A NEW INFINITE CLASS OF SUSY AdS_3 SOLUTIONS

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Interesting development in AdS/CFT was the discovery of $Y^{p,q}$.

JPG, Martelli, Sparks, Waldram

- Infinite # of $D=5$ Sasaki-Einstein metrics. $p>q>0 \quad p|q=1$
- Infinite # of type IIB Solutions

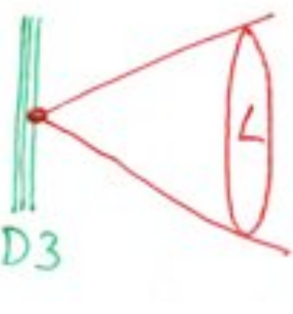
$$\begin{cases} ds^2 = ds^2(AdS_5) + ds^2(Y^{p,q}) \\ F_5 \neq 0 \end{cases}$$

- Infinite # $N=1$ SCFT's in $D=4$

L is S.E. $\Leftrightarrow$ cone $X \cong \mathbb{R}^+ \times L$

$$ds^2(X) = dr^2 + r^2 ds^2(L)$$

is Ricci-flat Kähler (Calabi-Yau)



- $Y^{p,q}$ generalised to $L^{a,b,c}$ Cvetic, Lu Page, Pope

- Dual SCFTs identified

  Bertolini, Bigazzi, Cotrone
  Benvenuti, Franco, Hanany, Martelli, Sparks

  - quiver gauge theories
  - periodic tilings, dimer models toric geometry

  Hanany et al

- Key tool in making detailed checks is "a-maximisation" Intriligator & Wecht

$$a \sim \frac{1}{\mathrm{Vol}(S.E.)}$$

- Geometric formulation of a-maximisation
  Martelli, Sparks, Yau

  Any S.E. has a Killing "Reeb" vector

$$\xi = J(r\partial_r)$$

  Dual to R-symmetry of SCFT

- ξ satisfies a variational procedure that depends on the complex structure of the cone $X \cong \mathbb{R}^+ \times S.E.$

- Knowing ξ gives the volume of S.E.

$$\mathrm{Vol}(S.E.) = \mathrm{Vol}(\xi)$$

New obstructions for S.E. metrics

JPG, Martelli, Sparks, Yau

$X \simeq \mathbb{R}^{+} \times L$

2n dimensions, "Gorenstein singularity"
no-where vanishing (n,0) form Ω

Qn. Is there a <u>conical</u> CY metric on X?
$\Leftrightarrow$ is there a S.E. metric on L?

"Bishop obstruction"

- Take putative Reeb vector ξ
- Calculate $\mathrm{Vol}(\xi)$
- If $\mathrm{Vol}(\xi) > \mathrm{Vol}(S^{2n-1})$

then there is no S.E. metric on L with this Reeb vector.

"Lichnerowicz obstruction"

- Holomorphic function on X

$$\mathcal{L}_\xi f = \lambda i f$$

- If $\lambda < 1$ there is no SE metric on L with Reeb ξ.

- Simple
- powerful

eg $\mathbb{C}^4$ $\{z_1, z_2, z_3, z_4\}$

$$X: \quad z_1^2 + z_2^2 + z_3^2 + z_4^k = 0$$

* Singular at $z_i = 0$
* Gorenstein
* obvious $\mathbb{C}^*$ action $\rightarrow \xi$

$k=1 \quad X = \mathbb{C}^3 \rightarrow S^5$

$k=2 \quad X = \text{Conifold} \rightarrow T^{1,1}$

$k=3 \quad ?$

$k>3$ obstructed

Note: $Y^{p,q}$ cohomogeneity one $SU(2)\times U(1) \times U(1)$

L_K cohomogeneity one $SU(2) \times U(1)$

Similarly can show infinite number of 4-fold singularities are obstructed to having conical Ricci-flat Kähler metrics

$Y^{p,q}$ have led to many interesting results in physics & mathematics.

$\boxed{\text{Can we find other infinite families of susy AdS Solutions especially in string theory?}}$

Recall how $Y^{p,q}$ were found:

① Classified most general susy solutions of D=11 SUGRA

$$\left\{\begin{matrix} AdS_5 \underset{w}{\times} M_6 \\ G_4 \neq 0 \end{matrix}\right. \rightarrow ds^2 = w(y)\left[ds^2(AdS_5) + ds^2(M_6)(y)\right]$$

② Assumed M_6 was compact, complex

$\rightarrow \infty$ families of explicit solutions

topology:

$$\begin{array}{cccc} S^2 \to M_6 & S^2 \to M_6 & S^2 \to M_6 & S^2 \to M_6 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ KE_4 & S^2 \times S^2 & S^2 \times H^2 & S^2 \times T^2 \end{array}$$

Dim reduction IIA

T-duality IIB $\rightarrow AdS_5 \times Y^{p,q}$

Can we repeat the success of this systematic approach?

a) Have classified JPG, Martelli, Sparks, Waldram

$$\left.\begin{array}{l} AdS_5 \times_w M_5 \\ \text{Fluxes} \neq 0 \end{array}\right\} \text{ in type IIB}$$

- generalises S.E.
- interesting geometries
-

b) Have classified JPG, MacConamhna, Mateos Waldram

$$\left.\begin{array}{l} AdS_4 \times_w M_7 \\ AdS_3 \times_w M_8 \end{array}\right\} \ D=11 \text{ SUGRA}$$

$G_4 \neq 0$ Magnetic G-flux only

- interesting class of geometries

No new explicit solutions as yet...

New infinite classes of AdS_3 solutions of D=11 SUGRA + type IIB SUGRA

D=11:

$$\begin{cases} ds^2 = \omega\left[ds^2(AdS_3) + ds^2(M_8)\right] \\ G_4 = \text{electric} + \text{magnetic} \end{cases}$$

topology:

$$\begin{array}{ccc} S^2 \to & M_8 \\ & \downarrow \\ & KE_4 \times KE_2 \end{array} \qquad \begin{array}{ccc} S^2 \to & M_8 \\ & \downarrow \\ & KE_2 \times KE_2 \times KE_2 \end{array}$$

KE $\leftrightarrow$ Kähler
Einstein $(R_{ij} = \lambda g_{ij})$

$KE_2 \leftrightarrow S^2, T^2, H^2$

$KE_4^+ \leftrightarrow \mathbb{C}P^2, S^2 \times S^2, dP_k \quad k = 3, \dots, 8$

Tian + Yau

More precisely: take canonical line bundle over $KE_4 \times KE_2$ or $KE_2 \times KE_2 \times KE_2$ base + add a "point at infinity" to each of the $\mathbb{C}$ fibres $\to$ S^2 fibre

$\boxed{\begin{array}{l}\text{Find infinite \# of compact susy} \\ \text{Solutions for} \\ \quad (KE_4^+) \times S^2 \qquad S^2 \times S^2 \times S^2 \\ \quad (KE_4^+) \times T^2 \qquad S^2 \times S^2 \times T^2\end{array}}$

type IIB

For $KE_4^+ \times T^2$ & $S^2 \times S^2 \times T^2$ cases

Dimensional reduction $\to$ IIA
T-duality $\to$ IIB

$$\begin{cases} ds^2 = \omega\left[ds^2(AdS_3) + ds^2(M_7)\right] \\ F_5 \neq 0 \end{cases}$$

$\boxed{\begin{array}{l}\text{For each } KE_4^+ = \mathbb{C}P^2, S^2 \times S^2, dP_k \\ \exists \text{ an infinite \# of new compact } M_7 \\ \text{labelled by integers } n, p, q > 0.\end{array}}$

$p, q \leftrightarrow$ geometry of M_7
$n \leftrightarrow$ flux over five-cycles in M_7

Topology:

$$\begin{array}{ccc} S^2 \to B_6 & \qquad & S^1 \to M_7 \\ \downarrow & & \downarrow \\ KE_4^+ & & B_6 \end{array}$$

Comment: Exactly the same topology as D=7 Sasaki-Einstein spaces!

Comments

1) Only $F_5 \neq 0 \Rightarrow$ configuration is near horizon limit of D3-branes
eg D3-brane wrapping $\Sigma_2 \subset CY_4$?

2) What is dual SCFT ?

- (2,0) susy in $D=1+1$
- isometries of $M_7 \to$ global symmetries of SCFT.

eg $KE_4^+ = S^2 \times S^2 \longrightarrow SO(3) \times SO(3) \times U(1) \times U(1)$

$KE_4^+ = \mathbb{C}P^2 \longrightarrow SU(3) \times U(1) \times U(1)$