivial scale dependence of coupling λ_α,
$\lambda_\alpha = (a/L)^{1-\Delta_\alpha} \Rightarrow$ **"frozen coupling"**.

B $\underline{1 > 2\sigma > 0}$:

a) $\alpha \neq 2\sigma$: frozen coupling λ_α, but: $\lambda \equiv \lambda_{2\sigma}$ gets turned on!

b) $\alpha = 2\sigma$: Coupling $\lambda \equiv \lambda_{2\sigma}$ flows according to

$$\boxed{\dot{\lambda} \equiv \frac{d\lambda}{dl} = \lambda y + E_\sigma \lambda^2 + \mathcal{O}(\lambda^3),} \qquad E_\sigma \equiv E^\sigma_{2\sigma,2\sigma}.$$

$$y \equiv 1 - \Delta_{2\sigma}$$

Interpretation:

- Coupling λ corresponds to **normalizable** states $\Psi^\sigma_{2\sigma}(x)|0\rangle$.

  $\Rightarrow$ RG flow describes **spontaneous** brane decay!

- Couplings λ_α, $\alpha \neq 2\sigma$ correspond to **non-normalizable** states $\lim_{x\to 0} \Psi^\sigma_\alpha(x)|0\rangle$.

  $\Rightarrow$ Frozen couplings $\lambda_\alpha = e^{l(1-\Delta_\alpha)}$ describe brane decay **forced** by strong external sources.

cf. Seiberg / Shenker

Minkowskian picture

$$\text{Ansatz}: \qquad S_B = \int_{\partial\Sigma} dx\, G(\nu X^0(x))\Psi^\sigma_{1-\nu}(x),$$
$$G(t) = e^t + \mathcal{O}(\kappa).$$

Determine $G(t)$ from requirement of conf. invariance.

$$0 = \lim_{\epsilon\downarrow 0} \left\langle \Psi \middle| \big(T(x+i\epsilon) - \bar{T}(x-i\epsilon)\big)\, e^{i\kappa S_B} \middle| B_\sigma \right\rangle$$

Contribution $\mathcal{O}(\kappa^2)$:

$$-\frac{1}{2}\kappa^2\, \mathrm{P}\int_{\partial\Sigma} dx_2 dx_1\, \big(e^{\nu X^0}\Psi^\sigma_{1-\nu}\big)(x_2)\big(e^{\nu X^0}\Psi^\sigma_{1-\nu}\big)(x_1)$$
$$\sim -c\frac{\kappa^2}{2\nu^2} E^\sigma_{1-\nu,1-\nu}\int_{\partial\Sigma} dx\, \big(e^{2\nu X^0}\Psi^\sigma_{2\sigma}\big)(x) + \ldots,$$

Violates conf. symmetry unless cancelled by correction to S_B.

Case $\alpha = 2\sigma$ (bound state):

$$\nu^2\dot{G}(t) = \nu^2 G + cE_\sigma G^2 + \ldots$$

So to this order:

$$\boxed{\text{(RG time)} \quad \nu^2 l = t \quad \text{(``real'' time)}}$$

Adiabatic approximation

$$(*) = (\omega\,|\,e^{i\kappa\int_{\partial\Sigma}dx\,G(\nu X^0(x))\Phi^\sigma_{1-\nu}(x)}\,|\,B_\sigma\,) \sim$$

$$\sim \int\frac{dt}{2\pi}\,(\omega\,|\,e^{i\kappa G(t)\int_{\partial\Sigma}dx\,\Phi^\sigma_{1-\nu}(x)}\,|\,B_\sigma(t)).$$

where

$$|\,B_\sigma\,) \equiv |B_{\rm N}\rangle\!\rangle \otimes |B_\sigma\rangle$$

$$|B_\sigma(t)) \equiv |B_{{\rm D},t}\rangle\!\rangle \otimes |B_\sigma\rangle$$

if $|B_{\rm N}\rangle\!\rangle$ and $|B_{{\rm D},t}\rangle\!\rangle$ realize Neumann and Dirchlet type boundary conditions for the X^0 CFT respectively.

Analyticity of $\Phi^\sigma_{1-\nu}$ around $\nu=0$:

$$\Phi^\sigma_{1-\nu} = \Phi^\sigma_1 + \mathcal{O}(\nu)$$

$$\Longrightarrow (*) \sim \int\frac{dt}{2\pi}\,(\omega\,|\,B_{G(t)})$$

$$|B_{G(t)}) \equiv |B_{{\rm D},t}\rangle\!\rangle \otimes |B_{G(t)}\rangle$$

$$\cos 2\pi G(t) = \frac{\pi\mu_B}{\sqrt{\pi\mu}}$$

Time-dependent cosm. constant ?

Observation: $(\omega\,|\,B_{G(t)})$ matches closed string emission amplitude from classical treatment of probe eigenval. in matrix model

J. McGreevy
H. Verlinde
J.T.

Endpoint of the decay

... from RG fixed point

$$\lambda^* = -\,{}^{y}\!/E_G\ ,\quad E_G = E^G_{2G,2G}$$

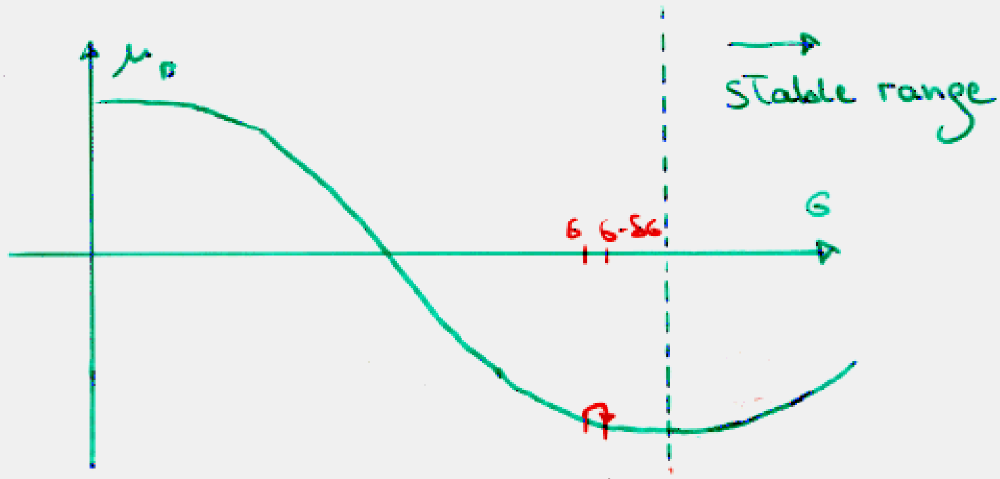
To identify BCFT at new fixed point use

$$\Phi^G_{1-\nu} = \Phi^G_1 + O(\nu)$$

(analyticity w.r.t. ν).

$\Rightarrow$ Decay of D-string with $0 < 1-2G \ll 1$ yields D-string with $G' = G + \delta G$,

$$\delta G = +\tfrac{1}{2}(2G-1)^2$$



A fast decay

Consider D-string, $G = 0$.

- Boundary state:

$$|B_{G=0}\rangle = |B_{G=1}\rangle + |B_{ZZ}\rangle$$

D-particle

- Spectrum:

$$\mathcal{H}_{0,0} = \int_0^\infty dP\, \mathcal{V}_P \oplus \mathcal{V}_i$$

$\Delta_i = 0$

- Decoupling:

$$\mathcal{V}_i \ni |i\rangle \equiv |0\rangle$$

$\Rightarrow$ **Open strings on the D-particle decouple from the rest!**

$\Rightarrow$ Bd. pert. $\lambda \int dx\, e^{x^0}$ corresponds to <u>normalizable</u> state

Amplitude for cl. string emission from decaying ($G=0$) D-string:

$$(\omega|B_0) = \mathcal{N} e^{-2i\tau\omega} e^{i\delta(\omega)} \left(1 + \frac{1}{2\sinh^2 2\pi\omega}\right)$$

- $|B_0) = |B_\lambda^{roll}\rangle_{x^0} \otimes |B_{G=0}\rangle_{Liou}$
- $\tau = \ln\lambda$

Summary:

Strong coupling

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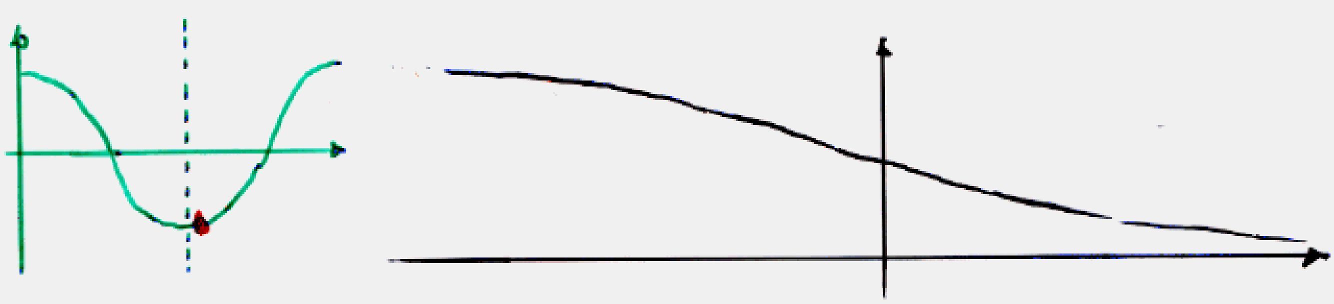
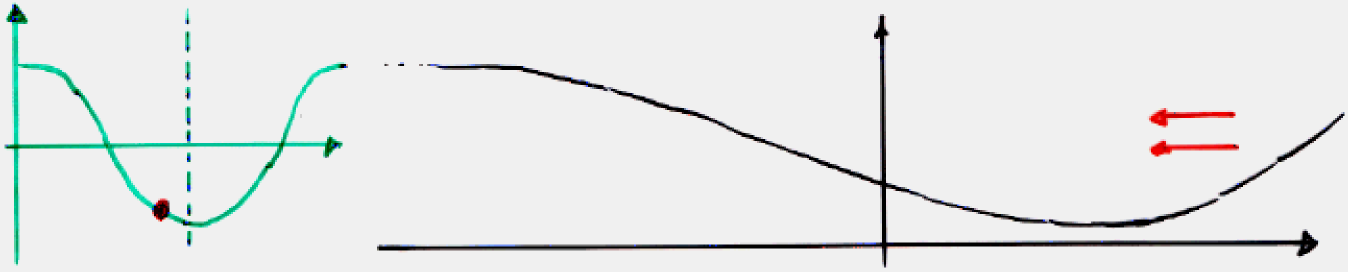
2 

Decay only if triggered by strong sources!

3 

Spontaneous, but slow decay back to 2

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(G=0) D-string = (G=1) D-string + D-particle

D-particle decays fast and violently, triggers decay of D-string.

Closing remarks

- Boundary perturbation that corresponds to classical probe eigenvalue creates non-normalizable states $\rightsquigarrow$ major perturbation of the system.

- Effective field theory for many $(G=0)$ D-string contains the matrix model.