A Classical

II A Landscape

Based on work with

O. DeWolfe (Princeton)

A. Giryavets (Stanford)

W. Taylor (MIT)

appearing in hep-th/0505160.

Closely related important works:

Derendinger, Kounnas, Petropoulos, Zwirner

S.K., Kashani-Poor

Grimm, Louis

Villadoro, Zwirner

Camara, Font, Ibanez

Saueressig, Theis, Vandoren

Extend our results in some directions of "real world" interest

I. Introduction

There is by now significant evidence that string theory gives rise to a vast "landscape" of metastable vacua, manifesting various possibilities for the gauge group, matter content, Λ_{4d}, and other quantities of physical interest.

Practically speaking, the techniques that go into proving this assertion, are identical to the techniques we use to find string compactifications with stabilized moduli and reasonable particle physics / cosmology.

Whatever one's "philosophical" feelings about the landscape are, those are obviously worthy goals, which motivate & justify work on this subject.

Thus far, the most detailed (though still very incomplete) understanding has been developed in the context of IIB Calabi-Yau orientifolds of D3|D7 type ("F-theory"). As we've heard from e.g. Denef:

- $\exists$ very large # vacua 10^N $N \sim 100$s (quite possibly sufficient to realize the Bousso-Polchinski idea for Λ)

③

- Fractions of vacua with

$$g_s \leq g^* \quad \sim \quad g^*$$

$$|W_0| \leq |W_0|^* \quad \sim \quad (|W_0|^*)^2$$

Douglas
Denef
...

→ possible to realize KKLT type constructions in very many cases (but, rare ... tuning!). Possibly other regimes of parameters also → interesting controlled vacua (c.f. Quevedo).

Two qualitative points to contrast to our IIA story:

- # maximally symmetric 4d IIB vacua may well be finite
- 'Control' over expansions is <u>numerical</u>, not <u>parametric</u>, here

In contrast, I will argue that IIA Calabi-Yau orientifolds with flux:

- Give rise to an infinite # of maximally symmetric 4d vacua
- Allow parametric control -- $\exists$ unconstrained flux N and $\left\{\begin{matrix} g_s \\ R \end{matrix}\right.$ get $\left\{\begin{matrix} \text{small} \\ \text{large} \end{matrix}\right.$ as powers of N.

II. Basic facts about IIA + Scaling argument

We'll be studying IIA on Calabi-Yau orientifolds. In that general setting, the ingredients one encounters in model building include:

- RR fluxes F_0, F_2, F_4, F_6 thread even cycles in geometry → potential for <u>kähler moduli</u>

- NS flux H_3 threads three-cycles in geometry → potential for <u>complex moduli</u>

∃ tadpole $\quad \int_{\Sigma_i} F_0 \wedge H_3 + N_{D6} = N_{O6}$

for each homology class of 3-cycles

→ in general if $F_0, H_3 \neq 0$

- O6 planes wrapping 3-cyles (arise from $\mathbb{Z}_2$ involution of CY)

⑥

Lets take an imaginary rigid Calabi-Yau with a single volume modulus (R) + dilaton (e^{ϕ}) as only moduli.

In 4d Einstein frame, energy cost of :

- $F_0 \rightarrow \quad f_0^2 \dfrac{e^{4\phi}}{R^6}$
- $F_4 \rightarrow \quad f_4^2 \dfrac{e^{4\phi}}{R^{14}}$

Differ by $\dfrac{1}{R^8}$ due to $g^{\cdot\cdot}$ factors in 10d

- $H_3 \rightarrow \quad h_3^2 \dfrac{e^{2\phi}}{R^{12}}$
- $O6 \rightarrow \quad -N_{O6} \dfrac{e^{3\phi}}{R^9}$

<u>Tadpoles</u> $\rightarrow$ in many models

$$f_0, h_3, N_{O6} \sim \mathcal{O}(1)$$

⑦

So schematically :

$$V = \frac{e^{4\phi}}{R^6} - \frac{e^{3\phi}}{R^9} + \frac{e^{2\phi}}{R^{12}} + f_4^2 \frac{e^{4\phi}}{R^{14}}$$

Important point : $\exists$ a scaling

$$f_4 \sim N$$

$$g_s = e^{\phi} \sim N^{-3/4}$$

$$R \sim N^{1/4}$$

that :

a) Makes all contributions equally important

b) Takes, at large N, R large, g_s small, and for any vacua

$$\frac{\left(\frac{1}{H}\right)^2}{R^2} \sim N$$

So vacua of this system quite plausibly exist in infinite families parametrized by $\underline{N}$, with arbitrarily large radius, weak coupling, and large hierarchy between 4d curvature & KK scale.

c.f. Saltman / Silverstein Non SUSY

Corrections? Most obvious worry is large f_4 → higher derivative terms $\propto |F_4|^2$ may be large?

Not so. Such a correction is ~ to:

- f_4^2, manifestly
- 4 powers of $g^{..} \to R^{-8}$
- 2 powers of g_s (RR vertex ops)

Overall scales like

$$N^2 \cdot \frac{1}{N^2} \cdot \frac{1}{N^{3/2}} \rightarrow$$

$\lambda = N^{-3/2}$ controls corrections

Other conceivable corrections (back-reaction from O6 background fields, corrections to 4d EQFT) can also be easily argued to be controlled by $1/N$.

While quite schematic, this toy argument captures essence (+ right N scalings) of the class of <u>real</u> Calabi-Yau models we have constructed, with <u>all</u> <u>moduli</u> <u>fixed</u>.

III. 4d $\mathcal{N}=1$ formalism

[building on Grimm & Louis]

IIA on a CY three-fold M yields a 4d, $\mathcal{N}=2$ supergravity

$$\mathcal{M} = \underbrace{\mathcal{M}_{vector}}_{\text{kähler moduli}} \times \underbrace{\mathcal{M}_{hyper}}_{\text{C.S. + dilaton}}$$

Metric on each factor can be fixed by a kähler pot. given by dim. red. (valid at large radius).

- Must orientifold to get $\mathcal{N}=1$ model.

We consider orientifolds by

$$\underbrace{\Omega_p}_{\text{W.S. parity}} \; (-1)^{F_L} \; \underbrace{\sigma}_{\text{anti-hol. involution on M}}$$

The involution should act on the kähler form J, hol. 3-form Ω, as:

$$\sigma^* J = -J \,, \quad \sigma^* \Omega = \bar{\Omega}$$

- Fixed loci are special Lagrangian 3-cycles Σ_n:

$$J|_{\Sigma_n} = 0 \,, \quad \mathrm{Im}\, \Omega|_{\Sigma_n} = 0$$

O6 planes wrap the Σ_n.

After doing projection to orientifold invariant fields:

<u>Kähler moduli space</u>

Surviving modes from $H^{1,1}_{-}$ $\Rightarrow$

$$t_a = i v_a + b_a$$

(12)

$$J_c = \sum_{a=1}^{h^{1,1}} t_a W_a$$

$\{W_a\}$ basis for $H^{1,1}$

Dim'l reduction $\Rightarrow$

$$K^K(t,\bar{t}) = -\log\left[\tfrac{4}{3} \kappa_{abc} v^a v^b v^c\right]$$

$$\kappa_{abc} = \int_M W_a \wedge W_b \wedge W_c$$

Complex structure moduli space

Quaternionic $\mathcal{M}_{hyper}$ is cut to space of complex dim $h^{2,1}+1$.

Symplectic basis α_i, β_j for $H^3(M) \Rightarrow \mathbb{Z}_2$ even/odd bases

$\{\alpha_k, \beta_\lambda\}$ $\qquad$ $\{\alpha_\lambda, \beta_k\}$

$k = 0, 1, \ldots, \tilde{h}$ $\quad$ $\lambda = \tilde{h}+1, \ldots, h^{2,1}$

($\tilde{h}$ is basis dependent)

⑬

The $\mathbb{Z}_2$ invariant moduli can be summarized in the object

$$\Omega_c \equiv C_3 + 2i\,(\mathrm{Re}\,(C\Omega)$$
$$C \equiv e^{-D + K^{c.s.}/2}$$
$$e^D \equiv \frac{e^\phi}{[\mathrm{vol}(M)]^{1/2}}$$
$$K^{c.s.} \equiv -\ln\left[i \int \Omega \wedge \bar{\Omega}\right]$$

The surviving moduli are then the expansion of Ω_c in H_3^+ :

$$\left.\begin{aligned} N_k &= \tfrac{1}{2} \int \Omega_c \wedge \beta_k \\ T_\lambda &= i \int \Omega_c \wedge d_\lambda \end{aligned}\right\} \ h^{2,1} \text{ of them}$$

Their kähler pot. is :

$$K^Q = -2 \ln\left[2 \int \mathrm{Re}\,(C\Omega) \wedge * \,\mathrm{Re}\,(C\Omega)\right]$$

Fluxes & Superpotential

Can turn on nonzero NS & RR fluxes, consistent w/ orientifold projection:

- Allowed H_3, F_2 odd
- Allowed F_4 even

$$H_3 = q_\lambda \alpha_\lambda - p_\kappa \beta_\kappa$$

$$F_2 = -m_a \omega_a$$

$$F_4 = e_a \tilde{\omega}_a$$

$H_+^{2,2}$ is Poincaré dual to $H_-^{1,1}$

$$F_0 = m_0$$

$$\left(\text{and } \int_M F_6 \to e_0\right)$$

Reducing 10d action explicitly $\Rightarrow$

(15)

$$V = e^{K} \Big(\sum_{i,j = \{t_a, N_k, T_\lambda\}} g^{i\bar{j}} D_i W \overline{D_j W}$$

$$- 3|W|^2 \Big) + \underbrace{m_0 \, e^{K^Q} \, \mathrm{Im}\, W^Q}_{\text{Huh?}}$$

where:

$$K \equiv K^K + K^Q$$

$$W(t_a, N_k, T_\lambda) = W^Q(N_k, T_\lambda) + W^K(t_a)$$

$$W^Q = \int \Omega_c \wedge H_3$$

$$W^K = e_0 + \int J_c \wedge F_4 +$$

$$\frac{1}{2} \int J_c \wedge J_c \wedge F_2 - \frac{1}{6} m_0 \int J_c \wedge J_c \wedge J_c$$

The "extra" term Huh? <u>cancels</u> when one enforces tadpoles + includes contribution of O6 tensions.

- W depends on all CY moduli + dilaton classically, in contrast to IIB + Heterotic systems.

Its now straightforward to write down the eqns for SUSY vacua

$$D_{t^a} W = D_{N_k} W = D_{T_\lambda} W = 0$$

+ search for SUSY vacua.

Upshot: In a 'generic' IIA CY orientifold, fluxes can: (classically)

- Fix all complexified kähler moduli
- Fix all cplx str modes of M
- Fix dilaton
- Fix one linear combination of axion partners of complex / dilaton moduli

17

'Generic' means

- m_0, $\geq$ one H_3 flux $\neq 0$
- One F_2 or F_4 flux per kähler mode

Comments:

- Can we explicitly get <u>all</u> moduli?

<u>YES</u>. In paper, discuss orientifold of

$$\underbrace{\left(T^6/\mathbb{Z}_3\right)}_{\text{Z-manifold}} / \mathbb{Z}_3 \leftarrow \text{Further free } \mathbb{Z}_3 \text{ c.f. Strominger '86}$$

$h^{1,1}(M) = 12 \qquad h^{2,1}(M) = 0$

→ 9 $\mathbb{P}^2$s at $\mathbb{C}^3/\mathbb{Z}_3$ sings.

→ 3 "p" moduli of T^2s $\subset T^6$

Turning on $\mathcal{O}(1)$ F_0, H_3 &

- f units of F_4 in each $\mathbb{P}^2$
- N units of F_4 in each $(T^2)^2$ $\Rightarrow$

find vacua with:

- $g_s \sim N^{-3/4}$
- R of $T^2_s \sim N^{1/4}$

Same scaling as toy model!

- radius of each $\mathbb{P}^2 \sim f^{1/4}$

Large N, f regime ($f \lesssim N$ to stay $\in$ Kähler cone) $\Rightarrow$

Infinite # of vacua, arbitrarily weak g_s + large R, and

$$\frac{\left(\frac{1}{H}\right)^2_{4d}}{R^2} \sim N$$

$\Rightarrow \exists$ 4d EQFT, <u>unlike</u> in Freund-Rubin vacua (which also come in ∞ families).

- $\exists$ a "projected in" Euclidean D2 brane $\forall$ unfixed axions in models w/ $h^{2,1} > 0$. One expects, ala BBS, that such effects may fix all axions in such cases.
- Known WS instanton corrections to $\mathcal{N}=2$ prepotential, could be incorporated into RR flux W. We will not do this here because already <u>classical</u> W $\Rightarrow$ vacua at large R, where $e^{-\text{Area}} \to 0$.

IV. Concluding Remarks

Recent extensions of these results:

Camara, Font, Ibanez

- Present models in this framework with MSSM-like low energy thy + moduli fixing

Saueressig, Theis, Vandoren

- Demonstrate that inclusion of non-perturbative corrections to "Universal hyper" (dilaton) can → de Sitter vacua in this setting

Most <u>striking</u> contrast to IIB results: infinite # of vacua, parametric control:

$$\mathcal{N}_{vacua}\,(R \leq R_*) \sim (R_*)^4$$

in our simple e.g.

Obviously, one can restore finiteness if one wishes, by well motivated physical cuts ("no KK modes with mass < $(\mathrm{mm})^{-1}$").

However, one should then worry that results of any attempted statistical prediction could be cut-off dominated.

Much work remains to be done in fleshing out the basic physics + duality relations between IIB / IIA / Heterotic / ··· ensembles of vacua.