Strings from $\mathcal{N}$=1

Superconformal field theories

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Summary

Introduction:

$\mathcal{N}$=4 AdS/CFT: Strings $\leftrightarrow$ Long ops.

- Point-like strings $\leftrightarrow$ BPS ops.
- String excitations $\leftrightarrow$ insertions
- Long strings $\leftrightarrow$ spin chains

$\mathcal{N}$=1 AdS/CFT

- $AdS_5 \times X_5$; X_5 is Sasaki-Einstein
- X_5 : $T^{1,1}$ SU(2) x SU(2) x U(1)
  $Y^{p,q}$ SU(2) x U(1) x U(1)
  $L^{p,q|r}$ U(1) x U(1) x U(1)

$\underline{Y^{p,q}}$

- Massless geodesics and long operators

  Matching of R and flavor charges.

- Extended strings (Qualitative)

  Eff. action appears in f.t. and is similar but not equal to string side.

$\underline{L^{p,q|r}}$

- Massless geodesics and long operators

  Matching of R and flavor charges.

$\mathcal{N}$=4 SYM $\Leftrightarrow$ II B on $AdS_5 \times S^5$

Strings? Long operators $\leftrightarrow$ Strings

(BMN, GKP) an. dimension $\leftrightarrow$ Energy

Frolov-Tseytlin: Long strings

Minahan-Zarembo: Spin chains

Bethe Ansatz (BFST)

$X=\phi_1+i\,\phi_2\,;Y=\phi_3+i\,\phi_4\,;\ \mathcal{O}=\mathrm{Tr}(XY\ldots Y)$

Long chains ($J\to\infty$) are classical

$$S_{\mathrm{eff.}} = J\left[\int \cos\theta\,\dot{\phi} - \frac{\lambda}{J^2}\int\left(\partial_\sigma \bar{n}\right)^2\right]$$

$$\bar{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

S_{eff}: Action of spin chain and fast strings.

(M.K.; Ryzhov, Tseytlin, M.K.; …)

Strings as states of $\mathcal{N}$=4 SYM on $R \times S^3$

The operator $\mathcal{O} = \sum c_i$ [Tr(XY…Y) $]_i$ maps to a state of the field th. on S^3

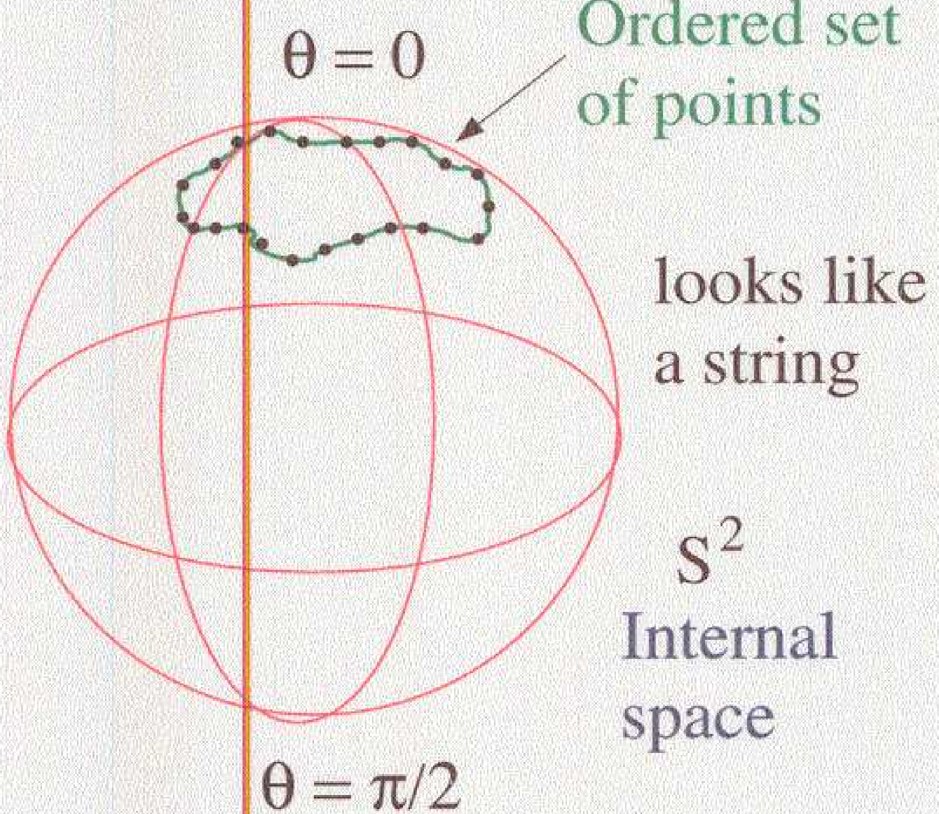
State: delocalized and has a large number of particles (X and Y).

Q.M. : $|\psi\rangle = \cos(\theta/2) \exp(i\,\phi/2)\ |X\rangle$

$$+ \sin(\theta/2) \exp(-i\,\phi/2)\, |Y\rangle$$

We can use $v_i = |\psi(\theta_i, \phi_i)\rangle$ to

construct: $\mathcal{O} = \mathrm{Tr}\,(v_1 v_2 v_3 \dots v_n)$
(Coherent state)



Strings are useful to describe states of a large number of particles (in the large N limit)

$\mathcal{N}=1$ AdS_5 x X_5 ; X_5 : Sasaki-Einstein

$$ds^2 = dx^2_{[4]} + \underbrace{dr^2 + r^2 \; dX_5^2}_{\Downarrow}$$

CY cone

Put D3 branes at $r = 0$ and take near horizon limit:

$$ds^2 = \boxed{r^2 dx^2_{[4]} + \frac{dr^2}{r^2}} + dX_5^2 \quad , \quad AdS_5 \text{ x } X_5$$

$T^{1,1}$ (conifold) Klebanov-Witten

$Y^{p,q}$ **Gauntlett, Martelli, Sparks, Waldram**
Benvenuti, Franco, Hanany, Martelli, Sparks

$L^{p,q\,|\,r}$ **Cvetic, Lu, Page, Pope; Benvenuti, M.K.; Franco, Hanany, Martelli, Sparks, Vegh, Wecht; Butti, Forcella, Zaffaroni**

$\underline{\mathrm{T}^{1,1}}$

$$ds^2 = \frac{1}{9}[d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2]^2$$

$$+\frac{1}{6}\sum_{i=1}^{2}\left[d\theta_i^2 + \sin\theta_i d\phi_i^2\right]$$

$\underline{\mathrm{Y}^{p,q}}$ **(Gauntlett, Martelli, Sparks, Waldram)**

$$ds^2 = w(y)[d\alpha + f(y)(d\psi - \cos\theta)]^2$$

$$+\frac{1-y}{6}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$+\frac{dy^2}{6p(y)} + \frac{q(y)}{9}(d\psi - \cos\theta\, d\phi)^2$$

$p(y) = \dfrac{a - 3y^2 + 2y^3}{3(1-y)}$; $y_1 < y < y_2$

$$y_{1,2} = \left(2p \mp 3q - \sqrt{4p^2 - 3q^2}\right)/(4p)$$

If $\alpha \to \beta = 6\alpha + \psi$; only p(y) appears.

Point-like strings (massless geodesics)

$$S=\frac{\sqrt{\lambda}}{2}\int d\tau\left(-\partial_\tau t\,\partial_\tau t+g_{ab}\partial_\tau x^a\partial_\tau x^b\right)$$

Const.: $-\partial_\tau t\,\partial_\tau t+g_{ab}\partial_\tau x^a\partial_\tau x^b=0$

$t=\kappa\tau$; $P_t=\Delta=\sqrt{\lambda}\;\kappa$

$$p_a=\frac{\delta\mathcal{L}}{\delta(\partial_\tau x^a)};\quad H=g^{ab}p_a p_b$$

$$\Delta^2=\left(\frac{3}{2}Q_R\right)^2+\frac{1}{6p(y)}\left(P_\alpha+3yQ_R\right)^2$$
$$+6p(y)P_y^2+\frac{6}{1-y}\left(J^2-P_\psi^2\right)$$

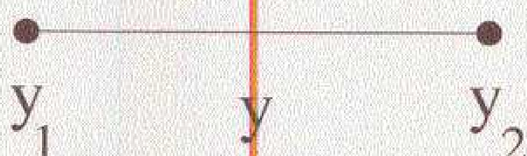
$\Delta\geq(3/2)\,Q_R$; $Q_R=2P_\psi-(1/3)\,P_\alpha$

BPS geodesics

$\Delta = (3/2)\, Q_R \quad \Rightarrow \; P_y = 0\, ,\; J = P_\psi\, ,$

$$y_0 = -\frac{P_\alpha}{3 Q_R}$$

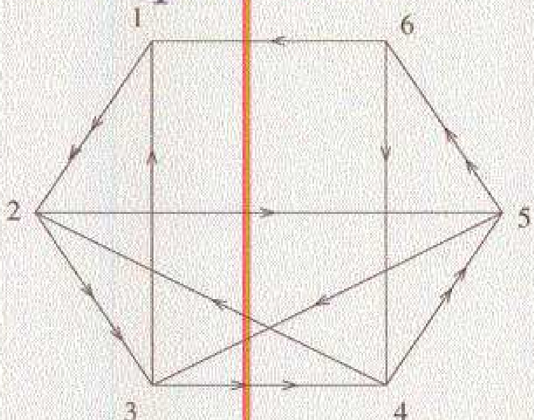
y: Ratio between U(1) and R-charges.



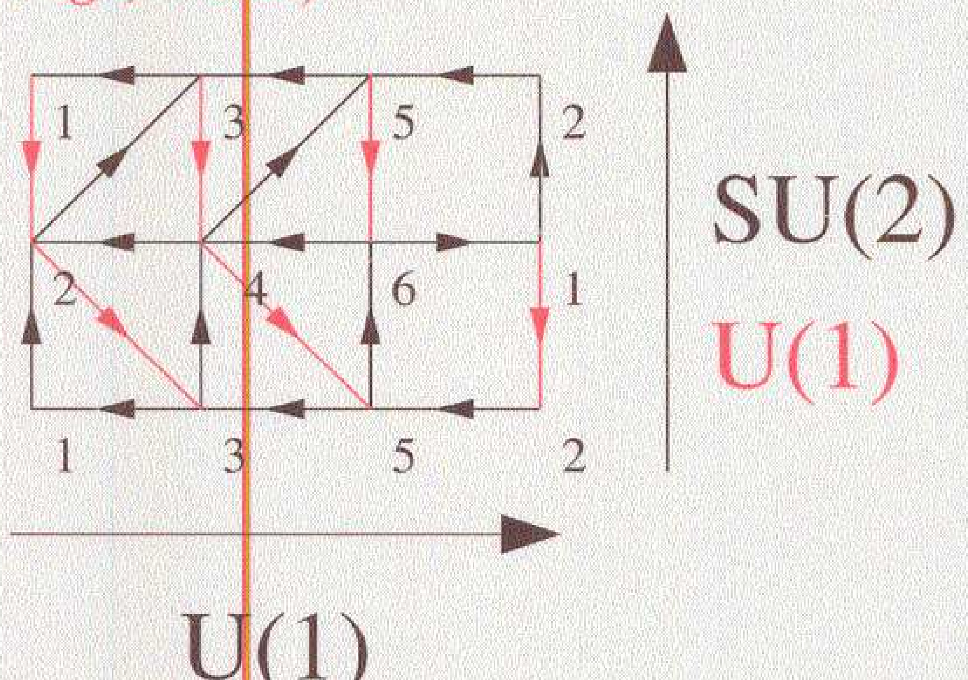
Also: $J = \frac{1}{2}(1 - y_0)\, Q_R$ (from $J = P_\psi$)

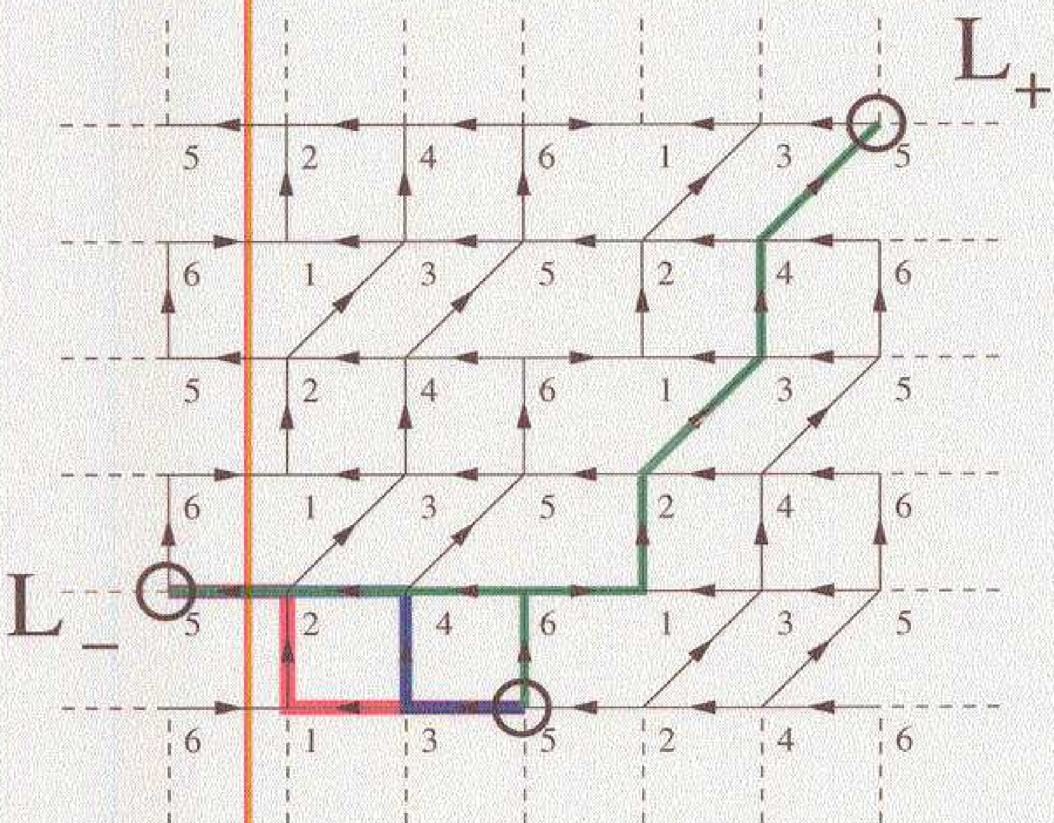
Dual gauge theories
(Benvenuti, Franco, Hanany, Martelli, Sparks)

Example: $Y^{3,2}$



Periodic rep.: **(Hanany,Kennaway, Franco, Vegh, Wecht)**



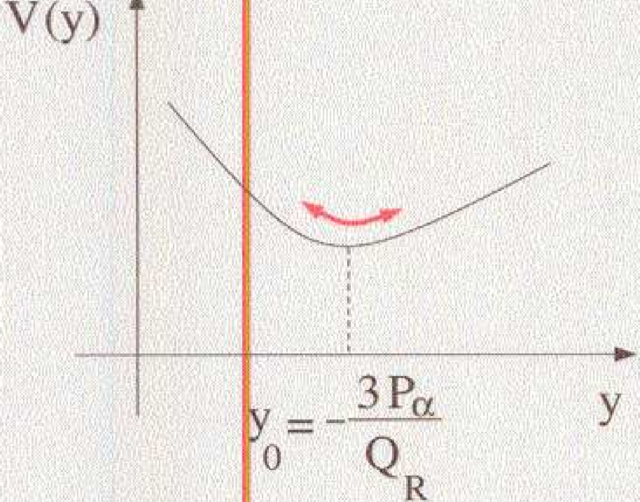


Operators of maximum and minimum slopes can be identified with the geodesics at $y=y_1$ and $y=y_2$

R-charges and flavor charges match.

(Also: Berenstein, Herzog, Ouyang, Pinansky)

Small Fluctuations (BMN, GKP, Gomis-Ooguri)



Quantize: $\delta\Delta=2n$, $\delta Q_R=0$, $\delta P_\alpha=0$, $\delta J=0$

Also ($J>P_\psi$): $\delta\Delta=2n$, $\delta Q_R=0$, $\delta P_\alpha=0$, $\delta J=n$

Agree with quantum numbers of U(1) and SU(2) currents $\Rightarrow$ we identify these non-BPS geodesics with insertions of the currents.

Extended Strings

We consider closed extended strings

such that each point moves

approximately along a BPS geodesic.

Effective action for such strings:

$$ds^2 = -dt^2 + \frac{1}{6} g_{ij} dx^i dx^j + \frac{1}{9}(d\psi + A_j dx^j)^2$$

We introduce a coordinate $\psi_1 = \psi - 3t$

$$ds^2 = \frac{2}{3} dt (d\psi_1 + A_j dx^j) + \frac{1}{6} g_{ij} dx^i dx^j$$

$$+ \frac{1}{9}(d\psi_1 + A_j dx^j)^2$$

Now we use ansatz: $t = \kappa \tau$

and take the limit:

$\partial_\tau X \to 0$, $\kappa \to \infty$, $\kappa \partial_\tau X$ fixed.

We get a reduced action:

$$S = \frac{1}{3}\int \kappa\left(\partial_\tau \psi_1 + A_i\, \partial_\tau x^i\right) - \frac{1}{12} g_{ij} \partial_\sigma x^i \partial_\sigma x^j$$

In our case:

$$S = \frac{Q_R}{4\pi}\left\{\int\left(\partial_t \psi_1 - y\,\partial_t \beta - (1-y)\cos\theta\,\partial_t \phi\right) - \frac{4\pi^2}{9}\frac{\lambda}{Q_R^2}\left[(1-y)(\partial_\sigma n)^2 + \frac{(\partial_\sigma y)^2}{p(y)} + p(y)\left(\partial_\sigma \beta - \cos\theta\,\partial_\sigma \phi\right)^2\right]\right\}$$

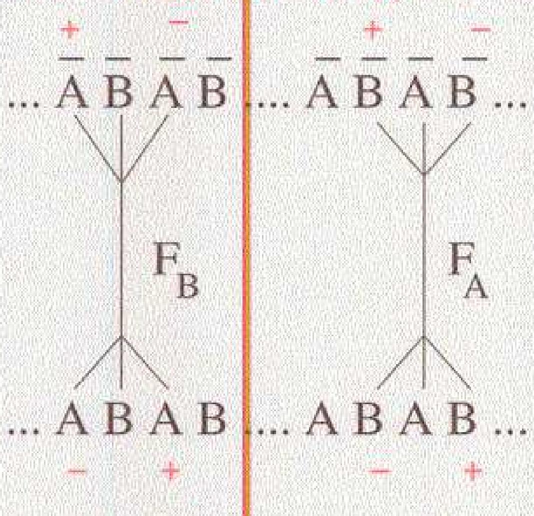
Example of $T^{1,1}$ (Angelova, Pando-Zayas, M.K. ; Strassler et al.)

$$S_{\text{eff.}} = J\left\{ \int \cos\theta_1 \, \dot{\phi}_1 + \cos\theta_2 \, \dot{\phi}_2 - \frac{\lambda}{J^2} \int \left[\left(\partial_\sigma \bar{n}_1 \right) + \left(\partial_\sigma \bar{n}_2 \right) \right] \right\}$$

Field theory:

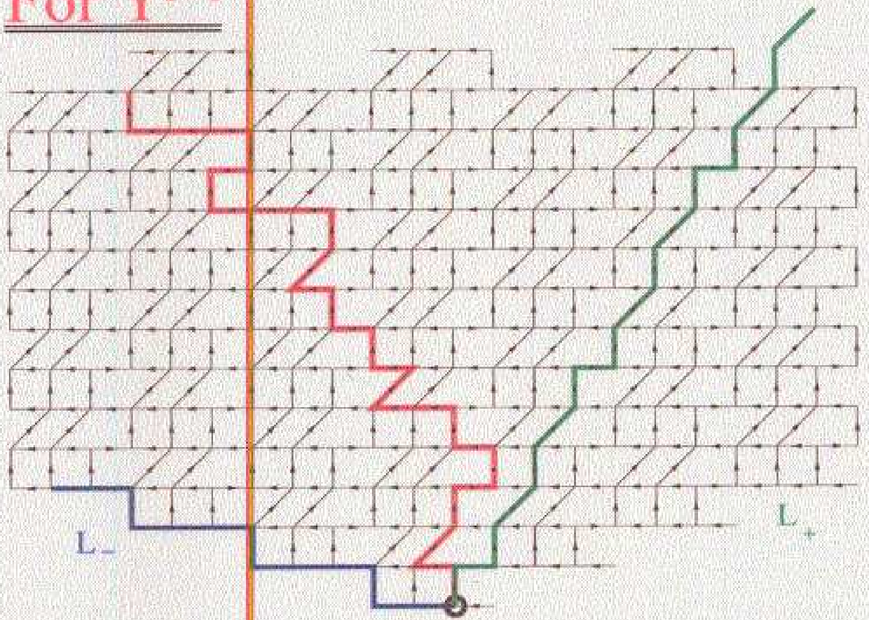
Tr(ABABAB…ABAB) ; SU(2)xSU(2)

$\mathcal{W} = \varepsilon_{ab}\, \varepsilon_{cd}\; \mathrm{Tr}\,(A^a B^c A^b B^d)$



+ , − : doublet of SU(2)

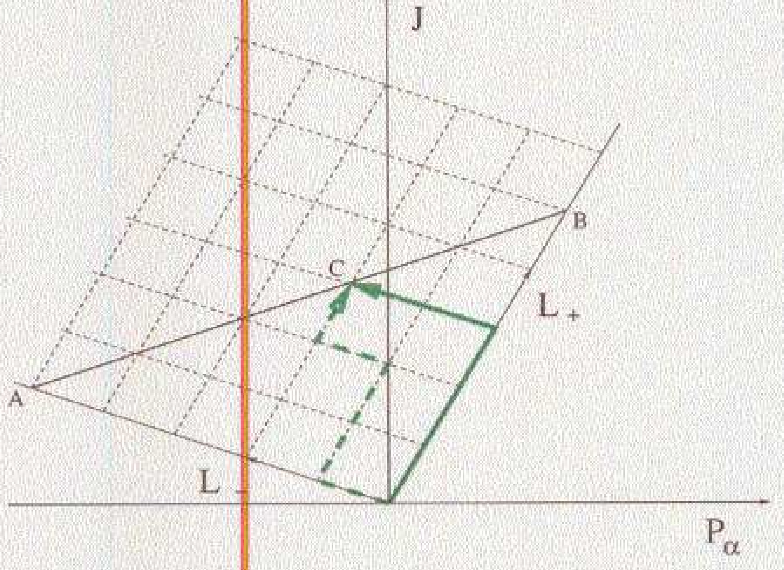
For $Y^{p,q}$



$y(\sigma) \to \text{slope}(\sigma)$; β: conjugate of y

$+$ spin $\to (\theta, \phi)$

Effective model: (max and min slopes)



$H = h_{eff.} \sum (1 - P_{i\,i+1})$

Coherent States:

$$O = \rho_{1i} \exp(iP_{\alpha}^{(1)}\alpha_i)\, U(\theta_i, \phi_i, \psi_i)\, L_1 + \rho_{2i} \exp(iP_{\alpha}^{(2)}\alpha_i)\, U(\theta_i, \phi_i, \psi_i)\, L_2$$

$$\rho_1^2 + \rho_2^2 = 1 \; ; \quad \rho_1^2 y_1 + \rho_2^2 y_2 = y$$

The coherent state action is

$$S = \int d\tau \, \mathrm{Im} \langle O | \partial_\tau | O \rangle - \int d\tau \, \langle O | H | O \rangle$$

which gives:

$$S=\frac{Q_R}{4\pi}\left\{\int\left(\partial_t\psi_1-y\partial_t\beta-(1-y)\cos\theta\,\partial_t\phi\right)\right.$$

$$-2\pi^2\frac{h_{eff}}{Q_R^2}\left[(1-y)(\partial_\sigma n)^2+\frac{(\partial_\sigma y)^2}{p(y)}\right.$$

$$\left.\left.+p(y)\left(\partial_\sigma\beta-\cos\theta\,\partial_\sigma\phi\right)^2\right]\right\}$$

where $\quad p(y) = (y_2 - y)(y - y_1)$

instead of $\quad p(y)=\dfrac{a-3y^2+2y^3}{3(1-y)}$

It is interesting that a string picture (and action) for the operators emerges from the analysis.

$L^{p,q|r}$

Metric: **(Cvetic, Lu, Page, Pope)**

$$ds^2 = \left(\frac{1}{3}\mathrm{d}\psi_R + \mathrm{A}\right)^2 + \frac{1}{4}\mathrm{d}s^2_{[4]}$$

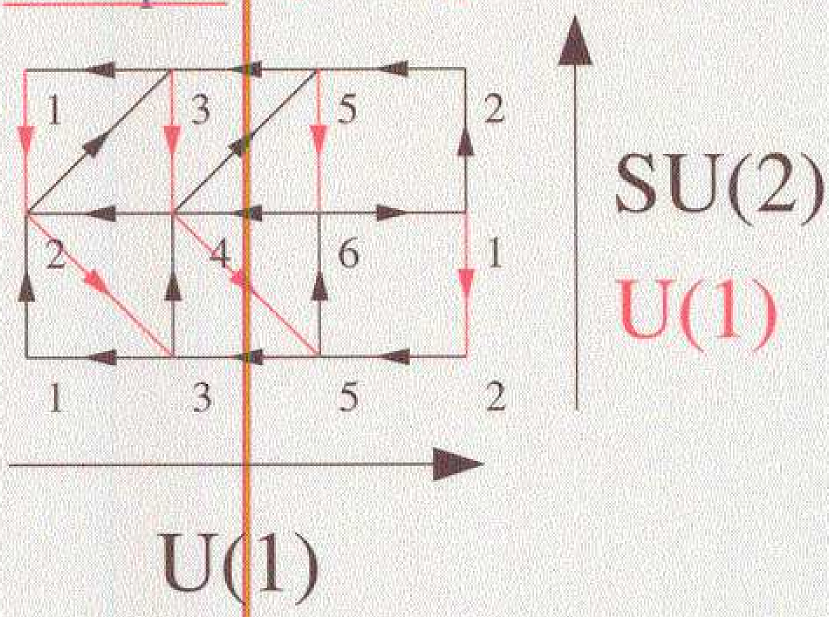
$$A = \frac{\alpha - 3\alpha y - 3x + 3xy}{6\alpha}\mathrm{d}\phi + \frac{\beta + 3\beta y - 3x - 3xy}{6\alpha}\mathrm{d}\psi$$

$$\mathrm{d}s^2_{[4]} = \frac{\rho^2}{f(x)}\mathrm{d}x^2 + \frac{f(x)}{\rho^2}\left(\frac{1-y}{\alpha}\mathrm{d}\phi + \frac{1+y}{\beta}\mathrm{d}\psi\right)^2 + \frac{\rho^2}{g(y)}\mathrm{d}y^2 + \frac{g(y)}{\rho^2}\left(\frac{\alpha - x}{\alpha}\mathrm{d}\phi - \frac{\beta - x}{\beta}\mathrm{d}\psi\right)^2$$

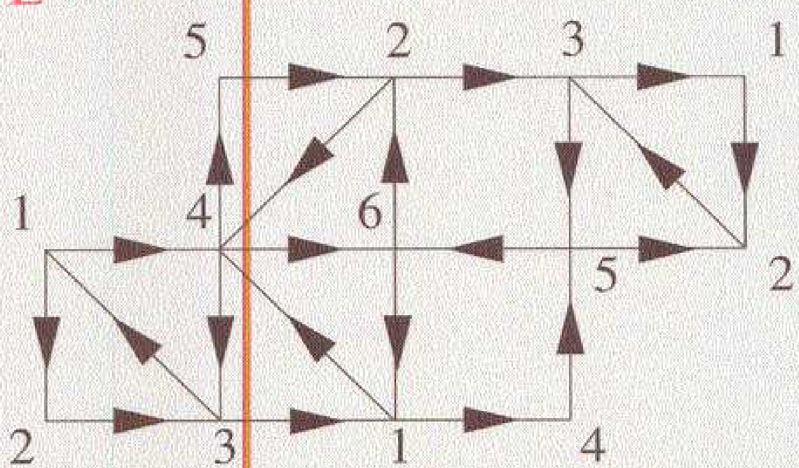
$$\rho^2 = \alpha + \beta - y(\alpha - \beta) - 2x$$

Field Theory: **Benvenuti, M.K.; Franco, Hanany, Martelli, Sparks, Vegh, Wecht; Butti, Forcella, Zaffaroni**

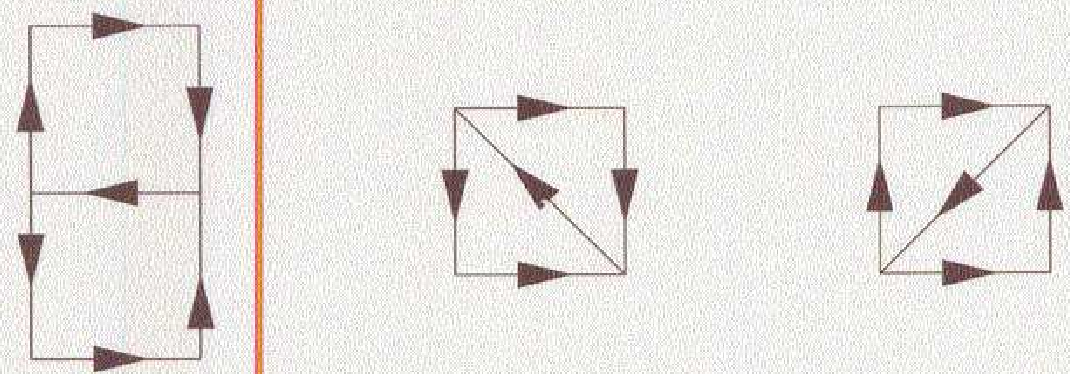
Example: $Y^{3,2} = L^{1,5\,|\,3}$



$L^{1,5\,|\,4}$



In general

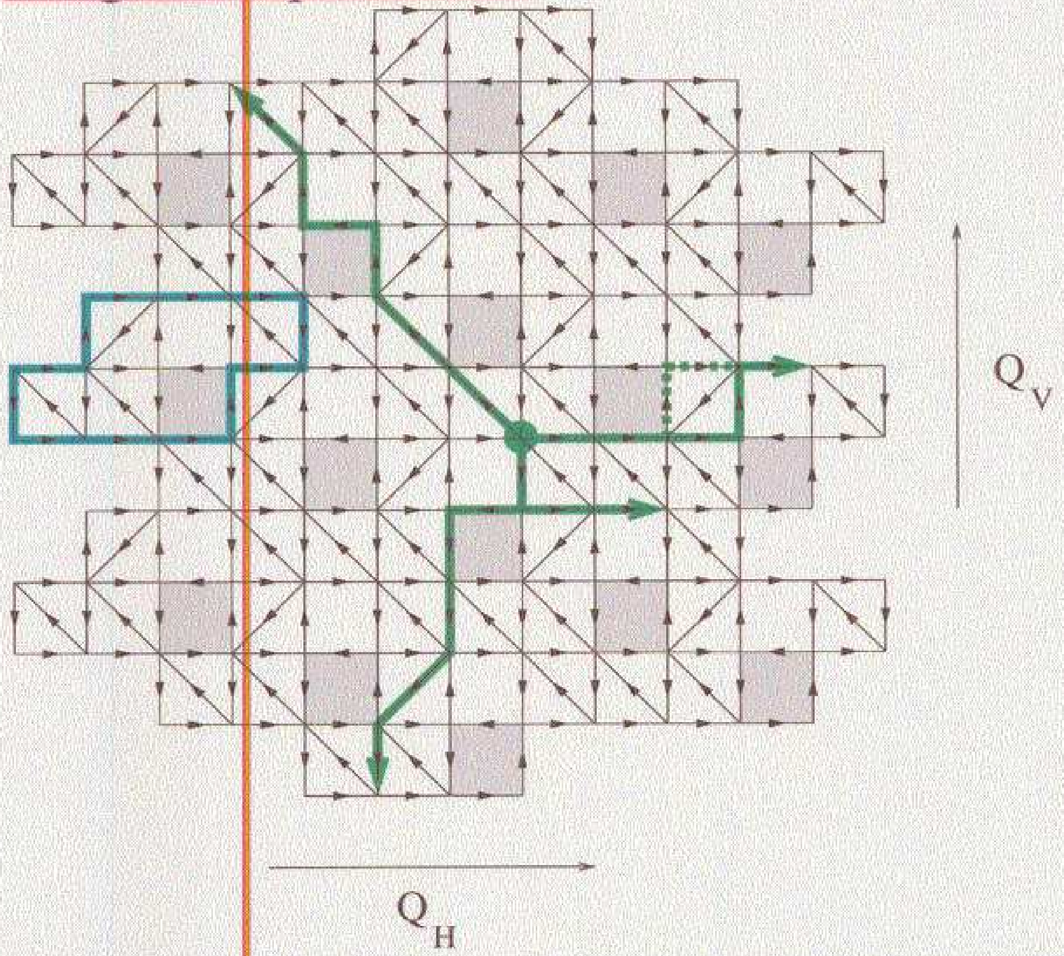


p r−p q−r

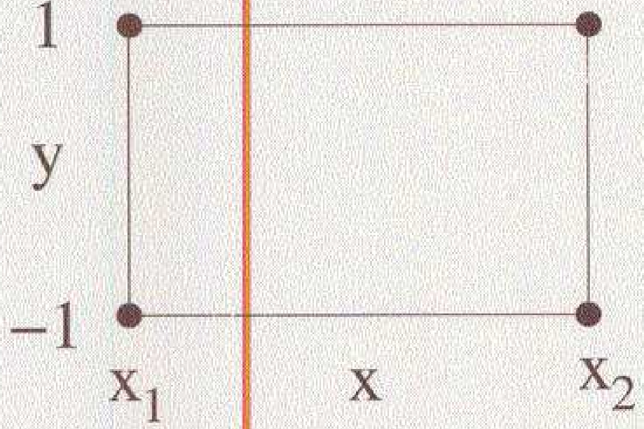
gives $L^{p,q|r}$.

Using a-max one can compute the R-charges and the central charge. Everything matches. The parameters can be mapped from one description to the other.

Long chiral primaries:



can be matched to geodesics at the "corners":



Conclusions

$Y^{p,q}$

- Massless BPS geod. $\leftrightarrow$ Chiral primaries.
- Non-BPS geodesics $\leftrightarrow$ Current insertions.
- Extended strings $\leftrightarrow$ Long operators.
- Effective action emerges in f.t. Similar but not equal to bulk eff. action.

$L^{p,q|r}$

- Found dual gauge theories.
- Matched BPS geodesics with long chiral primaries.