Instability with Chern-Simons Terms

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Strings2010
MARCH 15-19, 2010 COLLEGE STATION, TEXAS

based on

arXiv:0911.0679 with
Shin Nakamura and Chang-Soon Park

and work in progress with Chang-Soon Park.

(a related work by S. Domokos and J. Harvey, arXiv:0704.1604.)

Plan

1. Instability of electric field by CS terms

2. Instability of charged black holes in AdS_5

3. Phase transition and non-linear solutions

A constant electric field is a solution to the vacuum Maxwell equations.

$$\partial^\mu F_{\mu\nu} = 0$$

A constant electric field is a solution to the vacuum Maxwell equations.

The Chern-Simons terms abhor the constant electric field.

3 dimensions

$$\mathcal{L} = -\frac{1}{4} F \ast F + \frac{\alpha}{2} A \wedge F$$

$$\mathcal{L} = -\frac{1}{4} F {*F} + \frac{\alpha}{2} A \wedge F$$

$$\Rightarrow \quad d{*F} + \alpha F = 0$$

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{\alpha}{2} A \wedge F$$

$$\Rightarrow \quad d * F + \alpha F = 0$$

A constant electric field is not a solution.

$$d^*F + \alpha F = 0$$

$$\begin{aligned}\Box F &= d^* d^* F \\ &= -\alpha\, d^* F \\ &= \alpha^2 F\end{aligned}$$

$$\mathcal{L} = -\frac{1}{4} F \, {}^*F + \frac{\alpha}{2} A \wedge F$$

$$\Rightarrow \quad (\Box - \alpha^2) F = 0$$

Gauge field becomes massive.

Deser, Jackiw and Templeton (1982)

5 dimensions

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{1}{3!} \alpha \, A F F$$

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Gauge field in the vacuum is massless.

A constant electric field is a solution.

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{1}{3!} \alpha A F F$$

A constant electric field is a solution.

But, it is **unstable** (as I will show).

$$\mathcal{L} = -\frac{1}{4} F {}^*F + \frac{1}{3!} \alpha A F F$$

$$\Rightarrow \quad d {}^*F + \frac{1}{2} \alpha F \wedge F = 0$$

$$d * F + \frac{1}{2} \alpha F \wedge F = 0$$

$$F = F^{(0)} + f$$

Linearize.

$$d * f + \alpha F^{(0)} \wedge f = O(f^2)$$

Linearized equation with the Chern-Simons term:

$$d * f + \alpha F^{(0)} \wedge f = 0$$

Take: $$F^{(0)}_{01} = E$$

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$$d * f + \alpha F^{(0)} \wedge f = 0$$

Take: $F^{(0)}_{01} = E$

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4 \alpha E \epsilon_{ijk} \partial_j f_k = 0$$

$$\left(\begin{array}{c} \mu, \nu = 0, 1 \quad , \quad i, j, k = 2, 3, 4 \\ f_i = \epsilon_{ijk} f_{jk} \end{array} \right)$$

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4 \alpha E \, \epsilon_{ijk} \partial_j f_k = 0$$

$$(f_i = \epsilon_{ijk} f_{jk})$$

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momentum eigenstate $\quad p_{0,1} \quad k_{2,3,4}$

circular polarization

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circular polarization

$$(p_0)^2 - (p_1)^2 = k^2 \pm 4 \alpha E \, k$$

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4\alpha E \, \epsilon_{ijk} \partial_j f_k = 0$$

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momentum eigenstate $\quad p_{0,1} \quad k_{2,3,4}$

circular polarization

$$(p_0)^2 - (p_1)^2 = k^2 \pm 4\alpha E \, k$$

$$= (k \pm 2\alpha E)^2 - 4\alpha^2 E^2$$

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4 \alpha E \, \epsilon_{ijk} \partial_j f_k = 0$$

$$(f_i = \epsilon_{ijk} f_{jk})$$

$$(p_0)^2 - (p_1)^2 = (k \pm 2\alpha E)^2 - 4\alpha^2 E^2$$

The fluctuarion is tachyonic for

$$0 < k < 4 \alpha E$$

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{1}{3!} \alpha \, A F F$$

A constant electric field is unstable for

$$0 < k < 4 \alpha E$$

In contrast, a constant **magnetic** field is stable.

Gauge field fluctuations around it is massive.

The Chern-Simons terms abhor constant electric field.

In odd dimensions, the Chern-Simons term is induced by a massive electron in one-loop.

It is exact in the limit of large mass.

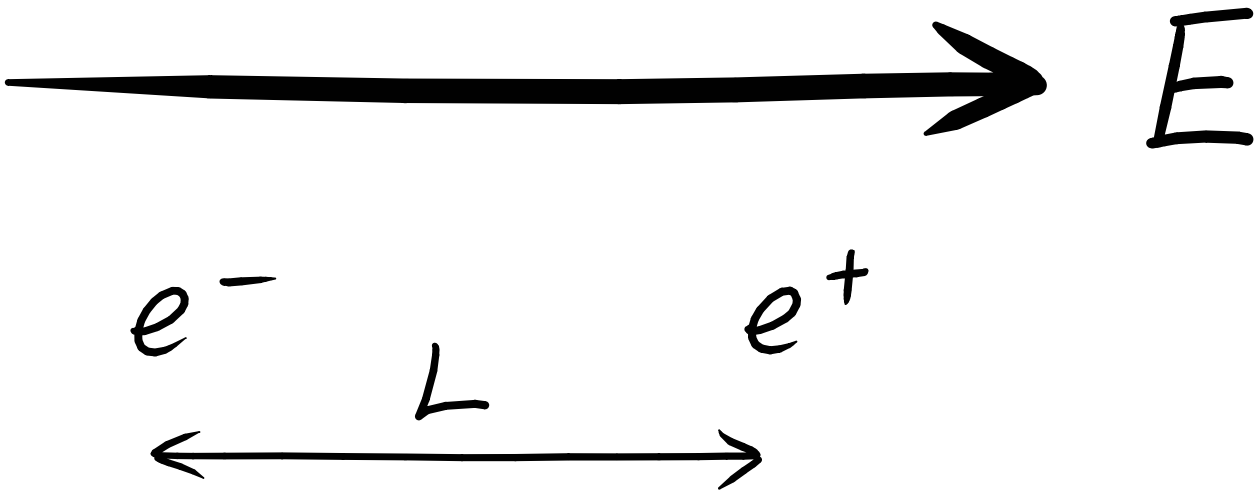
$$\log \det (i \not\partial + \not A + m)$$

$$= \frac{m}{|m|} \int_{\mathbb{R}^{2d,1}} A \wedge F^d + O\left(\frac{1}{m^2}\right)$$

Redlich (1984)

Schwinger mechanism:

A constant electric field can be screened by electron-positron pair creation.



$$qEL > 2mc^2$$

AdS/CFT Correspondence

A charged black hole in AdS_5

is dual to

a conformal field theory in 4 dimensions

at non-zero temperature and chemical potential.

Condensed Matter Physics Meets High Energy Physics

February 8 - 12 at IPMU

In the extremal limit (temperature = 0),

the near-horizon geometry of
the charged black hole in AdS_5

is

AdS_2 x R^3

with electric field ~ volume form of AdS_2.

Chern-Simons terms abound
in supergravity theories in AdS.

Instability of black holes in AdS

corresponds to

phase transition in dual CFT.

Things to be careful about:

(1) Stability conditions in AdS are different.

(2) Mixing of photons and graviton.

(1) Stability conditions in AdS are different.

Breitenlohner-Freedman bound

(1) Stability conditions in AdS are different.

instability range:

$$0 < k < 4\alpha E \quad \text{for } \mathbb{R}^{4,1}$$

$$\Downarrow$$

$$4\alpha E \gamma < k < 4\alpha E (1-\gamma)$$

$$\text{for } AdS_2 \times \mathbb{R}^3$$

$$2\gamma = 1 - \sqrt{1 - \frac{1}{16\alpha^2 E^2 R^2}}$$

AdS_2 radius

(1) Stability conditions in AdS are different.

Instability happens at non-zero momenta.

$$4\alpha E\gamma < k < 4\alpha E(1-\gamma)$$

for $AdS_2 \times \mathbb{R}^3$

$$2\gamma = 1 - \sqrt{1 - \frac{1}{16\alpha^2 E^2 R^2}}$$

AdS$_2$ radius

(1) Stability conditions in AdS are different.

In the near horizon geometry, $E\mathcal{R} = \sqrt{2}$.

Instability requires $\gamma < \frac{1}{2} \Leftrightarrow \alpha > \frac{1}{4\sqrt{2}}$

$$4\alpha E\gamma < k < 4\alpha E(1-\gamma)$$

for $AdS_2 \times \mathbb{R}^3$

$$2\gamma = 1 - \sqrt{1 - \frac{1}{16\alpha^2 E^2 \mathcal{R}^2}}$$

AdS$_2$ radius

(1) Stability conditions in AdS are different.

The Maxwell + Chern-Simons system in the near horizon geometry of the extremal charged black hole is unstable

if $\alpha > \frac{1}{4\sqrt{2}}$.

c.f., for the minimal gauged supergravity in 5 dimensions,

$$\alpha = \frac{1}{2\sqrt{3}} > \frac{1}{4\sqrt{2}}$$

Things to be careful about:

(1) Stability conditions in AdS are different.

(2) Mixing of photons and graviton.

With the background electric field,

the gauge kinetic term causes the mixing.

$$g^{IJ}\, g^{KL}\, F_{IK}\, F_{JL}$$

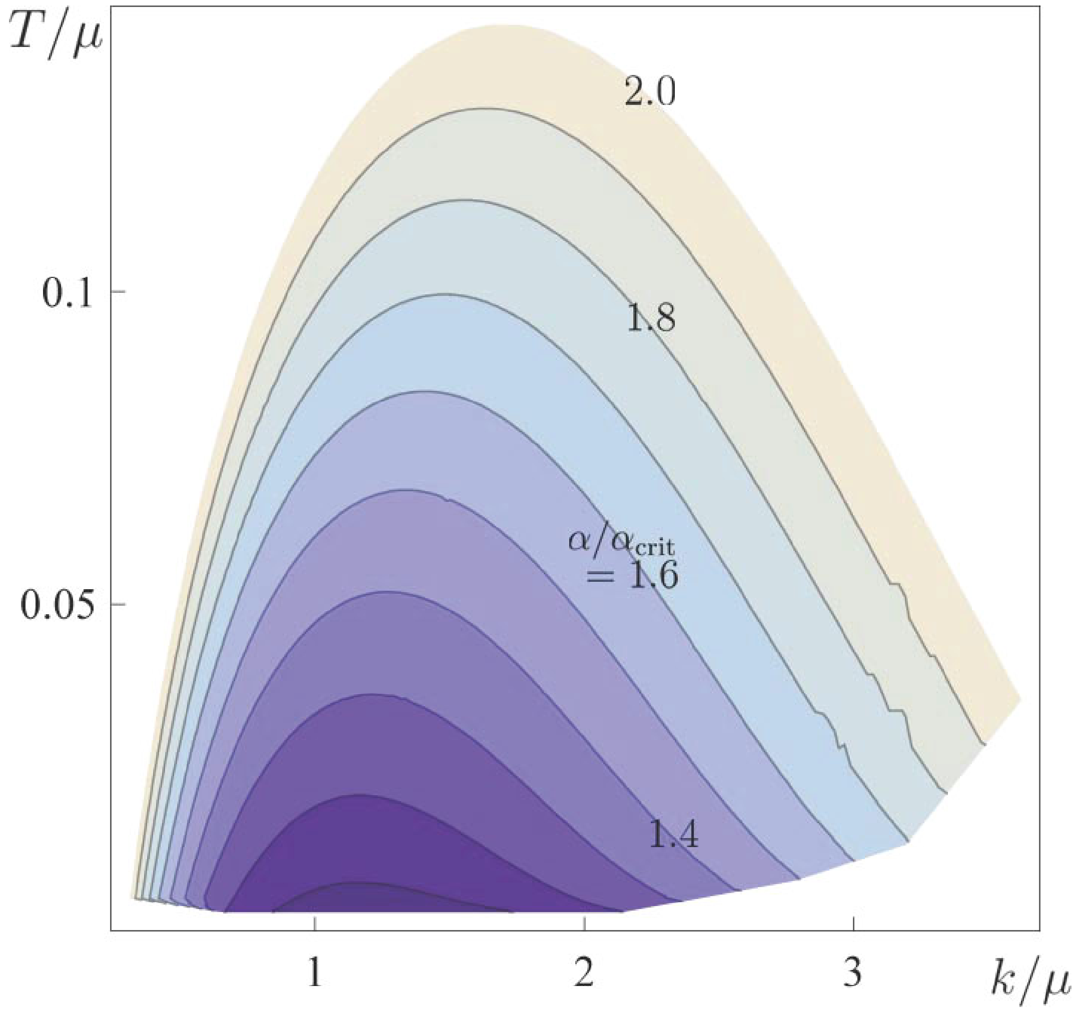
The mixing raises the critical value of the Chern-Simons coupling:

$$\alpha > 0.2896\cdots$$

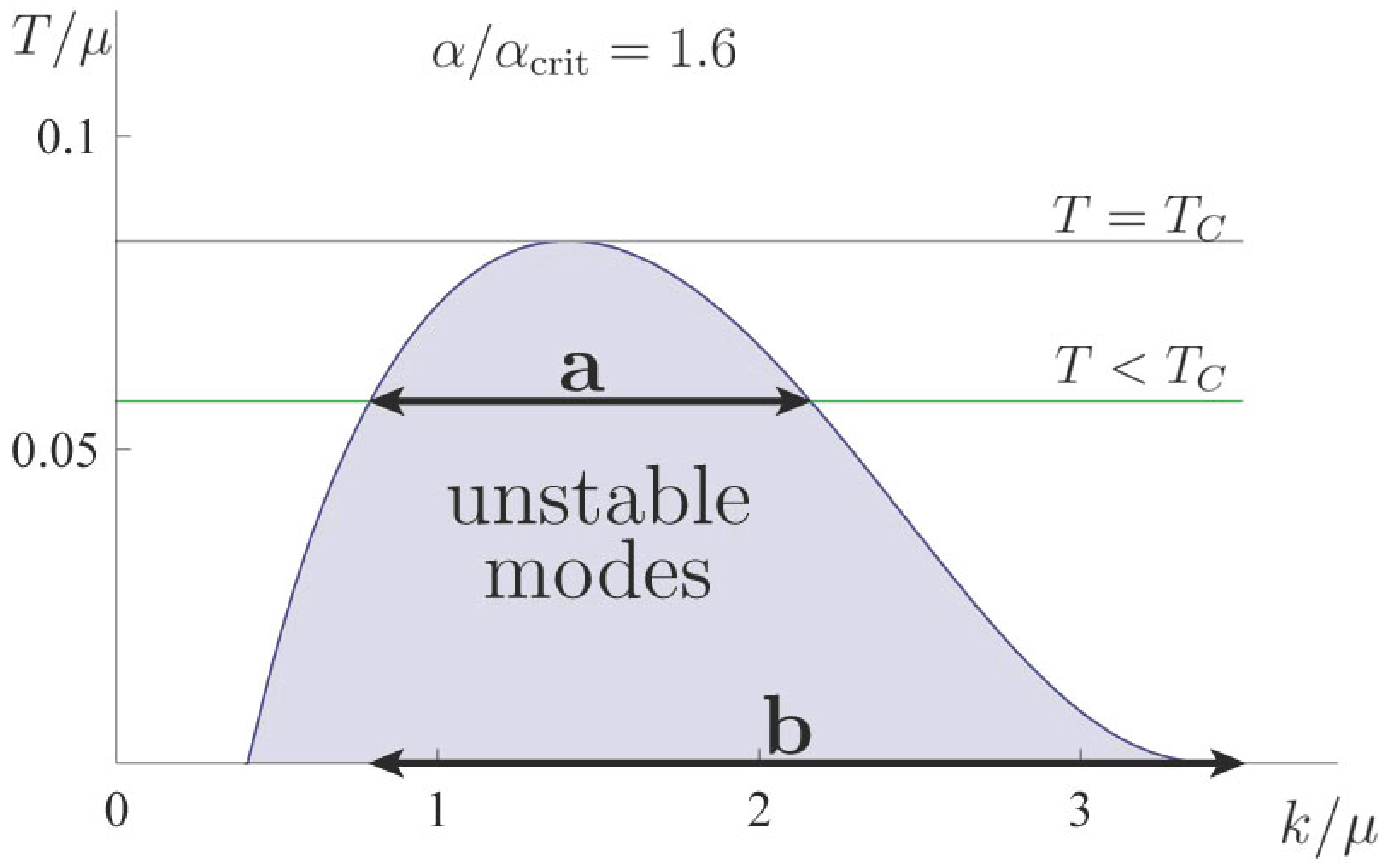
$$\text{c.f.} \quad \frac{1}{2\sqrt{3}} = 0.2887\cdots$$

AdS_2 x R^3 is the near horizon geometry of the extremal black hole (T=0).

We also analyzed stability of charged black holes with T > 0.

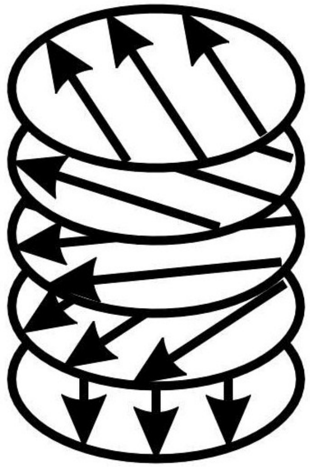


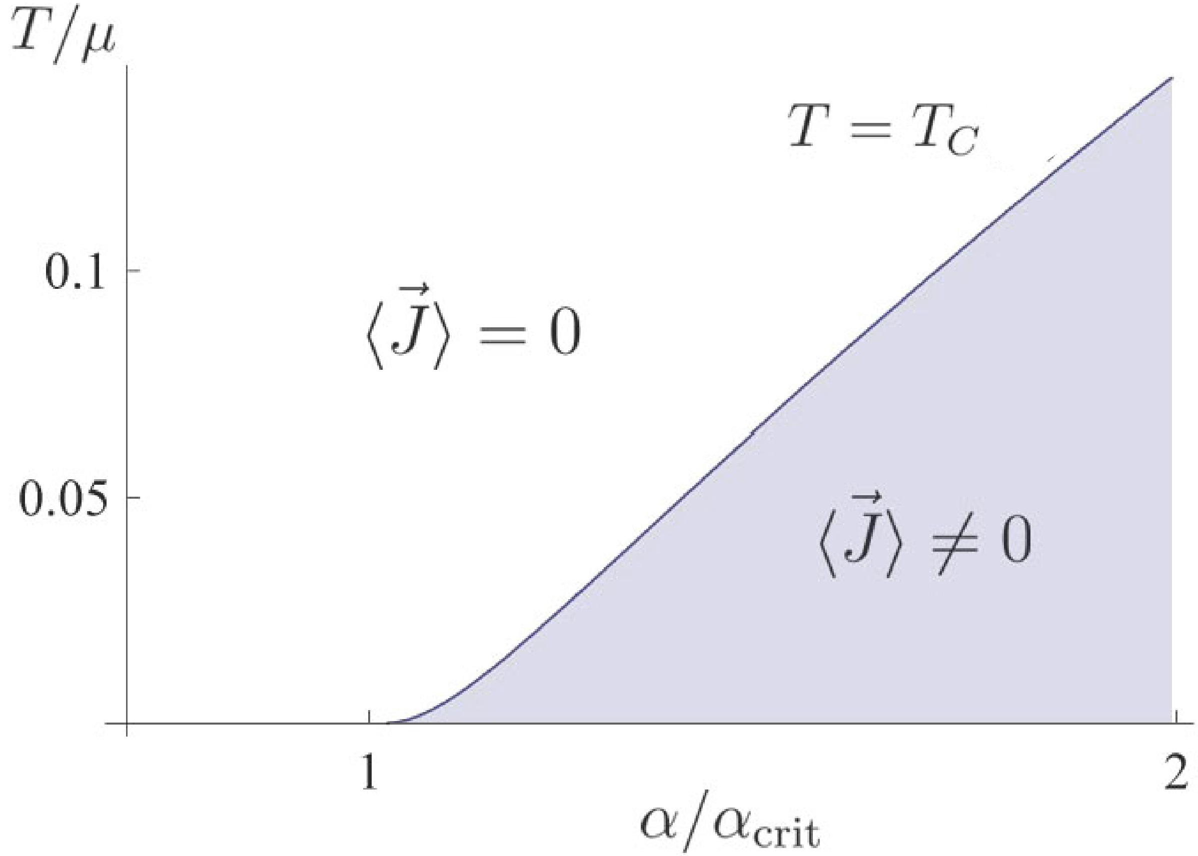
Unstable momenta at various values of the Chern-Simons coupling and temperature



Unstable momenta at \alpha = 1.6 \alpha_crit.

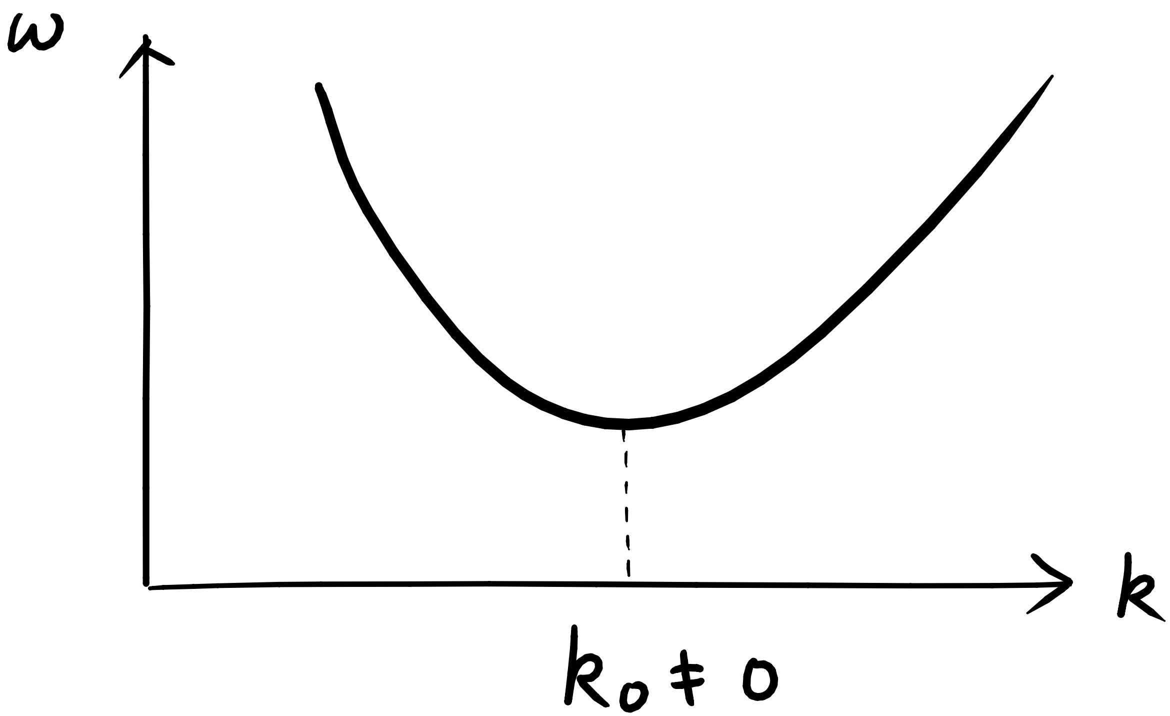
The range b is by the near horizon analysis at T=0.





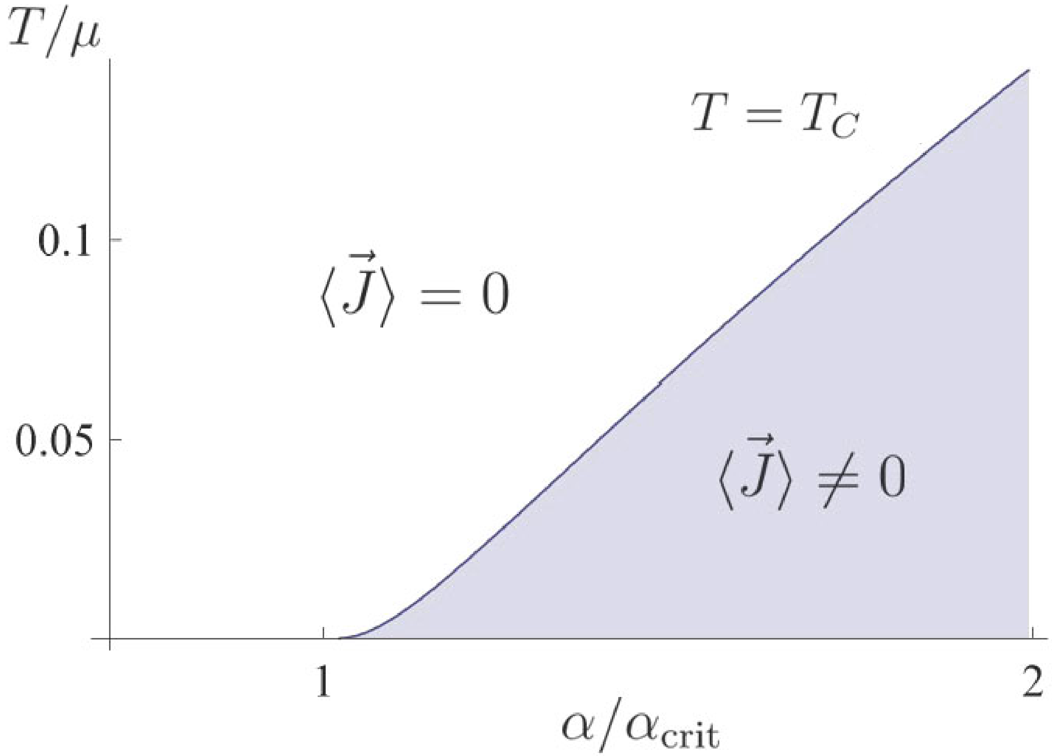
The instability of the gauge field means that the corresponding current in the dual CFT acquires a vacuum expectation value.

Even below the critical Chern-Simons coupling, effects of the spatially modulated phase can be seen in the dispersion relation.



Van Hove singularity.

To understand the nature of the new phase, we should study non-linear solutions.



To understand the nature of the new phase, we should study non-linear solutions.

Finding non-linear solutions including gravity is not easy, but there is a certain limit where it is possible.

Since the gauge field part of the Lagrangian is of the form,

$$-\frac{1}{4} F * F + \frac{1}{3!} \alpha A F F$$

backreaction to the metric can be made small by taking $\alpha \gg 1$

$$-\frac{1}{4} F * F + \frac{1}{3!} \alpha A F F$$

$$= \frac{1}{\alpha^2} \left(-\frac{1}{4} \tilde{F} * \tilde{F} + \frac{1}{3!} \tilde{A} \tilde{F} \tilde{F} \right)$$

where $\tilde{F} = \alpha F$.

For this limit to make sense, we should scale the background electric field:

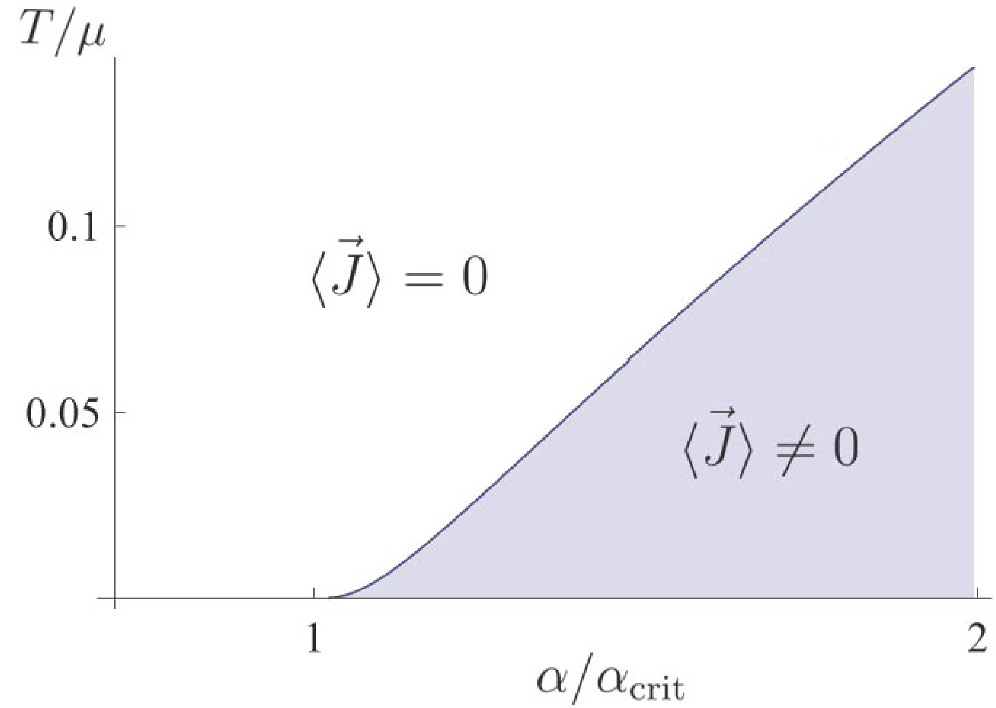
$$\alpha \to \infty \quad , \quad \alpha E : \text{finite}$$

In the charged black hole, this means:

$$\alpha \mu : \text{finite}$$

$$(\mu = \text{chemical potential})$$

This is perfect since we can probe the phase transition in this limit.



$$\alpha \sim \frac{1}{\mu} \longrightarrow \infty$$

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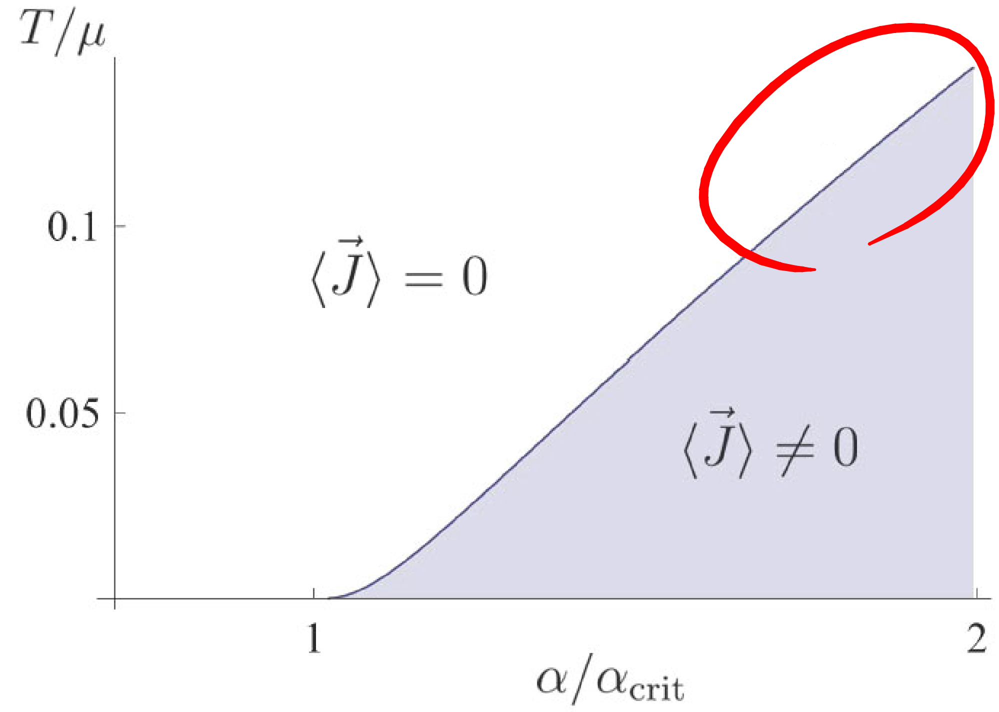


$$\alpha \sim \frac{1}{\mu} \longrightarrow \infty$$

Since $\mu \ll T$,

we can use the Schwarzschild AdS black hole as the initial solution.

$$ds^2 = -H(r)\,dt^2 + \frac{dr^2}{H(r)} + r^2 d\vec{x}^2$$

$$\vec{x} = (x^1, x^2, x^3)$$

$$ds^2 = -H(r)\,dt^2 + \frac{dr^2}{H(r)} + r^2 d\vec{x}^{\,2}$$

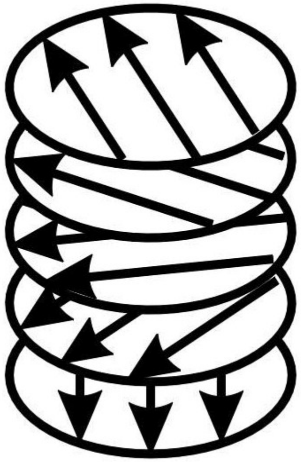
We look for a non-linear solution to

$$d{}^*F + \frac{1}{2}\,\alpha\; F \wedge F = 0$$

in the black hole geometry.

Translational invariance along the boundary will be broken spontaneously.

We assume that the following combinations to be unbroken:



$$\mathcal{P}_1 + k\,\mathcal{R}_{2,3}$$

$$\mathcal{P}_2\ ,\ \mathcal{P}_3$$

$$A_0(r), \quad A_r(r), \quad A_1(r)$$

$$A_{2+i3} = \phi(r)\, e^{ikx^1}$$

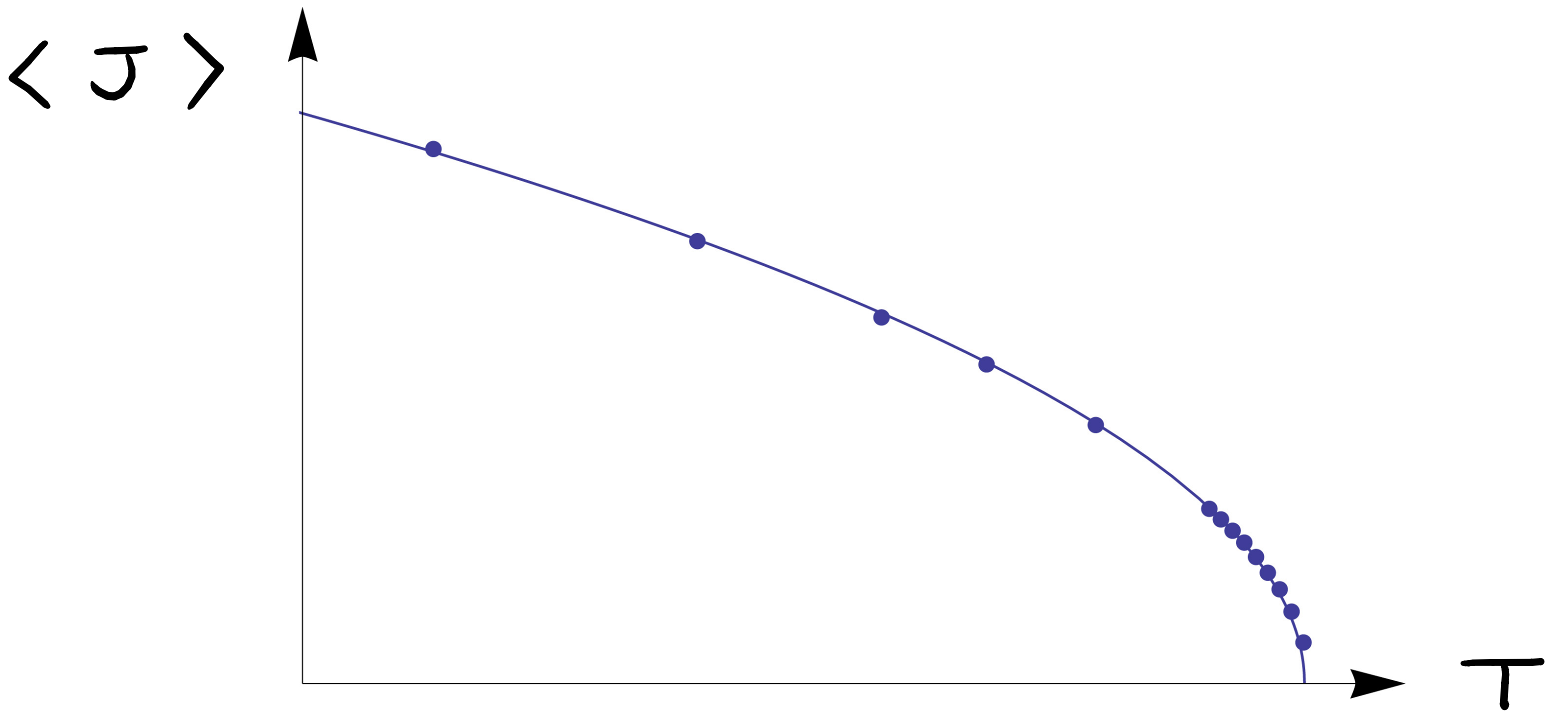
$$\Downarrow$$

$$\frac{d}{dr}\left(H(r)\, r \frac{d}{dr}\phi(r)\right)$$

$$-\left(\frac{k^2}{r} - \frac{4\alpha E k}{r^3}\right)\phi(r) - \frac{8\alpha^2 k^2}{r^3}\phi(r)^3 = 0$$

$$\frac{d}{dr}\left(H(r)\, r \frac{d}{dr}\phi(r)\right) - \left(\frac{k^2}{r} - \frac{4\alpha E k}{r^3}\right)\phi(r) - \frac{8\alpha^2 k^2}{r^3}\phi(r)^3 = 0$$

- ϕ^2 term absent
- Non-trivial solution for $T < T_c$.



2nd order phase transition
with the mean field exponent:

$$\langle J \rangle \sim (T_c - T)^{1/2}$$

2nd order phase transition with the mean field exponent:

$$\langle J \rangle \sim (T_c - T)^{1/2}$$

The order of the phase transition may change by 1/N effects.

An analogous problem was studied by

Brazovskii, JETP 41 (1975) 85,

(also by Hohenberg and Swift)

to describe spatially modulated phases, *e.g.* cholestric phase in liquid crystal.

Brazovskii model

$$V = \frac{1}{2}\left(\tau + (p-p_0)^2\right)\phi^2 + \frac{\lambda}{4}\phi^4 .$$

Classically, a 2nd order phase transition at $\tau = 0$.

For $\tau < 0$,

$$\langle \phi(x) \rangle \sim e^{i\vec{p}\cdot\vec{x}} \times (-\tau)^{1/2}$$

$$|\vec{p}| = p_0$$

Brazovskii model

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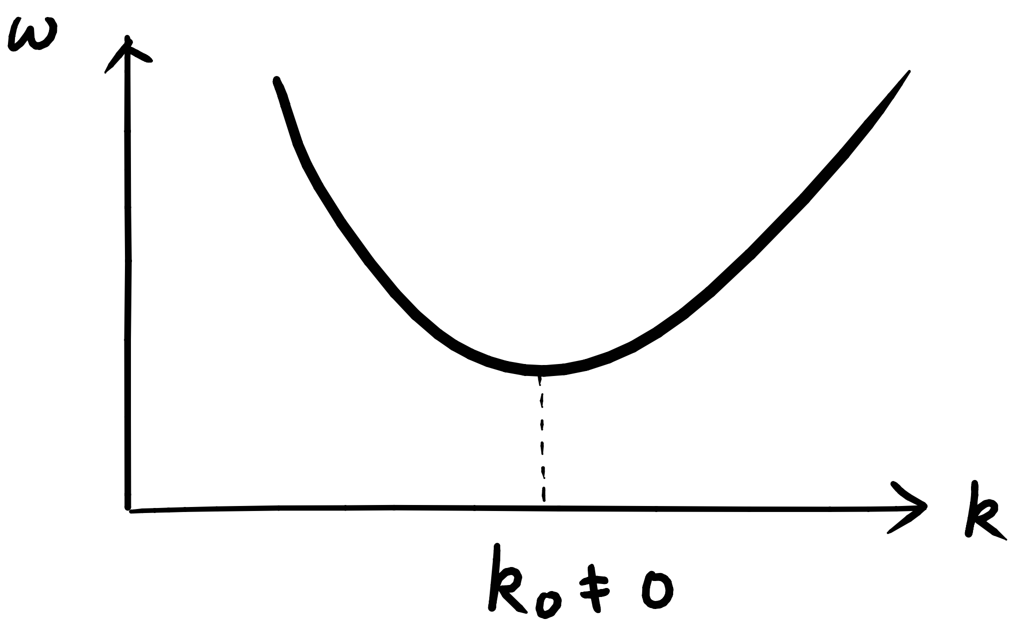


The Van Hove singularity enhances quantum fluctuations.

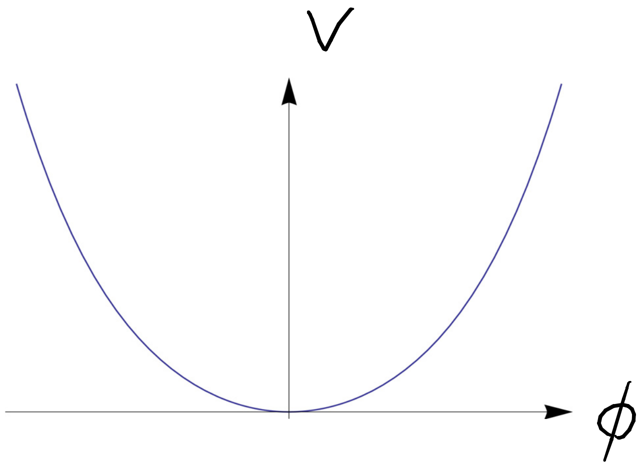
Brazovskii model

$$V = \frac{1}{2}\left(\tau + (p-p_0)^2\right)\phi^2 + \frac{\lambda}{4}\phi^4 .$$

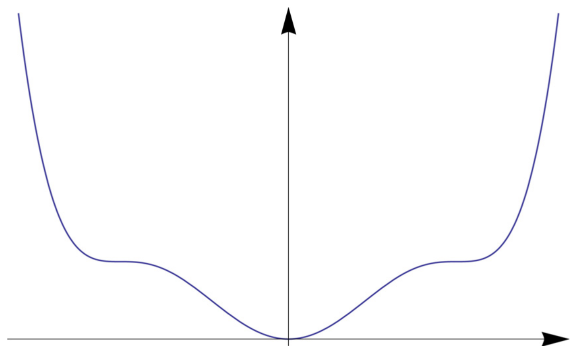
Classically, a 2nd order phase transition at $\tau = 0$.

Quantum fluctuation delays the phase transition

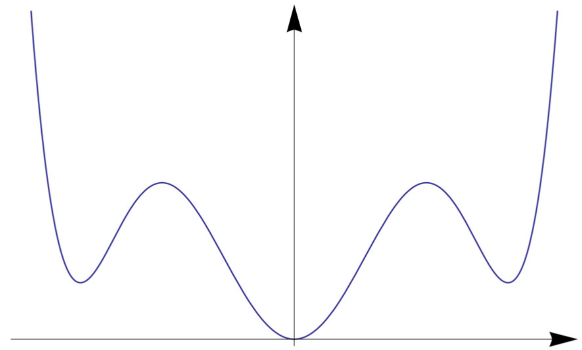
The 1st order phase transition at $\tau = \tau_1 < 0$.



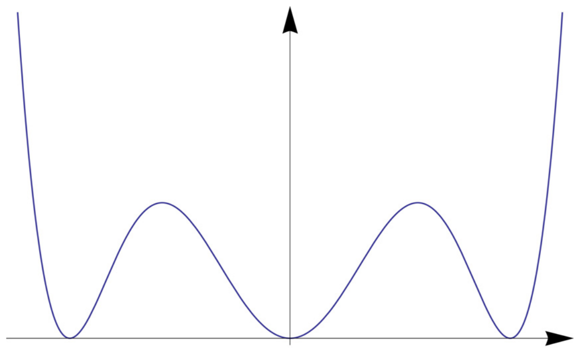
$\tau = 0$



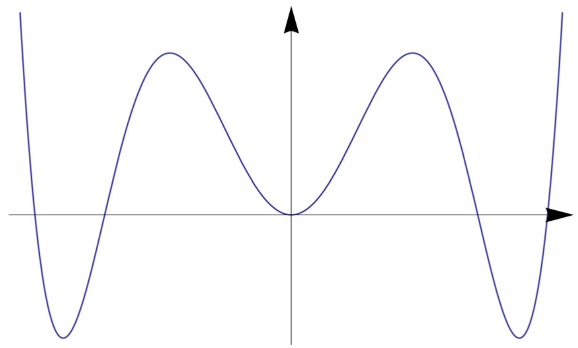
$\tau = \tau_0 < 0$



$\tau_1 < \tau < \tau_0$



$\tau = \tau_1$



$\tau < \tau_1$

Similarly, quantum effects in AdS may generate the first order phase transition in our model.

Spatially modulated phases are known in condensed matter physics and in QCD.

e.g., Fulde-Ferrell-Larkin-Ovchinnikov

involving Cooper pair of two species of fermions with different Fermi momenta.

The Chern-Simons coupling in the bulk

corresponds to

the chiral anomaly in the dual CFT.

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This correspondence has turned out
to have important implications on
the hydrodynamical regime of the CFT.

The Chern-Simons coupling in the bulk

corresponds to

the chiral anomaly in the dual CFT.

How does the chiral anomaly cause the spatially modulated phase transition?