M-THEORY COSMOLOGY

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Based on work with
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M-THEORY COSMOLOGY

OUTLINE

① Introduction: cosmological solutions with one scalar field with exponential potential (power law inflation)

② Assisted inflation : Cosmological solutions with multiple scalar fields

③ M5-brane dynamics and assisted inflation: By identifying the distance between M5-branes with the inflaton we find a realization of assisted inflation.

④ Observational constraints

⑤ Conclusions + Outlook.

Lucchin, Matarrese (1985)

INTRODUCTION : POWER-LAW INFLATION

Inflation is defined as a period in the evolution of our universe during which the scale factor of the 4D FRW universe satisfies

$$\underbrace{\ddot{a}(t)}_{t:\ \text{time}} > 0$$

Solution can be realized:

Assume the presence of one scalar φ with

inflaton

$$\boxed{U(\varphi) = U_0 \, e^{-\sqrt{\frac{2}{p}} \frac{\varphi}{M_{pl}}}}$$

POWER-LAW INFLATION

Such a potential leads to a power law solution

$$\boxed{a(t) = a_0 \, t^p}$$

This solution is inflationary if $p > 1$, i.e. for sufficiently shallow potentials.

In power law inflation the inflaton evolves according to

$$\varphi(t) = \sqrt{2p}\, M_{Pl} \log\left[\sqrt{\frac{V_0}{p(3p-1)}}\, \frac{t}{M_{Pl}}\right]$$

This solution is also valid for $1/3 < p < 1$ but in this regime it is not inflationary as $\ddot{a}(t) < 0$

Power law inflation has simple (constant) slow roll parameters:

* $\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{1}{p} < 1$

  guarantees flat potential

* $\eta = M_{Pl}^2 \frac{V''}{V} = \frac{2}{p} < 1$

  guarantees inflation lasts long enough

M-THEORY AND GRACEFUL EXIT/REHEATING

* Constant slow roll parameters mean that there is no exit from power-law inflation.
When embedded into M-theory this presents however no problem as additional contributions will modify the exponential potential causing inflation to end.

* Exponential potentials arise naturally in M-theory from various non-perturbative effects which need to be included anyway for moduli stabilization and spontaneous SUSY breaking

* We are interested in Heterotic M-theory so that after reheating we can make contact with the MSSM.

HETEROTIC M-THEORY AND OPEN MEMBRANES

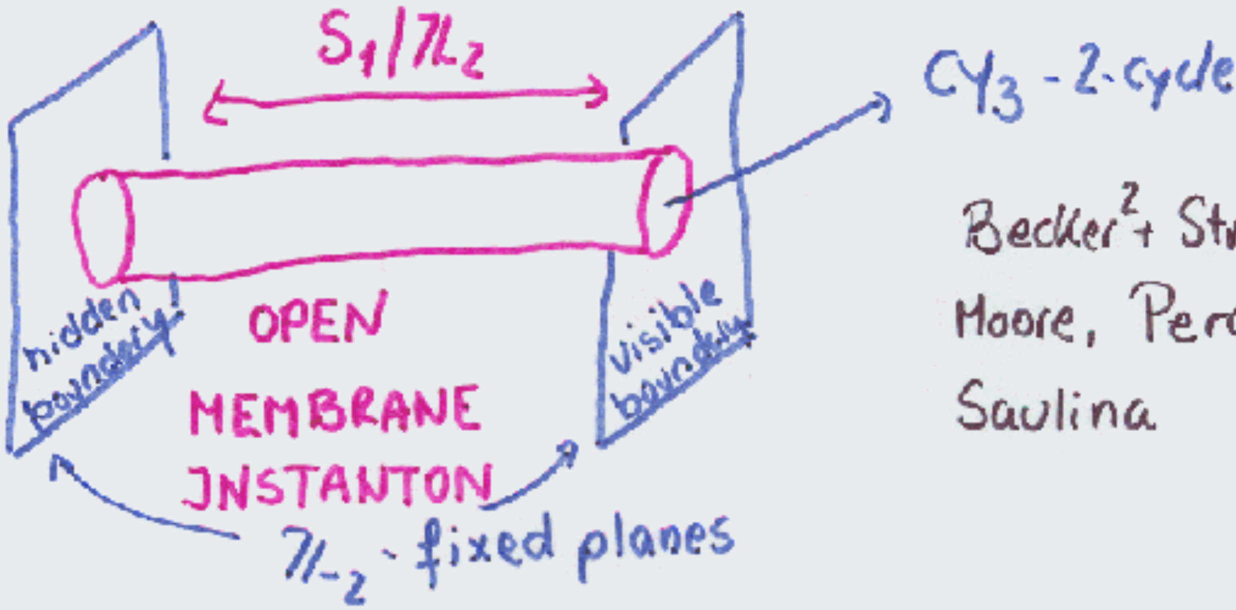
In Heterotic M-theory we compactify

$$\frac{\text{M-THEORY}}{M_6 \otimes S_1/\mathbb{Z}_2}$$

warped product

$M_6 \equiv CY_3$ for simplicity; $h_{11} = 1 \Rightarrow$
1 Kähler modulus T; in general M_6 is a manifold with torsion.

Compactifying on $S_1/\mathbb{Z}_2$ means:
the 11D space-time has 2 boundaries



Becker[2] + Strominger
Moore, Peradze + Saulina

Open membrane instantons cause the boundaries to repell each other, their interaction is being described in terms of the superpotential:

$$\boxed{W_{gg} = h\, e^{-T}}$$

volume of OM instanton

This interaction may look like a source for th exponential potentials needed in power-law inflation.
However 'T' does not have a canonically normalized kinetic term.

The field with a canonical kinetic term is:

$$\boxed{\varphi_T = M_{Pl} \sqrt{\frac{3}{2}} \log (T + \bar{T})}$$

The scalar potential for φ_T is a double exponential which is too steep to lead to inflation !

ASSISTED INFLATION (Liddle, Mazumdar + Schunck) 1998

This type of inflation is based on N scalar fields φ_i with a potential

$$U = U_0 \, e^{-\sqrt{\frac{2}{p}} \frac{\varphi_i}{M_{Pl}}} \qquad i = 1 \ldots, N$$

The e.o.m. have a late time attractor

$$\varphi_1 = \varphi_2 = \ldots = \varphi_N$$

The multifield problem can then be mapped to a single field problem that resembles power law inflation:

$$a(t) = a_0 \cdot t^{P(N)}$$

$$P(N) = N \cdot p$$

This system leads to inflation if

$$P(N) > 1$$

Even though a single exponential contribution may be too steep to support inflation individually, we can obtain inflation by choosing N to be large enough.

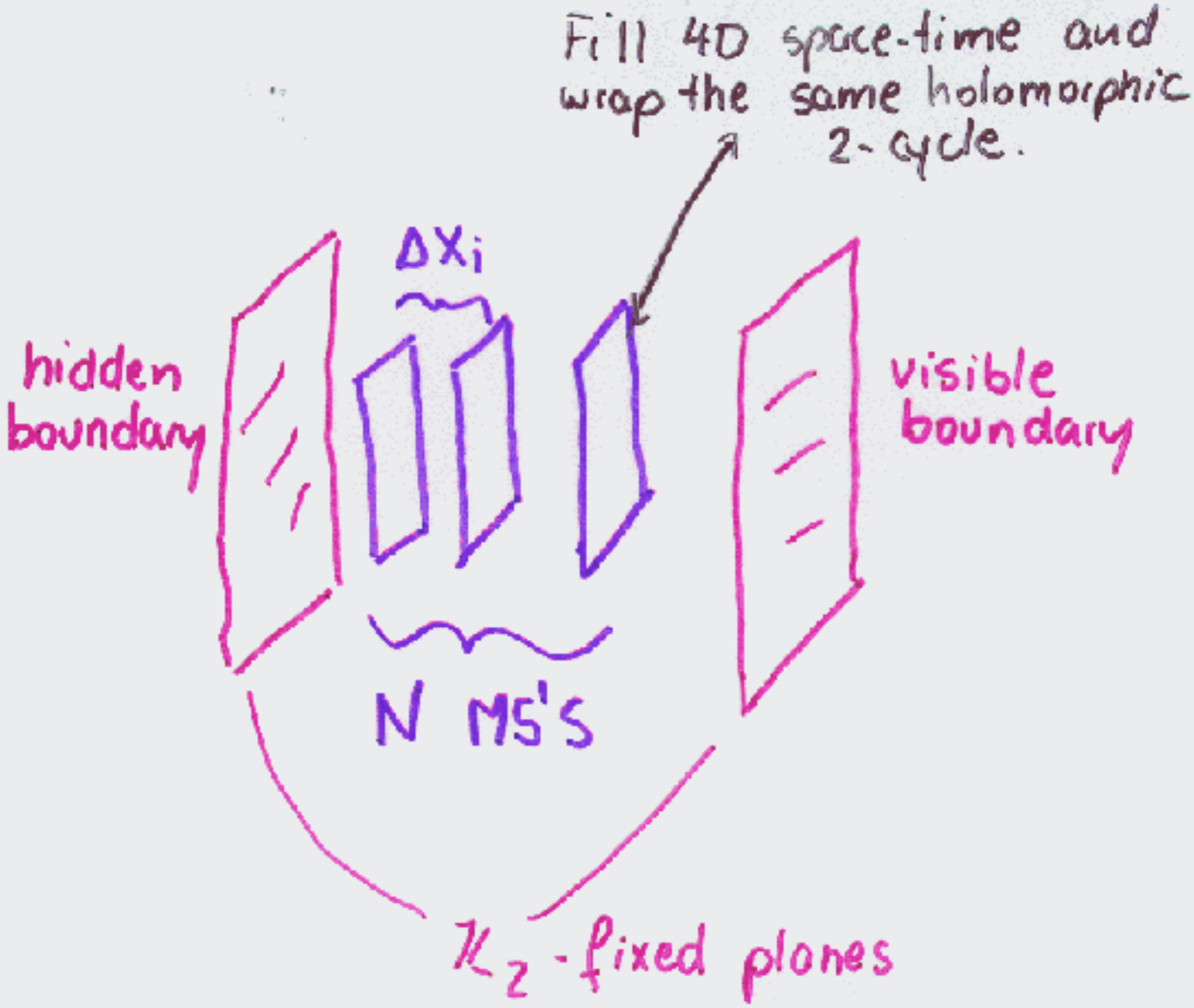
For large N, the slow roll parameters ε, η are small enough. !

How do we find assisted inflation in M-theory?

Scalar fields with an exponential potential (as needed for assisted inflation) naturally arise from the dynamics of N M5-branes in heterotic M-theory.

<u>GOAL</u>: Consider N M5's in heterotic M-theory ...

M5 - BRANES IN HETEROTIC M - THEORY



What are the fields of the 4D effective theory?

FIELDS IN THE EFFECTIVE 4D ACTION

The effective action has an $N=1$ susy in $D=4$. For simplicity take $h_{11}=1$. We get 1 Kähler modulus T:

* $T = V_{OM} + i\,\sigma_T$ volume of instanton. Kähler modulus.

Further moduli:

* $$S = V + V_{OM} \cdot \sum_{i=1}^{N} \left(\frac{X_i^{11}}{L} \right)^2 + i\sigma_S$$

  CY volume modulus

* $$Y_i = V_{OM} \left(\frac{X_i^{11}}{L} \right) + i\,\sigma_i \; ; \quad i = 1, \ldots, N$$

  M5 - position modulus

* h^{21} complex structure moduli Z^α

Kähler potentials and superpotentials?

The Kähler potential is given by

$$\boxed{K = K_S + K_Y + K_T + K_Z}$$ Kähler potential

$$K_S + K_Y = -\log\left(S + \bar{S} - \frac{\sum (Y_i + \bar{Y}_i)^2}{(T+\bar{T})} \right)$$

$$K_T = -\log\left(\frac{d}{6} (T+\bar{T})^3 \right)$$

$$K_Z = -\log\left(i \int \Omega \wedge \bar{\Omega} \right)$$

SUPERPOTENTIAL

The contributions to the superpotential come from membranes wrapping 2-cycles of the CY_3 and streching between

$$\boxed{W = W_{99} + W_{55} + \underbrace{W_{95} + W_{59}}_{}}$$

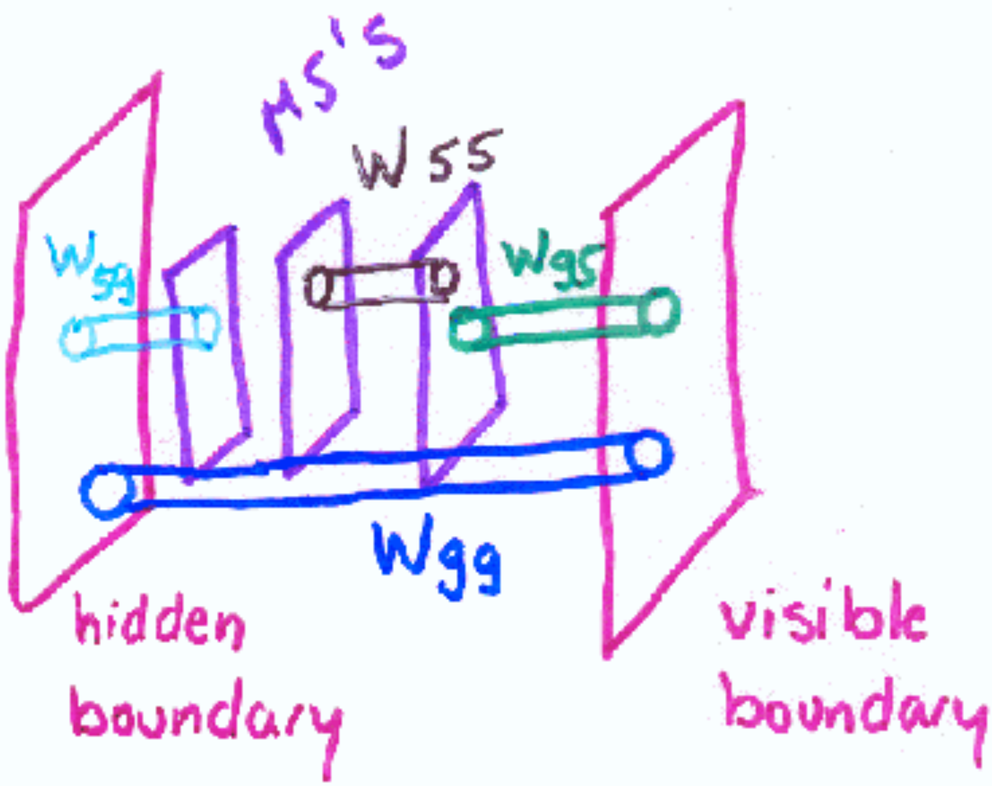
SUPERPOTENTIAL

the two $\mathbb{Z}_2$-fixed planes (M9-brane)

two M5'S

one $\mathbb{Z}_2$-fixed plane and one M5

OPEN MEMBRANE INSTANTONS



The explicit form of the superpotential is

$$W_{gg} = h\, e^{-T} \quad \text{(boundaries)}$$

$$W_{55} = h \sum_{i<j} e^{-Y_{ji}} \quad \text{(M5's)}$$

$Y_{ji} = Y_j - Y_i$

position of j-th M5

$$W_{g5} = h \sum_{i=1}^{N} e^{-Y_i} \quad \text{(M5 with boundary)}$$

$$W_{5g} = h \sum_{i=1}^{N} e^{-(T-Y_i)} \quad \text{(M5 with boundary)}$$

SCALAR POTENTIAL

In the following we would like to focus on the dominant nearest neighbor W_{55} interaction. This is correct as long as the distance between nearest M5 branes is smaller than the orbifold size. $\Rightarrow$ $W = W_{55}$ $\Leftarrow$

The scalar potential can be calculated from the $N=1$ SUGRA expression for F-terms:

$$U = M_{Pl}^4 \, e^K \left[\sum K^{\bar{I}J} D_{\bar{I}} \bar{W}_{55} D_J W_{55} - 3 |W_{55}|^2 \right]$$

APPROXIMATIONS

This potential is in general rather complicated but it can be mapped to assisted inflation using some natural approximations:

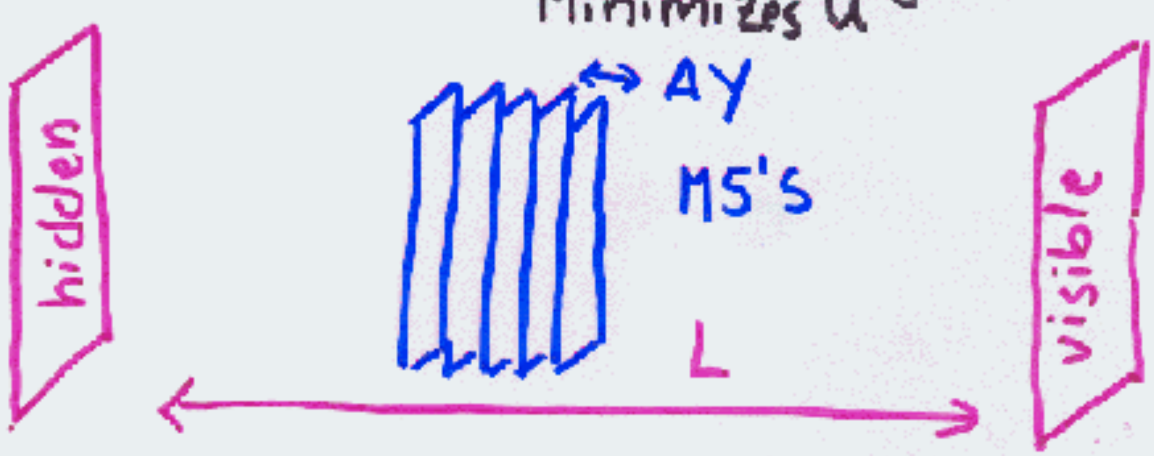
① $st \gg y^2 = \sum_{i=1}^{N} y_i^2$

$s = S + \bar{S}$

$t = T + \bar{T}$

$y_i = Y_i + \bar{Y}_i$

② $\Delta Y = Y_{i+1} - Y_i$: M5's are equidistant
Minimizes U



FIND PROPERLY NORMALIZED FIELDS

To map this potential to assisted inflation we have to find a set of fields that is canonically normalized.

$$S_{kin} = -M_{Pl}^2 \int d^4x \sqrt{-g}\, K_{i\bar{j}}\, \partial_\mu y^i\, \partial^\mu \bar{y}^{\bar{j}}$$

$$K_{ij} = \frac{4 y_i y_j + 2st\,\delta_{ij}}{s^2 t^2} \overset{st \gg y^2}{\simeq} 2\,\frac{\delta_{ij}}{st}$$

The canonically normalized MS-positions :

$$\phi_i = \frac{2 M_{Pl}}{\sqrt{st}}\, y_i$$ canonically normalized

1 single scalar !

Now map ϕ_i to $\Delta\phi = \phi_{i+1} - \phi_i$ (position difference) to obtain inflaton for assisted inflation.

The H5-position ϕ_i can be written in terms of $\Delta\phi = \phi_{i+1} - \phi_i$ and the center of mass coordinate ϕ_{cm}:

$$\boxed{\begin{array}{l} \phi_i = \phi_{cm} + \left(i - \frac{N+1}{2}\right)\Delta\phi \\[2em] \phi_{cm} = \frac{1}{N}\left(\phi_1 + \dots + \phi_N\right) \Rightarrow \text{not dynamical} \end{array}}$$

$$\frac{1}{2}\sum_{i=1}^{N} \partial_\mu \phi_i \, \partial^\mu \phi_i = \frac{N(N^2-1)}{12} \partial_\mu \Delta\phi \, \partial^\mu \Delta\phi$$

$$= \frac{1}{2} \partial_\mu \tilde{\varphi} \, \partial^\mu \tilde{\varphi}$$

INFLATON

$$\tilde{\varphi} = \sqrt{\frac{N(N^2-1)}{6}} \, \Delta\phi$$

Scalar potential?

Written in terms of the inflaton the scalar potential becomes:

$$U(\varphi) = \tilde{u}_0 \, (N-1)^2 \, e^{-\sqrt{\frac{3st}{2N(N^2-1)}} \, \frac{\varphi}{M_{Pl}}}$$

This 4D FRW universe is a solution to the equations of motion

$$H^2 = \frac{1}{3 M_{Pl}^2} \left(U(\varphi) + \frac{1}{2} \dot{\varphi}^2 \right)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dU}{d\varphi} = 0$$

which corresponds to power-law inflation:

$$a(\tilde{t}) = a_0 \, \tilde{t}^{\,p} \quad \text{with} \qquad p = \frac{4N(N^2-1)}{3st}$$

We have succeeded mapping the multi-field problem to power-law inflation!

LOWER BOUND FOR # OF M5'S :

From $$p = \frac{4N(N^2-1)}{3St}$$

we see that the bound $p > 1$ needed for inflation gives a lower bound for the number of M5's :

$$\frac{4}{3} N (N^2-1) > St$$

OBSERVATIONAL CONSTRAINTS

The dynamics of a set of N M5-branes can be mapped to assisted inflation provided we impose the constraints :

$$\underbrace{\frac{4}{3} N (N^2-1) > St}_{\substack{p>1 \\ \text{inflation}}} \overbrace{\gg y^2(N)}^{\text{large } S, t \text{ (simple exp.)}}$$

Restricts N : $19 < N \ll 195$

for $v = 341$, $v_{OM} = 7$ $\quad \frac{x_i}{L} \simeq O\left(\frac{1}{2}\right)$
typical values

EXPERIMENT

Within this range of parameters we can compare with recent cosmological data.

SPECTRAL INDEX

Quantum fluctuations of the inflaton field result in a spectrum of density perturbations which explain the large-scale structure of our universe.

$$\boxed{n = \text{spectral index}}$$

According to a recent summary of data published in astro-ph/0407372 (Seljak et. al) :

$$\boxed{n_{exp} = 0.98 \pm 0.02}$$

The spectral index for power law inflation is:

$$\boxed{n = 1 - \frac{2}{p}} \quad (*) \text{ Spectral Index}$$

$$n = 0.98 \overset{(*)}{\Longrightarrow} p \simeq 100 \overset{(*)}{\Longleftrightarrow} \boxed{N = 89}$$

$$p = \frac{4N(N^2-1)}{3st} \overset{(*)}{\simeq} \left(\frac{N}{19.3}\right)^3$$

$$19 < N \ll 195$$

Without having to invoke extremely large or unnatural values of N we can reproduce the experimental value of the spectral index !!

NUMBER OF e-FOLDINGS:

A number that characterizes the different inflationary scenarios is the number of e-foldings:

time when inflation ends

$$N_e = \log\left(\frac{a(\tilde{t}_f)}{a(\tilde{t}_i)}\right) = p \log\left(\frac{\tilde{t}_f}{\tilde{t}_i}\right)$$

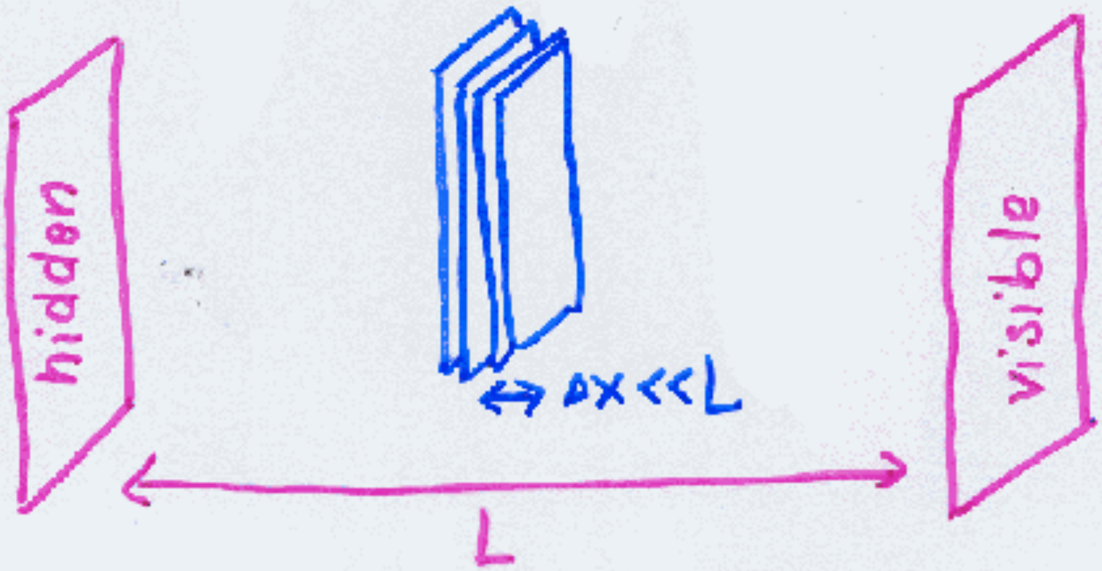
scale factor

time when inflation starts

$\tilde{t}_f$?

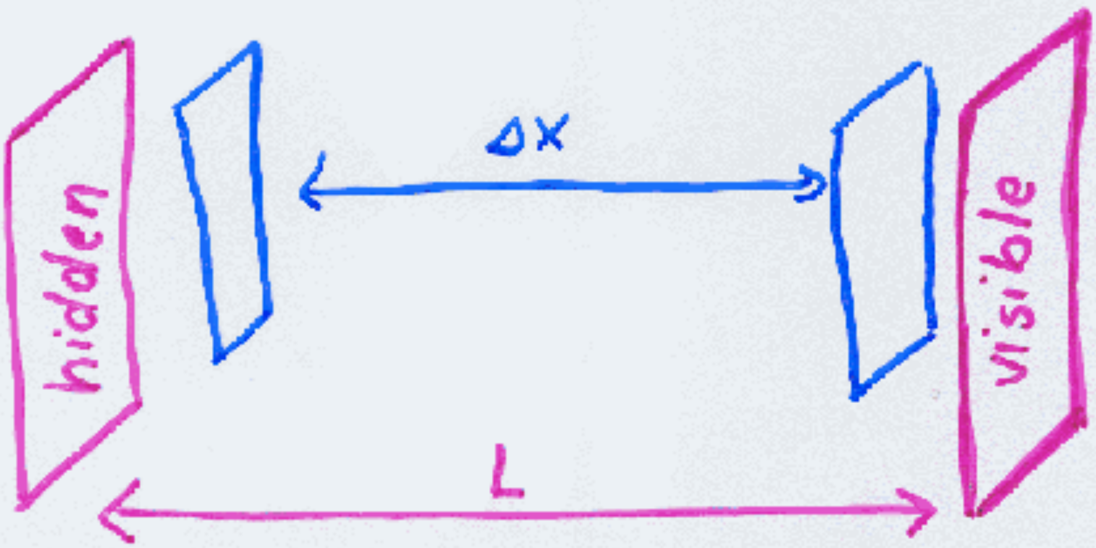
GOAL: We need to figure out why and when inflation ends...

Assisted inflation takes place as long as ..



But when ...

$\Delta x \approx L$



inflation comes to an end
as additional contributions to W
become relevant

At the beginning

$$\frac{\Delta X(t_i)}{L} \ll 1$$

At the end

$$\frac{\Delta X(t_f)}{L} \gtrsim \frac{1}{2}$$

Using the solution for the inflaton in the assisted inflation model

$$\varphi(t) = \sqrt{2p}\, M_{Pl} \log\left(\sqrt{\frac{u_0}{p(3p-1)}}\; \frac{t}{M_{Pl}}\right)$$

we get N_e in terms of ΔX : 

$$N_e = p \log\left(\frac{t_f}{t_i}\right) = \frac{t \cdot p}{2}\left[\frac{\Delta X(t_f)}{L} - \cancel{\frac{\Delta X(t_i)}{L}}\right]^{1/2}$$

$$N_e \approx \frac{t \cdot p}{4} \approx \left(\frac{N}{12.7}\right)^3$$

N_e :	50	60
N :	47	50

Conclusion

* We have found a realization of inflation in M-theory with $\eta \ll 1$
* Our model is based on N M5-branes in heterotic M-theory (important for reheating as we want MSSM).
  The inflaton is the distance between branes.
* Within the range where our approximations are valid we can obtain realistic values for the spectral index and the number of e-foldings.

OPEN PROBLEMS / FUTURE DIRECTIONS

* How generic is assisted inflation in string theory? Can it be found in type IIB?
* Torsion for M_6
* What exactly happens when the M5's disappear into the boundary? (small instanton transition) We need to understand reheating
* Does the end of inflation lead to strings (gauge theory solitons or cosmic strings)? Our heterotic model is highly constrained (if not unique). Signatures of these objects should be almost unique.