HOLOGRAPHY & QUANTUM FOAM

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INTRODUCTION

- 3 KEY PROBLEMS: GRAVITATIONAL SINGULARITY RESOLUTION, THERMODYNAMICS, INFO. LOSS

CLAIM

1. VERY HEAVY PURE STATES IN GRAVITY APPEAR MIXED TO ALMOST ALL PROBES
2. THE UNDERLYING PURE MICROSTATES FORM A TOPOLOGICALLY COMPLEX FOAM
3. THE EFFECTIVE LOW-ENERGY DESCRIPTION OF SUCH FOAMS IS VIA THE USUAL SINGULAR GEOMETRIES, PERHAPS WITH HORIZONS
4. THIS IS THE GENERAL MECHANISM FOR THE EMERGENT THERMODYNAMICS OF GRAVITY

PLAN

$\boxed{\text{I}}$ SCHWARZSCHILD — ARGUMENTS

$\boxed{\text{II}}$ HALF-BPS STATES — EXACT RESULTS

SCHWARZSCHILD

- $\Lambda < 0$: (i) BLACK HOLE IN A BOX — EQUILIBRIUM
  ($AdS_5 \times S^5$) (ii) $\mathcal{N}=4$, SU(N) SYM DUAL
- $ds^2 = -\left[1 + \frac{r^2}{\ell^2} - \frac{r_0^2}{r^2}\right]dt^2 + [\quad]^{-1}dr^2 + r^2\, d\Omega^2$
- $M \sim \frac{r_0^2}{G_5}$ $\quad \Delta \sim M\ell \sim N^2$ IN SYM
  $S = \frac{A}{4G_5} \sim N^2$
- $|\text{microstate}\rangle = \mathcal{O}|0\rangle$; $\Delta(\mathcal{O}) \sim N^2$

SUGRA STATES: $\Delta \sim \mathcal{O}(1)$

STRINGS: $\Delta \sim (g_s N)^{1/4}$

D-BRANES: $\Delta \sim N$

- STRUCTURE OF MICROSTATE OPERATORS:
  LONG POLYNOMIALS IN $\{A_\mu, \psi, X, Y, Z\}$

e.g., $\mathcal{O} \sim \mathrm{Tr}[XX^+YZ]\ \mathrm{Tr}[YYXZ^+]\ \mathrm{Tr}[\cdots]$

BUT? (i) TRACES OF $>N$ TERMS SPLIT UP
(ii) MIXING
(iii) NO SUSY — RENORMALIZATION?
[MAYBE NOT... NO g_{YM} DEP., $F_{weak} \xrightarrow{g\uparrow} \frac{3}{4} F_{weak}$]

SIMPLICITY: $\mathcal{O} \sim \mathrm{Tr}[XXY\bar{X}ZZ\bar{X}\cdots]$

Sprinkle Traces, Derivatives ...

CLAIM: LET $|\mathcal{O}\rangle = \mathcal{O}|0\rangle$; $\mathcal{O}_p$ = probe

$\langle \mathcal{O} | \mathcal{O}_p \cdots \mathcal{O}_p | \mathcal{O} \rangle$ DEPENDS ONLY ON Δ & GLOBAL CHARGES OF $\mathcal{O}$ & $\mathcal{O}_p$ UP TO $\mathcal{O}(e^{-N^2})$ CORRECTIONS FOR ALMOST ALL $\mathcal{O}$ & $\mathcal{O}_p$

WHY?

(a) ALMOST ALL LONG STRINGS $\in \left\{ \begin{smallmatrix} \text{TYPICAL} \\ \text{SET} \end{smallmatrix} \right\}$ OF STATISTICALLY RANDOM STRINGS

e.g. $\text{Prob}[\text{Random Letter} = X] = \frac{1}{|\text{ALPHABET}|} \equiv p(x)$

(b) SANOV'S THEOREM

$$\Pr\left[\begin{smallmatrix} \text{LETTER} \\ \text{DIST.} \end{smallmatrix} = q(x) \right] = e^{-\Delta D(p\|q)}$$

$$D(p\|q) = -\sum_i p(i) \ln \frac{p(i)}{q(i)}$$

ATYPICAL FRACTION $\to 0$

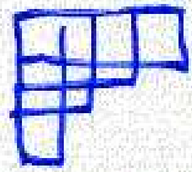
(c) STATISTICS CONTROLS CORRELATORS

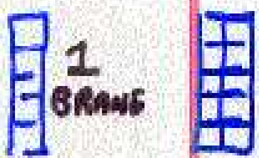
$$\langle 0 | \mathrm{Tr}(XYZXXYYX)^+ \, \mathrm{Tr}(XX)^+ \, \mathrm{Tr}(XX) \, \mathrm{Tr}(\cdots) | 0 \rangle$$

- PLANAR DIAGRAMS DOMINATED BY "PATTERN MATCHING"
- NON-PLANAR, INTERACTIONS etc. ALSO
- TRUE FOR <u>ALL</u> PROBES... STRINGS, D-BRANES AND EVEN OTHER BLACK HOLES

(d) UNLIKE A THERMAL GAS VERY HEAVY PROBES DO NOT DECOUPLE

HALF- BPS STATES ($\mathcal{N}=4$ SYM)

- ½- BPS STATES : $(0,p,0)$ REPS OF $SO(6)$ R-SYMMETRY $\Longrightarrow$ POLYNOMIALS IN X WITH $\Delta = J$
- $\mathcal{O}_T = \sum_{\sigma} a(\sigma)\, X^{i_1}_{i_{\sigma(1)}} \cdots X^{i_N}_{i_{\sigma(N)}}$ $\quad T =$  CJR

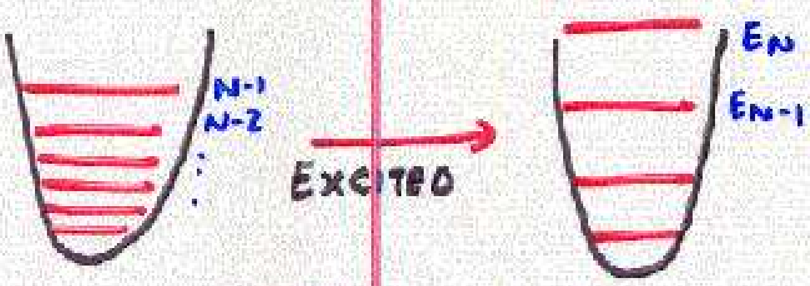
e.g. 

$\rightarrow$ EMERGENT GAUGE SYMMETRY ON BRANES (BBNS) (BBFH) (BHLN)

- ALSO: $\frac{1}{2}$ BPS- STATES $\Longleftrightarrow$ STATES OF N FERMIONS IN HARMONIC POT.
- WHY? $\Delta = J \Longrightarrow$ 1 SCALAR FIELD; ZERO MODE ONLY X, NOT $\bar{X}$; RX^2 POTENTIAL FROM CURVATURE COUPLING

(CJR Berenstein)

$\Longrightarrow$ HERMITIAN MATRIX MODEL $\rightarrow$ FREE FERMIONS

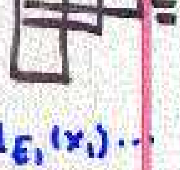


$\delta_N \equiv E_N - (N-1)$

$\delta_{N-1} \equiv E_{N-1} - (N-2)$

$\vdots$

$\delta_N \geq \delta_{N-1} \cdots \delta_1$

- $\{\delta_i\} =$  $= T$
- $\Psi = \begin{vmatrix} H_{E_1}(x_1) \cdots \\ H_{E_2}(x_2) \cdots \\ \vdots \end{vmatrix}$

$$\boxed{\begin{array}{c} \mathcal{O}_T|0\rangle \longleftrightarrow \Psi \\ \Sigma\,\delta_i = \Delta(\mathcal{O}) \end{array}}$$

(CJR Berenstein)

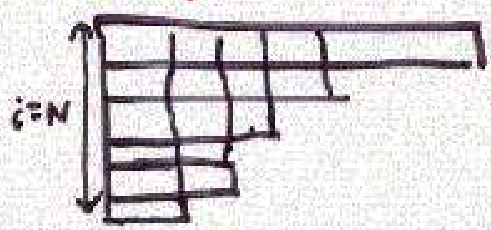
TYPICAL HALF-BPS STATES

- GIVEN $\Delta \sim N^2$ WHAT IS THE TYPICAL STATE?
  $\Longleftrightarrow$ TYPICAL PARTITION OF Δ INTO N INTEGERS
- SOLVE USING CANONICAL ENSEMBLE [Also See Suryanarayana, Buchel]

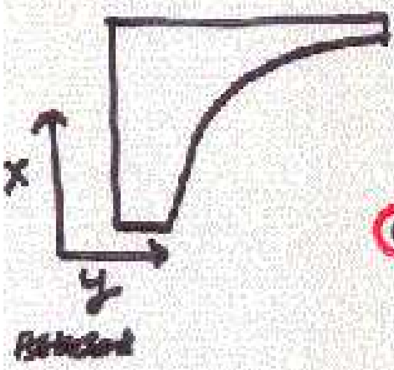
$$\langle c_i \rangle = \langle {\text{\# OF COLUMNS} \atop \text{OF LENGTH } i} \rangle = \frac{e^{-\beta i}}{1 - e^{-\beta i}} \Bigg\} \sim \text{Photon gas}$$

$$\Delta = N^2 \gamma \qquad \beta \sim \frac{1}{N\gamma}$$

$$\mathrm{Var}(c_i) = \frac{\langle c_i \rangle}{1 - e^{-\beta i}}$$



- SEMICLASSICAL LIMIT: $N \to \infty$; $N\hbar$ FIXED ($\hbar \to 0$)



LIMIT CURVE: $\boxed{e^{-\beta(N-x)} + t(N,\beta)\, e^{-\beta y} = 1}$

(a) SMALL x

$$x = \left(N - \frac{t}{\beta}\right) + t\, y$$

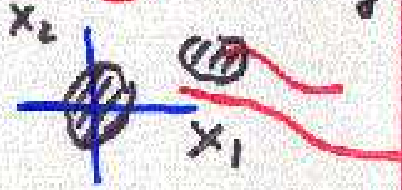
(b) LARGE x

$$y = \frac{1}{\beta} \ln t - \frac{1}{\beta} \ln \beta(N-x)$$

> ALMOST ALL STATES LIE CLOSE TO THE LIMIT CURVE. LIMIT STATE CAPTURES UNIVERSAL CORRELATORS UP TO SMALL CORRECTIONS

HALF-BPS STATES: CLASSICAL GRAVITY

- FOR OUR PURPOSE: (LLM
  - (i) COMPLICATED ASYMPT. AdS_5 SOLUTIONS
  - (ii) RADIAL COORD. y; METRIC FUNCTION $u(y)$
  - (iii) AS $y \to 0$ $\exists$ A PLANE (x_1, x_2) WITH COMPLEX TOPOLOGICAL STRUCTURE
  - (iv) $u(0, x_1, x_2)$ FIXES FULL SOLUTION
  - (v) $u \xrightarrow[y\to 0]{} \begin{smallmatrix} 0 \\ 1 \end{smallmatrix}$ $\Rightarrow$ NON-SINGULAR

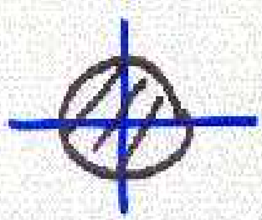


S_3 IN AdS_5 SHRINKS

S_3 IN S_5 SHRINKS

- FLUX QUANTIZATION $\Longrightarrow$

SYM		GRAVITY
$\hbar$	$\longleftrightarrow$	ℓ_p^4
ENERGY $= \Delta$	$=$	$\int_{R^2} \frac{1}{2}(x_1^2 + x_2^2)\, u(0, x_1, x_2)$
# OF FERM. $= N$ = # OF ROWS	$=$	$\int_{R^2} u(0, x_1, x_2)$



$= AdS_5 \times S^5$

$$\boxed{\begin{array}{c} x_1 \sim q \;;\; x_2 \sim p \\ u \sim \text{SEMICLASSICAL FERMION DENSITY IN SHO PHASE SPACE} \end{array}}$$

WHAT IS u FOR THE TYPICAL STATE?

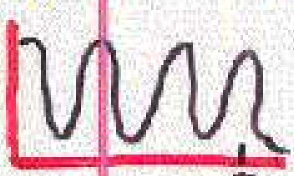
WIGNER DISTRIBUTION

- $\rho = |\psi(x_1 \cdots x_N)\rangle\langle\psi(y_1 \cdots y_N)|$
- INTEGRATE OUT FERMIONS $2 \cdots n$
- FOURIER TRANSFORM $x_1 - y_1 \to p$; $x_1 + y_1 = q$

$\Rightarrow$ $W(p,q)$

$$\langle \mathcal{O}(p,q)_{\text{Weyl}} \rangle = \int dp\,dq\; F(p,q)\; W(p,q)$$

- $\left| n^{th} \text{ SHO EIGENSTATE} \right\rangle \longrightarrow W_n(p,q) = \mathcal{L}_n$ = LAGUERRE POLYNOMIAL



(i) NOT A RING AT $p^2 + q^2 = E$

(ii) NEED NOT BE POSITIVE

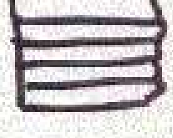
(iii) OSCILLATIONS AT $\Delta E \sim \hbar$

- SEMICLASSICAL LIMIT: $\hbar \to 0$, $N\hbar$ FIXED

(i) $\sum_1^N W_i =$

$\sim$ DISK

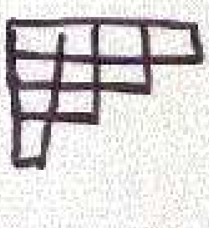
(ii)



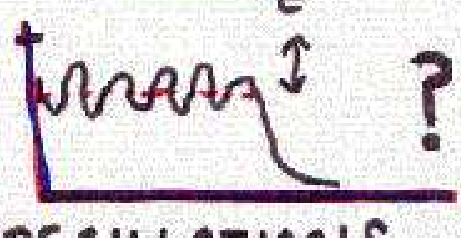
$= \sum_{i=k}^{k+N} W_i =$

$\sim$ RING ($k \sim N$)

(iii)



$= \sum_{i=1}^{N} W_{(a+bi)} =$



?

$\mathcal{O}(\hbar)$ OSCILLATIONS

$\boxed{\text{HOW TO COARSEN QUANTUM OSCILLATIONS?}}$

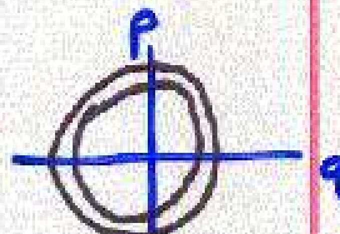
HUSIMI DISTRIBUTION

- SMEAR $W(p,q)$ BY A GAUSSIAN OF WIDTH $\hbar$

$\Rightarrow H(p,q)$ $\quad \int dp\, dq\, F(p,q)\, H = \langle \mathcal{O}(F)_{\text{NORMAL}} \rangle$

- $|\psi_n\ \text{SHO}\rangle \longrightarrow H_n = \frac{1}{2\pi\hbar\, n!} e^{-z} z^n$

$$z = \tfrac{1}{2}(p^2+q^2)\frac{1}{\hbar} = \frac{E}{\hbar}$$

$\Rightarrow$ 

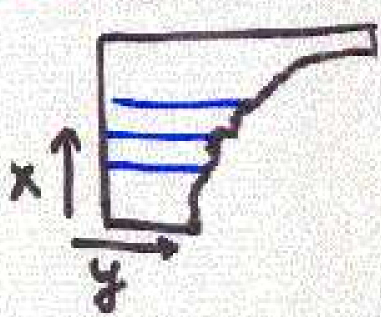
$$\left.\begin{array}{l} \text{WIDTH} \sim \sqrt{n}\,\hbar \sim \Delta E \sim \Delta(r^2) \\ \text{HEIGHT} \sim \frac{1}{\sqrt{n}\,\hbar} \end{array}\right.$$

$\Rightarrow$ NAIVE MAP TO AdS_5 (RINGS WITH $u=1$) $\Rightarrow$ CLASSICAL STRUCTU AT PLANCK SCALE

- SEMICLASSICAL OBSERVABLES $\mathcal{O}[\text{Poly}(p,q)]$ AVERAGE OVER $\hbar$ SCALE

SEMICLASSICAL DISTRIBUTION

- COARSENING SCALE ΔE $\quad \left(\frac{\Delta E}{\hbar} \xrightarrow[\hbar\to 0]{} \infty\right)$



$$g(E) = \left[\frac{\int_E^{E+\Delta E} dp\, dq\, H}{\int_E^{E+\Delta E} dp\, dq}\right] 2\pi\hbar$$

$$\Rightarrow g(E) \sim \frac{\Delta x}{\Delta E} \sim \frac{1}{(\Delta E/\Delta x)}$$

NOW: $E = x + y \Rightarrow \boxed{g(E) = \frac{1}{1+y'}}$

UP TO TINY CORRECTIONS, $g(E)$ COMPUTES ALMOST ALL OBSERVABLES

QUANTUM FOAM & THE HYPERSTAR

- MAP TO GRAVITY

$$(p, q) \longrightarrow (x^2, x^1)$$

$$E = p^2 + q^2 \longrightarrow r^2 = (x^2)^2 + (x^1)^2$$

$$\boxed{g(E) = g(p,q) = \frac{1}{1+y'} \rightarrow u(0, x_1, x_2)}$$

GRAYSCALE DISTRIBUTION $= u$

- TYPICAL STATES:

$$e^{-\beta(N-x)} + t(N,\beta)\, e^{-\beta y} = 1$$

$\Rightarrow u(0, x_1, x_2) \neq 0, 1$ SINGULAR GEOMETRY

$$\boxed{\text{SINGULAR EFFECTIVE GEOMETRY DESCRIBING SMOOTH QUANTUM STATE.}}$$

COMMENTS

(a) SINGULAR EFFECTIVE GEOMETRY FOR UNDERLYING SMOOTH FOAM

(b) CLASSICAL VS. QUANTUM FOAM
$\Rightarrow$ DEPENDS ON OBSERVABLE

(c) ALMOST ALL PROBES GIVE ALMOST NO INFORMATION

(d) GENERIC ORIGIN OF GRAVITATIONAL THERMODYNAMICS?
NO DECOUPLING IN GRAVITY $\Rightarrow$ INTEGRATING OUT ℓ_p LEADS TO MIXED STATE?

QUESTIONS

(1) D1/D5 STRING & MATHUR'S PROPOSAL?
[Emergent M=0 BTZ & Singular Black Rings (VB, Shigemori, Kraus)]

(2) RELATION TO TOPOLOGICAL STRING?

(3) α' CORRECTIONS & CLOAKING