Is there a closed string tachyon vacuum ?

-- Strings 2005 --

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In the presence of a tachyon two questions arise:

1. Is there a ground state of the theory without the instability, namely, is there a tachyon vacuum ?

2. What is the end-result of the physical decay process associated with the instability ?

Answers are known for open string theory tachyons that live on the world-volume of unstable D-branes:

(SEN)

1. The tachyon vacuum is a background without the D-brane and without open strings -- it is the vacuum state of closed strings.

2. The D-brane decays into an excited state of closed strings that carries the original energy.

The decay process is a transition from the unstable vacuum to an excited state of the stable vacuum.

A tachyon of closed string theory is said to be a bulk tachyon if it lives throughout spacetime. The answers of questions (1) and (2) are not known for the bulk tachyon of closed bosonic string theory.

This talk will present three main topics:

1. Developments in closed string field theory (CSFT)in 04-05: (a) construction of heterotic string field theory, (b) tools and computations in CSFT.

2. Effective field theory analysis of the bulk tachyon instability: the generality of a big crunch.

3. A *tentative* identification of a tachyon vacuum in bosonic closed string field theory.

Heterotic string field theory was developed in hep-th/0406212 (Y. Okawa and B.Z.) and completed in hep-th/0409018 (N. Berkovits, Y. Okawa, B. Z.).

Computational advances are in hep-th/0408067 (N. Moeller) and have been used to test CSFT in hep-th/0501142 and hep-th/0502161 (Haitang Yang and B. Z.)

The effective field theory analysis is in hep-th/0506076 (H. Yang and B. Z.).

The calculation of the universal closed string tachyon potential (in level truncation) and the search for a spacetime independent tachyon vacuum is in hep-th/0506077 (H. Yang and B. Z.).

Construction of Heterotic String Field Theory

Bosonic Open String Field Theory:

$$S = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Phi, Q\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi\star\Phi\rangle\right).$$

Bosonic Closed String Field Theory:

$$S = -\frac{2}{\alpha'\kappa^2}\left(\frac{1}{2}\langle\Psi, Q\Psi\rangle + \frac{1}{3!}\langle\Psi, [\Psi, \Psi]\rangle + \frac{1}{4!}\langle\Psi, [\Psi, \Psi, \Psi]\rangle + \cdots\right).$$

Open Superstring Field Theory (WZW)

$$S = -\frac{1}{2g^2}\int_0^1 dt\langle\partial_t(A_\eta A_Q) + A_t\{A_Q, A_\eta\}\rangle = -\frac{1}{g^2}\int_0^1 dt\langle(\eta A_t)A_Q\rangle$$

where $A_X = e^{-\Phi}(Xe^{\Phi})$. A_Q is a pure gauge field, and $A_t \equiv A_{\partial_t}$.

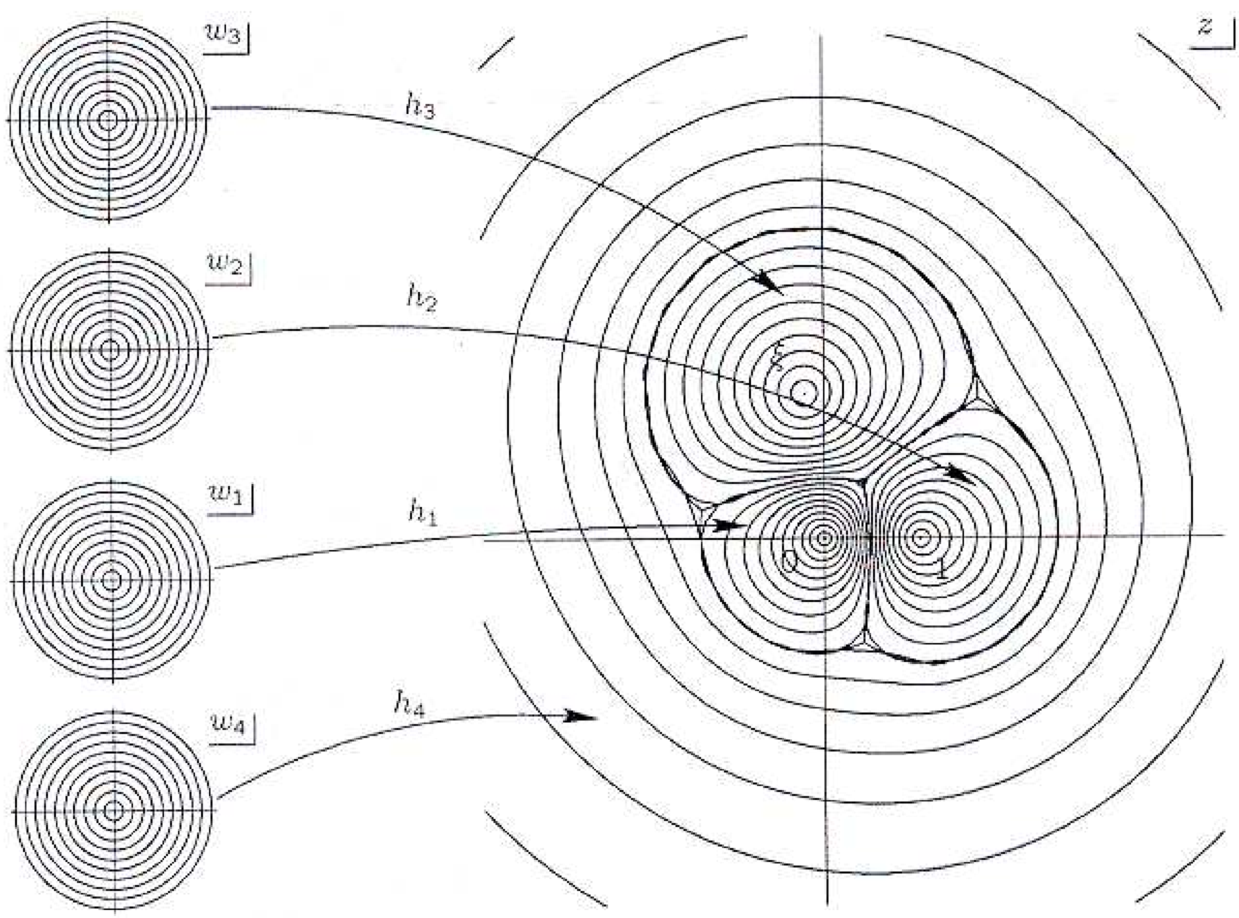
Heterotic String Field Theory (WZW-like)

$$S = \frac{2}{\alpha'}\int_0^1 dt\langle(\eta\Psi_t)\Psi_Q\rangle = \frac{2}{\alpha'}\left(\frac{1}{2}\langle\eta V, QV\rangle + \frac{\kappa}{3!}\langle\eta V, [V, QV]\rangle + \ldots\right)$$

where Ψ_Q is a pure-gauge closed string field.

The numerical fit of the function $a(\xi,\bar{\xi})$ that defines the quadratic differential $(\xi = x + iy)$:

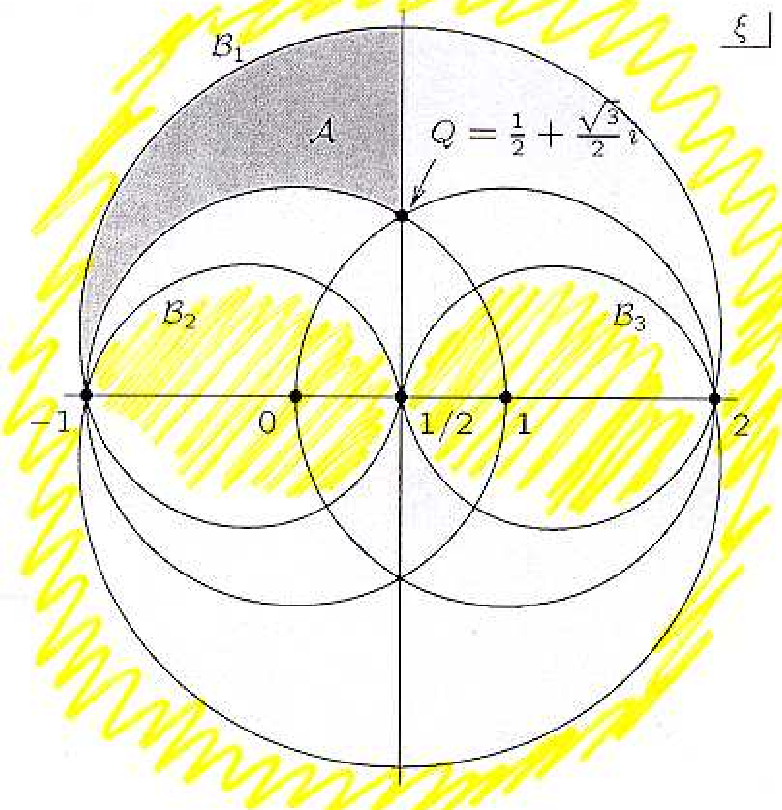
$$
\begin{aligned}
a^{\mathrm{fit}}(\xi) = {} & (0.67867 + 0.46345\,i) + (2.07710 + 0.49176\,i)\,x + (0.20000 + 0.58477\,i)\,y \\
& +(1.61074 - 0.64246\,i)\,x^2 - (0.09194 - 0.36467\,i)\,xy + (0.05965 + 0.38101\,i)\,y^2 \\
& -(0.33945 + 0.80519\,i)\,x^3 - (0.35448 - 0.94778\,i)\,x^2 y - (1.04747 + 0.85263\,i)\,xy^2 \\
& +(0.56049 - 0.43233\,i)\,y^3 - (0.63061 + 0.05352\,i)\,x^4 + (0.27490 + 0.76364\,i)\,x^3 y \\
& -(0.89712 + 0.75355\,i)\,x^2 y^2 + (0.70167 + 0.46374\,i)\,xy^3 - (0.50743 - 0.12128\,i)\,y^4 \\
& -(0.07990 - 0.08258\,i)\,x^5 + (0.32821 + 0.08618\,i)\,x^4 y - (0.08918 + 0.20755\,i)\,x^3 y^2 \\
& +(0.37843 + 0.20916\,i)\,x^2 y^3 - (0.15046 + 0.08495\,i)\,xy^4 + (0.11992 - 0.00719\,i)\,y^5
\end{aligned}
$$

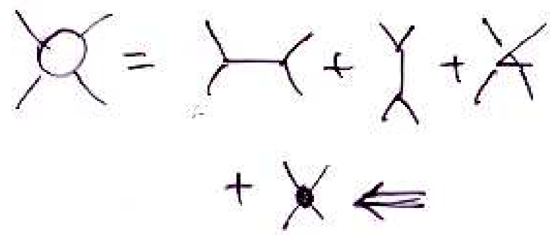


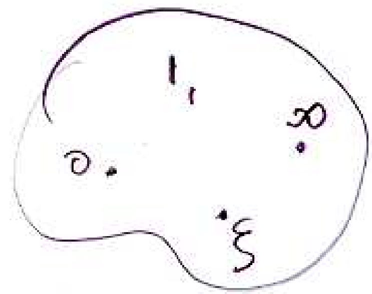
Local coordinates and map for $\xi = -0.2 + 1.5\,i$.

Computational tools in Closed SFT (Moeller)

The computation of 4-pt couplings requires finding the elementary vertex $\mathcal{V}_{0,4}$ that represents the ``missing four-punctured spheres (punctured at $0, 1, \xi, \infty$).







The moduli space $\mathcal{V}_{0,4}$ is shaded. Each point ξ represents one four-punctured sphere. The white regions around $0, 1$, and ∞ represent spheres produced by the Feynman graphs.

On each four-punctured sphere ξ we need local coordinates at the punctures $(0, 1, \xi, \infty)$. To get them we need the Strebel quadratic differential $\varphi = \phi(z)(dz)^2$

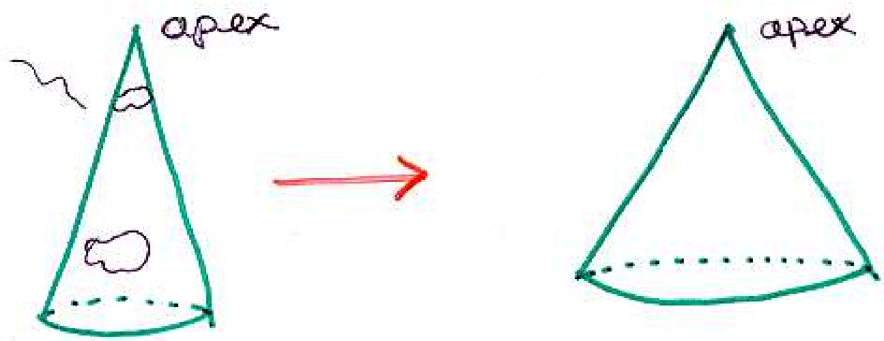
$$\phi(z) = \frac{-(z^2 - \xi)^2}{z^2(z-1)^2(z-\xi)^2} + \frac{a(\xi, \bar{\xi})}{z(z-1)(z-\xi)}.$$

It is determined by the function $a(\xi, \bar{\xi})$.

Computational tests

Twisted tachyon condensation in CSFT (Okawa + Z., hep-th/0403051, Bergman + Razamat hep-th/0410046)

Testing the Adams, Polchinski, Silverstein conjecture in the context of bosonic closed strings: The *twisted* tachyons that live on the apex of a cone signal an instability to decay into cones with smaller (or zero) deficit angle.



We formulate CSFT on the background of, say, $\mathbb{C}/\mathbb{Z}_3$ and calculate a potential in which one can identify vacua corresponding to $\mathbb{C}/\mathbb{Z}_2$ and flat space.

Interesting fact: The new vacua are created by the interplay of twisted fields at the apex and *untwisted* fields with vev's spread about the apex of the cone.

Figure of merit: with θ the deficit angle of the cone

$$\Delta\theta \simeq \kappa^2 \mathbb{V}(T) \qquad \text{(Dabholkar, hep-th/0111004)}$$

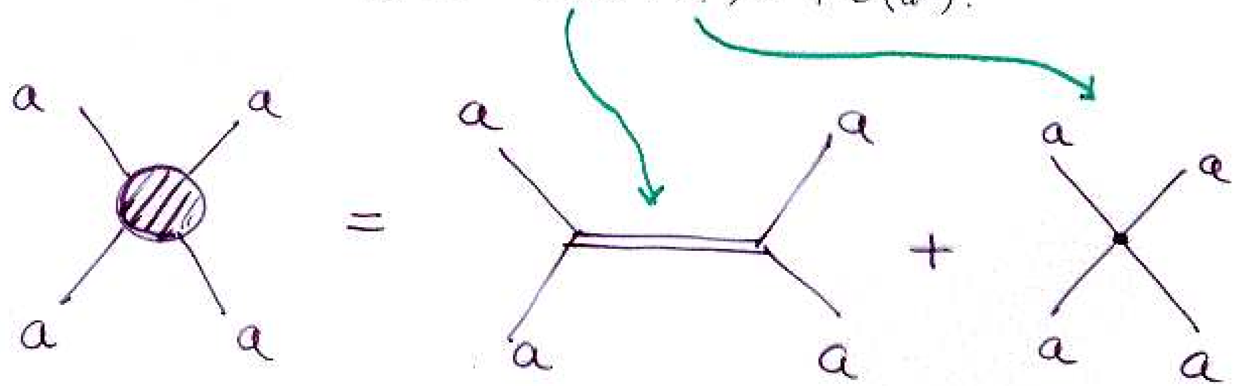
The lowest-level numerical computations (using cubic vertices only) give potentials that obey this relation to about 50%.

The mixing of twisted and untwisted fields (including gravity) suggests that $\mathbb{V}(T)$ is not truly well defined. Critical points of the full CSFT action, however, are.

Marginal directions in CSFT

Consider the marginal operator $c\bar{c}\partial X\bar{\partial}X$, and the associated marginal parameter a. The exact CSFT potential for a must vanish. There are no quadratic nor cubic terms in a. At quartic order

$$\kappa^2\,\mathbb{V}^{tot}_{(\ell)}(a) = (\mathcal{C}(\ell) + I_4\ )\,a^4 + \mathcal{O}(a^6)\,.$$



It turns out that $\mathcal{C}(\ell)$ is determined by the calculations that test the marginality of $c\partial X$ in OSFT (known to level 150! in Coletti, Sigalov, Taylor). With fits:

$$\mathcal{C}(\infty) \simeq 0.25587087$$

$$I_4 = -\frac{1}{\pi}\int_{\mathcal{A}} dx\,dy\,\Big|1 + \frac{1}{\xi^2} + \frac{1}{(1-\xi)^2}\Big|^2 \simeq -0.255\,872(\pm 2)\,.$$

Remarkable accurate confirmation of the region $\mathcal{V}_{0,4}$. Confirms the sign of the quartic term.

Dilaton deformations

The ghost-dilaton operator

$$\frac{1}{2}(c\partial^2 c - \bar{c}\bar{\partial}^2\bar{c}) \;\leftrightarrow\; (c_1 c_{-1} - \bar{c}_1\bar{c}_{-1})|0\rangle = |D\rangle$$

is **not primary** but the associated field d must have a vanishing potential in CSFT. Including also the field a, to cubic order

$$\kappa^2 V_{(0)} = -t^2 + \frac{6561}{4096}t^3 + \frac{27}{16}ta^2 - \frac{27}{32}td^2.$$

To find the effective potential for a and d, must integrate out t. The quadratic equation has a discriminant:

$$\text{discr} = \quad 1 + \frac{531441}{65536}\Big(\frac{d^2}{2} - a^2\Big)$$

a-deformations are bounded but d-deformations are not! This holds up to higher level.

$$\kappa^2\, \mathbb{V}^{tot}_{(\ell)}(d) = (\mathcal{C}_{d^4}(\ell) + I_{d^4}\,)\, d^4 + \mathcal{O}(d^5)\,.$$

Computing up to level 10 and with fits:

$$\mathcal{C}_{d^4}(\infty) \simeq 0.10584\,.$$

On the other hand:

$$I_{d^4} = \frac{1}{2\pi}\int_{\mathcal{A}} dxdy\; \langle\Sigma|\mathcal{B}\mathcal{B}^\star|D^4\rangle \simeq -0.1056\,.$$

Successful test of local coordinates for the surfaces in $\mathcal{V}_{0,4}$.

Tachyon driven rolling

Assume:

$$V(T) = -\tfrac{1}{2}m^2T^2 + \mathcal{O}(T^3)\,.$$

Build a rolling solution with $T(t \to -\infty) \to 0$:

$$T(t) = e^{mt} + \sum_{n\geq 2} t_n e^{n\,mt}\,.$$

The tachyon drives the rolling if:

$$\Phi(t) = \sum_{n\geq 2} \phi_n\, e^{n\,mt}\,, H(t) = \sum_{n\geq 2} h_n\, e^{n\,mt}\,.$$

The dilaton equation gives

$$\Phi(t) = \tfrac{1}{8}e^{2mt} + \ldots\,.$$

The dilaton begins to run towards *stronger* coupling.

On the other hand

$$\dot{H} \sim -\dot{T}^2 + 2\,\ddot{\Phi} = -m^2\,e^{2mt} + 2\cdot\tfrac{1}{8}\cdot(4m^2)e^{2mt} + \ldots = 0\cdot e^{2mt} + \ldots\,.$$

H vanishes to leading order: $h_2 = 0$. In fact, H vanishes to all orders, $h_n = 0$ for all n !

The string metric is <u>unchanged</u>.

With $H = 0$, the equations of motion become

$$\ddot{\Phi} = \tfrac{1}{2}T^2\,,$$

$\Phi(t)$ is a convex, so $\Phi(t)$ increases without bound.

The Einstein metric $g^E_{\mu\nu}$ tends to crunch:

$$g^E_{\mu\nu} = \exp\Big(-\frac{4}{d-1}\,\Phi\Big)\,g_{\mu\nu}$$

The remaining equations are

$$2\,\dot{\Phi}^2 = \tfrac{1}{2}\dot{T}^2 + V(T)\,, \qquad \ddot{T} - 2\,\dot{\Phi}\dot{T} + V'(T) = 0\,.$$

Can establish the following facts:

1. $\frac{d}{dt}(\frac{1}{2}\dot{T}^2 + V(T)) = 2\dot{T}^2\dot{\Phi} > 0$. "Energy" increases.

2. For non-positive potentials $V(T) \leq 0$, if $\dot{\Phi}(t_0) > 0$ at some time t_0, then $\dot{\Phi}(t_*) = \infty$ for some finite $t_* > t_0$. In fact

$$\Phi(t) = -\tfrac{1}{2}\ln(t_* - t) + \Phi_0 \qquad T(t) = \mp\ln(t_* - t) + T_0\,.$$

3. Even for $V(T) \to +\infty$ as $T \to \infty$, the crunch generally happens in finite time. It requires infinite time for $V(T) \sim \exp(nT)$ with $n \geq 2$.

General facts about the low energy model:

1) the string metric is constant

2) the dilaton rolls toward stronger coupling.

We get an Einstein-metric big crunch or flat string metric in the limit of infinite dilaton and tachyon vevs.

The generality of the evolution and the almost complete independence on the details of the tachyon potential suggest to us that the cosmological solutions presented here are relevant, modulo some stringy resolution of the big crunch singularity.

In cyclic universe models the crunch is induced by a scalar field rolling down a *negative* potential with a steep region -- the rest of the potential is largely undetermined.

Bulk closed string tachyons may induce a Big Crunch.

Searching for a closed string tachyon vacuum

In open bosonic string in the background of a spacefilling D-brane, the tachyon potential has a critical point with spacetime constant expectation values that represents spacetime without the D-brane and without physical open string excitations.

The closed bosonic string has a bulk tachyon with a tachyon potential. A critical point with spacetime constant expectation values could well represent a background with no closed string excitations.

Without any kind of excitations, for all intents and purposes, spacetime has dissappeared.

The existence of such a ``tachyonvacuum" background seems roughly compatible with the effective field theory analysis.

Quadratic and cubic terms in the closed string tachyon potential:

$$\mathbb{V}_0^{(3)} = -t^2 + \frac{6561}{4096}\,t^3 \simeq -t^2 + 1.6018\,t^3\,.$$

We find a critical point.

How about the quartic term in the potential ? Belopolsky (1994):

$$V_0^{(4)} = -3.017\,t^4\,.$$

This is negative and far too large; the critical point dissappears.

The ghost dilaton d is sourced by the tachyon -- it cannot be ignored. Lowest terms are cubic and quartic:

$$V(t,d) = -0.8438\,t\,d^2 + 3.8721\,t^3 d + \dots\,.$$

Integrating out the dilaton one finds

$$V(t,d) = 4.4422\,t^5 + \dots$$

This positive quintic term overwhelms the effect of the quartic term and restores the critical point.

The critical point, computed in level expansion using cubic and quartic terms, seems to survive.

Rolling Bulk Closed String Tachyon

A study of this process in the low energy effective action

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\, e^{-2\Phi}\Big(R + 4(\partial_\mu\Phi)^2 - (\partial_\mu T)^2 - 2V(T)\Big).$$

$g_{\mu\nu}$ is the string metric, Φ is the dilaton, and T is the tachyon.
The equations of motion are:

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi - (\partial_\mu T)(\partial_\nu T) &= 0\,, \\ \nabla^2 T - 2(\partial_\mu\Phi)(\partial^\mu T) - V'(T) &= 0\,, \\ \nabla^2\Phi - 2(\partial_\mu\Phi)^2 - V(T) &= 0\,. \end{aligned}$$

Use the cosmological ansatz

$$ds^2 = -(dt)^2 + a^2(t)(dx_1^2 + dx_2^2 + \ldots dx_d^2)\,,\, H(t) \equiv \frac{\dot a(t)}{a(t)}\,.$$

The equations of motion for the dilaton and the tachyon are:

$$\begin{aligned} \ddot\Phi + (d\,H - 2\,\dot\Phi)\,\dot\Phi + V(T) &= 0\,, \\ \ddot T + (d\,H - 2\,\dot\Phi)\dot T + V'(T) &= 0\,. \end{aligned}$$

$H > 0$ is the Hubble "friction", it opposes the field velocity.

$\dot\Phi > 0$ is *anti*-friction, it increases the field velocity

The dilaton is driven by $-V(T)$; it tends to go to strong coupling while $V(T) < 0$.

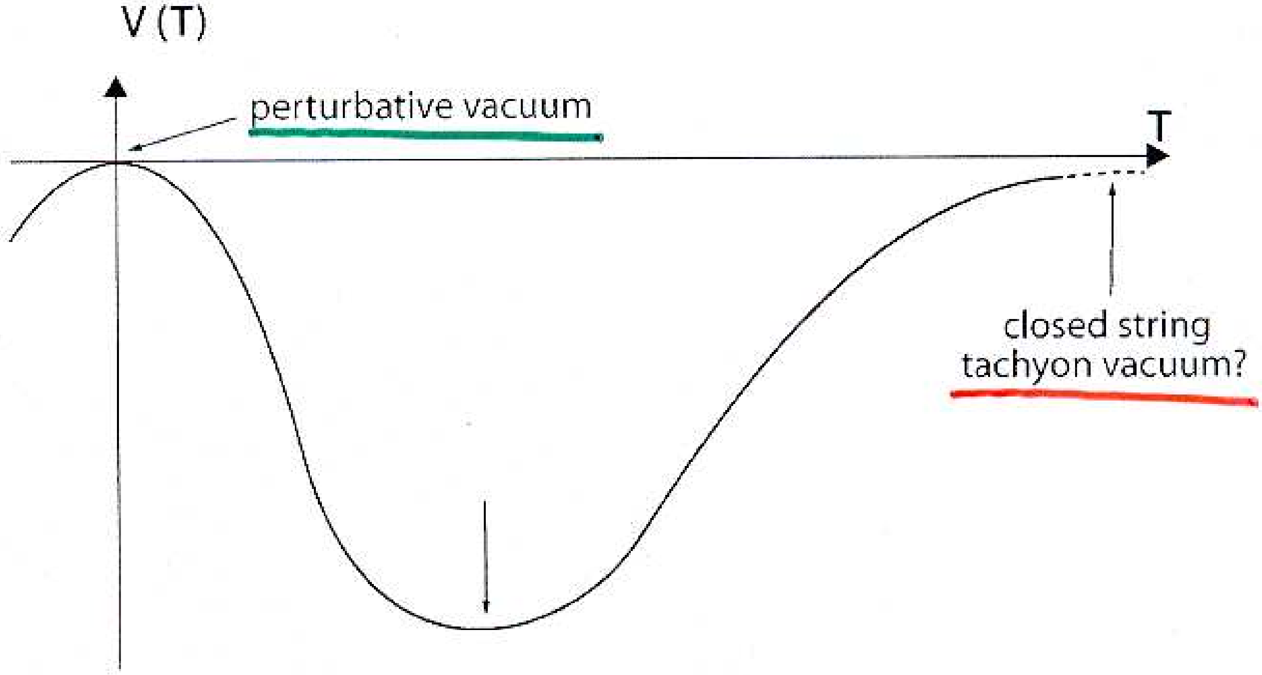
Gravity equation:

$$\tfrac{1}{2}(d-1)\,\dot H = -\tfrac{1}{2}\dot T^2 + \ddot\Phi - H\,\Phi\,.$$

We also find some weak evidence that the critical point is becoming more shallow.

In fact, the effective field theory EOM s, with constant vevs require that at the critical point:

$$V'(T_*) = 0, \qquad V(T_*) = 0$$



The perturbative vacuum is $T = 0$. The closed string tachyon vacuum would be the critical point with zero cosmological term, $T \to \infty$ (in CSFT this point corresponds to finite tachyon vev). A critical point with negative cosmological constant cannot provide a spacetime independent tachyon vacuum.

The universal closed string field relevant to this computation includes all scalars that are sourced by the tachyon.

These ghost number two states are built with the application of Virasoro matter operators $L_m, \bar{L}_m$, ghost, and antighost oscillators on the vacuum. They satisfy the conditions

1. The states are annihilated by $L_0 - \bar{L}_0$ and $b_0 - \bar{b}_0$.

2. The states are annihilated by $b_0 + \bar{b}_0$ (gauge fixing)

3. The states are even under world sheet parity (exchange of holomorphic and antiholomorphic labels).

   This gives (with level $\ell = L_0 + \bar{L}_0 + 2$.)

$$
\begin{aligned}
\longrightarrow \quad |\Psi_0\rangle \;=\;\; & t\, c_1\bar{c}_1|0\rangle && \text{level zero.}\\
& + d\,(c_1 c_{-1} - \bar{c}_1\bar{c}_{-1})|0\rangle && \text{level two}\\
& + \Big[f_1\, c_{-1}\bar{c}_{-1} + f_2\, L_{-2}c_1\,\bar{L}_{-2}\bar{c}_1 && \text{level four}\\
& + f_3\,(L_{-2}c_1\,\bar{c}_{-1} + c_{-1}\,\bar{L}_{-2}\bar{c}_1)\\
& + g_1\,(b_{-2}c_1\,\bar{c}_{-2}\bar{c}_1 \,-\, c_{-2}c_1\,\bar{b}_{-2}\bar{c}_1)\Big]|0\rangle\,.
\end{aligned}
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& + \Big[f_1\, c_{-1}\bar{c}_{-1} + f_2\, L_{-2}c_1\, \bar{L}_{-2}\bar{c}_1 && \text{level four} \\
& + f_3\,(L_{-2}c_1\bar{c}_{-1} + c_{-1}\,\bar{L}_{-2}\bar{c}_1) \\
& + g_1\,(b_{-2}c_1\,\bar{c}_{-2}\bar{c}_1 \;-\; c_{-2}c_1\,\bar{b}_{-2}\bar{c}_1)\Big]|0\rangle\,.
\end{aligned}
$$

We calculate the potential $\kappa^2 V$ to various levels and determine the critical point and its depth (the action density).

$\mathbb{V}_{12}^{(3)}$ includes quadratic and cubic interactions of all fields ($\ell \leq 4$).

$\mathbb{V}_{0}^{(4)}$ includes the above plus level zero quartic interactions. t^4

$\mathbb{V}_{2}^{(4)}$ includes level two quartic interactions. dt^3

$\mathbb{V}_{2}^{(4)}$ includes level four quartic interactions. $M_4 t^3,\ d^2 t^2$

Potential	t	d	Action density
$\mathbb{V}_{12}^{(3)}$	0.4371	0	−0.06338
$\mathbb{V}_{0}^{(4)}$	−−	−−	−−
$\mathbb{V}_{2}^{(4)}$	0.33783	0.4924	−0.05806
$\mathbb{V}_{4}^{(4)}$	0.2423	0.4596	−0.03382

The magnitude of the action density becomes smaller as we begin to include the effects of quartic couplings.

Points to note:

1. In this universal solution the string metric does not change.

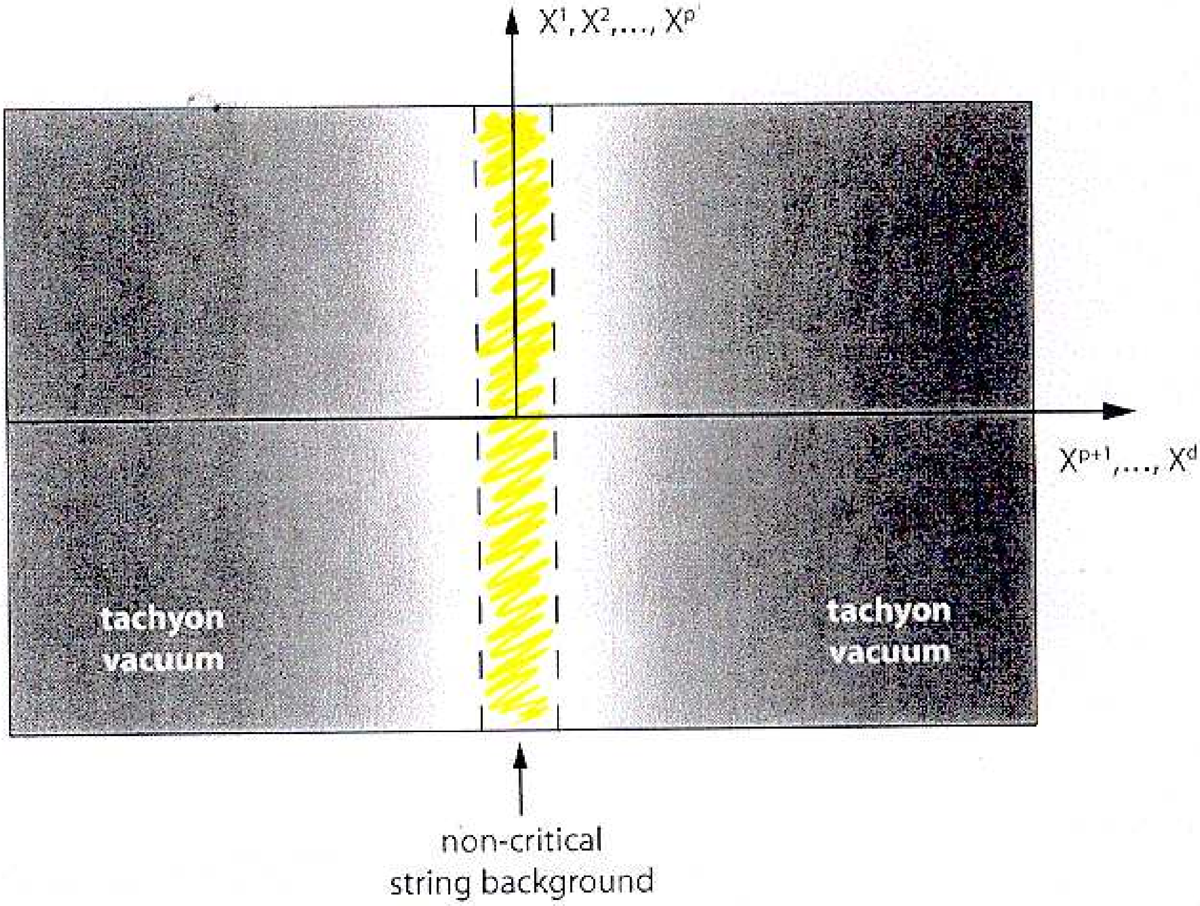
2. The value of the dilaton d corresponds to stronger coupling.

These two facts agree with the properties of the rolling solution.

Concern: The numerical evidence for a nontrivial critical point with zero action is still weak.

Indirect support for the existence of a tachyon vacuum.

1) The existence of sub-critical string theories.



A sub-critical string theory would correspond to a solitonic solution of critical string theory in which, the fields approach the closed string tachyon vacuum asymptotically .

This idea appears to work literally within p-adic open/closed string theory (Moeller and Schnabl, 2003).

2) Sigma model arguments (Tseytlin (2001), Andreev (2003)) suggest that the effective action for the tachyon has a prefactor e^{-T}. A tachyon potential of the form $-T^2 e^{-T}$ is thus natural, and would have a tachyon vacuum at $T \to \infty$.

Conclusions/Open Questions/Speculation

- Effective field theory suggests that a rolling closed string tachyon may crunch the spacetime.
- We have presented some evidence for the identification of a tachyon vacuum using CSFT.
- Additional computations needed to make the case stronger.
- Must develop suitable low-energy actions for the tachyon, dilaton, and gravity to explore the construction of sub-critical strings.
- Could closed string tachyons play a role in cyclic universe scenarios ? How to produce the transition from a big crunch to a big bang ?
- If the vacuum of the bulk closed string tachyon truly represents the demise of fluctuating spacetime, could it represent the state of a universe before the Big Bang ?