# String Theory of The Regge Intercept

Simeon Hellerman Kavli Institute for the Physics and Mathematics of the Universe Tokyo University Institutes for Advanced Study

S.H. and Ian Swanson, arXiv:1312.0999 S.H., J. Maltz, S. Maeda, I. Swanson, arXiv:1405.6197 S.H., and Ian Swanson, In Progress

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Image credit http://phys.columbia.edu/kabat/why\_strings/Regge.jpg

Image credit http://courses.washington.edu/phys55x/ %Physics20557\_lec11\_files/image064.jpg



The string theory of QCD was originally formulated to explain remarkable, robust patterns in hadronic spectral data.



All hadronic states appear to lie in a tower of resonances that can be plotted on a graph of mass-squared versus angular momentum, as straight lines with a common, universal slope.



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We know today that the string theory of QCD is JUST WRONG at distances  $\sim \sqrt{\alpha'}$ . However we can still treat string theory as a perfectly good effective theory at scales  $>> \sqrt{\alpha'}$ .



For a string with large angular momentum, its length is  $\simeq \sqrt{J\alpha'}$  so we should be able to use the effective theory of the string worldsheet when J >> 1.



This point of view predicts corrections to the Regge spectrum in the form

$$m^2 = rac{J}{lpha'} \cdot \left[ 1 + O\left( J^{-\kappa} \right) \right] , \qquad \kappa > 0 .$$



The leading large-J behavior represents a venerable story that motivated the development of string theory in the first place, during the 1970s. Since that time, no general theory of the subleading large-J corrections has ever been developed.



This talk will describe the development of such a theory.

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- You might ask: Why does this work at all, in any approximation?
- When the string is large, the short-distance structure should become irrelevant, in the technical sense of the renormalization group.
- The dynamics should be described by the most relevant terms one can write in a local action for a string, invariant under all the appropriate symmetries.
- The most relevant term invariant under the Poincaré symmetry of D-dimensional spacetime is the Nambu-Goto action:
  SNG = T<sub>string</sub> · Area<sub>worldsheet</sub> ,

$$\mathbf{T}_{\text{string}} \equiv \frac{1}{2\pi \alpha'}$$

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- The Nambu-Goto action describes the spectrum with arbitrarily good precision when the string is large, with typical size sale "R".
- ► Less relevant terms in the action should contribute with powers (perhaps including logarithms) of  $R/\sqrt{\alpha'}$ .
- ➤ An operator scaling as Length<sup>-p</sup> contributes to any observable at relative order R<sup>-(p+2)</sup> ("Relative" to the leading Nambu-goto contribution, that is).
- The coarse analysis of large-R corrections is easy to learn the power laws that appear rather than their coefficients, just classify possible invariant operators up to some order in inverse length.

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- In certain situations, the answer is yes assuming the theory is guantized consistently.
- ► The leading corrections to the NG action including the curvature-squared term scale as |X|<sup>-2</sup>
- ► Therefore these operators contribute to M<sup>2</sup><sub>meson</sub> at order J<sup>-1</sup> at most.
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- To carry out the analysis, we must pick a gauge.
- The two most commonly used gauges (for D not equal to the critical dimension) are orthogonal gauge and static gauge.
- The analysis in these two gauges has mostly been done disjointly, with little comparison between the two approaches. Recently, the two gauges, properly renormalized at the quantum level, have been found to be equivalent up to relative order (length)<sup>-6</sup>. (Aharony *et al.*; Dubovsky, Flauger, Gorbenko)
- The evidence for the agreement of gauges is overwhelming.

- In practice, orthogonal gauge is much simpler because it is free at leading order.
- Furthermore, we'll be interested in non-static situations, such as rotating strings, which makes static gauge complicated!
- I will not give a review of the old-fashioned approach to orthogonal gauge.
- I'll begin by constructing effective string theory in conformal gauge and placing it in a simplified framework by embedding it in the Polyakov formalism.

Let's begin by considering the usual Polyakov action for the bosonic string, but with an arbitrary number D of embedding coordinates.: The Polyakov string is defined by the path integral

$$\begin{split} Z &= \int \mathcal{D}\mathcal{M}^{\rm Polyakov}_{[g]} \exp\left(-S_{\rm Polyakov}\right) \ ,\\ \mathcal{D}\mathcal{M}^{\rm Polyakov}_{[g]} &\equiv \frac{\mathcal{D}_{[g]} X \, \mathcal{D}_{[g]} g}{\mathcal{D}_{[g]} \Omega} \\ S_{\rm Polyakov} &= \int d^2 \sigma \, \sqrt{|g_{\bullet \bullet}|} \mathcal{L}_{\rm Polyakov} \ ,\\ \mathcal{L}_{\rm Polyakov} &= \frac{1}{4\pi \alpha'} g^{ab} \, \partial_a X^{\mu} \, \partial_b X_{\mu} \ , \end{split}$$

The action  $S_{\rm Polyakov}$  is Weyl-invariant but the measure  $\mathcal{DM}_{[g]}^{\rm Polyakov}$  is not.

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Under a Weyl transformation  $g_{\bullet\bullet} \to \exp(+2\omega) g_{\bullet\bullet}$ ,

the individual factors of the integrand transform as:

 $egin{aligned} &\mathcal{S}_{ ext{Polyakov}} 
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ightarrow \exp\left(F_{ ext{anom}}[g,\omega]
ight) \mathcal{DM}^{ ext{Polyakov}}_{[g]} \end{aligned}$ 

$$Z[g] = \int \, \mathcal{DM}^{
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Under a Weyl transformation  $g_{\bullet \bullet} \rightarrow \exp(+2\omega) g_{\bullet \bullet}$ ,

the individual factors of the integrand transform as:

$$\begin{split} S_{\mathrm{Polyakov}} &\to S_{\mathrm{Polyakov}} \\ &\mathcal{D}\mathcal{M}_{[g]}^{\mathrm{Polyakov}} \to \exp\left[(D-26)F_{\mathrm{anom}}[g,\omega]\right]\mathcal{D}\mathcal{M}_{[g]}^{\mathrm{Polyakov}} \\ &\text{The form of the anomaly functional } F[g,\omega] \text{ is determined uniquely to be} \\ &F_{\mathrm{anom}}[g,\omega] \equiv \frac{1}{24\pi} \int \sqrt{|g|} d^2\sigma \left\{g^{\bullet\bullet} \partial_{\bullet} \omega \partial_{\bullet} \omega + \omega \mathcal{R}_{(2)}[g]\right\} \\ &\text{by the Wess-Zumino consistency condition.} \end{split}$$

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$$Z[g] = \int \mathcal{DM}^{\mathrm{Polyakov}}_{[g]} \exp\left(-S_{\mathrm{Polyakov}}
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$$\begin{split} S_{\rm Polyakov} &\to S_{\rm Polyakov} \\ \mathcal{D}\mathcal{M}^{\rm Polyakov}_{[g]} &\to \exp\left[(D-26)F_{\rm anom}[g,\omega]\right] \mathcal{D}\mathcal{M}^{\rm Polyakov}_{[g]} \\ F_{\rm anom}[g,\omega] &\equiv \frac{1}{24\pi} \int \sqrt{|g|} \, d^2\sigma \left\{g^{\bullet\bullet} \, \partial_{\bullet}\omega \partial_{\bullet}\omega + \omega \, \mathcal{R}_{(2)}[g]\right\} \end{split}$$

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So the path integral is not invariant unless D = 26:

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In order to obtain a Weyl-invariant partition function,

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 In order to obtain a Weyl-invariant partition function,

we can augment the action by a term  $S_{\text{anom}}$  that transforms under a <u>Weyl transformation</u> as

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$$S_{\text{anom}} \rightarrow S_{\text{anom}} + (D - 26) F[g, \omega].$$

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$$S_{\text{anom}} \rightarrow S_{\text{anom}} + (D - 26) F[g, \omega].$$

In the "linear dilaton theory", one cancels this anomaly by assigning a nontrivial Weyl transformation to one of the scalars  $X^{D-1} \equiv \frac{1}{|V|} V_{\mu} X^{\mu}$ :

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Weyl-invariance requires  $\Delta \mathcal{L} = \frac{1}{4\pi} \mathcal{R}_{(2)} V \cdot X$ .

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- In this gauge we have

$$\mathcal{L}_{\text{anom}} = \frac{\beta}{2\pi} \frac{(\partial^2 X \cdot \bar{\partial} X)(\partial X \cdot \bar{\partial}^2 X)}{(\partial X \cdot \bar{\partial} X)^2} , \qquad \beta \equiv \frac{26 - D}{12}$$

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- People familiar with the old covariant effective string formalism of Polchinski and Strominger (1989) in orthogonal gauge will recognize this Lagrangian as the *ad hoc* anomaly term...
- modulo terms proportional to the free equations of motion, which can be removed by field redefinitions.
- Here, we have derived the same term from a starting point with more gauge invariance by embedding it in the Polyakov path integral and canceling the Weyl anomaly. I will refer to this as the simplified covariant formalism.

- For rotating strings we know the length R should scale as  $\sqrt{\alpha' J}$  when J is large.
- ➤ So calculating the relative-order R<sup>-2</sup> corrections for rotating strings corresponds to calculating the relative-order J<sup>-1</sup> corrections.
- The leading-order value of the mass-squared for an open string in four dimensions is

$$m_{
m leading Regge}^2 = rac{J}{lpha'}.$$

In fact, this relationship defines the (asymptotic) Regge slope  $\alpha'$ .

Computing a relative order J<sup>-1</sup> term would corrspond to computing the asymptotic Regge intercept on the "leading trajectory" – that is, the set of states of lowest mass for a given angular momentum.

- Formally, the relative order  $J^{-1}$  correction to the dispersion relation on the leading trajectory is particularly simple, because the lowest state with given Noether charges is automatically Virasoro-primary, so the physical state conditions are automatically satisfied, except the mass-shell condition from  $L_0$ .
- The correction to the mass-squared of the string state is given by

$$\begin{split} \Delta M^2 \big|_{\substack{\text{closed}\\\text{first-order}}} &= \frac{2}{\alpha'} \Delta E_{\text{ws}} \big|_{\text{first-order}} ,\\ \Delta M^2 \big|_{\substack{\text{open}\\\text{first-order}}} &= \frac{1}{\alpha'} \Delta E_{\text{ws}} \big|_{\text{first-order}} ,\\ \Delta E_{\text{ws}} \big|_{\text{first-order}} &= \langle (P,J) \big|_{\text{free}} \hat{H}_{\text{first-order}} | (P,J) \rangle_{\text{free}} , \end{split}$$

- This in turn is given by the Casimir energy <sup>D-2</sup>/<sub>12</sub>, plus the classical value of the interaction Hamiltonian in the classical rotating solution with the appropriate angular momenta.
- The classical value of the perturbing Hamiltonian is equal to the negative of the classical value of the perturbing Lagrangian. This follows from elementary manipulations in classical mechanics and applies only to the lowest state of a system with fixed Noether charges.
- No higher loops or even one-loop diagrams involving interaction vertices contribute at NLO in J. Each additional interaction vertex, and each additional quantum loop, is suppressed by at least one additional power of J.

- Let's see how this works, concretely, for open strings in conformal gauge.
- The solution for the lowest-lying state with angular momentum J in a single plane is of the form

$$X^0 = 2 \alpha' P^0 \sigma^0 ,$$

$$Z = -i\sqrt{\alpha' J} \left( e^{i\sigma^+} + e^{i\sigma^-} \right) ,$$

with  $\sigma^1$  running from 0 to  $\pi$ . The classical solution satsifies the Neumann boundary condition at  $\sigma^1 = 0, \pi$ .

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- ► For this case, our analysis breaks down in its own terms.
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- ► For this case, our analysis breaks down in its own terms.
- The Lagrangian is singular near σ<sup>1</sup> = 0, π in the classical solution.
- This is a non-integrable singularity. The integral diverges:

$$\mathcal{L}_{_{\mathrm{rotating solution}}} = -rac{eta}{2\pi^2} \, rac{\sin^2(2\sigma_1)}{(1-\cos(2\sigma_1))^2} \; .$$

- For the open string, this singularity is present because the boundary is moving at the speed of light and there is a curvature singularity in the Lorentzian induced metric.
- For the closed string, there is a singularity representing a fold in the string.
- ► In both cases, the integrated anomaly term diverges.
- We will first consider a model calculation that avoids this singularity.
- This breakdown of the theory is a short-distance singularity, to be removed by renormalization.
- But first, let us consider a simpler case, where there is no such singularity.

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## Closed strings with rotation in two planes

- Let us now perform a calculation in a simple case to illustrate the general idea of large-J universality at subleading order.
- ► The properties of rotations are different in higher dimensions. So we consider closed strings rotating in D ≥ 5, which need not have folds: The Polchinski-Strominger denominator is nonvanishing everywhere.
- ► We consider closed strings in D ≥ 5, with nonzero classical angular momenta J<sub>1,2</sub> in two planes simultaneously.
- ▶ In terms of the  $SO(4) = SU(2)_+ \times SU(2)_-$  subgroup of the SO(D-1) little group, the total angular momenta are  $J_{\pm} \equiv \frac{1}{2}(J_1 \pm J_2)$  where we assume WLOG that  $J_1 > J_2 > 0$ .

#### Closed strings with rotation in two planes

The classical solution is

$$X^0 = \alpha' P^0 \sigma^0 ,$$

$$Z_i = -i \sqrt{\frac{\alpha'}{2}} \left( \alpha_{-1}^{Z_i} e^{i\sigma^+} + \tilde{\alpha}_{-1}^{Z_i} e^{i\sigma^-} \right) ,$$

$$\bar{Z}_i = i \sqrt{\frac{\alpha'}{2}} \left( \alpha_1^{\bar{Z}_i} e^{-i\sigma^+} + \tilde{\alpha}_1^{\bar{Z}_i} e^{-i\sigma^-} \right) ,$$

Here, the mode amplitudes are

$$\alpha_{-1}^{Z_1} = \alpha_1^{\bar{Z}_1} = \tilde{\alpha}_{-1}^{Z_1} = \tilde{\alpha}_1^{\bar{Z}_1} = \sqrt{J_1} ,$$

$$a_{-1}^{Z_2} = \alpha_1^{\bar{Z}_2} = -\tilde{\alpha}_{-1}^{Z_2} = -\tilde{\alpha}_1^{\bar{Z}_2} = \sqrt{J_2} \ .$$
# Closed strings with rotation in two planes

 Evaluated in this rotating solution, the contribution of the PS anomaly term, evaluated in the rotating ground state, takes the form

$${\cal L}_{_{
m rotating \ solution}} = - rac{eta J_{-}^2}{2\pi^2} \, rac{\sin^2(2\sigma_1)}{(J_+ - J_- \cos(2\sigma_1))^2} \; .$$

- ▶ This Lagrangian density becomes singular at the endpoints  $\sigma_1 = 0$  and  $\pi$ , in the limit  $J_+ = J_-$ . This limit is imposed automatically in D = 4, as the little group SO(D 1) has rank one, and  $J_2$  must vanish.
- But for generic biplanar angular momenta, this density is smooth.

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## Closed strings with rotation in two planes

The resulting mass shift is

$$egin{split} \mathcal{M}_{ ext{closed}}^2 &= rac{1}{lpha'} \left[ 2(J_1+J_2) - rac{D-2}{6} 
ight. \ &+ rac{26-D}{12} \, \left( \left(rac{J_1}{J_2}
ight)^rac{1}{4} - \left(rac{J_2}{J_1}
ight)^rac{1}{4} 
ight)^2 
ight] + O(J^{-1}) \;. \end{split}$$

► The contribution from the PS term is nonzero unless  $J_1 = J_2$ , or D = 26.

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- When  $J_2$  is taken to zero, this diverges as a fold develops.
- At present, we do not understand how to renormalize the singular Hamiltonian at the fold.

- Since we don't understand that, let us return to our original focus on strings with boundaries.
- Our approach is to regulate and renormalize the boundary singularities in the standard way.

► This works, because all UV-divergences are local terms.

The classical solution is

$$X^0 = 2\alpha' P^0 \sigma^0$$

$$\bar{Z}_1 = i\sqrt{\frac{\alpha'}{2}}\alpha_1^{\bar{Z}_1}\left(e^{-i\sigma^+}+e^{-i\sigma^-}\right) \,,$$

$$\bar{Z}_2 = i \sqrt{\frac{\alpha'}{2}} \frac{\alpha_2^{\bar{Z}_2}}{2} \left( e^{-2i\sigma^+} + e^{-2i\sigma^-} \right) ,$$

$$Z_1 = -i\sqrt{\frac{\alpha'}{2}}\alpha_{-1}^{Z_1}\left(e^{i\sigma^+}+e^{i\sigma^-}\right),$$

$$Z_2 = -i\sqrt{\frac{\alpha'}{2}}\frac{\alpha_{-2}^{Z_2}}{2}\left(e^{2i\sigma^+}+e^{2i\sigma^-}\right),$$

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► Here,

$$\begin{split} \alpha_1^{\bar{Z}_1} &= \sqrt{2J_1} & \alpha_{-1}^{Z_1} &= \sqrt{2J_1} \\ \alpha_2^{\bar{Z}_2} &= 2\sqrt{J_2} & \alpha_{-2}^{Z_2} &= 2\sqrt{J_2} \; . \end{split}$$

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- Remember that we can modify our choice for the composite Liouville field φ.
- We would like to do so so that it our choice is smooth near the boundary.
- Such a choice is

$$\phi \equiv -rac{1}{4}\ln({\cal I}_{11}^2-\epsilon^4\,lpha'\,\hat{\cal I}_{22})\;,$$

$$\hat{\mathcal{I}}_{22} \equiv \mathcal{I}_{22} - \frac{\mathcal{I}_{12}\mathcal{I}_{21}}{\mathcal{I}_{11}} , \qquad \qquad \mathcal{I}_{pq} \equiv \partial_+^p X \cdot \partial_-^q X$$

► Near the boundary, this behaves as Î<sub>22</sub> ≃ -I<sub>22</sub>, which is nonzero and smooth.

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Now, modulo terms that do not contribute, the density of the PS term is

$$\mathcal{L}_{\rm PS, reg} \equiv \frac{\beta}{2\pi} \frac{\mathcal{I}_{12}\mathcal{I}_{21}}{\mathcal{I}_{11}^2 + \epsilon^4 \, \alpha' \, \mathcal{I}_{22}} \; .$$

- ▶ Note that wherever and whenever  $\mathcal{I}_{11} \neq 0$ , we have  $\mathcal{L}_{PS, reg} \rightarrow \mathcal{L}_{PS}$ . The short-distance modification is irrelevant, whenever the leading-order action is nonzero. The short-distance modification kicks in only at boundaries and folds.
- The integral is

$$\Delta M_{
m open}^2 = rac{1}{\epsilon} rac{26-D}{24lpha'} \left(J_1 + 8J_2
ight)^{1/4} + ({
m finite}) \; .$$

The short-distance singularity can be cancelled by a local term at the boundary, of the form

$$\mathcal{O}_{\rm quark} \equiv (\mathcal{I}_{22})^{+\frac{1}{4}} = (-\hat{\mathcal{I}}_{22})^{+\frac{1}{4}}$$

This operator corresponds to an infinitesimal change in a renormalized quark mass.

This may seem like a peculiar operator, but in fact all boundary operators for open strings with Neumann boundaries are nonsingular operators \$\mathcal{I}\_{pq}\$ dressed with powers of \$\mathcal{I}\_{22}\$.

After renormalization, we find

$$\begin{split} \mathcal{M}_{\rm open}^2 &= \frac{1}{\alpha'} \left[ J_1 + 2J_2 - \frac{D-2}{24} \right. \\ &\left. + \frac{26 - D}{24} \left( -4 + \frac{3J_1 + 4J_2}{J_1^{\frac{1}{2}}\sqrt{J_1 + 8J_2}} \right) \right] + \mathcal{O}(J^{-1}) \; . \end{split}$$

- For angular momenta lying in a single plane (i.e., when  $J_2 = 0$ ), the mass-squared equals  $M_{\text{open}}^2 = (J_1 1)/\alpha'$ , independent of *D*. Of course, when D = 26, we obtain  $M_{\text{open}}^2 = (J_1 + 2J_2 1)/\alpha'$ .
- This is the case in which the bosonic string theory is well-defined microscopically, and the singular PS anomaly term is absent.

- ► It is worth emphasizing that we have fine-tuned the coefficient of the quark mass operator O<sub>(quark)</sub> so that there is no term of order J<sup>1/4</sup> in the mass-squared formula.
- Generically we should expect a J<sup>1/4</sup> term in the open-string mass-squared, unless the mass of the quark at the endpoint is light compared to the scale of the string tension.

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- In real QCD there will be additional degrees of freedom at the endpoints, carrying spin and flavor degrees of freedom.
- ► These degrees of freedom carry symmetries that constrain the allowed operators. In particular, chiral symmetry forbids quark masses, which are associated with the J<sup>+1/4</sup> term in the boundary action. We therefore speculate that in the correct effective boundary CFT of the real QCD string, the J<sup>+1/4</sup> term in the action may be completely fixed when exact chiral symmetry holds.

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- Several questions now arise.
- One might ask, why is the answer universal at all?
- And why do the boundary operators appear containing these strange quarter-integer powers I<sub>22</sub>?

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- This is one of the more surprising features of the effective string theory with Neumann boundary conditions.
- ► Let us consider any short-distance modification of the theory such that the bulk of the worldsheet is an ordinary effective string theory with an organization of operators such as we have described, with operators dressed with powers of I<sub>11</sub> – generically negative integer ones.
- ► For such a theory, the boundary operators always appear dressed with powers of I<sub>22</sub> – generically negative integer ones.
- This is so for artificial short-distance cutoffs preserving the symmetries – such as the one we have considered – but also for real short-distance effective theories.

- ► The result is that all boundary operators are of the form (∏<sub>pq</sub> I<sub>pq</sub>)/I<sup>k</sup><sub>22</sub>.
- ► We can quickly see that there are no marginal boundary operators of vanishing X-scaling.
- First, use the EOM to reduce all derivatives of X to the form  $\partial_0^p X$  or  $\partial_0^p \partial_1 X$ .

Then use <u>Neumann</u> boundary conditions to eliminate the <u>latter</u>.

- ▶ Now, all bilinear invariants of X at the boundary are of the form  $B_{(pq)} \equiv \partial_0^p X \cdot \partial_0^q X$ .
- ► All all boundary operators are of the form  $(\prod_{pq} B_{(pq)})/B_{(22)}^k$ .
- Now consider only marginal boundary operators.
- ▶ If the "undressed" operator (the numerator) has dimension  $\Delta \equiv \sum_{pq} p + q$ , then the dressing is  $B_{(22)}^{-(\Delta-1)/4}$ .

- Then in order to have positive or zero X-scaling, the undressed operator must have Δ ≤ 5.
- The operators B<sub>11</sub> and B<sub>12</sub> vanish as independent operators because they are proportional to free-field stress tensors and first derivatives thereof.
- ► The only marginal operator with  $\Delta = 5$  is  $B_{(23)}/B_{(22)}$  which is a total derivative along the boundary.

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So after modding out by Virasoro descendants, the only marginal operator with nonnegative X-scaling is the quark mass term, corresponding to Δ = 4.

- The remaining question is:
- Why should operators be organized in this form

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- The remaining question is:
- Why should operators be organized in this form with only B<sub>22</sub> appearing to negative or fractional powers?
- This is indeed counterintuitive but it appears to be true, for every good short-distance regulator we have examined that preserves all the symmetries of the system.

Let us begin by examining a "naturally occurring" regulator



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- ► The authors start out in D + 1 dimensions where the D + 1<sup>st</sup> direction φ is anisotropic with the others, due to the effect of a dilaton gradient, a tachyon profile, and a massive stringy condensate.

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The worldsheet Lagrangian is

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- The worldsheet Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\mathrm{free}} + \mathcal{L}_{\mathrm{tachyon}} + \mathcal{L}_{\mathrm{massive}}_{\mathrm{stringy}}$$

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$$egin{aligned} \mathcal{L} &= \mathcal{L}_{ ext{free}} + \mathcal{L}_{ ext{tachyon}} + \mathcal{L}_{ ext{massive stringy}} \ \mathcal{L}_{ ext{tachyon}} &\equiv \mu^2 \exp\left(+\gamma\,\phi
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$$\begin{split} \mathcal{L} &= \mathcal{L}_{\rm free} + \mathcal{L}_{\rm tachyon} + \mathcal{L}_{\rm massive}_{\rm stringy} \\ \mathcal{L}_{\rm tachyon} &\equiv \mu^2 \exp\left(+\gamma \phi\right) \\ \mathcal{L}_{\rm massive}_{\rm stringy} &\equiv {\mu'}^{-2} \exp\left(\gamma' \phi\right) \left(\mathcal{I}_{11}^2 + O(D^{-1})\right) \,, \end{split}$$

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- We will analyze the system in the limit  $D \to -\infty$ .
- Finite-D corrections do not appear to change the qualitative structure of the organization of operators.

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To understand how the Liouville direction gets integrated out,

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- To understand how the Liouville direction gets integrated out, first put the *D* Lorentz-invariant directions into an arbitrary nonsingular configuration X<sup>μ</sup>(σ).
- Then solve for the Liouville field  $\phi$  classically.
- For a nonsingular configuration of the the closed string, we find

$$\phi = -\frac{1}{2}\ln(\mathcal{I}_{11}) \; ,$$

where we have ignored quantum corrections.

Let's look at this calculation in a bit more detail.


- Let's look at this calculation in a bit more detail. It will be illuminating.
- The semiclassical Lagrangian for  $\phi$  is

$$\mathcal{L}_{\phi} \simeq \frac{|D|}{12\pi^2} \, (\vec{\partial}\phi)^2 + \mu^2 \exp\left(-2\phi\right) + {\mu'}^{-2} \exp\left(+2\phi\right) \, \mathcal{I}_{11}^2$$

 Restrict for the moment to the case where *I*<sub>11</sub> is time-independent, with a dependence only on the spatial worldsheet coordinate σ<sup>1</sup>.

► Then ooo

- Ignoring quantum corrections is strictly justified at  $D = -\infty$ .
- ▶ But of course we want to consider finite (and positive) *D*.
- For many purposes the 1/D expansion is not very useful at finite positive D, but for some purposes it is useful.
- ► In particular, it is not sufficient or necessary for deriving the power laws to which I<sub>11</sub> occurs in the effective action

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- For many purposes the 1/D expansion is not very useful at finite positive D, but for some purposes it is useful.
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 Nor is the large-D expansion useful for deriving the coefficients with which effective operators appear in the effective string theory. However the large-D expansion is good for one particular thing:

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- We always have  $M_{\phi}^2 \propto \mathcal{I}_{11} + \text{lower order in } X$
- Now one can compute the exact effective action for the string at the quantum level.

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► Thus it is *I*<sub>11</sub> and only *I*<sub>11</sub> that ever appears in the denominator of an effective operator.

#### Structure of boundary operators

- When the string has a Neumann boundary the only difference is that the classical solution has φ = -<sup>1</sup>/<sub>4</sub>ln(I<sub>22</sub>) + (const) + lower order in X near the boundary.
- ► The solution is still  $M_{\phi}^2 \propto \mathcal{I}_{11} + \text{lower order in } X$  in the bulk of the worldsheet.
- ► As a result, bulk operators are dressed with powers of I<sub>11</sub> and boundary operators are dressed with powers of I<sub>22</sub>.
- Thus the organization of operators is as we have said.
- ► This is also true for every other regulator we have examined.
- The set of allowed operators in a given effective theory as opposed to the coefficients of those operators – should be universal. So this operator dressing rule should hold in every UV completion.

We have found that the effective string theory framework is predictive for large-J corrections to the spectrum of rotating strings.

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