Exploring the Kähler potential

Michael R. Douglas
Rutgers and IHES

Strings 2007, Madrid

Abstract

Based on hep-th/0606261, 0704.4001 and to appear with:

Robert Karp
Semen Klevtsov
Sergio Lukic
Rene Reinbacher
Jessie Shelton
Gonzalo Torroba

1. Introduction

A central problem of fundamental physics:
From more fundamental (shorter distance) theory,

- Gauge theory (with specified matter, couplings)
- String compactification ,

derive effective long distance theory:

- low energy degrees of freedom
- effective potential and couplings

In supersymmetric theory, this amounts to

- gauge group and chiral multiplets
- superpotential and Kähler potential (and FI terms).

Example: $N = 1$ super Yang-Mills theory

$$\mathcal{L} = \tau \int d^2\theta \ \mathrm{tr}\ W_\alpha^2 + \mathrm{c.c.}$$

$$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$$

$\longrightarrow$

$$S \equiv \mathrm{tr}\ W_\alpha^2 = \mathrm{tr}\ \chi_\alpha \chi^\alpha + \dots \qquad \text{gaugino condensate}$$

$$W_{V-Y} = \tau S + S \log \frac{S^N}{\Lambda^{3N}\ e^N} \qquad \begin{array}{c}\text{Veneziano} - \text{Yankielowicz} \\ \text{superpotential}\end{array}$$

argument: look at $U(1)_R$ anomaly and phase rotation of S.

Describes N supersymmetric vacua $\partial W = 0$, with exponentially small gaugino condensate.

$$S = \Lambda^3 \exp - \frac{\mathrm{Im}\ \tau}{N}$$

Many, many generalizations: just for example,

- Matrix model solution for general gauge theories (Dijkgraaf-Vafa, CDSW)

$$W_{eff} = W_{V-Y} + \int_{S=\oint \mathrm{Tr}\ \frac{dz}{z-\phi}} [D\phi]\ e^{\hat{N}W_{bare}}$$

Justified using generalized Konishi anomaly.

Large N limit of matrix integral $\rightarrow$ (perturbative). planar diagram expansion. Minimizing in S reproduces the (non-perturbative) instanton expansion.

- Flux superpotential.

  Compactify IIb string on Calabi-Yau M.

$$W_{flux} = \int_M \Omega \wedge (F^{(3)} - \tau H^{(3)}) = \sum \Pi_i \left(N^i_{RR} - \tau N^i_{NS}\right).$$

Flux superpotential near conifold point:

- $\Pi_A = S$ volume of three-cycle

- $\Pi_B = S \log S + \mathrm{const} + \ldots$ volume of conjugate cycle

so $W_{flux} = W_{V-Y}$ in this case.

Many arguments that this is the gravity dual to pure SYM gauge theory (we review Klebanov-Strassler later).

2. Kähler potential

Much less is known. Details not needed to find global supersymmetric vacua, but needed

- to get normalized couplings and masses

$$\int d^4\theta K(\phi,\bar\phi) = g_{i\bar j}\partial_\mu\phi^i\partial_\mu\bar\phi^{\bar j} + \ldots$$

- to find supersymmetry breaking vacua, and scale of breaking.

$$\begin{aligned} V_{global} &= g^{i\bar j}\partial_i W \partial_{\bar j} W^* \equiv g^{i\bar j} F_i \bar F_{\bar j} \\ M_{3/2} &= \frac{M^2_{susy}}{M_{Pl}} = \frac{|F|}{M_{Pl}} \\ V' = 0 &\leftrightarrow Z^{\bar j k}\bar F_{\bar j} = 0; \qquad Z_{jk} = D_j\partial_k W \end{aligned}$$

- in supergravity:

$$V = e^K\left(g^{i\bar j}D_i W D_{\bar j} W^* - 3\frac{|W|^2}{M^2_{Pl}}\right)$$

where $D_iW = \partial_i W + \frac{\partial_i K}{M_{Pl}} W$.

More generally, metric on configuration space is just as important as potential energy, in forming picture of "landscape."

- distances are important for understanding dynamics, especially inflation:

  $$N_{e-folds} = \int \frac{d\phi}{V'(\phi)} g_{\phi\phi}$$

  Lyth bound: tensor/scalar perturbations $\leq$ distance travelled by slow roll. For more, see talk here by Kallosh.

- volume of region (in Planck units):
  - estimate for number of vacua of generic supergravity potential.
  - possible contribution to measure factor (Horne and Moore 94)

What are simple examples where we know metric?

- weak coupling gauge theory – inherited from bare metric, determined to be canonical metric by renormalizability.

- $N = 2$ supersymmetry, for which the vector multiplet metric is determined by a holomorphic prepotential $\mathcal{F}$ as

$$K = \sum_i \bar{\phi}^{\bar{i}} \partial_i \mathcal{F}(\phi) + \text{c.c.}$$

These include $N = 2$ gauge theory and $N = 2$ string compactifications.

The basic example is $U(1)$ gauge theory with charged matter, for which

$$\mathcal{F} = \frac{c}{2} a^2 + \frac{Q^2}{2\pi i} a^2 \log \frac{a}{\Lambda} K = c a \bar{a} + \frac{Q^2}{2\pi} |a|^2 \log \frac{|a|^2}{\Lambda^2}$$

reflecting the 1-loop beta function.

The same prepotential (with $a \to S$ and $Q = 1$) describes the conifold limit, since wrapped D3's provide a charged hypermultiplet.

3. Simple model of susy breaking

$N = 1$ super Yang-Mills theory has many embeddings into string theory. A simple choice is to start with $N = 2$ compactification of IIb, and wrap N D5-branes on a rigid two-cycle (so, no brane moduli and no chiral multiplets). As before, this theory has N supersymmetric vacua.

What if we take N wrapped anti D5-branes? In an $N = 2$ compactification, these are also supersymmetric, preserving a different $N = 1$ subalgebra of $N = 2$ supersymmetry. The resulting SYM field theory is the same. By combining branes and antibranes, or orientifold planes and antibranes, we can break supersymmetry.

One advantage of using D5 instead of D3 branes is that if they wrap a rigid cycle, they have no further moduli to stabilize. A concrete example of a model breaking supersymmetry in this way (combining D5 and anti-D5-branes) was given by Diaconescu, Garcia-Raboso and Sinha in hep-th/0602138.

What is physics, *e.g.* scale of susy breaking? What is the effective potential?

A simple guess: flip sign of N in V-Y superpotential:

$$W_{anti\ V-Y} = \tau S - NS \log \frac{S}{\Lambda^{3N}\ e^N}$$

Although a little strange from gauge theory point of view, this makes sense in the gravity dual, as the dual to antibranes is antiflux.

This was proposed and analyzed by Aganagic et al 0610249. The equation $\partial W = 0$ still has a solution,

$$S = \Lambda^3 \exp + \frac{\text{Im}\ \tau}{N} \ >> M_{Pl}^3,$$

however this is outside the regime of validity of the conifold expression.

On the other hand, taking $K = |S|^2 \log |S|/\Lambda^3$ for the conifold (as we argued earlier using $N = 2$ susy), the potential

$$V_{main} = g^{S\bar{S}} |\partial_S W|^2 = \frac{|N \log S + \beta|^2}{|\log |S|^2|}$$

has an $N = 1$ supersymmetry breaking minimum at

$$S \sim \Lambda^3 \exp - \frac{\text{Im}\ \tau}{|N|}.$$

So, we seem to have a very simple way to break susy in string theory. Furthermore, the scale $S \sim \Lambda^3 \exp -\frac{\mathrm{Im}\ \tau}{|N|}$ is low.

However, we still need to compute the actual scale of susy breaking. It is the norm of

$$F_S = \partial_S W \sim N \log S + \mathrm{Im}\ \tau \sim 2\mathrm{Im}\ \tau \sim \mathcal{O}(1),$$

using the inverse metric $g^{S\bar{S}} \sim 1/|\log |S|^2| \sim N/\mathrm{Im}\ \tau$. Thus

$$M^4_{susy} = g^{S\bar{S}} |F_S|^2 \sim N \mathrm{Im}\ \tau$$

and the breaking is at a high scale, despite the exponentially small value of S.

Putting the supergravity corrections into V does not change this. For example, the e^K prefactor is dominated by the constant term in K, which is set by bulk physics (the volume of the CY).

Actually this result is reasonable: $N \mathrm{Im}\ \tau \sim N/g_s$ is the expected tension for N anti D5-branes.

So we get high scale breaking. Is this correct, or is 4d effective theory is not applicable (in which case what is?), or did we make a mistake in using it?

The basic problem is that the effective potential we used, constructed using the large volume Kähler potential, does not take into account the effects of warping. From the gravity point of view, the warping is essential to reduce the scale of susy breaking from the antibrane tension to a low scale. While it is formally correct to leave this out in the large volume limit (since fluxes are quantized in units of α'), we lose the possibility of low scale breaking.

One can do better by using a Kähler metric and Kähler potential which takes warping into account (Giddings and collaborators, 2003–2005). Instead of the Kähler potential of Calabi-Yau special geometry,

$$K = -\log i \int_M \Omega \wedge \bar{\Omega},$$

we need to use

$$K = -\log i \int e^{-4A} \Omega \wedge \bar{\Omega},$$

where the warped metric is

$$ds^2 = e^{2A} dx^2_{Mink} + e^{-2A} g_{mn} dy^m dy^n$$

and Ω is the holomorphic three-form. The simplest argument for this is that the contribution to the e^K factor $1/(\text{volume of } CY)^2$ turns into the warped CY volume.

$$K = -\log i \int e^{-4A} \Omega \wedge \bar{\Omega},$$

Actually, there are questions regarding this claim, which are not fully addressed in the literature. For example, there is an independent computation of the modified Kähler metric on complex structure moduli space,

$$G_{\alpha\bar{\beta}} = \int e^{-4A} \chi_\alpha \wedge \bar{\chi}_{\bar{\beta}},$$

where χ_α is a basis of $(2,1)$ forms. Consistency of 4d supergravity requires that

$$G_{\alpha\bar{\beta}} = \partial_\alpha \bar{\partial}_{\bar{\beta}} K,$$

but this is not very obvious, as the two sides appear to differ by factors like

$$\int \Omega \wedge \bar{\Omega} \ \partial_\alpha \bar{\partial}_{\bar{\beta}} e^{-4A}.$$

And since the factor e^{-4A} is determined by an equation of motion

$$\Delta(e^{-4A}) = G^2 + \text{other sources},$$

it depends on the moduli.

Another general question (Burgess et al 2006) is that in this warped background, one can expect to find KK modes at the scales we are discussing, so it is not obvious that we can integrate them out.

4. Ten dimensional analysis

Because of these questions, we studied the problem in 10d as well as in 4d. This is based on the Klebanov-Strassler solution, which is somewhat complicated, for example

$$e^{-4A} = \frac{\alpha 2^{2/3}}{4} \int_{\log r} dx \frac{x \coth x - 1}{\sinh^2 x} (sinh(2x) - 2x)^{1/3}$$
$$\sim c + \alpha \left(\frac{g_s N}{2\pi} \right)^2 \frac{\log(r/|S|^3)}{r^4}.$$

One point which is easy to see from the KS solution, is that flipping $N \to -N$ does give a new solution, as it depends only on N^2 (except in the RR fluxes themselves). This reflects the fact that $N < 0$ preserves a different $N = 1$ subalgebra, and tells us that the supersymmetry breaking solution will be stable (at least in 10d supergravity).

One can now compute the coupling of the fermionic component of the S field to the bulk $N = 1$ gravitino, and see that this is supported down in the throat. Thus, the warp factor $1/r^4$ suppresses this coupling, leading to low scale breaking.

Actually, the content of this computation can be translated into 4d effective field theory terms. The 10d modes corresponding to the S field are constructed from the corresponding $(2,1)$ form,

$$\chi_S = \omega_3 - 2i\frac{dr}{r} \wedge \omega_2,$$

where ω_3 and ω_2 are volume forms on S^3 and S^2. Its coupling to the gravitino is found by normalizing it, and computing its integral against the flux G. Since the flux sits in the GVW superpotential, while the correct normalization is obtained by computing the norm

$$g_{S\bar{S}} = \int e^{-4A} \chi_S \wedge \bar{\chi}_S,$$

the result is simply the standard formula

$$m^2_{3/2} \sim g^{S\bar{S}} |D_S W|^2$$

which would be obtained by granting the validity of 4d effective supergravity.

Then, after some work, we have demonstrated in some generality that

$$G_{\alpha\bar{\beta}} = \partial_\alpha \bar{\partial}_{\bar{\beta}} K,$$

so we have addressed at least the most obvious objection to 4d effective supergravity. One might thus claim that the effective potential at a critical point and the supersymmetry breaking parameters will satisfy the constraints of 4d $N = 1$ supergravity in these problems. This seems at first reasonable since these are zero momentum quantities which can be defined by integrating out all the other fields.

However, having continued with the explicit computation of the potential by KK reduction, we have some doubts about the exactness of this formula, even in the supergravity limit $g_s, \alpha' \to 0$ and the flux G fixed. The existing derivation makes the assumption that the fluxes are imaginary self-dual or anti-self-dual.

We are studying the perturbation theory around these limits and (in our work so far, which is not yet complete) we find corrections to the DeWolfe-Giddings formula, however they have the same dependence on the warping as the original formula.

Thus, let us now assume that the essential part of the 10d computations can be reduced to an $N = 1$ 4d effective field theory, with the "flipped" (IASD) superpotential

$$W = N(c + S \log S) + \beta S + \ldots$$

the warped Kähler potential,

$$K = -\log i \int e^{-4A} \Omega \wedge \bar{\Omega},$$

and the corresponding metric

$$G_{\alpha\bar{\beta}} = \partial_\alpha \bar{\partial}_{\bar{\beta}} K = \int e^{-4A} \chi_\alpha \wedge \bar{\chi}_{\bar{\beta}}.$$

The presence of the warp factor means that the last of these can be computed to a good approximation just by integrating over the throat,

$$G_{S\bar{S}} \sim \int [a_0 + b_0 (g_s N)^2 \frac{\log(r/|S|^3)}{r^4}]\, \omega_3 \wedge \frac{dr}{r} \wedge \omega_2 \,.$$

Integrating over the angular variables and over $|S|^{1/3} \leq r \leq \Lambda_0$, we obtain

$$G_{S\bar{S}} \approx c_1 \log \left| \frac{\Lambda_0^3}{S} \right|^2 + c_2 (g_s N)^2 \frac{1}{|S|^{4/3}} . \qquad (1)$$

The new term $\sim |S|^{-4/3}$ is quite a surprise, since such singularities are in general forbidden in moduli spaces with special geometry. Note that in the regime of interest $|S| \to 0$, this extra term dominates the behavior of $G_{S\bar{S}}$.

It can be checked by computing K and differentiating as well. In this computation, it is a term

$$K = \ldots + (g_s N)^2 |S|^{2/3}.$$

Amusingly, such a term was suggested in the original work of Veneziano and Yankielowicz, since the gaugino condensate S has canonical dimension 3. Of course K gets large corrections and it is not really known what it looks like in weakly coupled gauge theory. But it comes naturally out of supergravity; actually it is just the warped volume at the end of the throat,

$$\text{warped volume} \sim \int dr r^5 \, \frac{(g_s N)^2}{r^4} \sim r^2 \sim |S|^{2/3}.$$

It is now straightforward to evaluate the scale of supersymmetry breaking:

$$M^4_{susy} := V_{N<0} = \frac{c}{g_s^2}\left|\frac{\beta^{NS}}{g_s N}\right|^2 \exp\left(-\frac{8\pi}{3}\left|\frac{\beta^{NS}}{g_s N}\right|^2\right)\Lambda_0^4 .$$

This has the desired exponential suppression in the semiclassical limit $\beta^{NS}/g_s N \gg 1$. Without taking into account the warping, the result would have been $V \sim |N\beta^{NS}/g_s|$. Thus we can understand this effect of warping in 4d effective supergravity terms. Interestingly, the important effect is not in the e^K factor, but in the inverse metric $g^{S\bar{S}}$.

Unfortunately, since the new potential vanishes as $S \to 0$, this supersymmetry breaking vacuum appears not to be stable. And, the computation we just did shows that any S-dependent supersymmetry breaking energy would scale the same way.

However, this is just the global supersymmetry part of the potential. In fact, the $-3e^K|W|^2$ term of supergravity makes a difference here.

As $S \to 0$, it is dominated by the constant term in $W = N(c + S \log S)$,

$$V \sim |S|^{4/3}|N \log S - \beta|^2 - 3N^2|c + S \log S|^2.$$

Note: this term in the energy of the RR flux will be cancelled by the no-scale structure of the potential, unless we stabilize the Kähler moduli. So the precise form of this term depends on non-perturbative stabilization a la KKLT, or otherwise.

Anyways, if we grant the form $-3|W|^2$, its contribution will lower the energy as we move away from $S = 0$, and stabilize the vacuum away from $S = 0$. By suitable choice of c and the other parameters, the resulting vacuum can break supersymmetry at a low scale. The actual value of c depends on the bulk, and we don't yet know whether this is achievable in real compactifications.

The main conclusion of this section is that we can compute the effect of warping on the Kähler potential, and it is drastic in this example, making possible low scale susy breaking by fluxes.

The resulting breaking scale is exponentially small, as in the dual gauge theory. We suspect that to get the precise susy breaking scale and effective field theory description, the KK reduction to $d = 4$ may need to be done more carefully than in existing work, and we are pursuing this.

5. Brane and bundle moduli metrics

The previous discussion was for closed string modes (complex structure moduli, etc.) What about open string modes, brane positions, bundle moduli and the like?

The "original" problem of this type is to compute the Kähler potential in $N = 1$ heterotic string compactification. In the original models of Candelas et al 1985, with spin connection embedded in the gauge group, it could be inferred from $N = 2$ supersymmetry of a related type II theory.

But again, this is not the general case. All current work on quasi-realistic heterotic strings (*e.g.* that of the Penn group) uses the more general "$(0, 2)$ compactifications," *i.e.* one takes the Yang-Mills connection to solve the hermitian Yang-Mills equations

$$0 = F^{(0,2)} = F^{(2,0)} = g^{i\bar{j}} F^{(1,1)}_{i\bar{j}}$$

for a vector bundle V which is different from the tangent bundle TM.

For these models, the problem of computing the Kähler potential for matter fields (and bundle moduli), amounts to the problem of computing the Weil-Petersson metric $G_{\alpha\bar{\beta}}$ on the moduli space of hermitian Yang-Mills connections. To compute this metric at a point, one must first find the connection A solving the hermitian Yang-Mills equations.

A tangent vector to a solution is then a solution δA to the linearized HYM equations. After gauge fixing, these are harmonic forms,

$$0 = (d + A)\delta A = (d + A) * \delta A$$

One needs to find a basis $\delta_\alpha A$ of these solutions, and compute their inner product,

$$G_{\alpha\bar{\beta}} = \int_M \mathrm{dvol}\ \delta_\alpha A \wedge \delta_{\bar{\beta}} \bar{A} \wedge \omega^{d-1}.$$

While doing all this is a well-posed mathematical problem, it is not easy, and essentially no $N = 1$ examples have been worked out. (for $N = 2$ these are hyperkähler metrics, where more is known.)

One reason for this is that the straightforward approach requires knowing the Ricci flat metric on the Calabi-Yau manifold M. While this was proven to exist by Yau, closed form expressions for it are not known and not believed to exist, except in trivial cases (tori and orbifolds).

One might hope for a non-straightforward approach. For example, perhaps the metric on open string moduli space, satisfies the constraints of some unknown "open string special geometry." While there are arguments that such a structure exists (see *e.g.* work of Lerche et al), the existing discussions such as BCOV suggest that they do not determine the moduli space metric.

We would argue that, even if they did, they would not be any easier to solve, than the equation $R_{ij} = 0$ which determines the Ricci flat metric. After all, the various bundle and brane problems of this type, are related by T-duality to the problem of finding the metric on the moduli space of D0-branes, which (in the large volume limit) is just the Ricci flat metric.

So, basic problem in this area is to compute, or get qualitative picture of, Ricci flat metric on Calabi-Yau manifold (and G_2, etc.)

Explicit solutions are known for noncompact CY, for example resolved and deformed conifold (Candelas-de la Ossa) as used in KS solution.

One simple approach – patch noncompact solutions into bulk, use rough approximation for bulk metric. OK if conifold throat is responsible for all anomalously large or small numbers, rest is order one.

Quintic in $\mathbb{CP}^4$:

$$0 = f(z^1, z^2, z^3, z^4, z^5) = \sum_{1 \le i \le 5} (z^i)^5 + \psi z^1 z^2 z^3 z^4 z^5.$$

$$K = \log \sum_{1 \le i \le 5} |z^i|^2.$$

Actually, this is not great even away from the conifold point (order one errors in $R_{ij} = 0$).

6. Balanced metrics

The approximation we just gave can be systematically improved. For example, we could use an ansatz for the Kähler potential with parameters,

$$K_1 = \log \sum_{1 \le i, \bar{j} \le 5} c_{i\bar{j}} z^i \bar{z}^{\bar{j}}.$$

This has 25 parameters. If we want more, we can take

$$K_2 = \log \sum_{1 \le i_1, i_2, \bar{j}_1, \bar{j}_2 \le 5} c_{i_1 i_2 \bar{j}_1 \bar{j}_2} z^{i_1} z^{i_2} \bar{z}^{\bar{j}_1} \bar{z}^{\bar{j}_2},$$

etc.

The idea would then be to find the parameter values which in some sense minimize the deviation from Ricci flatness. This can be done numerically.

One difficulty here is that there are many definitions of the "deviation from Ricci flatness." We could integrate $|R_{ij}|^2$, sum it over points, do the same with another norm, etc.

Now one of the main points in doing this is to discover new formal structure, and this arbitrariness is likely to obscure it.

$$K_2 = \log \sum_{1 \le i_1, i_2, j_1, j_2 \le 5} c_{i_1 i_2 \bar{j}_1 \bar{j}_2} z^{i_1} z^{i_2} \bar{z}^{\bar{j}_1} \bar{z}^{\bar{j}_2},$$

There is a canonical and pretty mathematical prescription called the "balanced metric," advocated by Donaldson, which approximates the Ricci flat metric. It can be defined mathematically as the solution of an integral equation,

$$(c^{-1})^{I\bar{J}} = \int_M d\mathrm{vol} \frac{s^I \bar{s}^{\bar{J}}}{c_{I\bar{J}} s^I \bar{s}^{\bar{J}}},$$

which can be efficiently found on a computer.

These techniques can be generalized to compute hermitian-Yang-Mills metrics and thus the moduli space metric for $(0, 2)$ compactification (work to appear with Karp,Lukic,Reinbacher). With Braun,Ovrut,Reinbacher we are pursuing this computation for a quasi-realistic model.

One can also start with the balanced metric and systematically improve the approximation scheme (work of Donaldson and Keller).

What does this mean physically? One can motivate it as a "maximal entropy" condition, which may be relevant for black holes, for example.

Consider a magnetic field F on the CY. Quantum mechanics of a particle in the magnetic field has a finite dimensional space of lowest energy solutions, the lowest Landau level.

$$(\partial_i - iA_i)^2\psi = 0..$$

If $F^{(0,2)} = F^{(2,0)} = 0$, by complex gauge transformation $\psi \to g\psi$, one can rewrite this as

$$(\partial - iA)(\bar{\partial})\psi = 0$$

so the LLL corresponds to holomorphic sections of a line bundle.

This enables contact with algebraic geometry.

What is the "maximal entropy" state of such a particle?

1. Equal probability for each state. Choose a basis for the LLL, $|\psi_\alpha\rangle$, then one has a density matrix

$$\rho = \frac{1}{N} \sum_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$$

2. Equal probability in position space.

$$P(z) = \langle z|\rho|z\rangle = \text{constant}.$$

In general, the two conditions are incompatible. Given ρ, one can compute $P(z)$, and it is not constant. However, it could be that for a special choice of F, it is constant.

In fact, this will be so if and only if $F = \omega$ (the Kähler form) and ω is the balanced metric, satisfying

$$(c^{-1})^{I\bar{J}} = \int_M \mathrm{dvol} \frac{s^I \bar{s}^{\bar{J}}}{c_{I\bar{J}} s^I \bar{s}^{\bar{J}}}$$

This follows from

$$P(z) = \frac{C_{I\bar{J}} s^I \bar{s}^{\bar{J}}}{c_{I\bar{J}} s^I \bar{s}^{\bar{J}}},$$

where

$$(C^{-1})^{I\bar{J}} = \langle \bar{J} | I \rangle$$

is the matrix of inner products. For the balanced metric, $C^{-1} = c$.

Thus, the balanced metric is the unique metric for which the two definitions of maximal entropy (the quantum definition using the density matrix, and the classical definition using the probability distribution) are in agreement.

Why is the balanced metric a good approximation to the Ricci flat metric?

There is an expansion (Tian-Yau-Zelditch) for the diagonal of the projector on the LLL, in the strength k of the magnetic field,

$$\begin{aligned} P(z) &= \frac{1}{N}\sum_\alpha |\psi_\alpha(z)|^2 \\ &= 1 + \frac{c_1}{k}R(z) + \frac{c_2}{k^2}\partial^2 R(z) + \ldots \end{aligned}$$

Thus, as $k \to \infty$, the balanced metric approaches the constant curvature (here, Ricci flat) metric, with corrections in powers of $1/k$. On the next pages, we graph $\omega^3/\Omega\wedge\bar{\Omega}$ on an S^2 to illustrate.