Aug '08 ①

Really important questions

1. Foundations of gauge/string duality

2. „Geometry" at „∞ curvature

3. Gauge/strings and de Sitter

4. Cosmological constant

Properties and paradoxes of the deSitter.

1. From $S \Rightarrow dS$

S) $\vec{n}^2 + n_0^2 = 1$

$n_0 \Rightarrow i n_0$

dS) $\vec{n}^2 - n_0^2 = 1$

The propagator

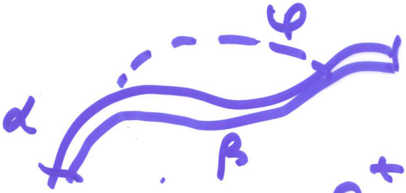
$$\langle \varphi(n_1)\, \varphi(n_2) \rangle$$

$$= \frac{1}{\sin \pi \nu} P_\nu\left(-(n_1 n_2)\right)$$

/or $\frac{1}{\sin \pi \nu} C_\nu^{d/2}(-n_1 n_2)$ if

$D = d+1$ / (Chernikov Tagirov '67, Bunch-Davies ...

Geodesic observer /Unruh detector/:



$$W_{\alpha\to\beta} \sim \int_{-\infty}^{+\infty} ds\, e^{-i(\mathcal{E}_\beta-\mathcal{E}_\alpha)s} \langle \varphi(u(s))\,\varphi(u(0))\rangle$$

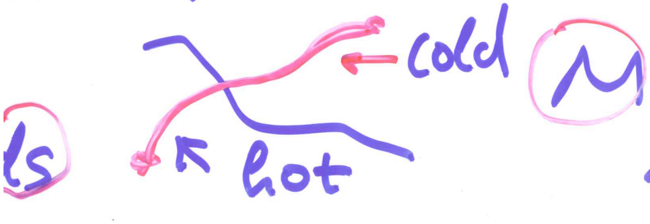
The B.-D. Green function

$$G \sim A e^{-imL} + B e^{imL}$$

⇒ energy can be taken from (dS).

(L - geodesic distance)

Paradox: patch of dS and Minkowsky



"Equivalence principle violated"

2d Model

$$\mathcal{L} = \psi_-^* (\partial_+ - h_{++} \partial_-) \psi_-$$
$$+ \psi_+^* \partial_- \psi_+ + m(\psi_+^* \psi_- + c.c)$$

dS space $h_{++} = R(x^-)^2$

/analogous gauge model:

$$\mathcal{L} = \psi_-^* (\partial_+ + iA_+) \psi_- + \dots$$
$$A_+ = E x^-$$

Bosonize

$$\mathcal{L} = (\partial\varphi)^2 + m^2(1-\cos\varphi) + R\,(x^-)^2 (\partial_-\varphi)^2 \quad \text{(grav)}$$

$$\mathcal{L} = (\partial\varphi)^2 + m^2(1-\cos\varphi) + E x^- \partial_-\varphi \quad \text{(gauge)}$$

L.C. quantization

$$P_+ = \int dx^- (1-\cos\varphi) -$$
$$- \int dx^- (x^-)^2 \, R \cdot (\partial_- \varphi)^2$$

(gravity)

$$P_+ = \int dx^- (1-\cos\varphi)$$
$$- \int dx^- \, E \, \varphi$$

Both unbounded below

Back-reaction cures this by screening E & R: $\langle T_{\mu\nu} \rangle - \Lambda g_{\mu\nu} = 0$

Nature abhors (positive) curvature

Stability of the curved space (in general)

How to quantize?

Proposal

In the stable space-time we should use Feynman's principle, defining

$$G_F(x,x') = \sum_{(P_{xx'})} e^{-imL(P_{xx'})}$$

$$G_F(x,x') \underset{x-x'\to\infty}{\sim} e^{-imL(x,x')}$$

This is NOT the case in Bunch-Davies vacuum.

Now we formulate the eternity condition:

Feynman = Schwinger - - Keldysh

Define

$$G_{++} = G_F(x, x', M^2 - i0)$$

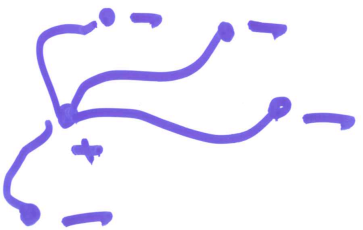
$$G_{+-} = G(x + i\eta, x', M^2)$$

(η - time-like, infinitesimal)

$$G_{-+} = G(x - i\eta, x', M^2)$$

Stability condition

No spiders



$= 0$

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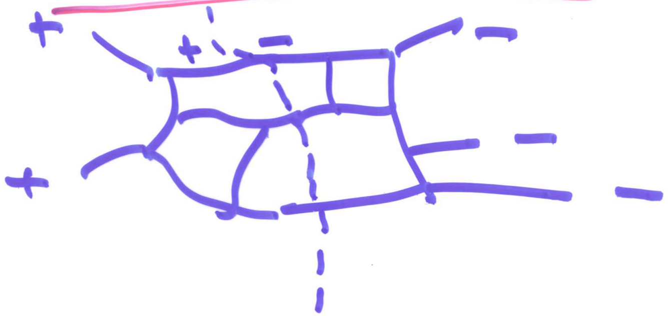
No spiders $\Rightarrow$ usual cutting rules:



Also stability requires

$$\operatorname{Im} G_F(x,x) = 0$$

This implies that

$$\langle 0 | 0 \rangle_{\substack{\\ in}}^{\!\!\!\!\!\!\!\!\!\!\!out} = e^{iW} ; \quad \operatorname{Im} W = 0.$$

In the (dS) case, the Feynman G-function is analytic continuation from AdS: (NOT from a sphere)

$$G = Q_\nu(n_1 n_2)$$

In (dS) (and FRW) the stability for the partial Green function, satisf.

$$\left(\partial_t^2 - \mu^2 + u(t)\right) G = \delta(\,)$$

$/ds^2 = dt^2 - a^2(t)(d\vec{x})^2/$

implies that the Jost functions have the property:

$$\int \phi^N(t)\,dt = 0 \quad / \text{zero spider}/$$

(here

$$G_F(t,t') \sim \phi(t_>)\,\chi^*(t_<)$$

$$\phi(t) \to e^{-i\mu t} \qquad t \to -\infty$$

$$\chi \longrightarrow e^{-i\mu t} \qquad t \to +\infty$$

Novikov's equation for $u(t)$

In ~~odd~~ dimension
(dS) potential $U(t) \infty \frac{1}{\cosh^2 t}$
is a soliton for the
KdV and Novikov eqs.
(reflectionless property
of $U(t)$ in odd dim
was already noticed by
Strominger et al.)
General questions:
are there other stable
FRW spaces?
Relation to the
~~H~~ Huygens
Principle ?

The B.-D. vacuum is unstable not by itself, but by sensitivity to perturbations:

$$\langle E | T e^{-i\lambda \int \varphi^4} | E \rangle = e^{iW}$$

$$\approx \exp\{-\lambda^2 \}$$

$$\operatorname{Im} W \propto \lambda^2 \cdot (\text{Volume})$$

$${}_{out}\langle E | E \rangle_{in} = 0$$

We expect to find at the time t a non-zero occupation numbers $\{f_p(t)\}$.

They satisfy (for small λ) the Boltzmann eqs!

$$\dot{f}(p) = \int |\gamma(p, p_1, p_2)|^2 \cdot \delta(p+p_1+p_2)$$

$$\{(1+f(p_1))(1+f(p_2))(1+f(p))$$

$$- f(p_1)\,f(p_2)\,f(p)\} + \ldots$$

$$\gamma(p, p_1, p_2) \propto \int \phi\,\phi\,\chi^*$$

(in terms of Jost funct.; for (dS) $\phi \propto H^{(1)}_{i\mu}(p\tau)$)

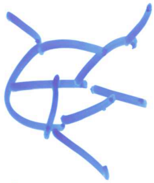
<u>The key</u>: No energy conservation, hence <u>no detailed balance</u>

f(p) blows up until stopped by back reaction

From the different
point of view, particle
production in (dS)
and possible screening
of the cosm. const
was considered by many
people starting from
'80. Very incomplete list
includes Antoniadis, Mazur,
Mottola, Myrhvald,
Bros, Epstein, Moschella
~~M.~~ Woodard, Tsamis.
Very different version
of dS/CFT was suggested
by Strominger.
Similar analytic cont.
AdS → dS discussed
by Maldacena.

String theory side

Planar diagrams diverge as c^k



(t' Hooft).

Scaling lim: $F = \sum \lambda^k F_k$

$\lambda \to \lambda_{cr}$. they become dense and can be viewed as a world sheet. This is what happens in the matrix models for minimal gravity, but NOT in the usual g/s duality.

We expect λ_{cr} to be complex

As we analytically continue from (AdS) to (dS) /getting the Q-propagator/ we have to continue the gauge theory to complex λ.

Conjecture $\lambda = \lambda_{cr}$,

de Sitter space is described by the dense Feynman diagrams

The theory is non-unitary Σ(probabilities) < 1 since the vacuum may disappear.

The sigma model

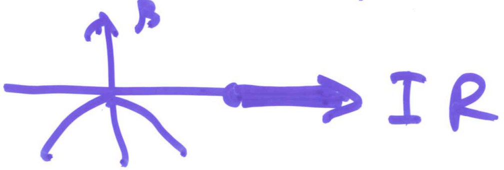
$$S = \frac{1}{2\alpha_0} \int (\partial n)^2 \, d^2\xi$$

Feynman graphs don't change as we go from Sphere $\Rightarrow$ de Sitter

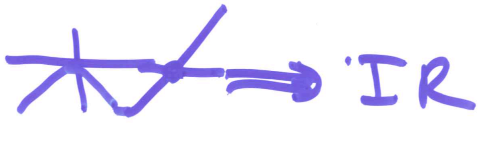
But the theories are different:

(S) — Mass gap, (discrete sp.) due to the compactness of (S)

The β - function — asympt. free



(Sph.)



(dS)

Large D expansion

$$\mathcal{L} = (\partial n)^2 + \lambda(n^2 - 1)$$

$$D \alpha_0 \int \frac{d^2 k}{k^2 + \lambda} = 1$$

But this can't be true in (dS). Take 1d sigma Model first. We have on finite circle:

$$D \alpha_0 \sum_n \frac{i}{\omega_n^2 + \lambda + i0} = 1$$

$$\omega_n = 2\pi n / T \qquad \lambda = m^2$$

$$mT = \frac{D \alpha_0 T}{\tanh(mT)}$$

The wrong saddle point

The right one: $m \Rightarrow i m$

$$mT = \frac{-\mathcal{D}\alpha_0 T}{\tan(mT)}$$

as $T \to \infty$ $\quad mT \to \pi/2$

The mass $m \propto 1/T$.

The same is true in (2d).

The spectrum of dimensions (?)