GEOMETRY QUANTIZATION from SUPERGRAVITY

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STRINGS 05 -TORONTO-

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GOAL

MODULI SPACE QUANTIZATION OF SUGRA SOLUTIONS

↳ "BUBBLING ADS"

PLAN

1. MINISUPERSPACE QUANTIZATION. CRNKOVIĆ – WITTEN – ZUCKERMAN METHOD

   ↳ SYMPLECTIC CURRENT

2. "BUBBLING ADS" EXAMPLE

   - WAVY LINE APPROXIMATION (AROUND PLANE WAVE)

   - GENERAL DROPLET CASE

QUESTION

- HOW TO QUANTIZE FAMILIES OF GRAVITY SOLUTIONS ?

1. 

FLUXES ARE QUANTIZED

$\mathcal{F} = \mathcal{F}_0 n \,, \quad n \in \mathbb{Z}$

(DIRAC QUANTIZATION)

2. 

MODULI SPACE, ALL FLUXES CONSTANT

?

RECENT EXAMPLES

MICROSCOPIC

BUBBLING ADS GEOMETRIES $\Longleftrightarrow$ FREE FERMION DROPLETS

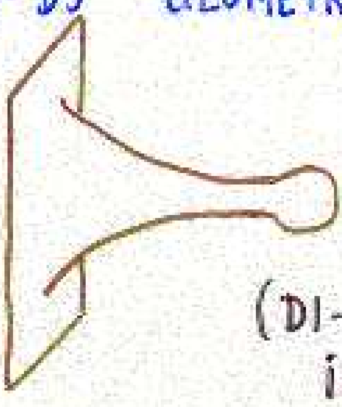
LIN, LUNIN, MALDACENA

CORLEY, JEVICKI, RAMGOOLAM
BERENSTEIN

D1-D5 GEOMETRIES WITH ANGULAR MOMENTUM

LUNIN, MALDACENA, MAOZ

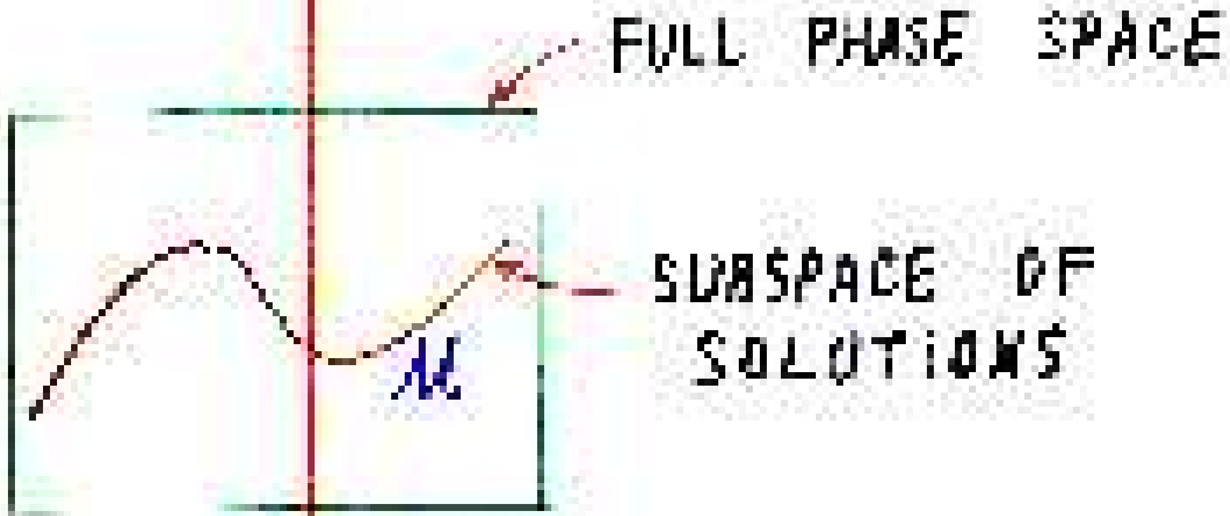
$\Longleftrightarrow$ CHIRAL FUND. STRING



$F_i(v)$
$i = 1, \ldots, 4$

(D1-D5 BH MICROSTATES IN MATHUR'S PROGRAM)

GENERAL IDEA



RESTRICT :

$$\omega = \Omega|_{\mathcal{M}}$$

Ω - SYMPLECTIC FORM OF GIVEN THEORY

EXAMPLE

CHIRAL SECTOR OF A BOSON

$$S = \tfrac{1}{2} \int dt\, dx\, (\partial\varphi)^2$$

- SYMPLECTIC FORM:

$$\Omega = \int_{t=\mathrm{const}} dx \quad \delta\dot{\varphi} \wedge \delta\varphi$$

δ - EXTERIOR DIFFERENTIAL

- MODULI SPACE OF SOLUTIONS:

$$\mathcal{M} = \{ \varphi = f(t+x) \ , \ f \text{ arbitrary} \}$$

- RESTRICTION:

$$\omega = \Omega|_{\mathcal{M}} = \int du \ \delta f'(u) \wedge \delta f(u)$$

$\Rightarrow$ POISSON BRACKETS

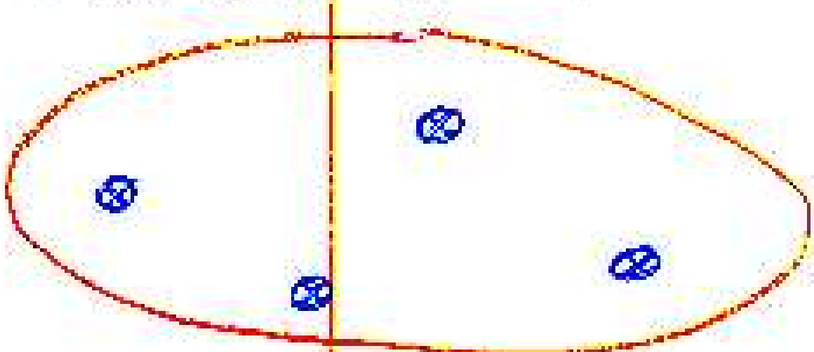
$$\{ f(u_1), f'(u_2) \} = \delta(u_1 - u_2)$$

$\Rightarrow$ QUANTIZATION

THIS IS NOT MODULI SPACE QUANTIZATION à la MANTON

MANTON STORY:

MANTON
ATIYAH, HITCHIN
GIBBONS, RUBACK
FERRELL, EARDLEY
...



$3n$-DIM MODULI SPACE

- MULTI-CENTERED STATIC EXTREMAL BH SOLUTIONS
- SLOW SCATTERING DESCRIBED BY GEODESICS IN (CALCULABLE) METRIC $ds^2 = g_{ij}(q)\, dq^i dq^j$

- PHASE SPACE IS $6n$-DIM
- SYMPLECTIC FORM DEGENERATE ON THE ORIGINAL $3n$-DIM MODULI SPACE

→ NO BH CENTER QUANTIZATION

SYMPLECTIC FORM OF GRAVITY

- REGULAR SOLUTIONS (NO HORIZON)
- CAUCHY SURFACE Σ

CANONICAL FORMALISM

- 3+1 SPLITTING

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

ARNOWITT, DESER, MISNER

- PHASE SPACE VARIABLES (h_{ij}, Π^{ij})

$$\Pi^{ij} = \sqrt{h}\left(K \cdot h^{ij} - K^{ij}\right)$$

- CONSTRAINTS
- SYMPLECTIC FORM

$$\boxed{\Omega = \int_\Sigma d^3x \; \delta \Pi^{ij}(x) \wedge \delta h_{ij}(x)}$$

COVARIANT APPROACH
CRNKOVIĆ - WITTEN CURRENT

$$\Omega = \int d\Sigma_\mu J^\mu$$

SYMPLECTIC CURRENT:

$$J^\mu_{CW} = \delta\Gamma^\mu_{\nu\lambda} \wedge \delta[\sqrt{g}\, g^{\nu\lambda}] - \delta\Gamma^\nu_{\nu\lambda} \wedge \delta[\sqrt{g}\, g^{\mu\lambda}]$$

- $\partial_\mu J^\mu = 0 \Rightarrow \Omega$ INDEPENDENT OF Σ VARIATIONS
- Ω INV. UNDER $\delta g_{\mu\nu} \to \delta g_{\mu\nu} + \nabla_{(\mu}\xi_{\nu)}$

ON SOLUTIONS

CRNKOVIĆ, WITTEN 86

GENERALIZATION TO ANY LAGRANGIAN THEORY

$$\mathcal{L} = \mathcal{L}(\varphi_A, \partial_\mu \varphi_A)$$

$$\Omega = \int d\Sigma_\mu \, J^\mu$$

$$J^\mu = \delta\left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi^A}\right] \wedge \delta \varphi^A$$

$$\left.\begin{array}{l} \bullet \ \partial_\mu J^\mu = 0 \\ \bullet \ \Omega \text{ GAUGE INV.} \end{array}\right\} \text{ON SOLUTIONS OF E.O.M.}$$

ZUCKERMAN
LEE, WALD

FIXED FLUXES RESTRICTION

GAUGE FIELD QUANTIZATION IN NONTRIVIAL TOPOLOGY

E.g. ELECTROMAGNETISM

$$S = \int \sqrt{g}\, F_{\mu\nu}^2 \, d^4x$$

SYMPL. CURRENT:

$$J^\mu = \delta[\sqrt{g}\, F^{\mu\nu}] \wedge \delta A_\nu$$

USES δA_ν !

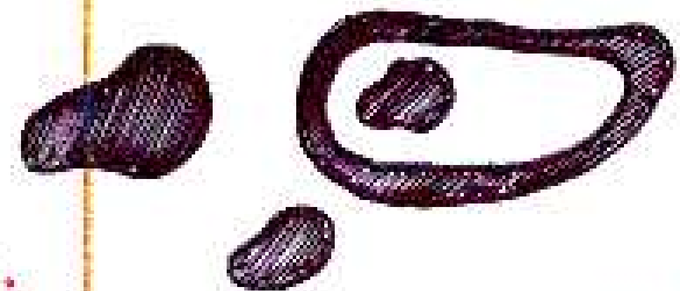
→ MUST CHOOSE $\delta A_\nu(x)$ REGULAR EVERYWHERE ON Σ

→ IMPOSSIBLE UNLESS $\delta F = d(\delta A)$ HAS ZERO FLUX ON ALL 2-CYCLES

(11)

WORKED EXAMPLE: "BUBBLING ADS"

MODULI SPACE:



PROBLEM:

FIND SYMPL. FORM RESTRICTED ON MODULI SPACE

- WHICH VARIATIONS TO CONSIDER?

  ALL FLUXES FIXED $\Leftrightarrow$

  ALL BLACK & WHITE AREAS FIXED

RELEVANT ACTION $(g_{\mu\nu}, F_5)$ SECTOR

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left(R - 4|F_5|^2\right)$$

SYMPL. CURRENT:

$$J^\mu = -\delta\Gamma^\mu_{\nu\lambda} \wedge \delta[\sqrt{g}\, g^{\nu\lambda}] + \delta\Gamma^\nu_{\lambda\nu} \wedge \delta[\sqrt{g}\, g^{\mu\lambda}]$$

(GRAVITY PART)

$$-8\, \delta[\sqrt{g}\, F^{\mu\cdots}] \wedge \delta A_{\cdots}$$

(GAUGE PART)

$$(F = dA)$$

- $$ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx_i^2) + y\, e^{G} d\Omega_3^2 + y\, e^{-G} d\tilde{\Omega}_3^2$$

$i = 1, 2$

$$h^{-2} = \frac{y}{\sqrt{1/4 - z^2}} \quad ; \quad e^{2G} = \frac{1/2 + z}{1/2 - z}$$

$$z = \frac{1}{\pi} \frac{y^2}{(x^2+y^2)^2} * Z$$

$$V_i = \frac{\varepsilon_{ij}}{\pi} \frac{x_j}{(x^2+y^2)^2} * Z$$

$$\boxed{Z = \begin{cases} -1/2, & x \in \bullet \\ 1/2, & x \notin \bullet \end{cases}}$$

- $$F_5 = F \wedge d\Omega + \tilde{F} \wedge d\tilde{\Omega}$$

$$F = dB \qquad \tilde{F} = d\tilde{B}$$

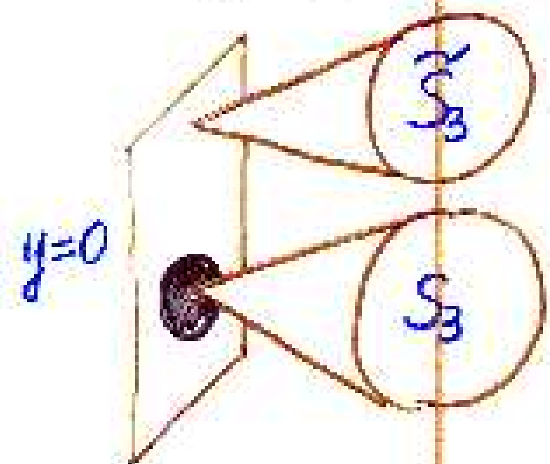
$$B_i = -\frac{y^2 V_i}{4(1/2 - z)} - \frac{\varepsilon_{ij}}{4\pi} \frac{x_j}{x^2+y^2} * Z + \frac{x_1}{4} \delta_{i,2}$$

$$\tilde{B}_i = -\frac{y^2 V_i}{4(1/2 + z)} - \frac{\varepsilon_{ij}}{4\pi} \frac{x_j}{x^2+y^2} * Z - \frac{x_1}{4} \delta_{i,2}$$

$$B_y = \tilde{B}_y = 0$$

SUBTLETY

- $A_4 = B \wedge d\Omega + \widetilde{B} \wedge d\widetilde{\Omega}$ IS SINGULAR ON $y=0$ PLANE:



$B, \widetilde{B} \to$ CONST $(y \to 0)$

$S_3\ (\widetilde{S}_3)$ SHRINKS

- δA_4 CAN BE MADE REGULAR BY GAUGE TRANSF. (DROPLET DEP.)

$$\delta B_i = \partial_i \lambda \quad (y=0,\ Z=-1/2)$$
$$\delta \widetilde{B}_i = \partial_i \widetilde{\lambda} \quad (y=0,\ Z=1/2)$$

FULL SYMPLECTIC FORM

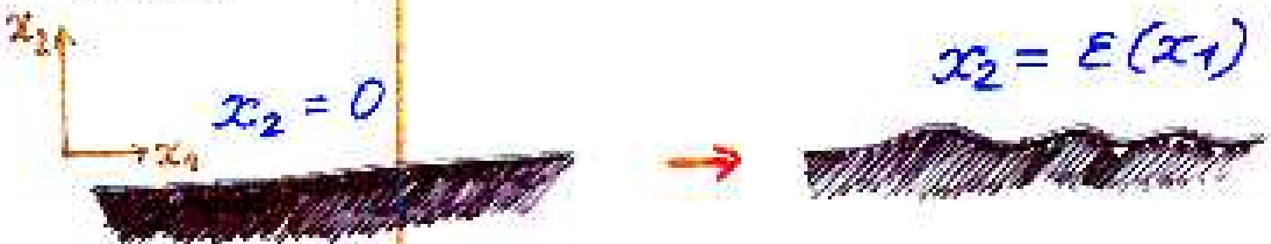
$$\Omega = \int_{t=\text{const}} J^t \qquad \text{(BULK)}$$

$$+8\int_{y=0,\, Z=-1/2} \lambda \wedge \delta \widetilde{F}_{12} \qquad \left(\begin{array}{c}\text{BOUNDARY}\\ \text{COMPENSAT.}\end{array}\right)$$

$$-8\int_{y=0,\, Z=1/2} \widetilde{\lambda} \wedge \delta F_{12}$$

SIMPLEST COMPUTATION:
PERTURBATION AROUND PLANE WAVE

WAVY LINE APPROXIMATION

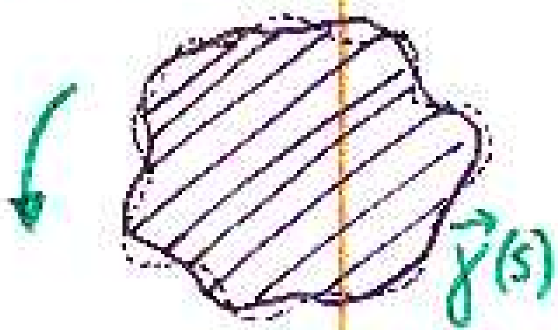


1. EXPRESS VARIATIONS OF ALL FIELDS VIA $\varepsilon(x_1)$
2. COMPUTE SYMPL. CURRENT
3. INTEGRATE TO GET SYMPL. FORM

ANSWER (IN MOM. REP.)

$$\boxed{\omega = \frac{(2\pi^2)^2}{K_{10}^2} \int \frac{dp}{2\pi} \frac{i}{p} \varepsilon(p) \wedge \varepsilon(-p)}$$

DUAL PICTURE — FREE FERMIONS



SEMICLASSICAL QUANTIZATION (LARGE N LIMIT)
HYDRODYNAMIC APPROACH

$$H_{TOT} = \int_{\text{DROPLET}} \frac{dp\, dq}{2\pi\hbar} \, \frac{p^2+q^2}{2} = H_{TOT}[\vec{\gamma}(s)]$$

$\hbar$ — 1D PLANCK CONSTANT

$$\frac{d\vec{\gamma}}{dt} = \{\vec{\gamma}, H_{TOT}\}$$

P.B. GENERATED BY SYMPLECTIC FORM:

$$\omega = \frac{1}{8\pi\hbar} \oint \oint ds\, ds' \, \text{Sign}(s-s') \, \delta\gamma_\perp(s) \wedge \delta\gamma_\perp(s')$$

$$\left(\oint ds \, \delta\gamma_\perp(s) = 0 \right)$$

POLYCHRONAKOS
DHAR
G.M.M.P.R.

→ AGREES WITH WAVY LINE GRAVITY COMPUTATION

$$\left(\hbar = \frac{k_{10}}{4\pi^{5/2}} \right)$$

GENERAL DROPLET CASE

SYMPL. CURRENT VIA FIELD VARIATIONS:

$$J^t = y^3 \frac{3/4 + z^2}{(1/4 - z^2)^2} \, \delta(V_i \partial_i z) \wedge \delta z$$

$$+ y^3 \, \delta\left(h^{-4} V_j \partial_{[i} V_{j]}\right) \wedge \delta V_i$$

$$+ 8 \varepsilon^{ij} \left(\delta \tilde{B}_i \wedge \partial_y \delta B_j - \delta B_i \wedge \partial_y \delta \tilde{B}_j \right)$$

NEED TO INTEGRATE:

$$\omega \sim \int dy \, dx_1 \, dx_2 \; J^t + \int_{y=0} (\dots)$$

CAN SHOW THAT

$$J^t = \partial_y \left[\varepsilon_{ij} \, \delta\left(\frac{y^4 (1/4 + z^2) z V_i}{(1/4 - z^2)^2} \right) \wedge \delta V_j \right]$$

$$+ \partial_i \left[\frac{2 y^3 z}{1/4 - z^2} \, \delta z \wedge \delta V_i \right]$$

BOUNDARY CONTRIBUTION

DOES NOT CONTRIBUTE

ADDING $y=0$ BOUNDARY CONTRIBUTIONS, ONE GETS

$$\omega \sim \int_{y=0,\, Z=-1/2} \varepsilon^{ij}\, \delta C_i \wedge \delta C_j$$

$$\delta C_i = \varepsilon_{ij} \frac{x_j}{x^2} * \delta Z = \partial_i \lambda$$

$$\omega \sim \int_{\partial \mathcal{D}} \lambda \wedge \delta C_{\parallel}$$

$\partial\mathcal{D} \leftarrow$ DROPLET BDRY

$$\delta C_{\parallel} \to \delta \gamma_{\perp}(s)$$
$$\lambda \to \int^{s} \delta\gamma_{\perp}(s')\, ds'$$

on $\partial\mathcal{D}$

$$\Rightarrow \boxed{\omega \sim \oint ds \;\; \delta\gamma_{\perp}(s) \wedge \int^{s} \delta\gamma_{\perp}(s')\, ds'}$$

AGREEMENT

(ALSO COEFFICIENT)

CWZ vs. EFFECTIVE ACTION

Crnković Witten Zuckerman

QUANTIZING QUADRATIC FLUCTUATIONS

$S_{II}(\delta g, \delta A, \dots)$ – quadratic action

- COULD FIRST QUANTIZE S_{II}, THEN SPECIALIZE.
  - TRACTABLE IN CERTAIN CASES (AdS, PLANE WAVE)
  - HARD (IMPOSSIBLE ?) IN GENERAL
- TRUNCATING ACTION TO MODES FROM $\mathcal{M}$ IS IMPOSSIBLE IN GENERAL.

E.g. FOR **LLM** AROUND $AdS_5 \times S^5$

$\Delta = J$ MODES COUPLE TO $\Delta = -J$ IN S_{II} (ANGULAR MOMENTUM CONSERVATION)

CWZ BYPASSES THESE PROBLEMS

CONCLUSIONS

- MINISUPERSPACE QUANTIZATION
  → CHECK / FIND MICROSCOPIC DUALS OF SUGRA SOLUTION FAMILIES
- COVARIANT CWZ METHOD
- QUANTUM EQUIVALENCE "BUBBLING ADS" ⇔ FREE FERMIONS

FUTURE

- D1-D5 SOLUTIONS OF LUNIN-MALDACENA-NAOZ
  → EXPECT $\omega \sim \int dv\, \delta F_i'(v) \wedge \delta F_i(v)$
- FURTHER APPLICATIONS TO **BH** MICROSTATE GEOMETRY COUNTING
  → SEE BENA'S TALK
- 1/4 BPS SOLS OF LIU-VAMAN
  → FRACTIONAL STATISTICS ?
- BETTER UNDERSTAND CORRECTIONS/ ROLE OF SUSY