Disorder operators in gauge theories and duality

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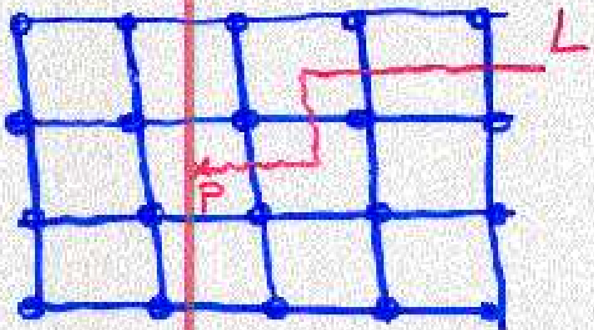
I. Disorder operators.

A. 2d Ising model



Flip the sign of β for all links intersected by L.

$\Downarrow$

Local operator μ at P

$$E = -\beta \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

$\mu \equiv$ disorder operator

σ = order operator

Low-temp phase: $\langle \sigma \rangle \neq 0$, $\langle \mu \rangle = 0$

High-temp. phase: $\langle \sigma \rangle = 0$, $\langle \mu \rangle \neq 0$

Continuum viewpoint:

$$S = \int d^2x \; \psi \not{\partial} \psi \qquad \text{(Majorana fermion)}$$

Twist operators:

$$\sigma(0)\, \psi(z) \sim \frac{1}{z^{1/2}}\, \mu(0)$$

σ and μ cannot be expressed as local functions of ψ.

B. Compact $U(1)$ gauge theory in 3d (Polyakov, 1976)

Monopole operators:

$$\int F = 2\pi n$$

$$\Downarrow$$

gauge field is singular at p.

n = "magnetic charge"

n does not determine the operator uniquely.

E.g., if $\exists$ a scalar field in the theory, it may also be singular at P.

Add SUSY ($N=2$ or $N=4$)

BPS monopole $\Rightarrow$ $\phi \sim \frac{1}{r}$ at P.

If there are matter fields, they also may be singular.

$\Rightarrow$ monopole operators may carry "non-obvious" flavor quantum numbers.

C. Topological vs. non-topological disorder operators.

Take $SU(2)$ gauge theory with matter in the fundamental rep.

Any $SU(2)$ bundle on S^2 is trivial

$\Rightarrow$ no top. disorder ops.

(If matter is in the adjoint, we may consider $G = SO(3)$ bundles which do not lift to an $SU(2)$ bundle, i.e. $w_2(S^2) \neq 0$

$\Rightarrow$ get $\mathbb{Z}_2$ monopoles carrying 't Hooft magnetic flux).

Are there non-topological disorder operators?

Such operators were argued to be dual to "normal" operators under 3d Mirror Symmetry (Borokhov, 2003)

I will discuss the analogue of this in 4d gauge theories.

4d case is simpler, in a sense, because $\exists$ gauge theories which are weakly coupled at all scales.

E.g. $N=4$ SYM at weak gauge coupling.

II. Disorder operators in 4d gauge theories.

Motivating question:

What is the "magnetic" analogue of the Wilson loop operator

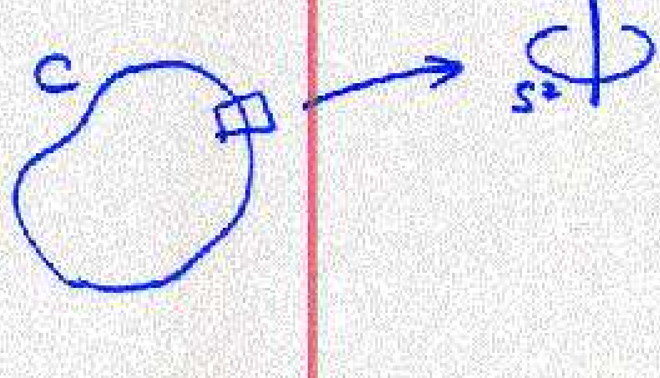
$$W_R(C) = \mathrm{tr}_R\, P\exp\left(i\int_C A\right)$$

? ? ?

't Hooft (1978):

Let us consider a gauge theory with fields in the adjoint only.

$G = SU(N)/\mathbb{Z}_N$.



$\sim \quad \mathbb{R}^2 \times S^2$

Require the $SU(N)/\mathbb{Z}_N$ bundle to be nontrivial on S^2.

Gluing on the equator

$\Downarrow$

$\pi_1(SU(N)/\mathbb{Z}_N) = \mathbb{Z}_N$ classifies $SU(N)/\mathbb{Z}_N$ bundles on S^2.

$\Downarrow$

't Hooft operators are labelled by $m \in \mathbb{Z}_N$ ('t Hooft magnetic flux).

Not satisfactory!

1). $N=4$ SYM with $G = SU(N)$ is believed to have S-duality. Since Wilson loops are labelled by R (an irrep. of $SU(N)$), so should the "magnetic" Wilson loops.

2). Consider $N=2$ SYM with $G = SU(2)$ and $N_f = 4$ hypermultiplets in the fundamental rep.

There is a lot of evidence that this theory has S-duality $\tau \to -\frac{1}{4\tau}$.

But since $\pi_1(SU(2)) = 0$, this theory, according to 't Hooft, does not have any disorder operators.

- Way out: consider non-topological disorder operators.

Consider a 4d CFT, for simplicity.

C

Wilson loop depends on C & R and breaks exactly the same symmetries as a geometric line C.

$\Rightarrow$ require "magnetic" Wilson loops preserve all symmetries which leave C invariant.

$$ds^2 = dt^2 + dr^2 + r^2 d\Omega_2^2$$

Weyl rescaling by $\frac{1}{r^2}$

$\Downarrow$

$$ds^2 = \underbrace{\frac{dt^2 + dr^2}{r^2}}_{AdS_E^2} + \underbrace{d\Omega_2^2}_{S^2}$$

$\Rightarrow$ a straight line leaves $SL(2,\mathbb{R}) \times SO(3)$ unbroken.

Problem: classify $SL(2,\mathbb{R}) \times SO(3)$-inv. bdry conditions for gauge fields on $AdS^2_E \times S^2$.

1. "Free" bdry conditions

$$A \simeq a(t) \frac{dt}{r} + \dots$$

+ attach Wilson loop at $r = 0$.

2. "Fixed" bdry conditions.

$$F \sim \frac{B}{2} \, vol(S^2) + \dots$$

where B is a covariantly-constant section of the adjoint bundle.

In other words:

$$F_{ij} = \frac{B}{2} \epsilon_{ijk} \frac{x^k}{r^3} + \dots$$

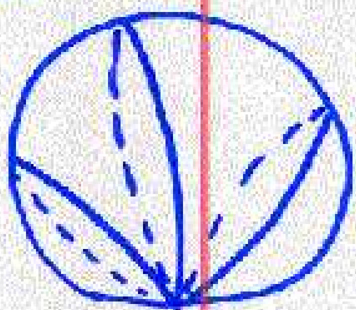
where B is covariantly constant.

Goddard, Nuyts, Olive (1977)
studied the same ansatz, but for $r \to \infty$. Can use their results:

1). B must satisfy

$$e^{2\pi i B} = id_G$$

Reason:



For each loop compute holonomy of the connection $\Rightarrow$ get a path γ in G

γ must be a loop $\Rightarrow$ B is "quantized".

(If the adjoint bundle is nontrivial, B is a section of this bundle.
In that case id_G should be understood as identity in $G/Z(G)$.
The class of γ in $\pi_1(G/Z(G))$ is the 't Hooft magnetic flux.

2). B is defined modulo constant gauge transformations.

Can always rotate B into a fixed Cartan subalgebra of $\mathfrak{g}$ (the Lie algebra).

Then $e^{2\pi i B} = 1$

$\Updownarrow$

$\alpha(B) \in \mathbb{Z}$ for any root of $\mathfrak{g}$

$\Downarrow$

B belongs to $\Lambda_r^* = \Lambda_{mw}$ (the lattice of magnetic weights).

Residual gauge transformations = Weyl reflections.

Thus "magnetic charge" takes values in $\Lambda_{mw} / \mathcal{W}$ (Weyl group). (GNO charge)

Remarks

1). Let $H_\alpha = \frac{2\alpha}{\langle \alpha, \alpha \rangle}$ (coroot)

H_α span a lattice $\Lambda_{cr} \subset \Lambda_{mw}$.

$\Lambda_{mw} / \Lambda_{cr} = Z(G)$

$\pi : \Lambda_{mw} \to \Lambda_{mw} / \Lambda_{cr}$ sends the GNO charge to the 't Hooft flux.

2). The lattice Λ_{mw} is the weight lattice of $\hat{\mathfrak{g}}$ (Langlands-dual Lie algebra)

$\Rightarrow$ GNO charges are in 1-1 correspondence with $\hat{\Lambda}_w / \mathcal{W}$, i.e. with Irreducible Reps. of $\hat{\mathfrak{g}}$.

GNO interpreted this as evidence of <u>electric-magnetic duality</u>.

Generalization: dyonic operators.

Let $B \neq 0$. B breaks the gauge group G $\Rightarrow$ "electric" charge is an irreducible rep. of the residual gauge group G_B.

Or: the Weyl group $\mathcal{W}$ is broken down to $\mathcal{W}_B \Rightarrow$ instead of specifying a weight modulo $\mathcal{W}$, have to specify a weight of $\mathfrak{g}$ modulo $\mathcal{W}_B$.

Or: dyonic operators are labelled by a pair

$$(\nu, \mu = B), \; \nu \in \Lambda_w, \; \mu \in \Lambda_{mw}$$

modulo the action of $\mathcal{W}$.

III. S-duality.

1. T-transformation.

$$\theta \to \theta + 2\pi$$

$$\Delta S_\theta = -i \int_C B \cdot A_0 \, dt$$

$$\Downarrow$$

$$(\nu, \mu) \mapsto (\nu + \mu, \mu)$$

(a form of Witten effect)

2. S-transformation?

$$(\nu, \mu) \mapsto (-\mu, \nu) \quad ?$$

S & T generate $SL(2, \mathbb{Z})$.

This works for simply-laced groups.

For non-simply-laced exceptional groups S is not well-defined.

However, ST^2S is well-defined for $G = F_4$; ST^3S is well-defined for $G = G_2$.

The group generated by T and ST^nS is called $\Gamma_0(n)$.

Problems with S arise because the S-duality group for F_4 and G_2 is not $SL(2,\mathbb{Z})$, but a subgroup of $SL(2,\mathbb{R})$ whose intersection with $SL(2,\mathbb{Z})$ is $\Gamma_0(2)$ or $\Gamma_0(3)$.

First noted by Dorey, Fraser, Hollowood, Kneipp (1995)

Also work in progress ...

IV. Further remarks.

We have argued that $\exists$ nontop. disorder ops. in 4d gauge theories.

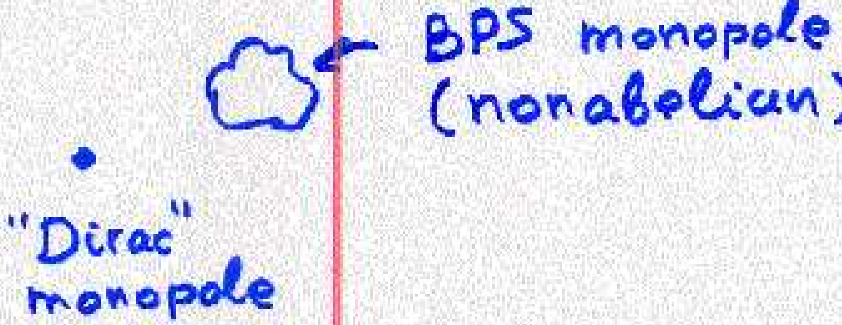
1) Do we get more order parameters?

Presumably, not: only topological classification should affect if a loop has an area or perimeter law.

E.g. a Wilson loop in the adjoint can be completely screened by a gluon.

We expect that if μ is the highest weight of the adjoint of $\hat{\mathfrak{g}}$, it can also be screened, ...

by what?



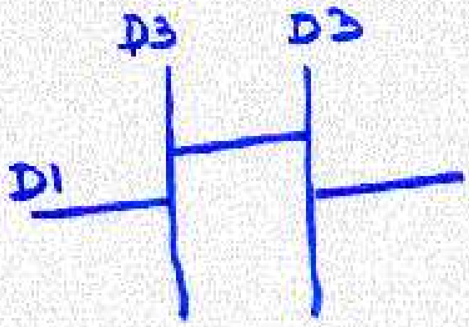
How can a fat nonabelian monopole screen a point-like singularity?

Answer: as it appoaches the Dirac singularity, it shrinks to zero size.

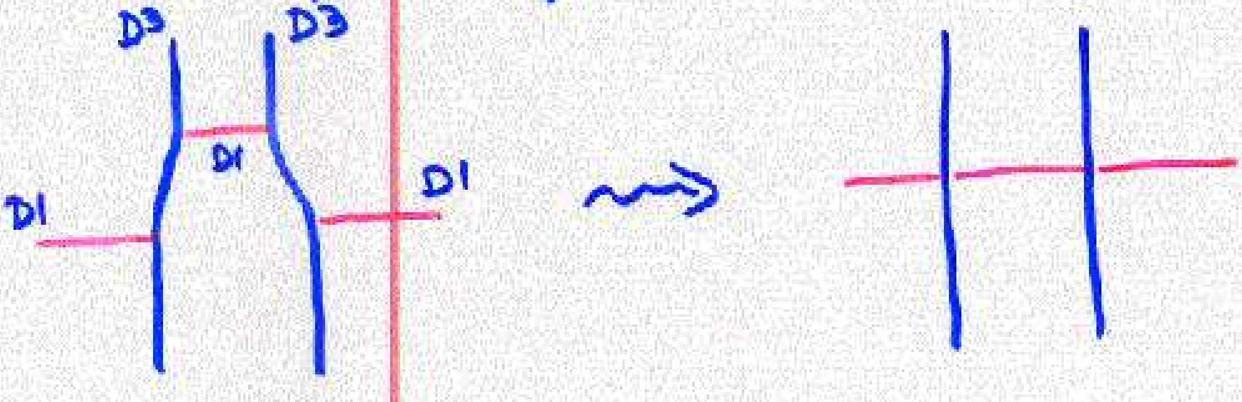
"Stringy" argument:

D3 1 2 3

D1 4



more precisely:



2) Disorder operators are new "probes" of duality.

E.g., what is the dual of the Wilson loop in the fundamental rep. of $SU(N_c)$ under Seiberg duality of $\mathcal{N}=1$ SUSY QCD?

Is it a "magnetic" Wilson loop?

(Note: no nontrivial 't Hooft flux is allowed in this theory).

?