

Constructions and distributions of flux vacua

Frederik Denef

Strings 2005

Motivation

The Landscape

The good news

The bad news

What can we do?

Construction of vacua

IIB KKLT vacua

IIB nonsusy AdS vacua with exponentially large volume

I/IIB with gauge fluxes

M-theory and IIA flux vacua

More models: heterotic, non-geometric, ...

de Sitter vacua

Statistics of vacua

Susy IIB

Nonsusy IIB

M-theory

Vacua with enhanced (R-)symmetries

Gepner and intersecting brane models

Open string flux vacua and the OSV conjecture

Motivation

Not everything that can be counted counts,
and not everything that counts can be counted.
– Albert Einstein

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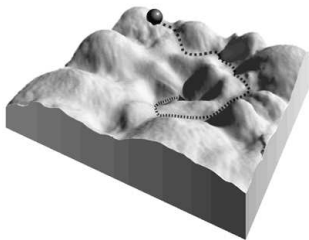
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Picture: **String theory Landscape** [Susskind]



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For K sufficiently large, $|\Lambda| < 10^{-120} M_p^4$ attainable in this simplified model.

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If sufficiently finely scanned, landscape picture offers at least possibility for a consistent explanation for a number of absurd finetunings of parameters in our universe!

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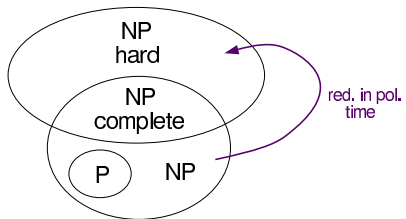
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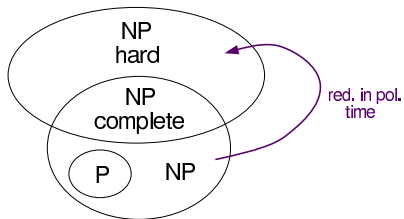
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Even in simple Bousso-Polchinski toy model, the problem to find the flux vectors N^α such that $0 < \Lambda(N) < \epsilon$ is *NP-hard*.

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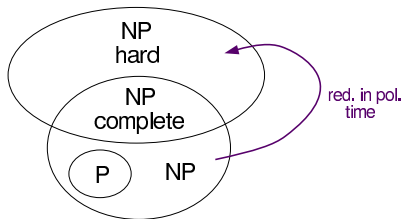


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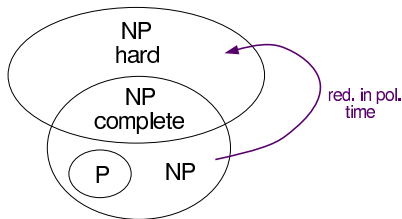
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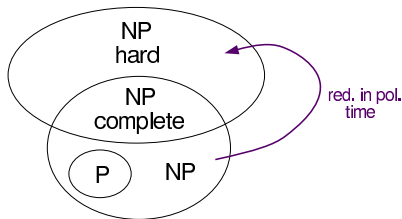
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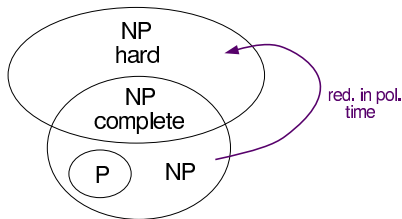
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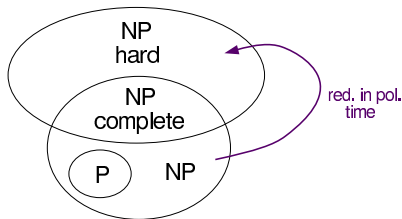
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\Rightarrow if you find a polynomial time algorithm to identify string vacua from parameter data, you're rich...

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Construction of vacua

Any intelligent fool can make things bigger and more complex...
It takes a touch of genius, and a lot of courage, to move in the opposite direction.
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The first part of the program includes

- ▶ Intersecting brane models, including Kähler potentials, Yukawa couplings, susy breaking soft terms [Aldazabal, Angelantonj, Antoniadis, Blumenhagen, Camara, Cremades, Cvetič, Dudas, Franco, Görlich, Graña, Grimm, Ibañez, Jockers, Körs, Langacker, Liu, Louis, Lüst, Mayr, Marchesano, Rabadan, Reffert, Richter, Sagnotti, Shiu, Stieberger, Taylor, Uranga, Wang]
- ▶ Heterotic constructions [Braun, Donagi, He, Ovrut, Pantev, Reinbacher]
- ▶ Gepner models [Aldazabal, Andres, Blumenhagen, Brunner, Dijkstra, Hori, Hosomichi, Huiszoon, Juknevich, Leston, Nuñez, Schellekens, Walcher, Weigand]

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But in this talk: **focus on moduli fixing.**

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KKLT [Kachru-Kallosh-Linde-Trivedi]: IIB on CY_3 orientifold Y/\mathbb{Z}_2 + RR flux F_3 + NS flux H_3

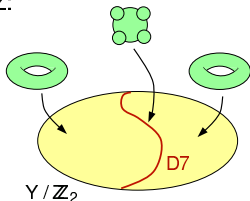
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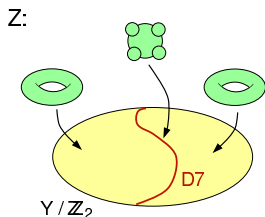


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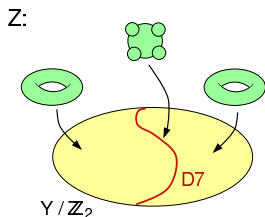
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\rightsquigarrow can fix all moduli in principle.

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By now, several examples known:

- ▶ [Denef-Douglas-Florea]: various constructions of models with a sufficient number of D3 instanton divisors with exactly 2 fermion zero modes ($h^{0,i}(M5) = 0$ [Witten]).
- ▶ [Denef-Douglas-Florea-Grassi-Kachru]: completely explicit, simple model: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$; *all* moduli (open, closed, untwisted *and* twisted) fixed.
- ▶ [Aspinwall-Kalosh]: Stabilize M-theory on $K3 \times K3$, making use of previous work of [Saulina, Kalosh - Kashani-Poor - Tomasiello] that had shown that the topological conditions on divisors to contribute to W are substantially relaxed in the presence of flux.

IIB nonsusy AdS vacua with exponentially large volume

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Rough idea: keep some divisor volumes $\rho_i \sim O(1)$ while sending overall vol to infinity, and balance nonperturbative $e^{-\rho_i}$ off against perturbative α' corrections.

\Rightarrow Volume stabilized at exponentially large value:

$$\text{Vol} \sim W_0 e^{c/g_s}$$

where W_0 and g_s are fixed by the fluxes.

IIB nonsusy AdS vacua with exponentially large volume

In [Balasubramanian-Berglund, Balasubramanian-Berglund-Conlon-Quevedo, Conlon-Quevedo-Suruliz] it was shown that, when taking into account α' corrections to the Kähler potential, a new branch of vacua can appear as nonsusy AdS minima of the potential.

Rough idea: keep some divisor volumes $\rho_i \sim O(1)$ while sending overall vol to infinity, and balance nonperturbative $e^{-\rho_i}$ off against perturbative α' corrections.

\Rightarrow Volume stabilized at exponentially large value:

$$\text{Vol} \sim W_0 e^{c/g_s}$$

where W_0 and g_s are fixed by the fluxes.

Unlike KKLT, apparently also in well-controlled regime for $O(1)$ values of W_0 .

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$$\text{Im}[e^{-i\theta} e^{\mathcal{F}+iJ}] = 0$$

(= mirror to slag cond.), and F-terms, constraining complex structure and open string moduli:

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Open string moduli should be easily fixable as well in this way.

[Gomis-Marchesano-Mateos, del Moral]

M-theory flux vacua

M-theory on G_2 hol. manifold X : turning on G_4 -flux in X gives

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Rather different compactification is obtained when taking G_4 along 4d spacetime \rightsquigarrow Freund-Rubin; $X = \text{weak } G_2$ (Einstein), e.g. $X = AdS_4 \times S^7$. Often moduli-free, and can support chiral fermions [Acharya-Denef-Hofman-Lambert], but typically $R_{KK} \sim R_{AdS}$.

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More general M-theory compactifications on weak G_2 + fluxes have been discussed e.g. by [Lambert].

Type IIA flux vacua

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Analysis of toroidal case with metric fluxes refined and generalized in [Camara-Ibañez-Font], including tadpole cancellation conditons involving metric fluxes, and inclusion of intersecting brane models.

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- ▶ [Maloney-Silverstein-Strominger, Silverstein]: Susy broken at string scale (noncritical string theories).

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- ▶ Now that we know zoo of possible constructions, more effort should start going in opposite direction: try to *eliminate* candidate solutions. For example metastability of dS vacua?

Statistics of vacua

We can't solve problems by using the same kind of thinking
we used when we created them
– Albert Einstein

Statistics: general idea

[Douglas, Ashok-Douglas, Denez-Douglas, Douglas-Shiffman-Zelditch]

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Vacuum characterized by discrete (compactification) data \vec{N} and critical point of effective potential $V_N(z)$:

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\rightsquigarrow not very practical; need some more structure \perp approx.

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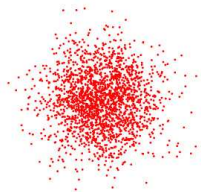
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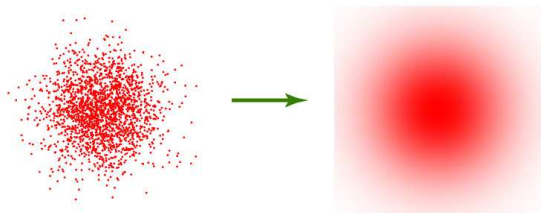


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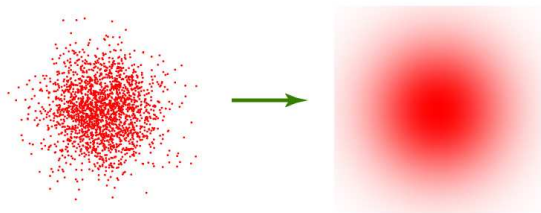


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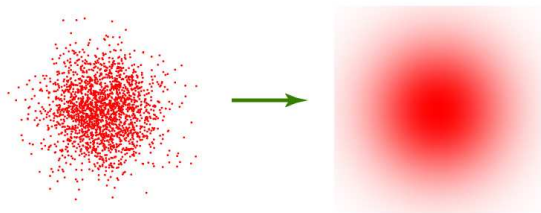
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Distributions of susy IIB flux vacua

- ▶ Number of flux vacua in region \mathcal{S} of moduli space

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Distributions of susy IIB flux vacua

- ▶ Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|z|] \sim \frac{d|z|}{|z|(\log |z|)^2}$$

→ Relation to dual YM coupling: $|z| \sim e^{-b/g_{YM}^2} \Rightarrow$ *uniform*:

$$d\mathcal{N} \sim dg_{YM}^2$$

Behavior near other singularities?

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All of above tested by Monte Carlo experiments

[Giryavets-Kachru-Trivedi, Conlon-Quevedo].

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⇒ $d\mathcal{N} \sim d\Lambda d|F|^6$.

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Reason large hierarchies are suppressed (as opposed to IIB): only as many fluxes as moduli \Rightarrow all scales set by V , no further discrete tuning possible once moduli are fixed.

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↔ possibility of environmental selection of symmetries. . .

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- ▶ [Dijkstra-Huiszoon-Schellekens] did very impressive systematic search for vacua with Standard Model chiral spectrum, among all simple current orientifolds of all Gepner models. They find almost 180,000 distinct solutions (not counting hidden sector degrees of freedom), and thoroughly analyze various distributions.

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(Skipping some details) \rightsquigarrow for small ϕ^0 :

$$Z_{osv} = \sum_q \Omega(p, q) e^{-\pi\phi^0 q_0 - \pi\phi^A q_A} \quad (1)$$

$$\approx \sum_{N, F} p_\chi(N) e^{\pi\phi^0(N - \frac{1}{2}F^2 - \frac{\chi}{24}) - \pi\phi^A J_A \cdot F} \quad (2)$$

$$\times \int_{\mathcal{M}} d^{2n}z \delta^{2n}(F^{2,0}) |\det \nabla_i F_j^{2,0}|^2 \quad (3)$$

where \mathcal{M} = divisor deformation moduli space.

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Using $\mathcal{N} = 1$ special geometry structure of [Lerche-Mayr-Warner], in small ϕ^0 approximation, and using techniques developed in [Ashok-Douglas,Denef-Douglas], this can be computed to be

$$Z_{\text{osv}} \approx \hat{\chi}(\mathcal{M}) \frac{\phi^0}{2} \exp \left(\frac{\pi}{\phi^0} \left(-\frac{1}{6} (P^3 + c_2 \cdot P) + \frac{1}{2} P \cdot \Phi \cdot \Phi \right) \right)$$

where

$$\hat{\chi}(\mathcal{M}) \equiv \int_{\mathcal{M}} \frac{1}{\pi^n} \det R$$

computed with metric $g_{i\bar{j}} = \int \omega_i^{2,0} \wedge \bar{\omega}_{\bar{j}}^{0,2}$.

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Note: small $\phi^0 = \text{large } g_{\text{top}}$. Also, instanton corrections suppressed in $\phi^0 \rightarrow 0$ limit, so don't expect to see them in this approximation.

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If we knew what it was we were doing,
it would not be called research, would it?

– Albert Einstein

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ICTP, Trieste

May 29 - June 3, 2006

organizing committee:

Bobby Acharya	Frederik Denef
Michael Douglas	Shamit Kachru
Dieter Lüst	Eva Silverstein