The Tachyon at The End of the Universe

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+ work in progress w/ Horowitz

+ 0502021 w/ Adams, Liu, Saltman

Consider a flat FRW solution to GR

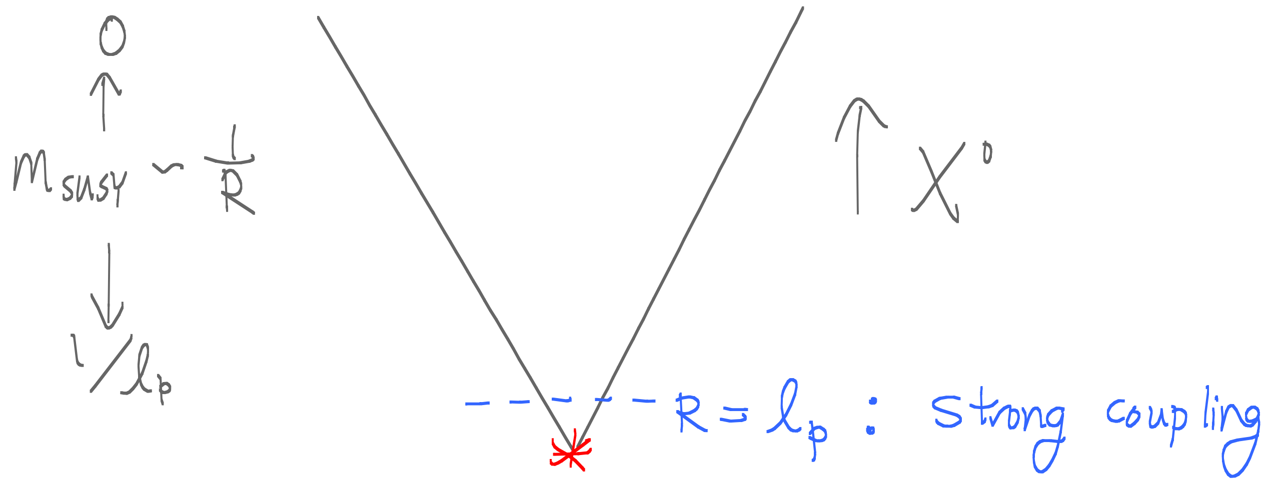
$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^{\,2} + ds_\perp^2$ with

- at least one circle $\vec{x} = \vec{x} + \vec{L}$ around which spacetime fermions have antiperiodic boundary conditions.

- $v(t) \equiv \dot{a}(t)|\vec{L}| \ll 1$ at time t_s at which the circle size is string scale $R \equiv |\vec{L}|\, a(t_s) = l_s$

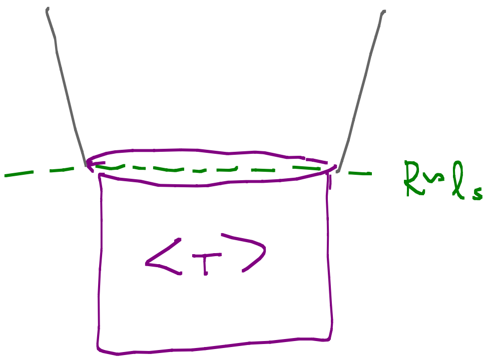
(obtained by sufficiently weak matter source)

This has a spacelike big bang singularity in the past in the GR solution:



with a level of SUSY breaking appropriate to that in early universe cosmology (& inside black holes)

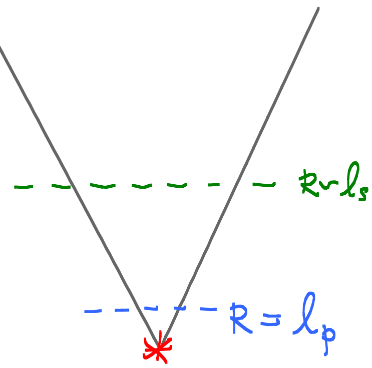
In string theory, a winding tachyon appears and condenses at the string scale



String theory

$$Z_{\text{worldsheet}} \sim \int DX \exp\left\{ G_{\mu\nu}\partial X^\mu \partial X^\nu - e^{-kX^0} V_T \right\}$$

tachyon background suppresses fluctuations in path integral



GR

$$Z_{\text{worldline}} \sim \int DX\, e^{i\int G_{\mu\nu}\dot{X}^\mu \dot{X}^\nu} \prod \theta$$

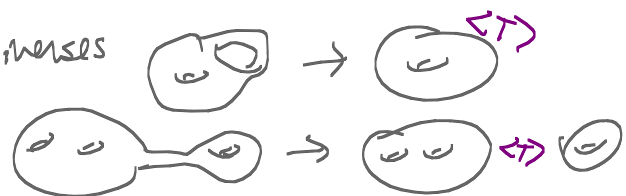
$G_{xx} \to 0$ singularity

from unsuppressed fluctuations in path integral

Remark: The problem of closed string tachyon condensation, often motivated by the question of the vacuum structure of string theory, is crucial to a basic question about gravity (spacelike singularity resolution).

We have seen winding tachyon condensation address questions of spacetime dynamics also in

- timelike conical singularities 

w/ Adams Polchinski ...
- topology change & baby universes 

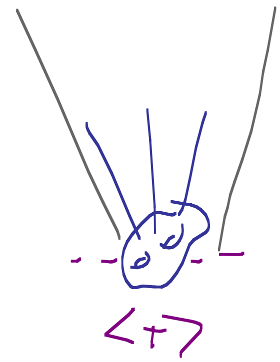
w/ Adams, Liu, McGreevy, Saltman
- Closed timelike curves Costa et al Null Singularities Berkooz et al
- Endpoint of Hawking decay for black strings Horowitz

Because the tachyon background

(semiclassically $\left(\int d^2\sigma\, \mu\, e^{-2\kappa X^0}\, \hat{T}\right)_r$ deformation of the worldsheet action)

damps contributions to the worldsheet path integral, it provides a possibility of <u>perturbatively</u> curing the singularity.

We will now verify this via systematic computations of perturbative amplitudes, applying <u>methods of Liouville theory</u>



will find <u>limited</u> support of amplitudes in X^0 direction:

$$\Delta X^0 = \frac{-\ln \mu/\mu_*}{\kappa}$$

$\ll$ time to would-be singularity

Work in superstring in critical dimension in conformal gauge, e.g. heterotic:

Formal Path Integral in Lorentzian signature

$$\mathcal{G}(\{V_n\}) = \int [d\mathbb{X}][d\Psi_-][d(\text{ghosts})]\, d(\text{moduli})\, e^{iS} \prod_n \left(i\int d\sigma\, d\tau\, V_n(\mathbb{X})\right)$$

$$\mathbb{X}^\mu = X^\mu + \theta^+ \psi_+^\mu \qquad \Psi_-^a = \psi_-^a + \theta^+ F^a$$

where S = semiclassical action $= \int d\sigma\, d\tau\, d\theta^+ \{ D_{\theta^+}\mathbb{X}^\mu \partial_- \mathbb{X}^\nu G_{\mu\nu}(\mathbb{X})$

$$+ \Psi_- D_{\theta^+} \Psi_- - \mu\, \underbrace{\Psi_- : e^{-\kappa \mathbb{X}^0} \cos(w\tilde{\theta}):}_{\text{winding tachyon semiclassically}} + (\text{ghost}) + (\text{dilaton}) \}$$

Timelike Liouville

and the other ingredients are:

vertex operators $\quad V_{\vec{k},n} \simeq \int d\theta^{+} e^{i\vec{k}\cdot\vec{X}} \hat{V}_{n}$

semiclassically

dilaton $\quad \Phi = \Phi_0 \simeq -\infty$

semiclassically.

* As in Liouville field theory, the path integral generates automatically the appropriate corrections to the semiclassical quantities.

Wick rotation

$$\tau \equiv e^{i\gamma}\tau_\gamma \quad \vec{X} \equiv e^{i\gamma}\vec{X} \quad \mu \equiv e^{-i\gamma}\mu_\gamma \quad \vec{k} = e^{-i\gamma}\vec{k}_\gamma$$

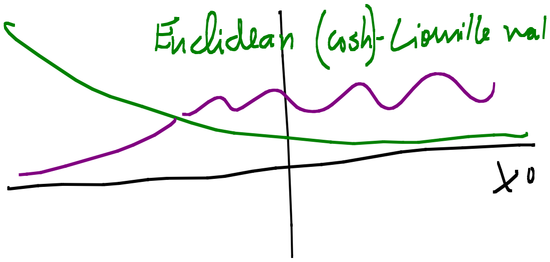
with $\gamma \to \frac{\pi}{2}$

produces a well-defined Euclidean continuation defining the worldsheet Path Integral in a standard way. $\rightarrow$

$$\int (DX_E)\cdots e^{-S_E}$$

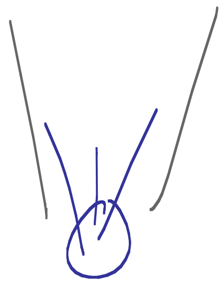
$*$ positive potential from tachyon suppresses contributions to amplitudes

$$S_E^{(Het)} = \text{positive kinetic terms} + \mu_E^2 e^{-2\kappa X^0}\cosh^2 w\hat{\Omega}_E$$
$$+ \text{fermions}$$



To start, focus on the "Euclidean" or "Hartle-Hawking" state: no excitations above $\langle T \rangle$ in the past. More on the status of other vacua later.

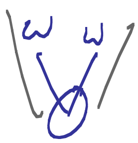
Interpretation of Amplitudes: (cf Polyakov '92)



Correlation functions of bulk vertex operators give components of the state of closed strings

(cf Gutperle, Strominger, Takayanagi, ...)

e.g. 2-point function: gives the Bogoliubov coefficients:



$$|\Psi\rangle = \mathcal{N} e^{\frac{\beta}{2\alpha} a^{+2}} |0\rangle$$

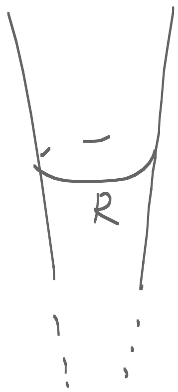
related to reflection amplitude in Euclidean continuation

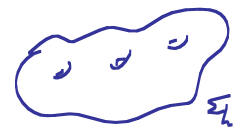
$$\rightarrow \frac{\beta}{\alpha} = \frac{\langle\Psi| a^{+2} |0\rangle}{\langle\Psi|0\rangle} = \langle \textstyle\int V_\omega \int V_\omega \rangle \leftrightarrow$$

(Mixing of positive & negative frequency modes $e^{i\omega t} \rightarrow \alpha e^{i\omega t} + \beta e^{-i\omega t}$)

(Mixing of positive & negative momentum modes)

Let us start with the vacuum amplitudes





we chose velocity $v = \frac{dR}{dX^0} \ll 1$

so that starting at string scale, the time to the GR-predicted singularity is

$$\Delta X^0_* \sim \frac{l_s}{v}.$$

Whereas flat space vacuum amplitudes are extensive in spacetime,

$$Z_{flat} = \delta(0)\,\hat{Z}$$

↖ volume of time from X^0_0 zero-mode integral

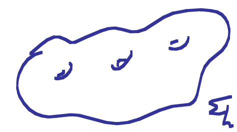
$*$

ours will only have support for

$$\left(\Delta X^0 \sim -\frac{\ln \mu/\mu_*}{\kappa}\right) \ll \left(\Delta X^0_* \sim \frac{l_s}{v}\right)$$

In our case the vacuum amplitudes are

$$Z_h = \int [dX_E][d(\ldots)]\; e^{-\int_{\Sigma_h} d\sigma_E^2\, d\theta^+ \left(\mathcal{L}^{(T=0)} + \mu_E e^{-\kappa X^0}\hat{T}\right)}$$



Split $X^0 = X^0_0 + \hat{X}^0(\sigma, \tau_E)$ and calculate

0-mode

$$\frac{\partial Z_h}{\partial \mu_E} = \int [d\vec{X}][d(\ldots)][d\hat{X}^0]\, \underbrace{dX^0_0\, e^{-\kappa X^0_0}}_{\frac{dy}{-\kappa}}\, \frac{C}{\mu_E}\, e^{-S_E^{(T=0)}}\, e^{-Ce^{-\kappa X^0_0}}$$

where $C = \int \mu_E\, e^{-\kappa \hat{X}^0}\, \hat{T}$. The 0-mode integral is of the form $\left(y \equiv e^{-\kappa X^0_0}\right)$

$$\int_{\simeq 0}^{\infty} dy\, e^{-Cy} \simeq \frac{1}{C}$$

← ($v \ll 1 \Rightarrow$ will be self-consistent)

This yields (cf Gupta, Trivedi, Wise; Bershadsky, Klebanov et al in LFT)

$$Z_h = \Big(\underbrace{-\frac{\ln \mu/\mu_*}{k}}_{} + \underbrace{i\,\frac{\pi}{2k}}_{} \Big)\, \hat{Z}_h(T=0)$$

* Range of X^0 restricted to bulk region where $T \to 0$

continuation back to $\mu = e^{-\frac{i\pi}{2}} \mu_E$

↳ thermal state: $X^0 \to X^0 + i\beta$ with $\beta = \frac{\pi}{k}$ will be corroborated below.

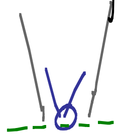
nonzero mode contribution to $T=0$ bulk theory

$\left(\Delta X^0_* \sim \frac{l_s}{v} = \text{time to would-be singularity} \right)$

$\Rightarrow$ • (sum over) closed string states lifted in $\langle T \rangle$ phase

• Back reaction from quantum stress-energy controllably small

The 2-point function $\langle \int V_\omega \int V_\omega \rangle = \frac{\beta}{\alpha}$



gives the Bogoliubov coefficients:

- The X^0 integral $\Rightarrow \langle \int V_\omega \int V_\omega \rangle \propto \mu_E^{-\sum_n \frac{\omega_n}{K}}$
- The magnitude of the result is 1 in the Euclidean continuation (total reflection off a Liouville wall)

$$\left| \frac{\beta_{\vec{k},n}}{\alpha_{\vec{k},n}} \right| = e^{-\omega(\vec{k},n)\frac{\pi}{K}} \quad \underset{|\beta|^2 \mp |\alpha|^2 = 1}{\Rightarrow} \quad \boxed{* \quad N_{\vec{k},n} = |\beta_{\vec{k},n}|^2 = \frac{1}{e^{\frac{\pi}{K}(2\omega)} \mp 1}}$$

Thermal distribution of created pairs at temp $\frac{K}{\pi}$ as above

Finally, one can assess the singularity structure of other perturbative amplitudes similarly. Again as in Liouville, find divergences only at $\sum \omega_n (\pm)_n = 0$ (where X_0^0 integral unsuppressed in bulk: these are expected divergences from physical states).

* In particular, the amplitudes self-consistently shut off as explained above far away from the planckian regime of black hole formation

Remarks: • $\langle T\rangle$ and the "Nothing Phase": Our result that $\langle T\rangle$ shuts off support of amplitudes lines up with many heuristic arguments, including:

- In matter sector, tachyon vertex operator is relevant → lose degrees of freedom: cf spatially localized cases 

- Worldline QFT analogue (a.k.a. minisuperspace) is an exponentially increasing mass (Strominger ...)

$$S_{\text{worldline}} = \int d\tau \left(-(\partial_\tau X^0)^2 + (\partial_\tau \vec{X})^2 - (m_0^2 + \mu^2 e^{-2\kappa X^0}) \right)$$

- There is some large ↔ small radius correlation between tachyonic systems and those with Witten "bubble of nothing" decays

* Note that this phase is not necessarily static, but is adiabatic

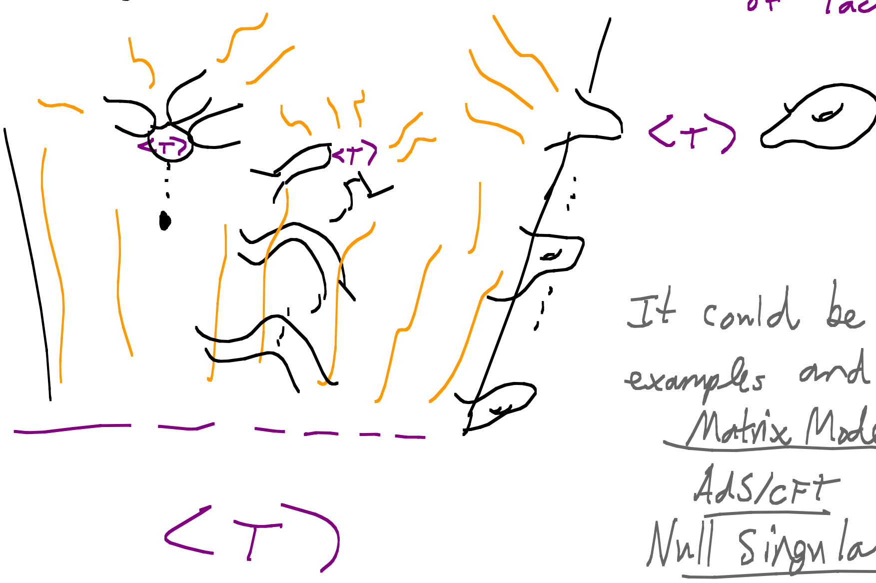
- These results indicate a perturbative, stringy mechanism for "starting time from nothing"

cf perturbative, stringy topology change & Baby universe production by winding tachyons (ALMSS 0502021)

These are subjects previously studied via Euclidean quantum gravity instantons (cf Hartle, Hawking, Linde, Vilenkin, Giddings, Strominger & more recently Tye; Dijkgraaf, Gopakumar, Gukov, Ooguri, Vafa, Verlinde ...)

A lesson I take from this is that although (because!) they are instabilities, Tachyonic modes play a useful role in addressing problems of gravity.

Big Picture that is suggested in this perturbative regime : Regions of bulk spacetime smoothly end at regions of Tachyon condensate $\langle T \rangle$

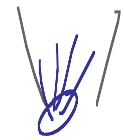


It could be interesting to relate to other examples and/or approaches e.g.

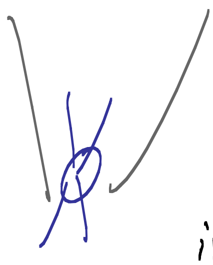
- Matrix Model $\langle T \rangle$: Polchinski, Karczmarek, Strominger, ...
- AdS/CFT Shenker et al, Hertog/Horowitz ...
- Null Singularities Liu Moore Seiberg ... Berkooz et al
  - ↳ matrix theory Craps Sethi Verlinde
- Euclidean Q.G. String FT Zweibach talk..

Future Directions (with Horowitz and McGreevy) *Preliminary

- Status of other putative vacua:

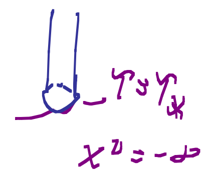


Above we considered a vacuum with no excitations in the $\langle T \rangle$ phase. An interesting potential BRST anomaly arises if we consider

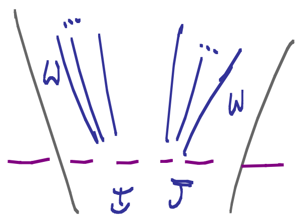


worldsheets that don't bounce back to the bulk, related to the non-self-adjointness of $L_0 = \frac{\partial^2}{\partial X^{0\,2}} + \cdots$ in configurations where $X^0 \to -\infty$ in finite worldsheet time τ.

(cf Berkooz, Hertog-Horowitz)



This potential BRST anomaly is cancelled if we require the worldsheet to bounce back to the bulk, but it can do so with any unitary mixing of modes



→ * Still thermal spectrum at $\beta = \frac{\pi}{k}$, but different phases in the state.

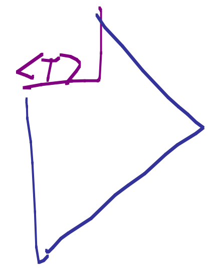
(as in multiple-trace deformations of AdS/CFT)

Aharony, Berkooz, Seva, ES, Sharon, Witten...

[In type II theories, the worldsheets may consistently end on D-branes: analogues of the ZZ branes in Liouville]

... Future Directions

- These states with unitary "reflection off $X^0 = X^0_*$" may allow us to microphysically check for some cases the "Black Hole Final State" proposal of Horowitz & Maldacena (cf Gottesman, Preskill...)



- In any case, these results may apply to the spacelike singularities in black hole physics.

cf Horowitz : $\langle T \rangle$ perturbatively mediates transition black string $\to$ bubble of nothing

• <u>Positive Curvature Spatial Slices:</u> (cf Polyakov)

The RG behavior in the matter sector of the $O(N) \cong S^{N-1}$ model is similar to the winding tachyon case discussed above (to analyze, can use large N and/or SUSY linear sigma model)

$$S_{ws} = \int d^2\sigma\, \partial X^\mu \partial X^\nu G_{\mu\nu} + \Lambda(X^0)^{2-\Delta}\, \mathcal{O}_{\Delta<2}$$

"Tachyon" term again suppresses fluctuations in worldsheet path integral

increasing function of X^0 as space shrinks

relevant operator (mass)

* However, the "velocity" with which the S^{N-1} shrinks is <u>not</u> tunably small in this case.