CHIRAL GRAVITY IN THREE DIMENSIONS

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So far, string theory is the only known consistent quantum theory of gravity complex enough to contain black holes. Simpler examples - e.g in 2 or 3 dimensions - would be useful. Oddly, despite 30 years of research, it remains unclear if such exist!

In this talk we explore a new possibility:
"CHIRAL GRAVITY"

At first, 3D Einstein gravity

$$S_E = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right)$$

appears trivial because

$$R_{abcd} = R_{ac}g_{bd} + \text{symm}$$

But then BTZ found ●.
In the late 80's an exact solution was proposed, but it did not explain ● entropy and was reconsidered last year.

OPEN QUESTION

$\boxed{\text{Is there a quantum theory of pure 3D Einstein gravity?}}$

No viable current proposal.

Another theory in 3D

$\boxed{\text{Topologically Massive Gravity}}$

Deser Jackiw & Templeton

$$S_{TMG} = \frac{1}{16\pi G}\int\left(R + \frac{2}{\ell^2} + \frac{1}{\mu} I_{CS}\right)$$

μ ← graviton mass

$$I_{CS} = -\frac{1}{2}\varepsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}\left[\partial_\mu \Gamma^{\sigma}_{\rho\nu} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}\right]$$

E.O.M.

$$G_{\mu\nu} + \frac{1}{\ell^2} g_{\mu\nu} = \frac{1}{\mu}\varepsilon_\mu{}^{\beta\alpha}\nabla_\alpha\left(R_{\beta\nu} - \frac{R}{4} g_{\beta\nu}\right)$$

Every solution of Einstein's equation

$$G_{\mu\nu} = -\frac{1}{\ell^2} g_{\mu\nu}$$

is a solution of TMG, but there are more....

MASSIVE GRAVITONS

Consider an expansion about flat space with $\frac{1}{\ell^2} = 0$. The higher derivative $\underline{I_{CS}}$ adds propagating degrees of freedom: a **massive graviton** with positive mass μ but negative energy $E<0$. Its like a wrong-sign scalar

$$S = +\int d^4x \left((\nabla \phi)^2 + m^2 \phi^2 \right)$$

~~Take $G<0$?~~ Instead $\rightarrow$

AdS_3 TMG

We wish to study the case of negative c.c. expanded about AdS_3

$$ds^2 = \ell^2(-\cosh^2\rho \, d\tau^2 + d\rho^2 + \sinh^2\rho \, d\phi^2)$$

with Brown-Henneaux b.c.s.
This has 6 $SL2R_L \times SL2R_R$ Killing vectors

$L_0 = \partial_\tau - \partial_\phi$ $\qquad$ $\bar{L}_0 = \partial_\tau + \partial_\phi$

$L_1 = \ldots$ $\qquad$ $\bar{L}_1 = \ldots$

$L_{-1} = \ldots$ $\qquad$ $\bar{L}_{-1} = \ldots$

Gravitons

Linearized soln's fall into $SL2R_R \times SL2R_L$ irreps (highest weight) for generic μ, ℓ.

HWS
$$L_0|\psi\rangle = h_L|\psi\rangle$$
$$\bar{L}_0|\psi\rangle = h_R|\psi\rangle$$
$$L_1|\psi\rangle = \bar{L}_{+1}|\psi\rangle = 0$$

After much work, one finds

(h_L, h_R)

$(0, 2)$ right boundary graviton

$(2, 0)$ left bdry graviton

$\left(\frac{1}{2} + \frac{3\mu\ell}{2}, \frac{\mu\ell}{2} - \frac{1}{2}\right)$ massive graviton

Note $\mu\ell \to 1$ degeneracy!!

Also $E \to 0$.

Asymptotic Symmetry Group

Given any set of b.c.s, in our case Brown-Henneaux

$$g_{tt} = -e^{2\rho} + \mathcal{O}(1)$$

$$\vdots \quad \cdots$$

there is a set of allowed diffeos which preserve them. Given some dynamics, a subset can be labelled as trivial if they vanish, including surface terms, when the constraints are applied. Then

$$\text{ASG} \equiv \frac{\text{allowed diffeos}}{\text{trivial diffeos}}$$

Physical states are in reps of the ASG, and are annihilated by trivial diffeos.

TMG ASG

a

Generically, the ASG for TMG is generated by

Left Virasoro $\times$ Right Virasoro

$$c_L = \frac{3\ell}{2G}\left(1 - \frac{1}{\mu\ell}\right) \times c_R = \frac{3\ell}{2G}\left(1 + \frac{1}{\mu\ell}\right)$$

Brown Henneaux
Kraus Larsen

with explicit generators

$$G(\varepsilon^+, \varepsilon^-) = \oint_\infty d\tau^+ h_{++} \varepsilon^+ \left(1 + \frac{1}{\mu\ell}\right)$$

metric fluctuation

diffeo parameter on boundary

$$+ \oint_\infty d\tau^- h_{--} \varepsilon^- \left(1 - \frac{1}{\mu\ell}\right)$$

$\varepsilon^\pm = \varepsilon^\pm(\tau \pm \phi)$

FOR $\mu\ell = 1$, ε^- TRANSFORMATIONS ARE TRIVIAL!

So

$\mu \ell \to 1$

ASG → Right Virasoro

Also

Left & Massive Gravitons → Null states

BTZ black holes → $E = J$

$c_L \to 0$

basis of LSS conjecture

TMG becomes chiral.

There is no left Virasoro for states to transform under.

"CHIRAL GRAVITY"

Open Question:

E+J>0???

Conjecture

Chiral gravity exists as a quantum theory and is dual to a $c_R = \frac{3\ell}{G}$ holomorphic boundary CFT.

N.B. If the CFT is unitary, its microstates explain the black hole entropy via the Cardy formulaas in A.S. '98.

A number of purported counterexamples appeared in the literature after the chirality conjecture was made[Li, Song, A.S.] but before it was proven,[A.S.] These all fail to be true counter-examples and do not take into account the fact that left Virasoro transformations are pure gauge at the chiral point.

ASIDE

salvation for $\mu\ell \neq 1$? 12

Warped AdS_3

For $\mu\ell \neq 1$, TMG on AdS$_3$ is unstable. Could there be another ground state? $\exists$ interesting warped WAdS$_3$ solutions

Warp factor

$$ds^2 = L^2\left(-\frac{dt^2 + dy^2}{y^2} + \Lambda^2\left(d\phi + \frac{dt}{y}\right)^2\right)$$

These have interesting black hole solutions which are quotients of WAdS$_3$ just as BTZ = AdS$_3/\mathbb{Z}$. A rich and interesting story is unfolding...

Gurses Nutku
Clement Leygnac
Padi Anninos Li Song A.S. Ait Moussa
Guennoune Bouchareb...

Relation to Witten's 2007 "3D Gravity Reconsidered"

Proposed end run for pure gravity on AdS_3

Consistency requirements	Assumptions
1. Modular invariance for $Z(\tau,\bar{\tau})$	1. $Z(\tau) = Z(\tau) Z(\bar{\tau})$
2. No primaries below lightest ● mass	

CHIRAL GRAVITY

$\Rightarrow$ exact formula for Z in terms of Hecke transforms of j-functions.

Sum over Euclidean 3-geometries can be done exactly and yields Yin Maloney Witten $Z \neq Z(\tau) Z(\bar{\tau})$ for pure gravity. For chiral gravity, though some details remain to be worked out, the Euclidean action is complex and appears to yield precisely the desired result. Moreover, the integrality of the coefficients in the q-expansion of Z is nontrivial evidence for the existence of quantum chiral gravity.

But what does this have to do with the real world?

The Kerr-CFT Correspondence

Monica Guica, Tom Hartman,
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in progress

An extreme 4D

Kerr  black hole

has angular momentum

$$J = GM^2,$$

Hawking temperature

$$T_H = 0,$$

and Bekenstein-Hawking entropy

$$S_{BH} = \frac{2\pi J}{\hbar}.$$

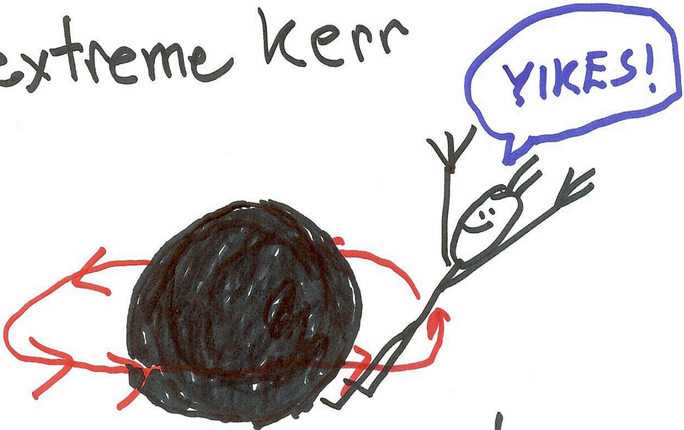
GRS 1915+105 has

$$M \sim 14 M_\odot$$

$$\frac{J}{GM^2} > .98$$

McClintock, Shafee, Narayan, Remillard, Davis & Li (2006)

The near-horizon region of extreme kerr



is like chiral gravity: all excitations must move counterclockwise at the speed of light!

Bardeen & Horowitz (99)

found the Near-Horizon Extreme-Kerr NHEK geometry

$$ds^2 = 2GJ\Omega^2\left[\frac{-dt^2+dy^2}{y^2}+d\theta^2 + \Lambda^2\left(d\phi+\frac{dt}{y}\right)^2\right]$$

$$\Lambda^2 = \frac{2\sin\theta}{1+\cos^2\theta}$$

$$\Omega^2 = \frac{1+\cos^2\theta}{2}$$

and found an $SL2R \times U(1)$ isometry. Cross sections at fixed polar angle theta are precisely the quotients of WAdS$_3$ geometries discussed earlier! Expect CFT...

We have found that for suitable b.c.s the ASG contains a Virasoro with

$$c_L = \frac{12 J}{\hbar}$$

Further, the Frolov-Thorne vacuum is a thermal state with energy precisely L_0 of the Virasoro at temperature

$$T_L = \frac{1}{2\pi}$$

the Cardy formula

$$S_{CFT} = \frac{\pi^2 c_L T_L}{3}$$

gives

$$S_{CFT} = \frac{2\pi J}{\hbar} = S_{BH}!!!$$

Conjecture

The black hole GRS 1915+105 is dual to a $c_L = 2.0 \times 10^{76}$ two-dimensional conformal field theory.