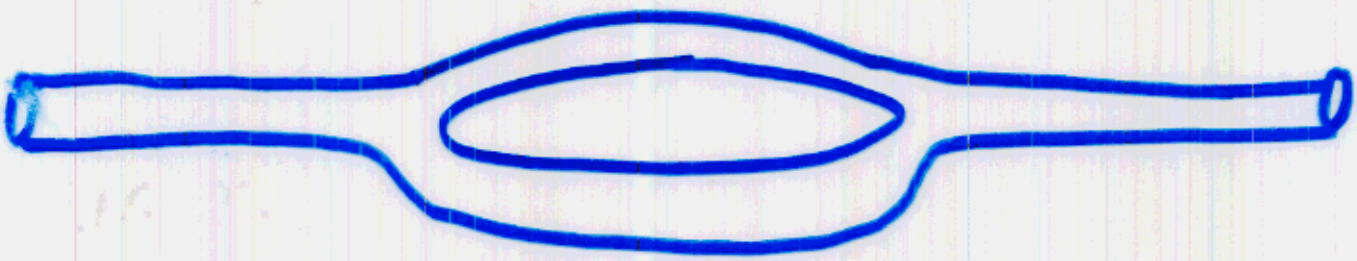
STRING THEORY IN PLANE WAVES & IN AdS



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KITP, SANTA BARBARA

ONE OF THE MOST POWERFUL TOOLS WE HAVE IN STRING THEORY IS AdS/CFT BUT...

STRING THEORY IN AdS IS HARD:

WE RESORT TO SUGRA APPROXIMATION

IN MY TALK:

* PLANE WAVE LIMIT OF AdS/CFT

  LIGHT-CONE SFT

FROM GAUGE THEORY

[based on my papers w/ He, Klebanov, Pearson, Roiban, Schwarz, Spradlin, Vaman, Verlinde]

* INTEGRABILITY OF AdS STRING THEORY

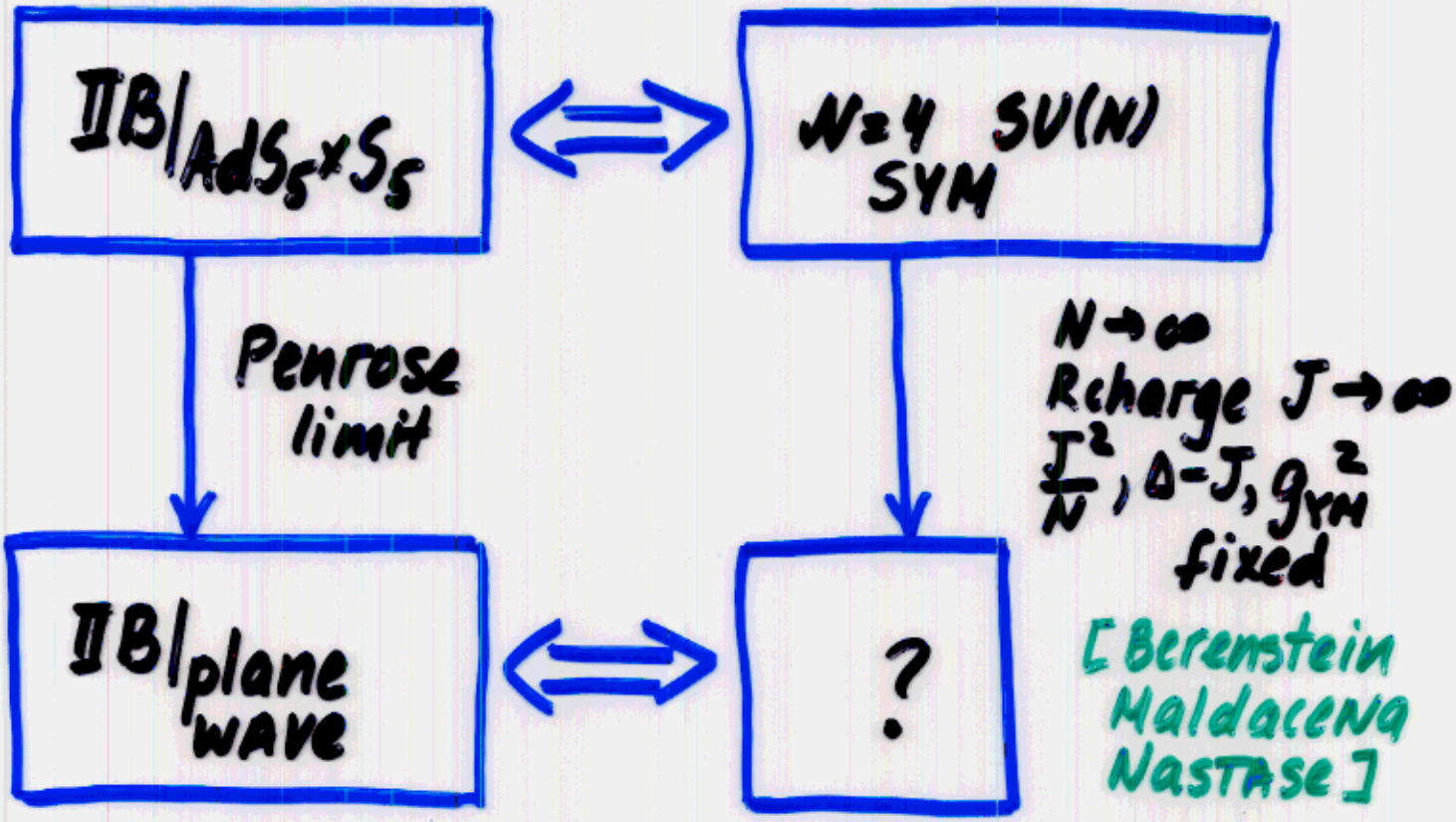
  CURRENTS

  S-MATRIX

  SPECTRUM FROM TBA

[very much work in progress with DeWolfe, Polchinski, Roiban]

AdS/CFT IN PLANE WAVE LIMIT



MOTIVATIONS

- EXACTLY SOLVABLE STRING THEORY [Metsaev]

- POSSIBLY PERTURBATIVE DUALITY

$g_2 = g_s (\mu p^+ \alpha')^2 = J^2/N$ YM genus

$\lambda' = 1/\mu p^+ \alpha' = \lambda/J^2$ YM loops

STATE/OPERATOR FOR FREE STRING

$$|0,p^+\rangle \to \mathrm{tr}\, Z^J$$

$$a_n^+ a_{-n}^+ |0,p^+\rangle \to \sum_{k=0}^{J} e^{\frac{2\pi i n k}{J}} \mathrm{tr}(\Psi Z^k \Psi Z^{J-k})$$

WHAT'S THE EVIDENCE?

- FREE STRING THEORY IN PLANE WAVE IS EXACTLY SOLVABLE IN LIGHT-CONE

$$P^- = \frac{1}{2p^+\alpha'} \sum_{n=-\infty}^{+\infty} N_n \sqrt{n^2 + (\mu p^+ \alpha')^2}$$

[Metsaev]

- THE SPECTRUM AGREES W/ PLANAR ANOMALOUS DIMENSIONS $(\Delta - J)_n$ IN GAUGE THEORY

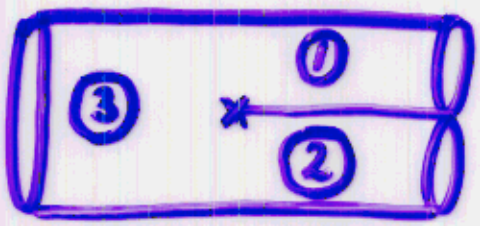
[Berenstein, Maldacena, Nastase] [Gross, Mikhailov, Roiban] [Santambrogio, Zanon]

- THIS ESTABLISHES $\frac{2}{\mu} P^- = \Delta - J$ @ $g_2 = 0$

WHAT ABOUT INTERACTIONS?

LIGHT-CONE SFT IN PLANE WAVES

- EXTENSIVELY DEVELOPED IN FLAT SPACE IN 80s BY GREEN, SCHWARZ, BRINK
- GAUGE THEORY NATURALLY GIVES A LIGHT-CONE QUANTIZED STRING



- SFT THREE-STRING VERTEX

$$|V\rangle \sim \mathcal{P}\, e^{a^+ N a^+}\, |0\rangle_3$$

$a^+ N a^+$: Neumann matrices

$\mathcal{P}$: prefactor

$e^{a^+ N a^+}$: delta-overlap

IS DETERMINED FROM PLANE-WAVE SUPERALGEBRA

DELTA-OVERLAP

- CONTINUITY OF THE WORLDSHEET

$$\Delta^2 [X_1(6) + X_2(6) - X_3(6)]$$

- OSCILLATOR EXPRESSION REQUIRES INVERTING (INFINITE)x(INFINITE) μ-DEPENDENT MATRIX [Spradlin, A.V.]

- DETERMINED N^{rs}_{mn} TO ALL ORDERS IN λ' [He, Schwarz, Spradlin, A.V.]

$$N^{12}_{mn} \sim \frac{1}{\omega_1^m \beta_1^+ + \beta_2^+ \omega_2^n} \sqrt{\frac{(\omega_1^m + \mu\alpha' p_1^+)(\omega_2^n + \mu\alpha' \beta_2^+)}{\omega_1^m \omega_2^n}}$$

WHERE $\omega_r^m = \sqrt{n^2 + (\mu\alpha' p_r^+)^2}$

↖ 1,2,3 string

- FRACTIONAL POWER (OF λ') APPEAR [Klebanov, Spradlin, A.V.]

- PERTURBATIVE DUALITY ONLY WHEN NUMBER OF IMPURITIES IS CONSERVED

- UNALLOWED INTERCHANGE OF LIMITS

$$J \leftrightarrow \lambda'$$

PREFACTOR

- SUSY REQUIRES THE INSERTION OF A LOCAL OPERATOR $\mathcal{P}$ AT THE INTERACTION POINT

- IN CONTINUUM BASIS $\mathcal{P}$ IS THE SAME AS IN FLAT SPACE [Spradlin, A.V.]

$$\mathcal{P} \sim p^2(\sigma) - \partial X^2(\sigma)\Big|_{\sigma\text{-interaction point}}$$

- IN OSCILLATOR BASIS IT IS SIMPLY

$$\mathcal{P} \sim \sum_{m=-\infty}^{+\infty} e(m) \frac{\omega_m^r}{p_r^+} a_m^{r+} a_m^r \Big|_{L_{\pm 1}}$$

- WE CONSTRUCTED 3-STRING VERTEX TO FIRST ORDER IN g_2 AND ALL IN λ'

- WITH THIS VERTEX ONE CAN IN PRINCIPLE DETERMINE ALL TREE LEVEL STRING AMPLITUDES (THOUGH TECHNICALLY DIFFICULT)

- NOW LET'S SEE HOW TO RECOVER SOME OF THESE AMPLITUDES FROM THE DUAL GAUGE THEORY ...

STRING INTERACTIONS FROM YANG-MILLS

- IN GAUGE THEORY STRING INTERACTIONS CORRESPOND TO NONPLANAR DIAGRAMS COMPUTED BY

[Constable, Freedman, Headrick, Minwalla, Motl, Postnikov, Skiba], [Gross, Mikhailov, Roiban], [Beisert, Kristjansen, Plefka, Semenoff, Staudacher], [Chu, Khoze, Travaglini]

- QUANTITATIVELY WE SHOULD CHECK [Gross, Mikhailov, Roiban]

$$\frac{2}{\mu} P^- = \Delta - J \qquad g_2 \neq 0$$

- GAUGE THEORY INNER PRODUCT IS [Vaman, Verlinde]

$$\langle \Psi_m | \Psi_n \rangle \sim (e^{g_2 \Sigma})_{mn}$$

$\langle \Psi_m |$: m-trace operator

$(e^{g_2 \Sigma})_{mn}$: permutation operator (vertex w/o prefactor)

- BASIS CHANGE TO ALL ORDERS IN g_2 [Pearson, Spradlin, Vaman, Verlinde, AV]

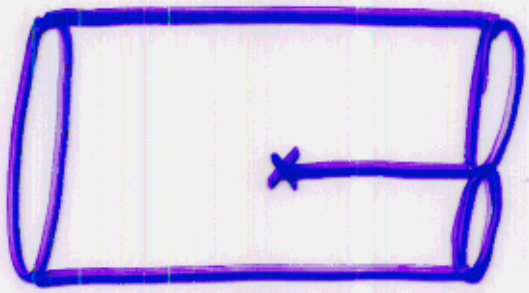
$$|\widetilde{\Psi_m}\rangle = \exp\left(-\frac{g_2 \Sigma}{2}\right) |\Psi_m\rangle$$

$|\widetilde{\Psi_m}\rangle$: m-string state

$|\Psi_m\rangle$: m-trace operator

COMPARE IN THE SAME BASIS

$a_n^{\psi+} a_{-n}^{\psi+} |0\rangle$



$a_m^{\psi+} a_{-m}^{\psi+} |0\rangle$

$|0\rangle$

$$\langle 1|K|2\rangle\langle 3|V\rangle \sim \left(\frac{\omega_m'}{p_1^+} + \frac{\omega_n^3}{p_3^+}\right)\left((N_{mn}^{13})^2 - (N_{-m-n}^{13})^2\right)$$

to first order in $g_s \lambda'$

$$\sim \sin^2 \pi n \frac{p_1^+}{p_3^+}$$

AGREEMENT WITH TWO POINT FUNCTION OF SINGLE/DOUBLE TRACE CORRELATOR IN GAUGE THEORY AFTER BASIS CHANGE

[Pearson, Spradlin, Vaman, Verlinde, AV]
also see [Gross, Mikhailov, Roiban]
[Gomis, Moriyama, Park]

MOVING ALONG TO ORDER g_2^2

- BASIS INDEPENDENT STATEMENT REQUIRES $\Delta - J$ AND $\frac{2}{\mu} P^-$ HAVE THE SAME EIGENVALUES

- IN GAUGE THEORY $(\Delta - J)$ HAS BEEN DIAGONALIZED WITHIN THE SUBSPACE OF TWO IMPURITY OPERATORS

$$(\Delta - J)_n = 2 + \lambda' \left[n^2 + \frac{g_2^2}{4\pi^2} \left(\frac{1}{12} + \frac{35}{32\pi^2 n^2} \right) \right]$$

[Beisert, Kristjansen, Plefka, Semenoff, Staudacher]
[Constable, Freedman, Headrick, Minwalla]

- WHAT ABOUT ONE-LOOP CALCULATIONS IN STRING FIELD THEORY?

CONTACT TERMS IN LC SFT

- ZERO MEASURE REGIONS IN ST MODULI SPACE PECULIAR TO SUPERSTRING

$$P^- = P_2^- + g_2 P_3^- + g_2^2 P_4^- + \dots$$
$$Q = Q_2 + g_2 Q_3 + g_2^2 Q_4 + \dots$$

- EIGENVALUES RECEIVE TWO CONTRIBUTIONS [Roiban, Spradlin, A.V.]

ITERATED CUBIC INTERACTION

$$a_n^+ a_{-n}^+ |0\rangle \longrightarrow a_n^+ a_{-n}^+ |0\rangle \int dT \langle n| P_3^- e^{T(P_2^- - E_n)} P_3^- |n\rangle$$

CONTACT TERM

$$\langle n| \{Q_3, \bar{Q}_3\} |n\rangle$$

- PERFORMING A TRUNCATED CALCULATION WE GET AN AGREEMENT

SUMMARY OF DEVELOPMENTS

- CALCULATIONAL EVIDENCE SUPPORTS AGREEMENT BETWEEN SFT AMPLITUDES & DUAL GAUGE THEORY FOR A VARIETY OF PROCESSES $\sim \lambda' g_2$
  - * MULTI-TRACE
  - * ARBITRARY MANY IMPURITIES
  - * NON-SCALAR IMPURITIES
  - * OPEN STRING
- $g_2 (\lambda')^n$: ALL n IN ST; $n=2$ IN GT
  factor of 2 disagreement
- $g_2^n \lambda'$: $n>2$ VERY HARD IN ST

GAUGE THEORY CAN BE STUDIED USING A SIMPLE QUANTUM MECHANICAL MODEL

THE MOST OPTIMISTIC HOPE IS THAT ALL AMPLITUDES (i.e. THE FULL S-MATRIX) CALCULATED IN THIS QUANTUM MECHANICS AGREE WITH THOSE IN LIGHT-CONE STRING FIELD THEORY

OPEN QUESTIONS

- ROLE OF HIGHER-POINT FUNCTIONS
- SOLVE QUANTUM MECHANICS (ALL g_2)
- DISCRETIZE SFT $\leftrightarrow$ BIT MODEL
- HOLOGRAPHY FOR PLANE WAVES

INTEGRABILITY OF STRING THEORY ON $AdS \times S$

- GREEN-SCWARZ STRING ON $AdS \times S$ IS A COSET SIGMA MODEL W/ WZ TERM & κ-SYMMETRY [Metsaev, Tseytlin]
- CLASSICALLY IT EXHIBITS INFINITELY MANY CONSERVED NONLOCAL CHARGES

  [BENA, POLCHINSKI, ROIBAN]

- GxG NONLINEAR SIGMA MODEL $\mathcal{L} \sim tr(\partial g^{-1} \partial g)$

  FLAT CONNECTIONS

$$A \sim (1 \pm ch\lambda)\, dg\, g^{-1} + sh\lambda * dg\, g^{-1}$$

  NONLOCAL CHARGES

$$Q(t) \sim P\exp \int_C A$$

  C: from $(-\infty, t)$ to $(+\infty, t)$

PSU(2|2), Osp(4|2) PRINCIPAL CHIRAL MODELS

in progess w/ DeWolfe, Polchinski, Roiban

- PCM ON BOSONIC GROUPS CAN BE SOLVED USING BETHE ANSATZ [Faddeev, Ogievetsky, Reshetikhin, Wiegmann]

- MASSLESS S-MATRIX (R-matrix) [Zamolodchikovs]

PARTICLES L_α, R_α w/ RAPIDITIES θ

$$L(\theta_1)L(\theta_2) = S_{LL}(\theta_1-\theta_2)L(\theta_2)L(\theta_1)$$

$$R(\theta_1)R(\theta_2) = S_{RR}(\theta_1-\theta_2)R(\theta_2)R(\theta_1)$$

SATISFIES YANG-BAXTER EQUATION, CROSSING, UNITARITY

- FOR SUPERGROUPS

$$S(\theta) = X(\theta)(i\mathbb{1}+\theta\Pi)\otimes(i\mathbb{1}+\theta\Pi)$$

$X(\theta)$: CDD factor; $(i\mathbb{1}+\theta\Pi)\otimes(i\mathbb{1}+\theta\Pi)$: projector

SPECTRUM FROM THERMODYNAMICAL BETHE ANSATZ

- FINITE SIZE EFFECTS

- SOLVE INTEGRAL EQUATION FOR $\epsilon(\theta)$

$$\epsilon(\theta) = e^{\theta} + \frac{1}{2\pi i}\int d\theta' \partial \ln S(\theta-\theta') \ln(1+e^{-\epsilon(\theta)})$$

...

- THE EMERGING INTERPLAY OF STRING THEORY AND INTEGRABLE SYSTEMS IS VERY EXCITING

- HOPEFULLY, WE'LL LEARN A LOT ABOUT STRING THEORY FROM IT IN THE NEAR FUTURE !