Anomalies, Hydrodynamics, and Nonequilibrium Phenomena

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Strings 2013, Seoul

Two topics on applied holography:

◎ manifestation of **anomalies** in hydrodynamics

edge current, Hall viscosity,
angular momentum generation

◎ **far out of equilibrium**

non-linear response
effective temperature

In this talk, I will use units where 2 = π = 1, *etc*.
Precise expressions will be given in our papers.

Anomalies and Hydrodynamics

based on

arXiv:1212.3666 (Phys.Rev.Lett.110.211601)
with Hong Liu, Bogdan Stoica and Nico Yunes,

and work in progress
with Hong Liu and Bogdan Stoica.

Anomalies have played important roles in high energy physics and string theory.

Recently, it has become clear that anomalies have significant manifestations in the long range behavior of many body systems and affect **transport and hydrodynamics**.

4d / 5d

$$\nabla_\mu j^\mu = \alpha F^* F \quad \Leftrightarrow \quad \alpha \int_{5d} A \wedge F \wedge F$$

New kinetic coefficients:

$$j^\mu = n u^\mu - \sigma T (g^{\mu\nu} + u^\mu u^\nu) \partial_\nu \left(\frac{\mu}{T}\right)$$
$$+ \alpha \, \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

Bhattacharya, et al. , 0712.2456

Erdmenger et al. , 0809.2488

Son and Surowka, 0906.5044

Chiral anomalies also generate stripe phases.

Domokos and Harvey, 0704.1604.
Nakamura, Park and H.O., 0911.0697.
Park and H.O., 1007.3737, 1011.4144.

The construction has been successfully embedded in string theory.

Donos and Gauntlett, 1106.2004.

reduction to 4d / 3d

$$\alpha \int_{4d} \theta \, F \wedge F \quad \Leftrightarrow \quad \alpha \int_{3d} \theta \, \Phi$$

pseudo
- scalar

Consider Reissner-Nordstrom black brane
with chemical potential μ

edge current

heuristic argument

With chemical potential, $\alpha \int \theta \, F \wedge F$ generates

$$\langle J^i(x)\, \Phi(y) \rangle \sim \alpha \mu \, \epsilon^{ij} \partial_j \delta^{(3)}(x-y)$$

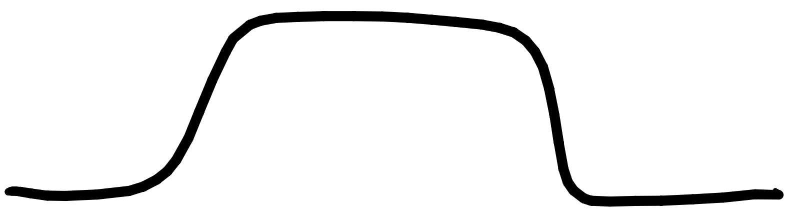
With slight inhomogeneity in boundary condition,

$$\langle J^i(x) \rangle \sim \alpha \mu \, \epsilon^{ij} \partial_j \theta$$

$i, j = 1, 2$

edge current: ρ

$$\langle J^i(x) \rangle = \epsilon^{ij} \partial_j \rho(x)$$



$\rho \neq 0$ $\rho = 0$

This gives an **edge current at the boundary** of the support of ρ.

edge current: ρ

$$\langle J^i(x) \rangle = \epsilon^{ij} \partial_j \rho(x)$$

$$\rho = \alpha \int dr\, F_{tr} \cdot \theta$$

$$= \alpha \mu \theta \quad \text{if} \quad \partial_r \theta = 0$$

i.e. Φ : marginal

angular momentum

heuristic argument

With chemical potential, $\alpha \int \theta \, F \wedge F$ generates

$$\langle T^{0i}(x)\, \Phi(y) \rangle \sim \alpha \mu^2 \epsilon^{ij} \partial_j \delta^{(3)}(x-y)$$

With slight inhomogeneity in boundary condition,

$$\langle T^{0i}(x) \rangle \sim \alpha \mu^2 \epsilon^{ij} \partial_j \theta$$

angular momentum density: ℓ

$$T^{0i} = \epsilon^{ij} \partial_j \ell$$

(*total angular momentum*)

$$= \int dx^2 \, \epsilon_{ij} \, x^i \, T^{0j} \sim \int dx^2 \, \ell$$

angular momentum density: ℓ

$$T^{0i} = \epsilon^{ij} \partial_j \ell$$

$$\ell = \alpha \int dr \, \theta \, (A_t - \mu) F_{rt}$$

$$= \alpha \, \mu^2 \, \theta \Big|_{r=\infty} \qquad \text{if } \partial_r \theta = 0, \quad \text{i.e. marginal.}$$

angular momentum generated by gravitational Chern-Simons: $\propto \int \theta \, R \wedge R$

angular momentum density:

$$\ell = \alpha \int dr \, \theta \, \partial_r \left(\frac{\left(\left(\partial_r - \frac{2}{r} \right) g_{tt} \right)^2}{g_{tt} \, g_{rr}} \right)$$

$$= \alpha \, T^2 \, \theta \Big|_{r=\infty} \qquad \text{if } \partial_r \theta = 0 ,$$

i.e. marginal

Hall viscosity

Avron, Seiler and Zograf ('95)

$$T^{\mu\nu} = (\mathcal{E} + P)\, u^\mu u^\nu + P g^{\mu\nu}$$
$$- \frac{1}{2} \eta_H \epsilon^{\mu\alpha\beta} u_\alpha \left(\partial_\beta u^\nu + \partial^\nu u_\beta - \delta^\nu_\beta \, \partial \cdot u \right)$$
$$+ (\mu \longleftrightarrow \nu)$$

generated holographically
by gravitational Chern-Simons: $\quad \propto \int \theta \, R \wedge R$

Holographically, the Hall viscosity requires scalar hair.

Saremi and Son, 1103.4851

$$\eta_H = \alpha \, \frac{\left(\partial_r - \frac{2}{r}\right) g_{tt}}{g_{tt}\, g_{rr}} \, \partial_r \theta \, \Big|_{horizon}$$

We found a class of holographic models
for which the Hall viscosity is non-zero.

There seems to be a connection between the angular momentum and the Hall viscosity.

$$\ell = \alpha \int dr\, \theta\, \partial_r \left(\frac{\left(\left(\partial_r - \frac{2}{r}\right) g_{tt}\right)^2}{g_{tt}\, g_{rr}} \right)$$

$$\eta_H = \alpha \left. \frac{\left(\partial_r - \frac{2}{r}\right) g_{tt}}{g_{tt}\, g_{rr}} \partial_r \theta \right|_{horizon}$$

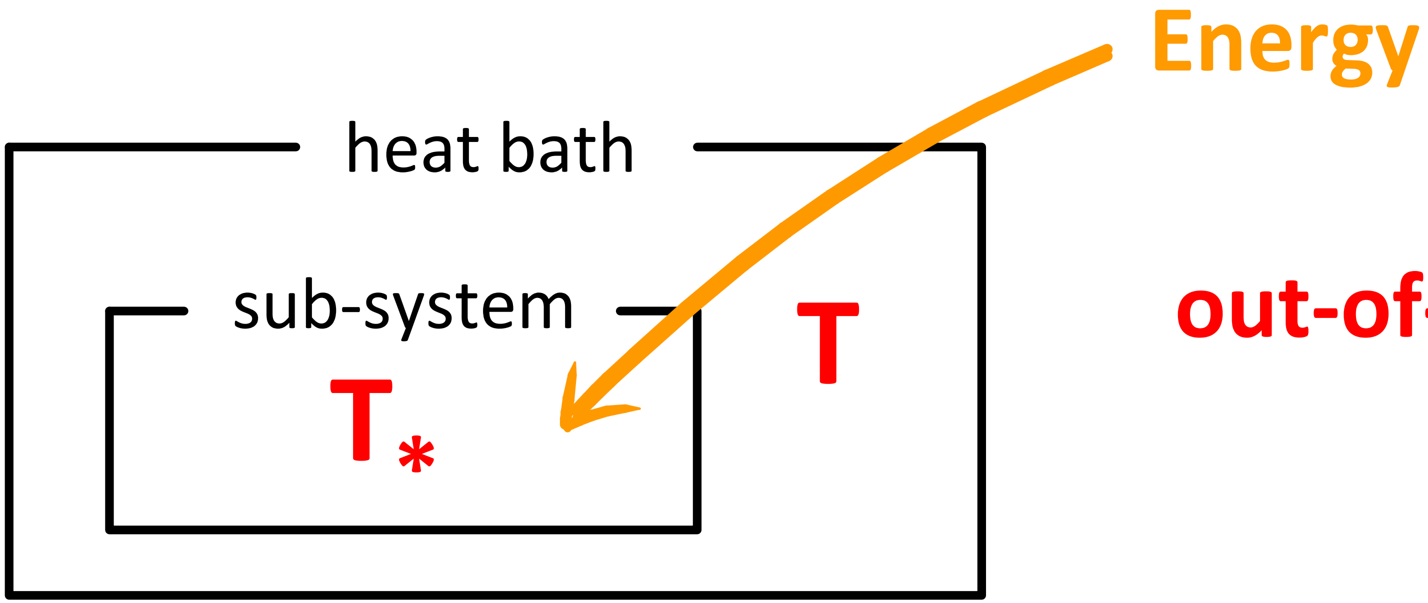
Chern-Simons terms in the bulk generate

★ Edge current
★ Angular momentum density
★ Hall viscosity

More to be learned from interplay
of anomalies, topology and hydrodynamics.

Out-of-equilibrium Phenomena

based on work in collaboration with Shin Nakamura.



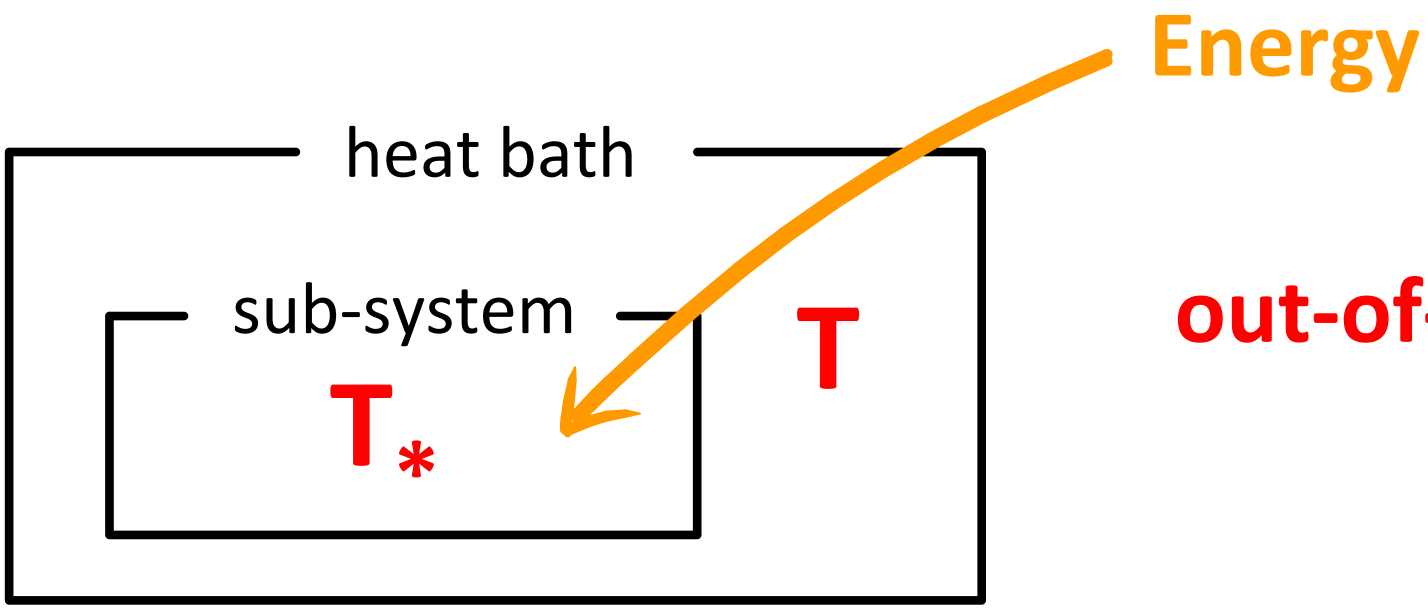
out-of-equilibrium

but **stationary**

I will discuss two types of non-linear responses:

◎ electric field $\Rightarrow$ current

◎ drag force $\Rightarrow$ brane motion



out-of-equilibrium
but **stationary**

We find:

◎ fluctuations are **universal and thermal**.

◎ **Hawking temperature** T_* is consistent with the fluctuation-dissipation theorem.

◎ **unexpected features** of T_*

($p+1$)-dim QFT at temperature T

… add ($q+1$)-dim defect.

holographically:

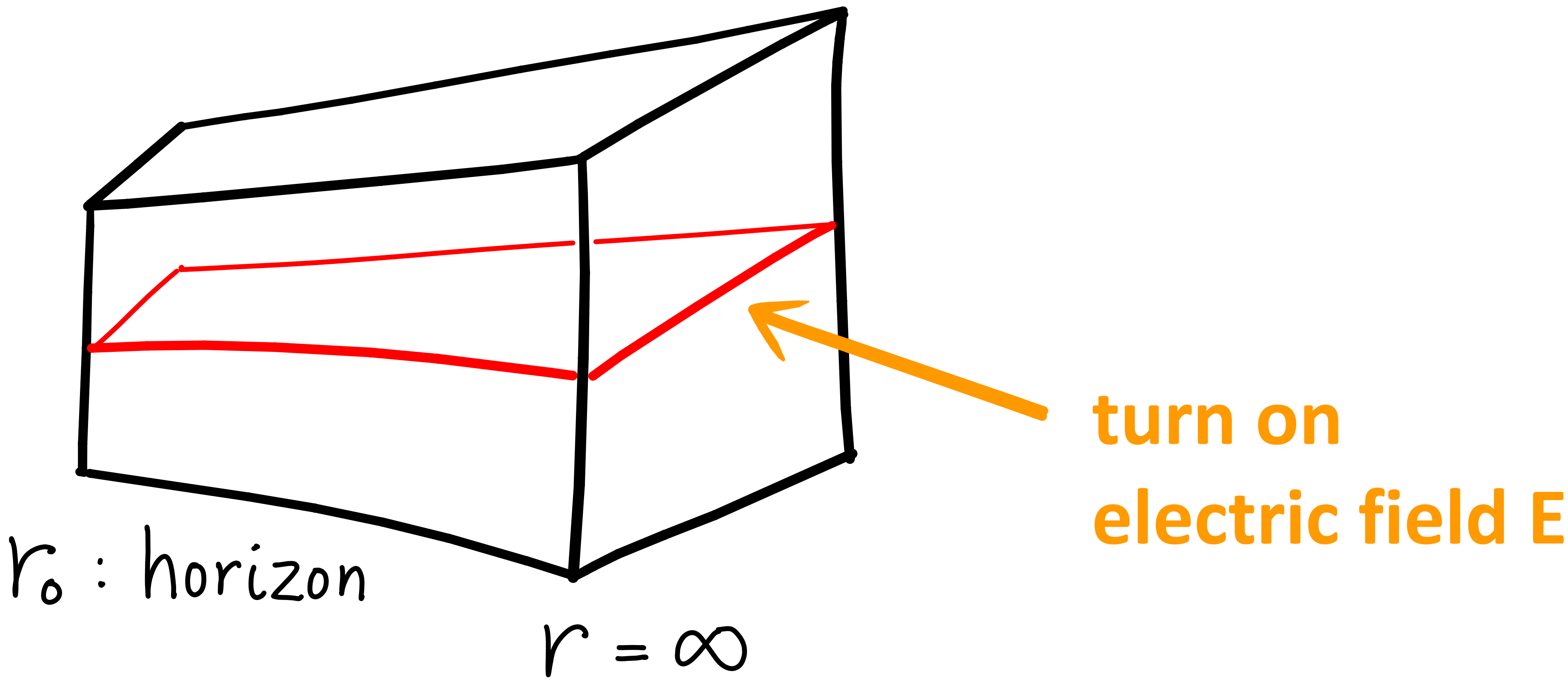
$$ds^2 = g_{tt}\,dt^2 + g_{xx}\underbrace{dx^2}_{p\text{-dim}} + g_{rr}\,dr^2 + g_{\theta\theta}\underbrace{d\Omega^2}_{\text{compact}}$$

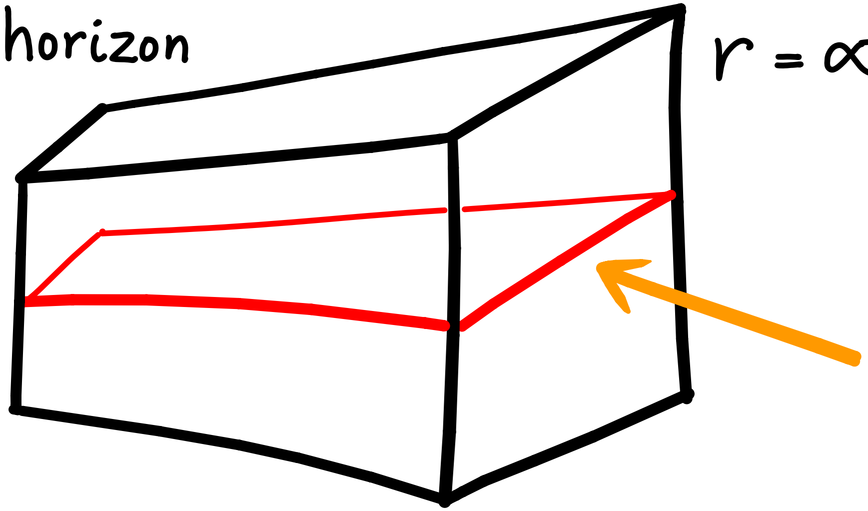
probed by a ($q+1+n$)-brane

wrapping a compact n-cycle.

$$ds^2 = g_{tt}\,dt^2 + g_{xx}\underbrace{dx^2}_{p\text{-dim}} + g_{rr}\,dr^2 + g_{\theta\theta}\underbrace{d\Omega^2}_{\text{compact}}$$

probed by a (*q*+*1*+*n*)-brane on a compact *n*-cycle.

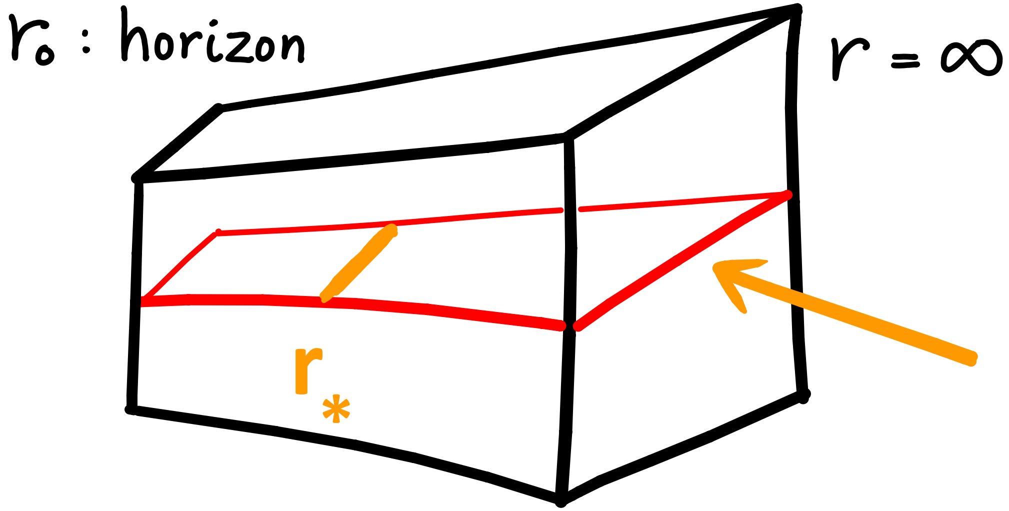




$$\text{e.o.m.}:\quad \partial_r\left(\frac{\partial \mathcal{L}}{\partial F_{rx}}\right) = 0 \;\Rightarrow\; \frac{\partial \mathcal{L}}{\partial F_{rx}} = \text{const} = J$$

$$\Rightarrow\quad (F_{rx})^2 \sim \frac{E^2 - |g_{tt}|\, g_{xx}}{J^2 - e^{-2\phi}|g_{tt}|\, g_{xx}^{q-1}}$$

$$r_0 < {}^{\exists} r_* < \infty, \qquad E^2 = |g_{tt}|\, g_{xx}(r_*)$$



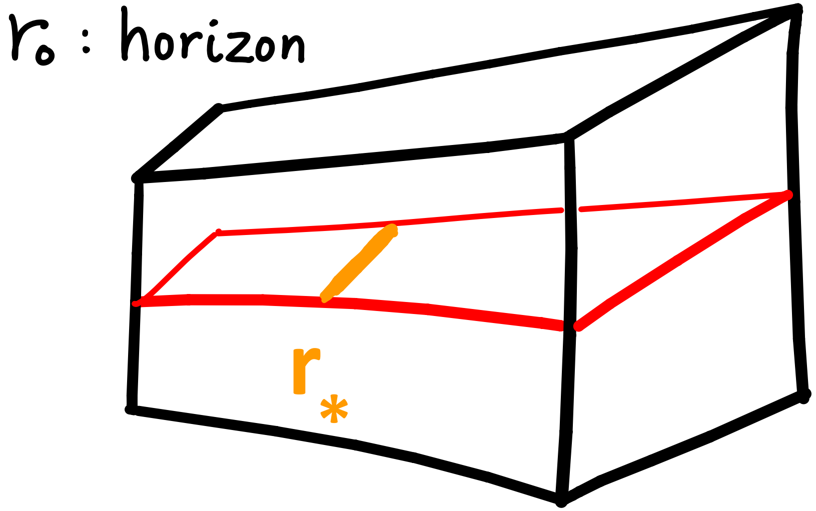
turn on electric field E

$$(F_{rx})^2 \sim \frac{E^2 - |g_{tt}|\, g_{xx}}{J^2 - e^{-2\phi}|g_{tt}|\, g_{xx}^{q-1}}$$

$$\begin{cases} E^2 = |g_{tt}|\, g_{xx}(r_*) \\ J^2 = e^{-2\phi}|g_{tt}|\, g_{xx}^{q-1}(r_*) \end{cases} \Longrightarrow \quad J = J(E)$$

$$r_0 < {}^{\exists} r_* < \infty$$

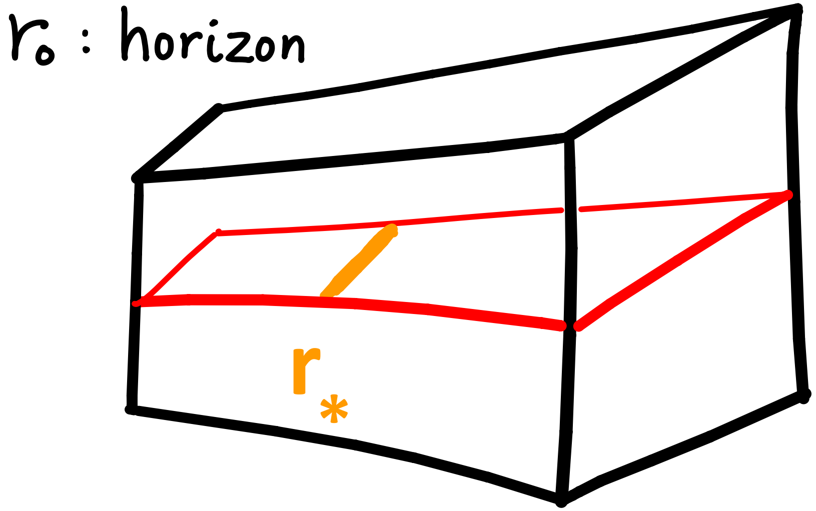
Karch and O'Bannon, 0705.3870.



Scalar and gauge field fluctuations feel **different effective metrics** on the brane,

But, both have a horizon at r_*
with the **same Hawking temperature** T_* .

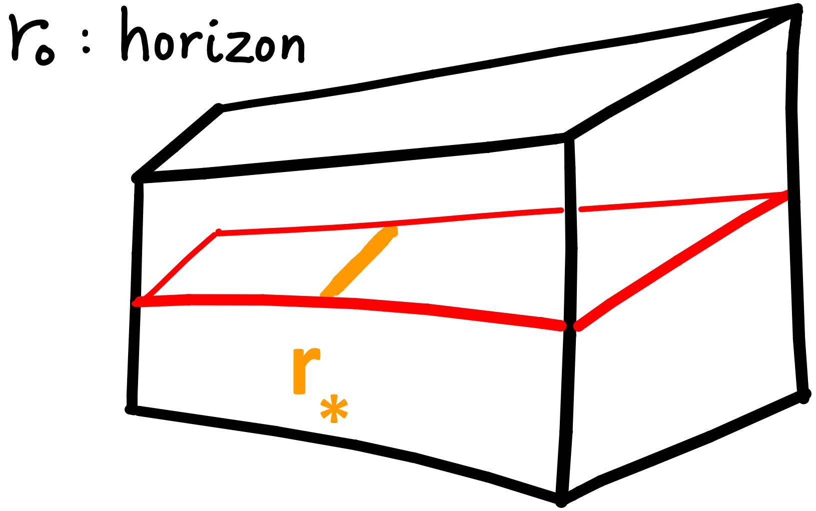
Assuming that the brane is static,
fluctuations should be thermalized at T_*.



For D*p* branes probed by D(*q+1+n*) branes wrapping a compace *n*-cycle,

$$T_* = \frac{\left(T^{\frac{14-2p}{5-p}} + C E^2\right)^{\frac{1}{2}}}{\left(T^{\frac{14-2p}{5-p}} + E^2\right)^{\frac{1}{7-p}}}$$

$$C = \frac{1}{2}\left(q + 3 - p + \frac{p-3}{7-p}\, n\right)$$



$$T_* = \frac{\left(T^{\frac{14-2p}{5-p}} + C E^2\right)^{\frac{1}{2}}}{\left(T^{\frac{14-2p}{5-p}} + E^2\right)^{\frac{1}{7-p}}}$$

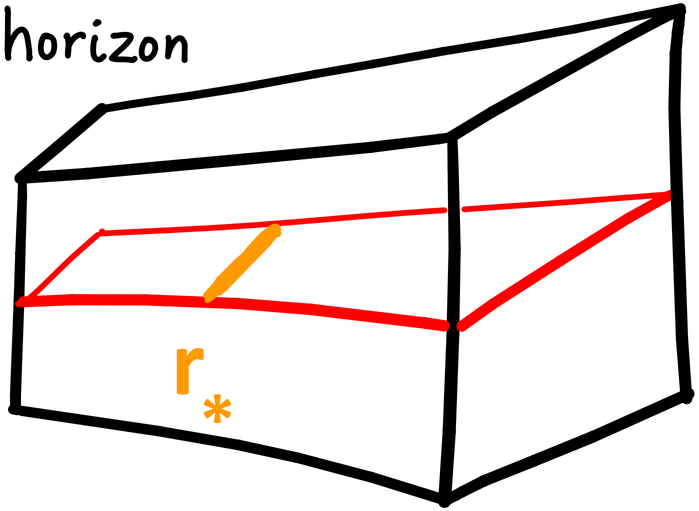
$$C = \frac{1}{2}\left(q + 3 - p + \frac{p-3}{7-p} n\right)$$

For example, for $p = 3$ and $(q, n) = (2, 3)$,

$$T_* = \left(T^4 + E^2\right)^{\frac{1}{4}}$$

reproducing Sonner and Green, 1203.4908.

r_0 : horizon



$$T_* = \frac{\left(T^{\frac{14-2p}{5-p}} + C E^2\right)^{\frac{1}{2}}}{\left(T^{\frac{14-2p}{5-p}} + E^2\right)^{\frac{1}{7-p}}}$$

$$C = \frac{1}{2}\left(q + 3 - p + \frac{p-3}{7-p} n\right)$$

$T_*(E)$ is monotonic in E^2:

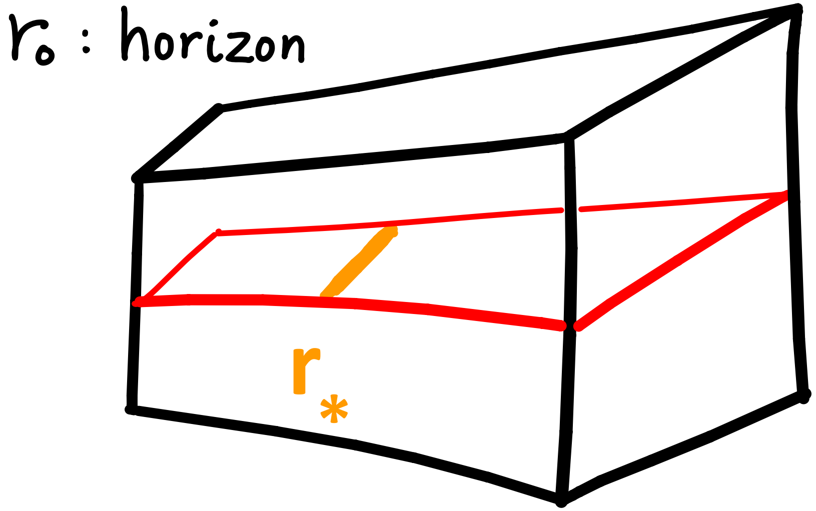
$$T_* = T + \left(\frac{1}{2} C - \frac{1}{7-p}\right) \frac{E^2}{T^{\frac{9-p}{5-p}}} + O(E^4)$$

$$T_* = T + \left(q + 3 - p + \frac{(p-3)n - 4}{7-p} \right) \frac{E^2}{4T^{\frac{9-p}{5-p}}} + O(E^4)$$

$$\mathbf{T_* < T \quad if} \quad q + 3 - p + \frac{(p-3)n - 4}{7-p} < 0$$

For example, for p = 4 and $(q, n) = (1, 0)$,

$$T_* = \frac{T^3}{(T^6 + E^2)^{1/3}}$$



$$T_* = \frac{\left(T^{\frac{14-2p}{5-p}} + C E^2\right)^{\frac{1}{2}}}{\left(T^{\frac{14-2p}{5-p}} + E^2\right)^{\frac{1}{7-p}}}$$

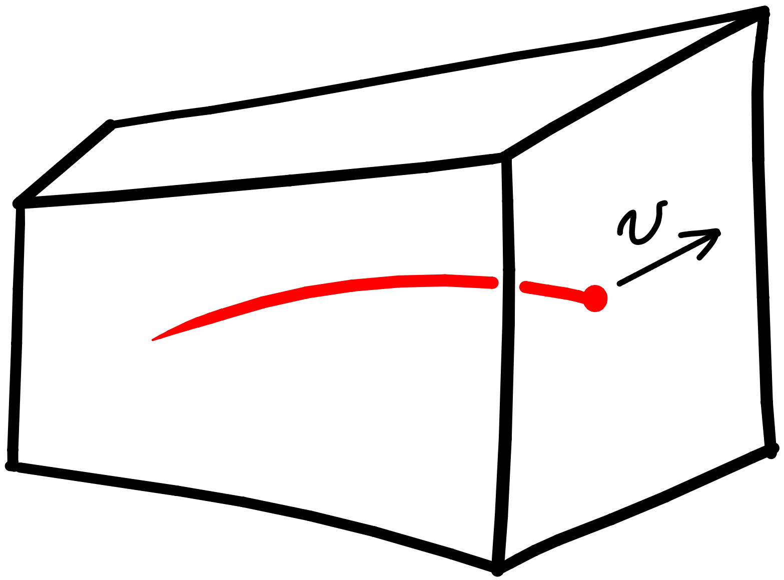
$$C = \frac{1}{2}\left(q + 3 - p + \frac{p-3}{7-p} n\right)$$

◎ one can lower the effective temperature T_* on the brane by turning on the electric field.

◎ T_* is the same for all fluctuation modes.

◎ linear response theory is for O(E).

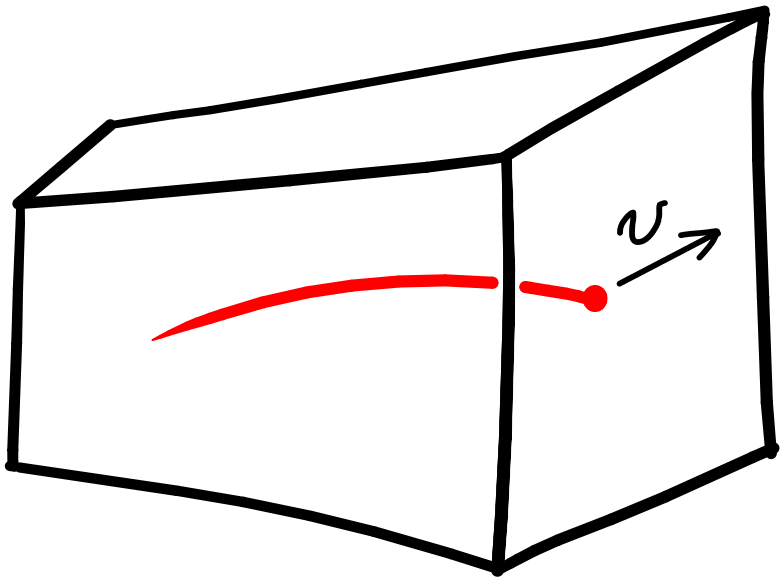
drag force



D*p* branes probed
by a D(*q+1+n*) brane.

pull the D(*q+1+n*) brane
with constant velocity.

drag force

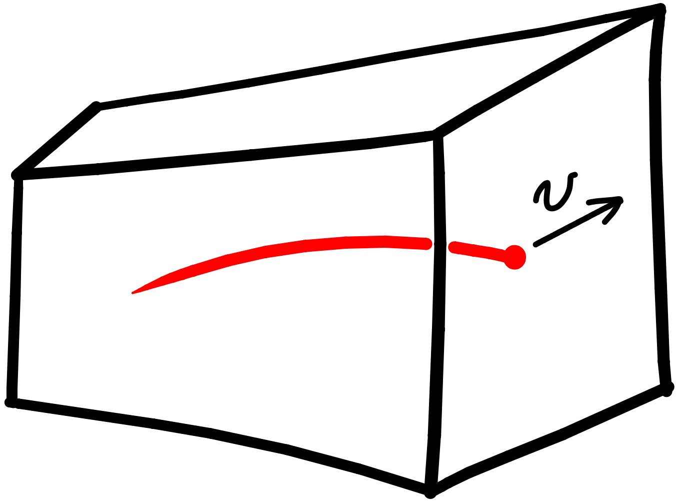


D*p* branes probed
by a D(*q+1+n*) brane.

pull the D(*q+1+n*) brane
with constant velocity.

$$T_* = (1 + c v^2)^{\frac{1}{2}} (1 - v^2)^{\frac{1}{7-p}}\, T$$

drag force



$$T_* = (1 + c v^2)^{\frac{1}{2}} (1 - v^2)^{\frac{1}{7-p}} T$$

For example, for p = 3 and q = 0,

$$T_* = (1 - v^2)^{\frac{1}{4}} T < T$$

◎ electric field ⇒ current

$$T_* = \frac{\left(T^{\frac{14-2p}{5-p}} + cE^2\right)^{\frac{1}{2}}}{\left(T^{\frac{14-2p}{5-p}} + E^2\right)^{\frac{1}{7-p}}}$$

$$= T + \left(\frac{1}{2}c - \frac{1}{7-p}\right)\frac{E^2}{T^{\frac{9-p}{5-p}}} + O(E^4)$$

◎ drag force ⇒ brane motion

$$T_* = (1+cv^2)^{\frac{1}{2}}(1-v^2)^{\frac{1}{7-p}}\,T$$

$$= T + \left(\frac{1}{2}c - \frac{1}{7-p}\right)v^2 T + O(v^4)$$

◎ electric field $\Rightarrow$ current

◎ drag force $\Rightarrow$ brane motion

In both cases, $\mathbf{T}_* < \mathbf{T}$ when

$$q + 3 - p + \frac{(p-3)n - 4}{7-p} < 0$$

For $T \to 0$,

◎ electric field ⇒ current

$$T_* = \frac{\left(T^{\frac{14-2p}{5-p}} + CE^2\right)^{\frac{1}{2}}}{\left(T^{\frac{14-2p}{5-p}} + E^2\right)^{\frac{1}{7-p}}} \sim E^{\frac{5-p}{7-p}}$$

for $p < 5$ & $C > 0$

◎ drag force ⇒ brane motion

$$T_* = (1 + cv^2)^{\frac{1}{2}} (1 - v^2)^{\frac{1}{7-p}} T \longrightarrow 0$$

comparison with the Langevin equation

$$\frac{dP}{dt} = -\eta P + \xi$$

For $p = 3$ and $q = 0$,

$$\eta \sim \frac{T^2}{m}, \qquad \langle \xi \xi \rangle_T \sim \frac{T^3}{(1-v^2)^{1/4}}$$

$$\langle \xi \xi \rangle_L \sim \frac{T^3}{(1-v^2)^{5/4}}$$

Gubser, Herzog, et al., Casalderry-Solana and Teaney,
Liu, et al., Giecold, et al.

$$\eta \sim \frac{T^2}{m}, \qquad \langle \xi \, \xi \rangle_T \sim \frac{T^3}{(1-v^2)^{1/4}}$$

$$\langle \xi \, \xi \rangle_L \sim \frac{T^3}{(1-v^2)^{5/4}}$$

$$\Downarrow$$ fluctuation-dissipation theorem

$$\langle (\delta p_T)^2 \rangle \sim \frac{mT}{(1-v^2)^{1/4}}$$

$$\langle (\delta p_L)^2 \rangle \sim \frac{mT}{(1-v^2)^{5/4}}$$

$$\sqrt{m^2+p^2} = \sqrt{m^2 + (p_0 + \delta p_L)^2 + (\delta p_T)^2}$$

$$\sim \sqrt{m^2+p_0^2} + \frac{(\delta p_T)^2}{2\sqrt{m^2+p_0^2}} + \frac{m^2(\delta p_L)^2}{2\left(\sqrt{m^2+p_0^2}\right)^3} + \cdots$$

$$\left\langle \frac{(\delta p_T)^2}{2\sqrt{m^2+p_0^2}} \right\rangle = \frac{1}{2}(1-v^2)^{\frac{1}{4}} T$$

$$\left\langle \frac{m^2(\delta p_L)^2}{2\left(\sqrt{m^2+p_0^2}\right)^3} \right\rangle = \frac{1}{2}(1-v^2)^{\frac{1}{4}} T$$

Consistent with $T_* = (1-v^2)^{\frac{1}{4}} T$

Comments:

◎ $r_0 < r_*$ does <u>not</u> imply $T < T_*$ since the metrics in the bulk and on the brane are different.

◎ known theorems are mostly in O(E);
$T > T_*$ happens at $O(E_2)$.

◎ there are lattice models with $T > T_*$,
but rather artificial.
... more robust examples by holography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감사합니다