FROM SPACETIME TO WORLDSHEET: FOUR POINT CORRELATORS

RAJESH GOPAKUMAR

HRI, ALLAHABAD

BASED ON:

(i) hep-th/0606078

(JUSTIN R. DAVID + R.G)

(ii) hep-th/0504229 (R.G.)

ALSO O. AHARONY, Z. KOMARGODSKI & S. RAZAMAT

hep-th/0602226

IN THIS TALK:

(1) DESCRIBE A PRECISE PRESCRIPTION TO WRITE LARGE N FIELD THEORY AMPLITUDES AS CLOSED STRING CORRELATORS

$$\Big\langle \overbrace{\mathcal{O}_1(x_1)}^{\text{Gauge Inv.}}\, \mathcal{O}_2(x_2) \ldots\ldots \mathcal{O}_n(x_n) \Big\rangle^{\text{genus } g}_{\substack{\text{space}\\ \text{time}}} = \int_{\mathcal{M}_{g,n}} \Big\langle V_{x_1}(\xi_1) \ldots\ldots V_{x_n}(\xi_n) \Big\rangle_{\substack{\text{world}\\ \text{sheet}}}$$

(2) IMPLEMENT THIS PRESCRIPTION AND EXTRACT EXPLICIT CANDIDATE WORLDSHEET 4 PT. FUNCTIONS – FOR HIGHLY CURVED $AdS_5 \times S^5$.

e.g.

$$\Big\langle \mathrm{Tr}\, X^{J_1}(x_1)\, \mathrm{Tr}\, Y^{J_2}(x_2)\, \mathrm{Tr}\, Z^{J_3}(x_3)\, \mathrm{Tr}(\bar{X}^{J_1}\bar{Y}^{J_2}\bar{Z}^{J_3})(0) \Big\rangle$$

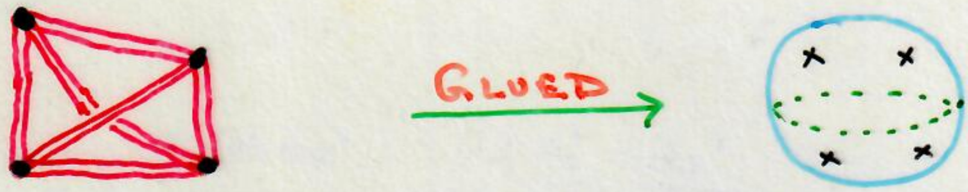
$$= \int d^2\eta \; G^{\{J_i\}}_{\{x_i\}}(\eta, \bar{\eta})$$

CROSS RATIO ON $\mathcal{M}_{0,4}$

* $G^{\{J_i\}}_{\{x_i\}}(\eta,\bar{\eta})$ SATISFIES CROSSING SYMMETRY ($\eta \to 1-\eta;\ \eta \to \frac{1}{\eta}$) AND HAS A SENSIBLE WORLDSHEET OPE ($h - \bar{h} \in \mathbb{Z}$)

* IN FACT, $G_{\cdot}(\eta,\bar{\eta})$ SEEMS LIKE A LEGITIMATE 2d CFT CORRELATOR — EXPRESSIBLE IN TERMS OF FOUR POINT FNS. OF ($c = \frac{1}{2}$) ISING MODEL!

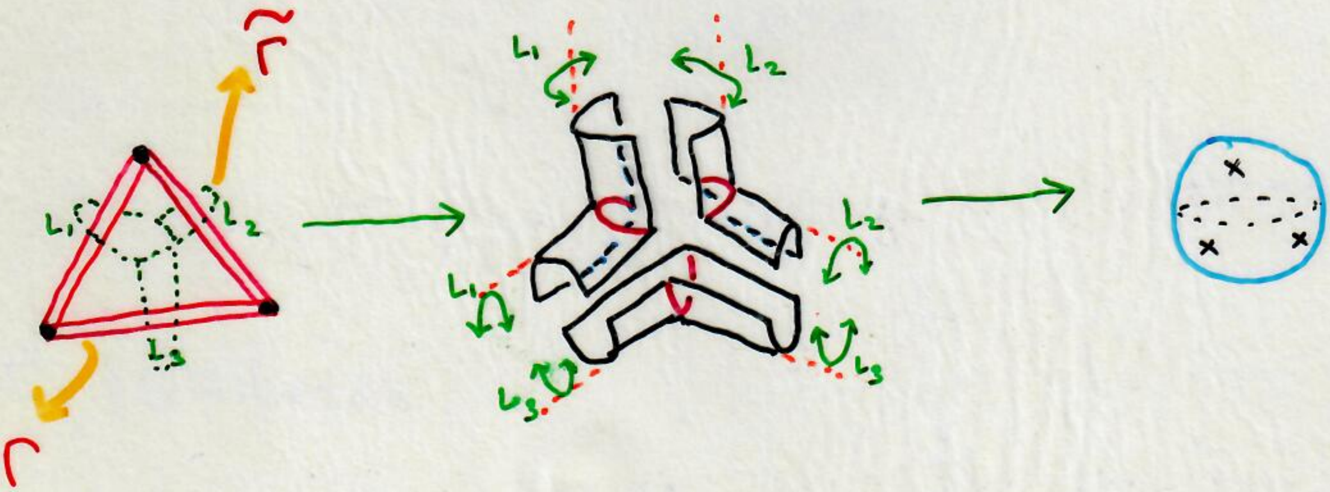
A MATHEMATICAL GLUING PROCEDURE



INPUT DATA: GENUS g
DOUBLE LINE GRAPH Γ
n VERTICES AND $l_r > 0$
FOR EACH EDGE.

OUTPUT: RIEMANN SURFACE
$\Sigma_{g,n} \in \mathcal{M}_{g,n}$
(and extra data $\in \mathbb{R}^n_+$)

ESSENTIALLY, ONE GLUES "OPEN STRING STRIPS" OF WIDTH L_r INTO CYLINDERS MEETING ON THE DUAL GRAPH $\tilde{\Gamma}$.



Key Ingredient In The Gluing: Strebel

Differentials – $\varphi_s(z)\,dz^2$ – Special Kind Of Holomorphic Quadratic Differential. Unique For $\Sigma_{g,n} \in \mathcal{M}_{g,n}$ + $\{p_a\} \in \mathbb{R}_+^n$

Residues

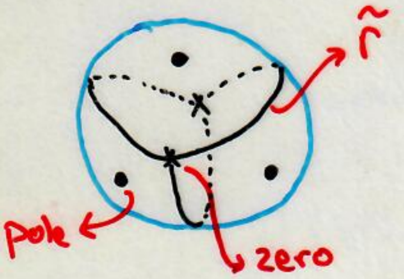
ⓐ Double Poles Near Punctures

$$\varphi_s(z)\,dz^2 \underset{z \to z_a}{\approx} -\frac{p_a^2}{(2\pi)^2}\frac{dz^2}{(z-z_a)^2}$$

ⓑ Strebel Lengths Between Zeroes Are Real

$$L_r = \int_{e_r} \sqrt{\varphi_s(z)}\,dz$$

In Fact, $\varphi_s(z) \longrightarrow$ Graph $\tilde{\Gamma}$ (with genus g + n vertices, and $L_r > 0$)



Obtained By Connecting Zeroes Of $\varphi_s(z)\,dz^2$ Along Trajectory Where $\sqrt{\varphi_s(z)}\,dz$ Is Real.

Each Face Of $\tilde{\Gamma}$ Contains A Double Pole.

In Terms Of The Locally Flat Metric $ds^2 = |\varphi_s(z)|\,dz\,d\bar{z}$, The Riemann Surface Built From A Bunch Of Strips (of width L_r) Glued Into Cylinders Meeting On $\tilde{\Gamma}$.

$$\boxed{\text{Varying } 0 < L_r < \infty \text{ And Graph Covers } \mathcal{M}_{g,n} \times \mathbb{R}_+^n}$$

THE PRESCRIPTION

FEYNMAN GRAPHS ARE GLUED AS ABOVE TO FORM WORLDSHEETS W/

$$\boxed{\sigma_r = \frac{1}{\tau_r} = L_r = \int_{e_r} \sqrt{\varphi_s(z, \eta_i, p_a)}\, dz}$$

residues

cplx. moduli

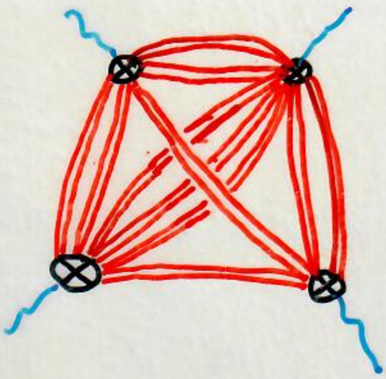
(INVERSE) SCHWINGER TIMES = STREBEL LENGTHS

THEREFORE, PROPER TIME INTEGRALS IN FIELD THEORY $\longrightarrow$ INTEGRALS OVER MODULI SPACE.

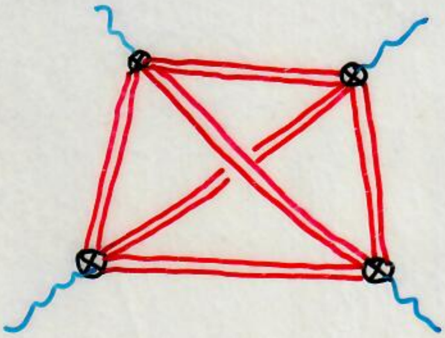
$\Rightarrow$ INTEGRANDS ARE WORLDSHEET CORRELATORS

USE THIS TO GLEAN INFORMATION ABOUT THE WORLDSHEET CFT.

IN FACT, A NON-TRIVIAL CHECK OF THE PROPOSAL FOR THIS COMPLICATED CHANGE OF VARIABLES ON FIELD THEORY AMPLITUDES TO YIELD SOMETHING WHICH MIGHT BE A 2d CFT CORRELATOR.



Start w/ Feynman Graph Of Genus g w/ n Vertices, In Schwinger Parametrised Form.

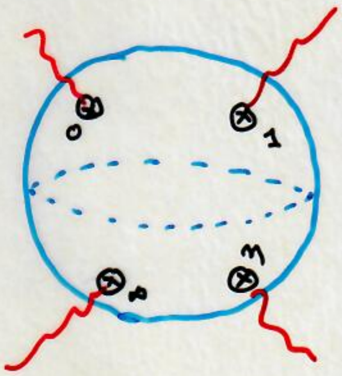


Partially Glue Into A Skeleton Graph - All Homotopic Wick Contractions Glued Together

$$\sigma_r = \sum_i \sigma_{r(i)}$$

Homotopic Edges.

Matches The Additivity Of Strebel Lengths L_r.



Carry Out Loop Integrals

$$\sum_{\text{graphs}} \int \prod_r d\sigma_r \, I^{\{I_i\}}(\sigma_r, x_i)$$

Use Strebel Dictionary

$$= \int_{\mathcal{M}_{g,n} \times \mathbb{R}_+^n} \prod_i d^2\eta_i \prod_a dp_a \, K^{\{I_i\}}_{\{x_i\}}(\eta_i, p_a) \cdot$$

Integrate p_a

$$= \int_{\mathcal{M}_{g,n}} \prod_i d^2\eta_i \, \underbrace{G^{\{I_i\}}_{\{x_i\}}(\eta_i, \bar{\eta}_i)}$$

Candidate worldsheet correlator.

Implementing The Prescription

Non-Trivial Candidate Correlators Arise Even For Planar Four Point Functions In Free Field Limit – Highly Curved AdS.

$\Gamma =$  Six Schwinger Parameters $\sigma_{(ij)} = L_{(ij)}$

In Correspondence w/ η, $\{p_a\}$ $(a=1,\dots 4)$

Dictionary via Strebel Differential For Four Punctured Sphere

Four Zeroes (vertices of $\tilde{\Gamma}$)

$$\varphi_s(z)\,dz^2 = -\frac{C\,(z^2-1)(z^2k^2-1)\,dz^2}{\prod_{a=1}^{4}(z-z_a)^2}$$

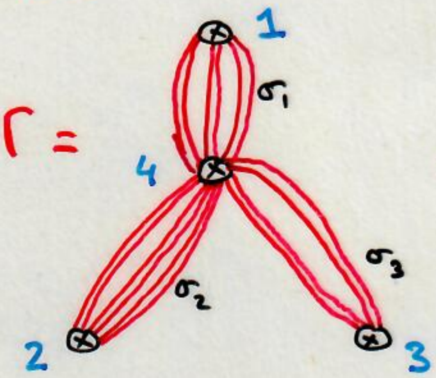
Four Double Poles (vertices of Γ)

Note That Unique Strebel Differential Fixed By $\eta = \frac{z_1-z_2}{z_1-z_3}\cdot\frac{z_3-z_4}{z_2-z_4}$ And Residues $\{p_a\}$ $(a=1\dots 4)$.

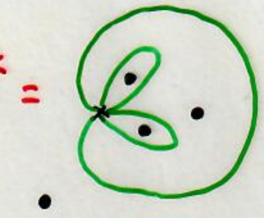
$$\sigma_{(ij)} = L_{(ij)} = \int_{z_i}^{z_j}\sqrt{\varphi_s(z,\eta,p_a)}\,dz$$

Transcendental Relation. But Can Be Carried Out In Terms Of Elliptic Function. Though Not Explicit Enough At The Moment.

SIMPLIFICATION

$\Gamma =$ 

DUAL

$\tilde{\Gamma} =$ 

CONSIDER SPECIAL FEYNMAN GRAPHS (Aharony, Komargodski + Ragamat)

ZEROES OF $\varphi_s(z)\,dz^2$ COINCIDE – FOURTH ORDER ZERO. RELATION BETWEEN $\sigma_{(ij)}$ AND η BECOMES ALGEBRAIC.

$$\varphi_s(z)\,dz^2 = -\frac{C\,(z-a)^4\,dz^2}{\prod_{a=1}^{4}(z-z_a)^2}$$

SOLVE FOR $\eta(\sigma)$.

$$\eta = \left(\frac{\sqrt{(\sigma_1+\sigma_2+\sigma_3)\sigma_2} + i\sqrt{\sigma_1\sigma_3}}{\sigma_1+\sigma_2}\right)^2$$

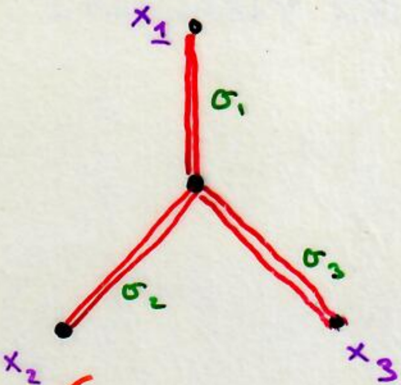
$\Rightarrow$

$$\sigma_1 = a\left(1 - |\eta| + |1-\eta|\right)$$

$$\sigma_2 = a\left(1 + |\eta| - |1-\eta|\right)$$

$$\sigma_3 = a\left(-1 + |\eta| + |1-\eta|\right)$$

CAN DIRECTLY APPLY THIS TO CORRELATORS



LIKE $\left\langle \mathrm{Tr}\, X^{J_1}(x_1)\, \mathrm{Tr}\, Y^{J_2}(x_2)\, \mathrm{Tr}\, Z^{J_3}(x_3)\, \mathrm{Tr}\,(\bar{X}^{J_1}\bar{Y}^{J_2}\bar{Z}^{J_3})(0) \right\rangle$

WHICH GET FREE FIELD CONTRIBUTION

ONLY FROM Y DIAGRAM

$$\frac{1}{x_1^{2J_1}\, x_2^{2J_2}\, x_3^{2J_3}} = \int d\sigma_1\, d\sigma_2\, d\sigma_3\, \sigma_1^{J_1-1}\sigma_2^{J_2-1}\sigma_3^{J_3-1}\, e^{-(\sigma_1 x_1^2 + \sigma_2 x_2^2 + \sigma_3 x_3^2)}$$

THE Y DIAGRAM IS ALSO ONE OF THE CONTRIBUTIONS TO THE EXTREMAL CORRELATOR

$$\left\langle \mathrm{Tr}\, Z^{J_1}(x_1)\, \mathrm{Tr}\, Z^{J_2}(x_2)\, \mathrm{Tr}\, Z^{J_3}(x_3)\, \mathrm{Tr}\, \bar{Z}^{J}(0) \right\rangle$$

(d'Hoker, Freedman, Mathur, Matusis, Rastelli)

WHICH ARE NOT RENORMALISED FROM THE FREE FIELD VALUE.

(Additional contributions being evaluated by Aharony et. al.)

INTERESTING CLASS FOR WHICH TO KNOW CANDIDATE WORLDSHEET ANSWER.

IN ANY CASE, THE CORRELATOR

$$\int d\sigma_1 d\sigma_2 d\sigma_3\, \sigma_1^{J_1-1} \sigma_2^{J_2-1} \sigma_3^{J_3-1}\, e^{-(\sigma_1 x_1^2 + \sigma_2 x_2^2 + \sigma_3 x_3^2)}$$

$$= \int d^2\eta\, G^{\{J_i\}}_{\{x_i\}}(\eta,\bar\eta)$$

(USING ABOVE CHANGE OF VARIABLES AND INTEGRATING OVERALL $\mathbb{R}_+$)

WHERE

$$\boxed{G^{J_1 J_2 J_3}_{x_1 x_2 x_3}(\eta,\bar\eta) = \frac{\left[1+|\eta|+|1-\eta|\right]^{1/2}}{|\eta||1-\eta|} \times \frac{\left[1-|\eta|+|1-\eta|\right]^{J_1-1/2}\left[1+|\eta|-|1-\eta|\right]^{J_2-1/2}\left[-1+|\eta|+|1-\eta|\right]^{J_3-1/2}}{\left[x_1^2(1-|\eta|+|1-\eta|)+x_2^2(1+|\eta|-|1-\eta|)+x_3^2(-1+|\eta|+|1-\eta|)\right]^{J}}}$$

$(J = J_1+J_2+J_3)$

THIS IS THE CANDIDATE WORLDSHEET CORRELATOR FOR

$$\left\langle V^{J_1}_{x_1}(0)\, V^{J_2}_{x_2}(1)\, V^{J_3}_{x_3}(\infty)\, \nabla^{J}(\eta,\bar\eta)\right\rangle_{\text{world sheet}}$$

DOES IT HAVE THE FEATURES EXPECTED OF A WORLDSHEET CORRELATOR ?

$G^{\{J_i\}}_{\{x_i\}}(\eta,\bar\eta)$ HAS A NUMBER OF NICE FEATURES:

★ OBEYS CROSSING SYMMETRY

(i) $G^{J_2 J_1 J_3}_{x_2 x_1 x_3}(1-\eta,\, 1-\bar\eta) = G^{J_1 J_2 J_3}_{x_1 x_2 x_3}(\eta,\bar\eta)$

(exchanging labels $1 \leftrightarrow 2$)

(ii) $G^{J_3 J_2 J_1}_{x_3 x_2 x_1}\left(\frac{1}{\eta}, \frac{1}{\bar\eta}\right) = |\eta|^4\, G^{J_1 J_2 J_3}_{x_1 x_2 x_3}(\eta,\bar\eta)$

(exchanging $1 \leftrightarrow 3$)

CHANGE OF VARIABLES FROM σ TO η HAS CROSSING SYMMETRY BUILT IN.

THE WEIGHT $|\eta|^4$ CORRESPONDS TO $(h,\bar h) = (1,1)$

★ HAS AN EXPANSION IN η CONSISTENT w/ A STRING THEORY.

$$G^{\{J_i\}}_{\{x_i\}}(\eta,\bar\eta) = \sum_{h,\bar h} C^{\{J_i\}}(x_i)\,\eta^h\,\bar\eta^{\bar h}$$

HERE $h, \bar h \in \mathbb{Z}+1/2$ BUT $(h-\bar h) \in \mathbb{Z}$.

(Checked at leading order by Aharony et. al. Here manifest to all orders).

☆ FORM OF $G^{\{J;\}}_{\{x;\}}(\eta, \bar{\eta})$ SUGGESTIVE OF A CORRELATOR IN A 2d CFT.

IN FACT, BUILDING BLOCKS SEEM TO BE ISING MODEL CORRELATORS.

$$\langle \sigma(1)\,\sigma(2)\,\sigma(3)\,\sigma(4) \rangle \propto \frac{1}{|\eta|^{1/2}\,|1-\eta|^{1/2}} \times \left[1 + |\eta| + |1-\eta|\right]^{1/2}$$

$$\langle \sigma(1)\,\mu(2)\,\sigma(3)\,\mu(4) \rangle \propto \frac{1}{|\eta|^{1/2}\,|1-\eta|^{1/2}} \left[-1 + |\eta| + |1-\eta|\right]^{1/2}$$

PERMUTATIONS OF LOCATIONS OF DISORDER OPERATORS μ GIVES RISE TO THE OTHER TWO COMBINATIONS: $\left[1 - |\eta| + |1-\eta|\right]^{1/2}$ AND $\left[1 + |\eta| - |1-\eta|\right]^{1/2}$.

PERHAPS THE APPEARANCE OF ISING CORRELATORS IS A REFLECTION OF HIGHER SPIN SYMMETRY IN SPACE TIME. (cf. W_3 SYMMETRY IN 2d NONCRITICAL STRING FOR ISING MODEL.)

SO GOOD REASON TO TAKE $G^{\{J_i\}}_{\{x_i\}}(\eta,\bar{\eta})$ SERIOUSLY AS A WORLDSHEET CORRELATOR.

⇝ CONFORMAL BLOCKS OF ISING MODEL CORRELATORS LEADS TO FACTORISED $\sum |\ |^2$ STRUCTURE FOR $G^{\{J_i\}}_{\{x_i\}}(\eta,\bar{\eta})$. USE THIS TO STUDY THE CFT.

⇝ NEED MORE EXAMPLES SUCH AS EXTREMAL CORR.

$$\langle \mathrm{Tr}\, Z^{J_1}\, \mathrm{Tr}\, Z^{J_2}\, \mathrm{Tr}\, Z^{J_3}\, \mathrm{Tr}\, \bar{Z}^{J_1+J_2+J_3} \rangle$$

WHICH CAN BE COMPARED TO LARGE λ RESULTS.

⇝ NEED TO ALSO UNDERSTAND BETTER EXAMPLES WHERE CORRELATOR SEEMS LOCALISED ON A SUBSPACE OF $\mathcal{M}_{0,4}$ e.g. $\langle (\mathrm{Tr}\, \Phi^2)^4 \rangle$ (Aharony et.al.)

⇝ ALSO NEED TO UNDERSTAND CHOICE OF WORLDSHEET GAUGE WHICH MAKES SPACETIME SPECIAL CONFORMAL SYMMETRIES NON-MANIFEST ON THE WORLDSHEET.

⇝ SIMILARITIES AND DIFFERENCES WITH $AdS_3 \times S^3$ CORRELATORS.

SUMMING UP:

- WELL DEFINED PRESCRIPTION THAT TRANSFORMS SPACETIME AMPLITUDES INTO WORLDSHEET CORRELATORS.

- YIELDS VERY SUGGESTIVE CANDIDATE CORRELATORS FOR FOUR POINT FUNCTIONS.

- TASK IS TO RECONSTRUCT WORLDSHEET CFT FOR AdS_5 (at least in tensionless limit) FROM THESE CLUES.

(Parallel to early days - before dual models became string theory)