On 6d SCFT's

Cumrun Vafa

Strings 2014
Princeton

June 23, 2014

Based on:

arXiv:1312.5746 On the Classification of 6D SCFTs and Generalized ADE Orbifolds
Jonathan J. Heckman, David R. Morrison, Cumrun Vafa

M. Del Zotto, J. Heckman, A. Tomasiello, C. Vafa, to appear.

arXiv:1310.1185 On orbifolds of M-Strings
Babak Haghighat, Can Kozcaz, Guglielmo Lockhart, Cumrun Vafa

arXiv:1406.0850 E + E → H
Babak Haghighat, Guglielmo Lockhart, Cumrun Vafa

6d SCFT's enjoy a unique status: They are the highest dimension known non-trivial CFT's.

Two types based on number of SUSY's:

(2,0): Believed to be classified by an ADE.

A: Stack of parallel M5 branes
ADE: type IIB at an ADE singularity

(1,0): More difficult to classify. Many examples.

Blum,Intriligator; Seiberg,Witten; Hanany,Zaffaroni; Aspinwall,Morrison; Ganor,Hanany; …

There are two aspects:

1-Classify the (1,0) theories

Will use IIB/F-theory realization

2-Develop tools to study each

Tensionless strings/suspended M2 branes

(update with new results since strings 2013)

There are two types of deformations:

Coulomb/Tensor branch:

$$(B^i_{\mu\nu}, \phi^i) \rightarrow String; \qquad Tension = n_i \phi^i$$

Higgs Branch:

$$\text{Hyperkähler geometry}$$

Examples:

M5 branes probing an An singularity

Higgs branch: moving away from singularity
Coulomb branch: separating M5 branes

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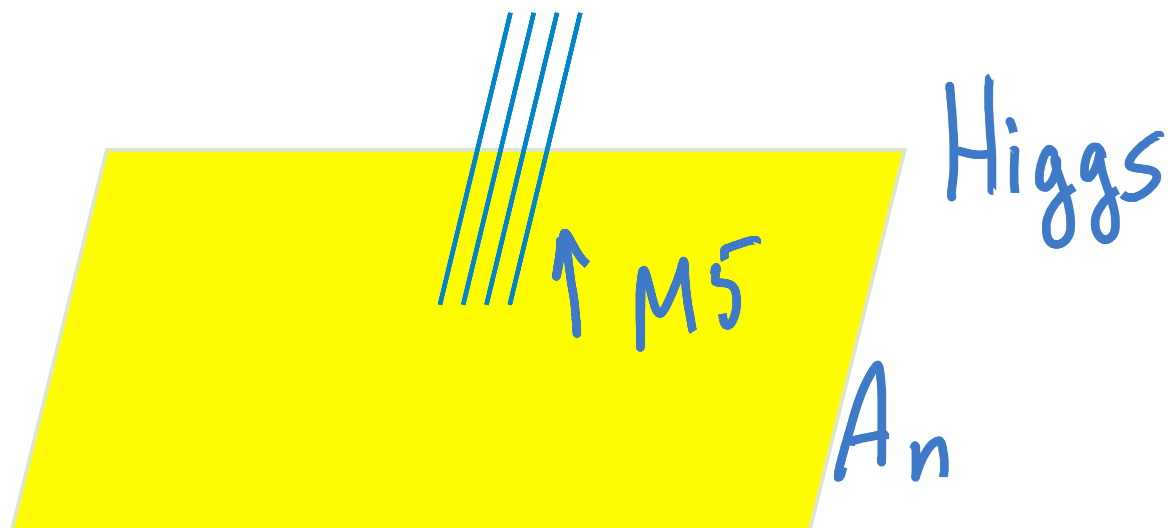

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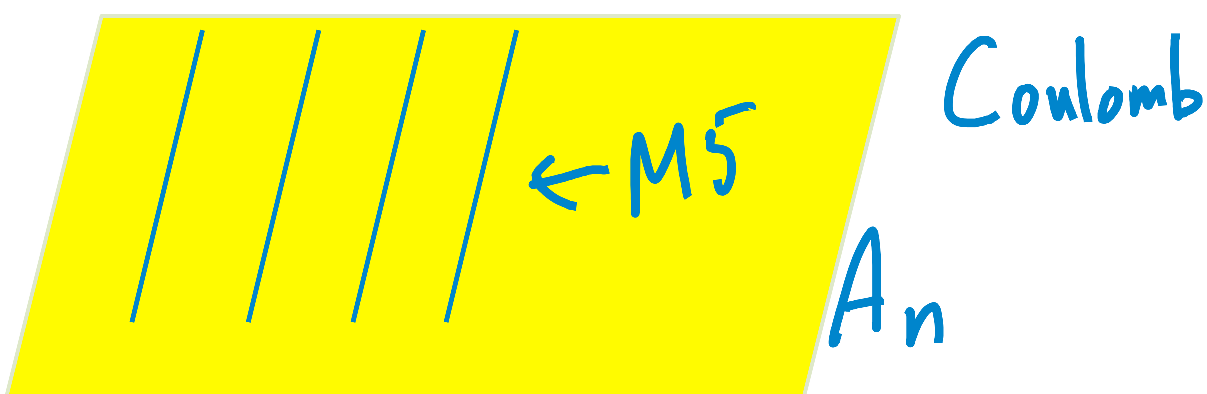

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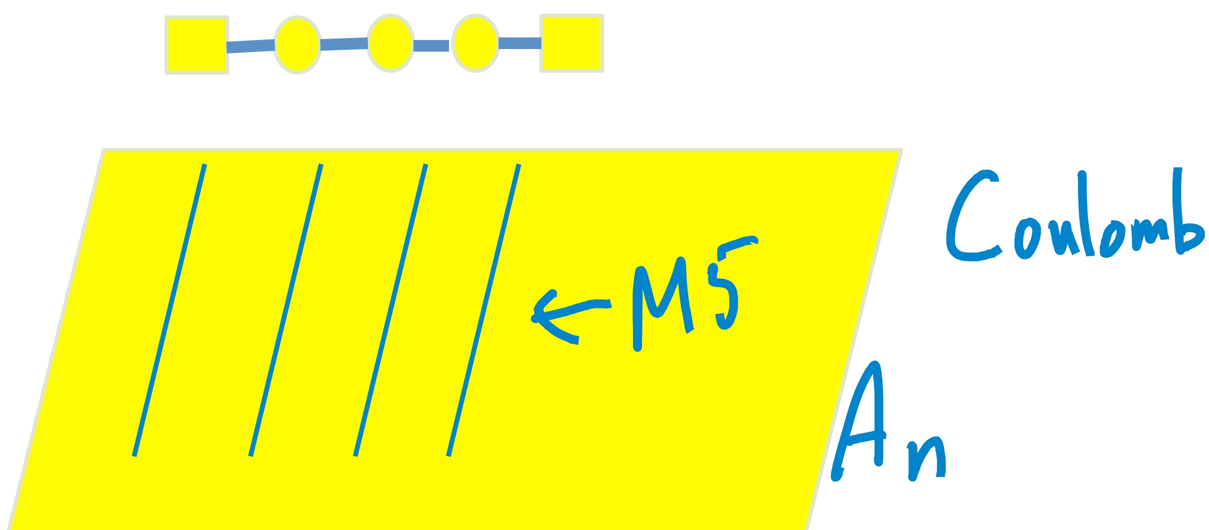
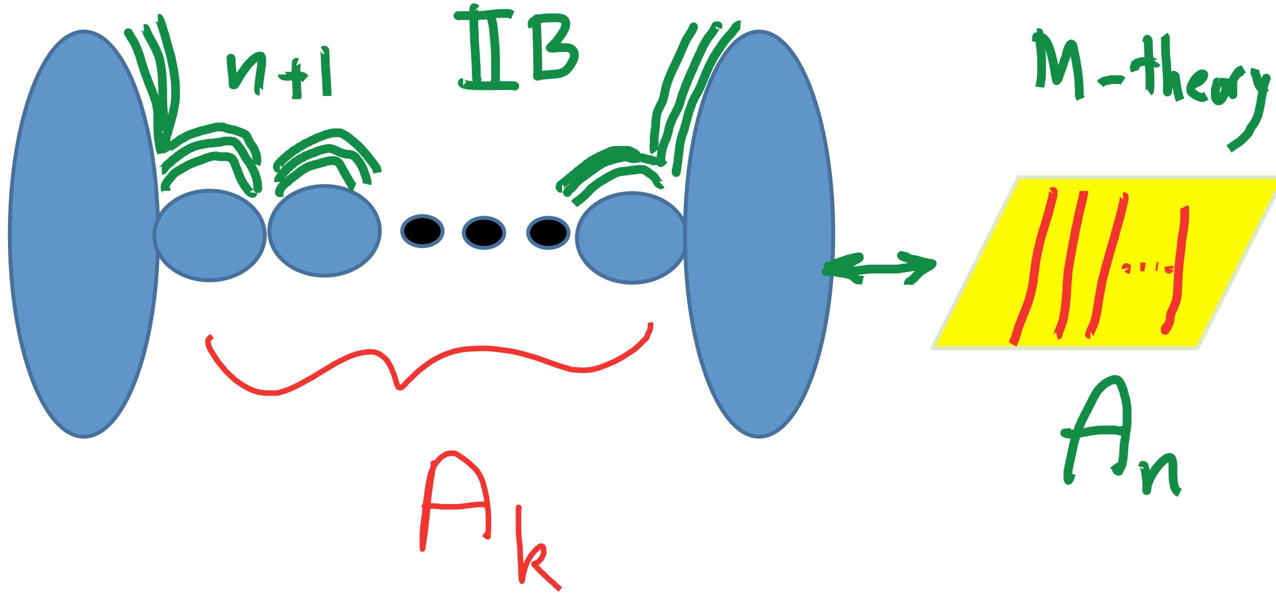
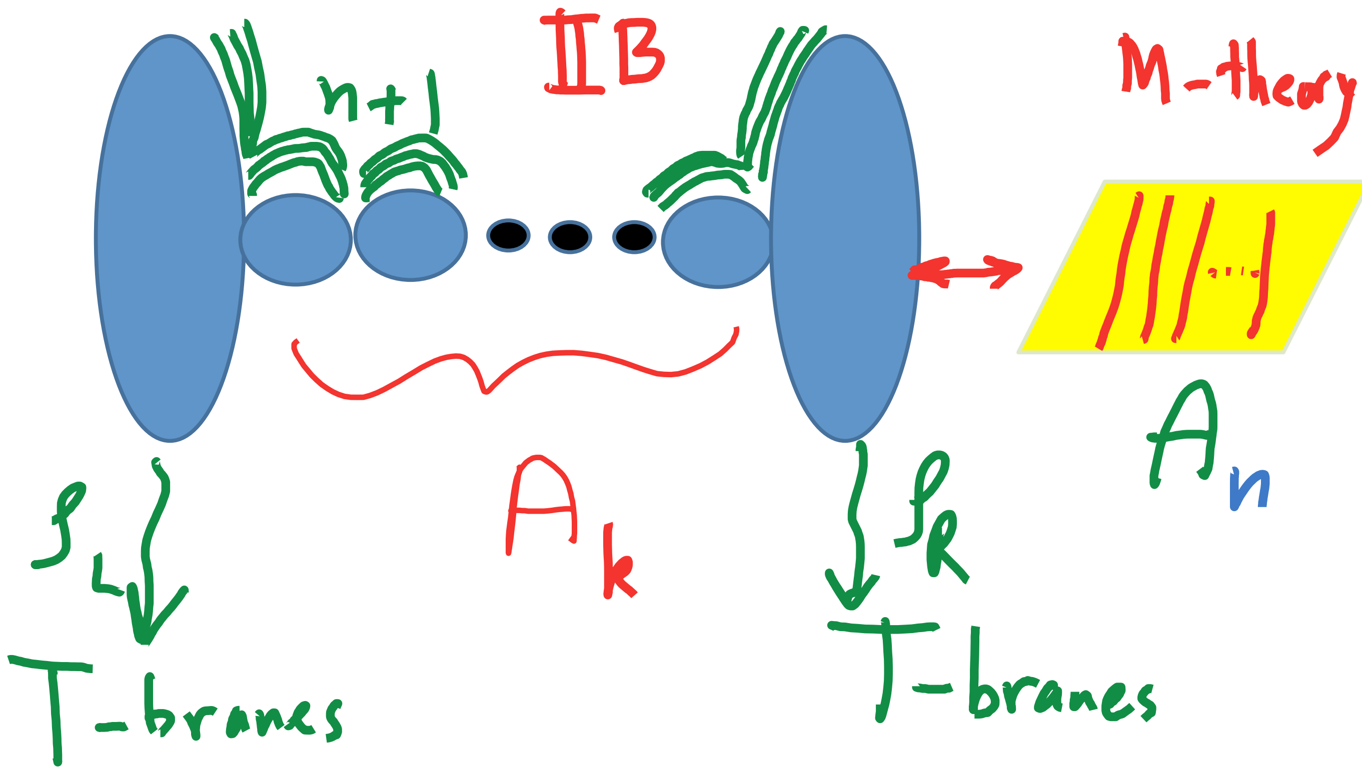




n+1
IIB
M-theory
A_k
A_n

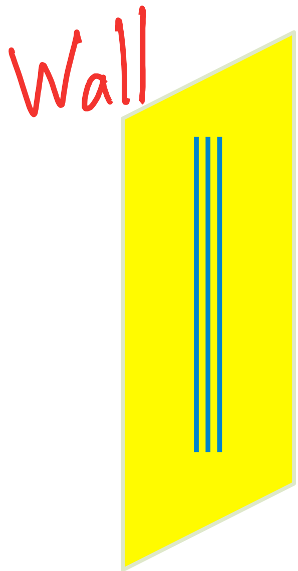


Equivalent to IIA constructions of [Gaiotto,Tomasiello]

M5 branes at the Hořava-Witten wall

Higgs branch: Dissolving branes in the wall giving finite size E8 instantons

Coulomb branch: Moving away from the wall and each other



E8 global symmetry

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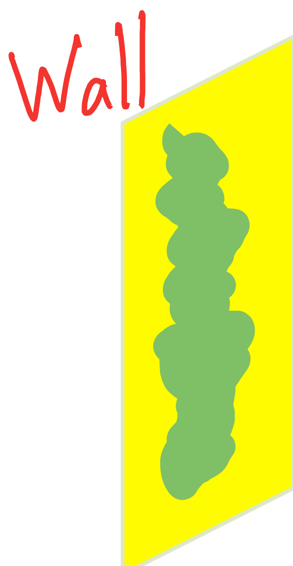

Higgs branch

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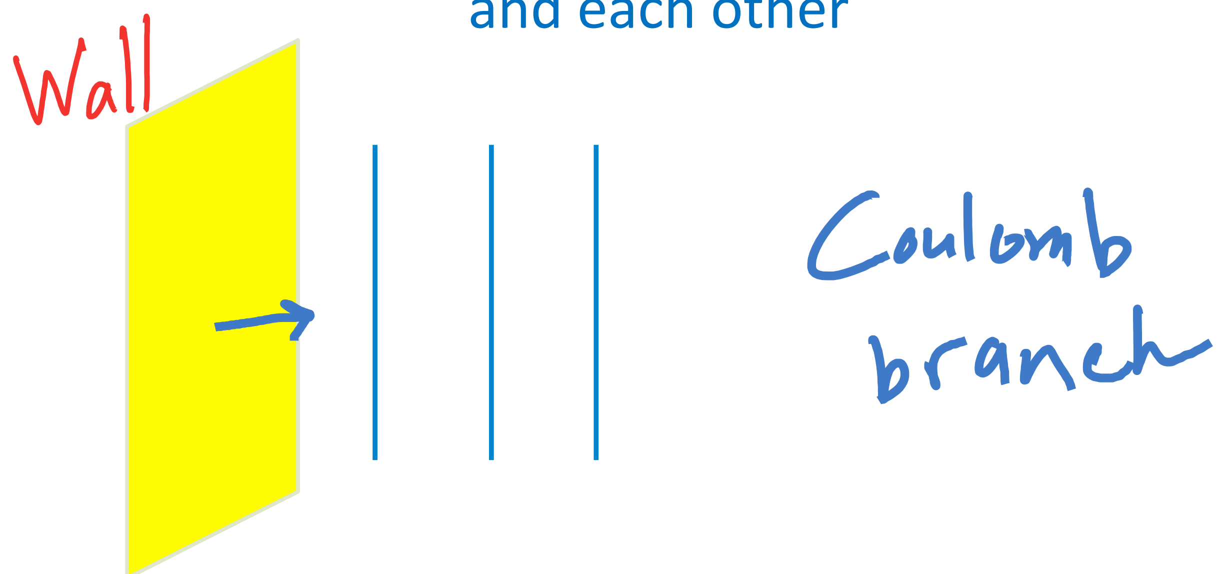
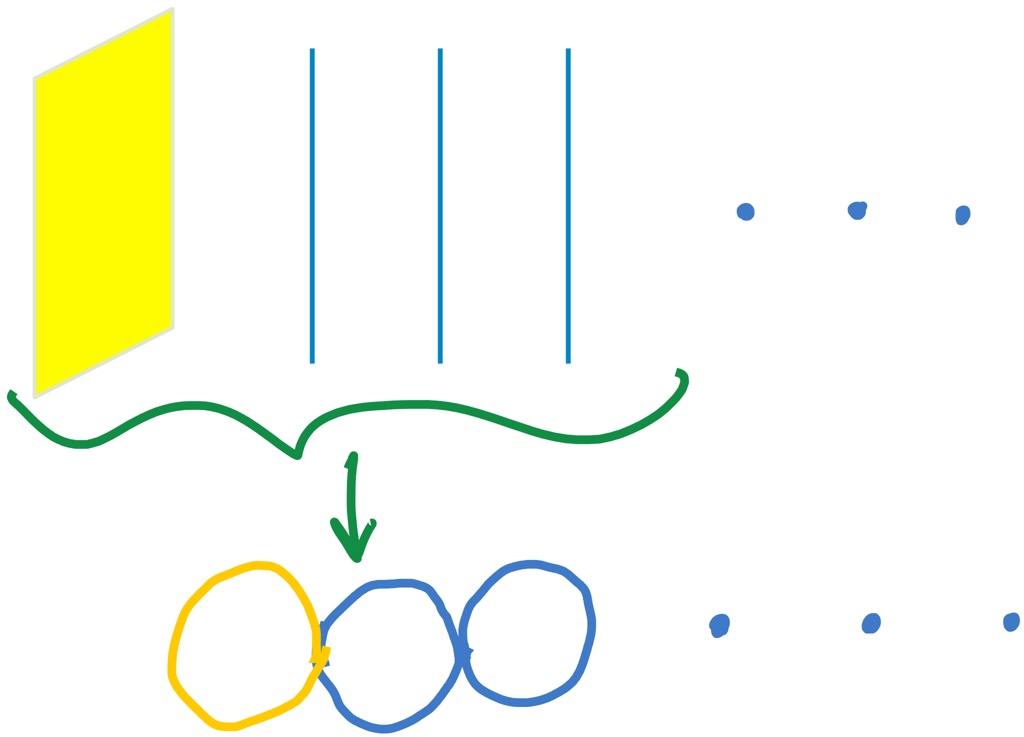


F-theory realization:



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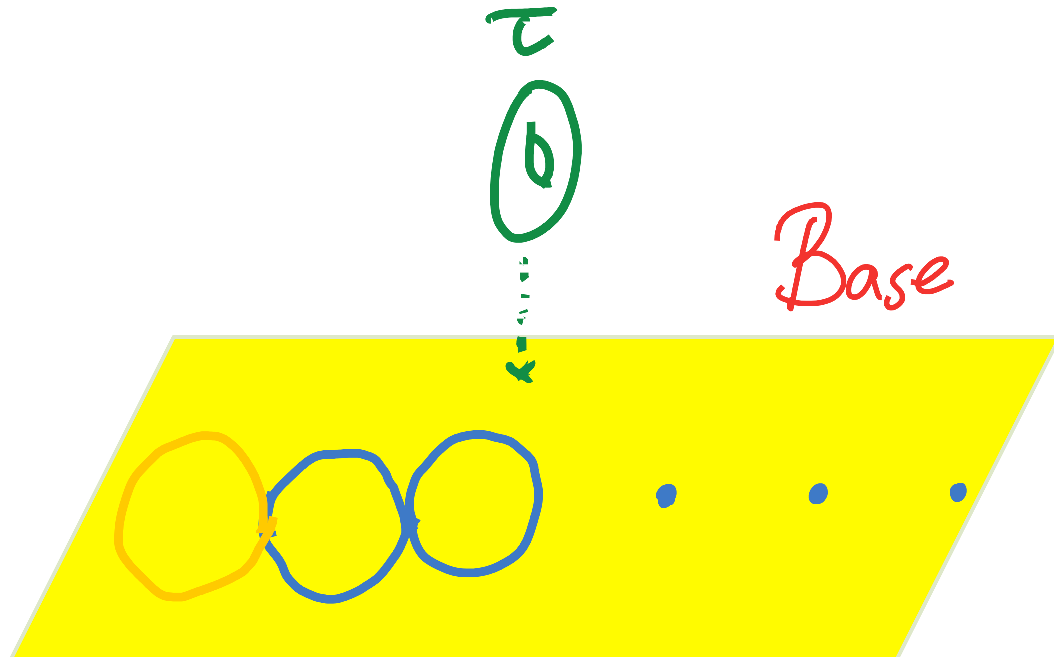
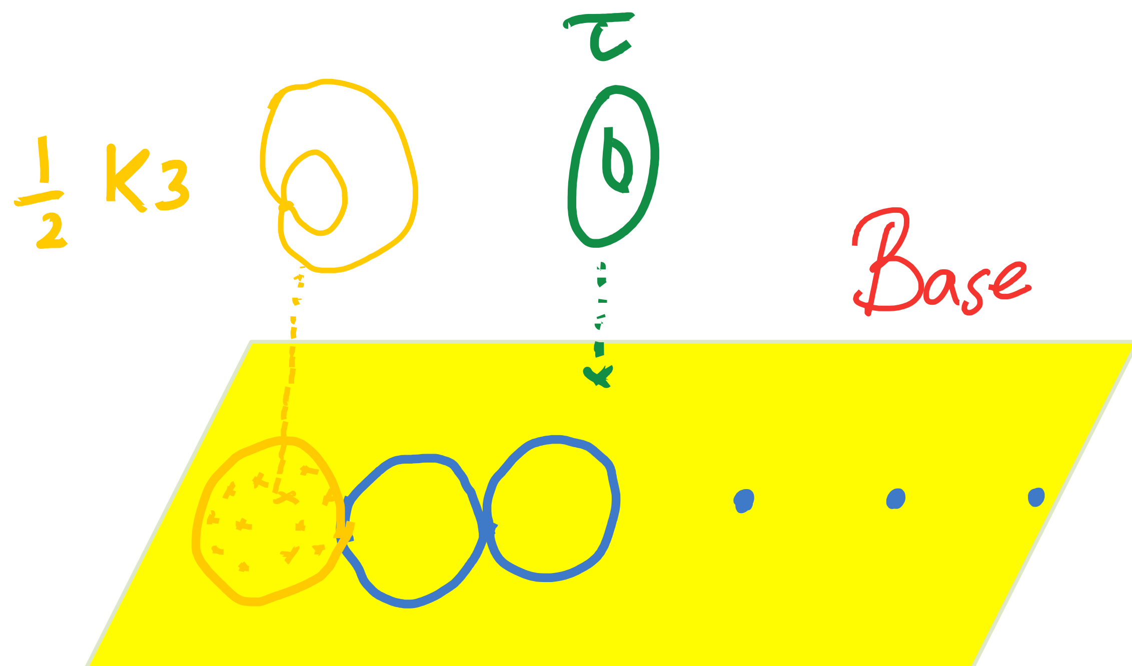
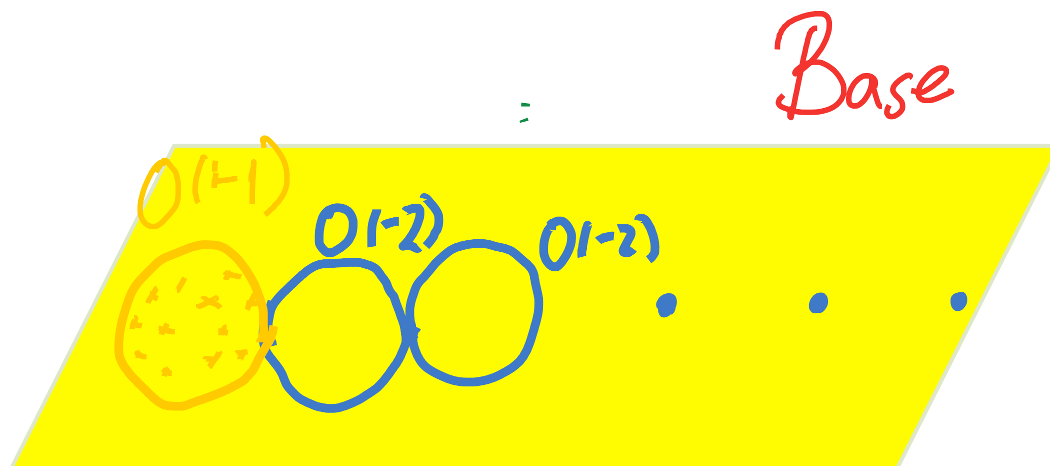
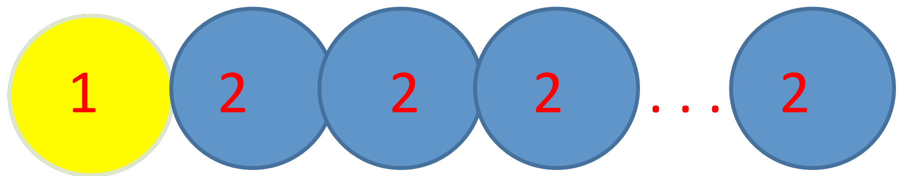




$\frac{1}{2}$ K3
τ
b
Base


Base
O(-1)
O(-2)
O(-2)



1 2 2 2 . . . 2

Whereas the A_n case was given by (7-brane dressing of)

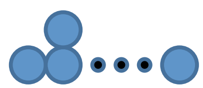
2 2 2 . . . 2

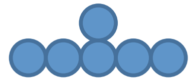
F-theory Classification

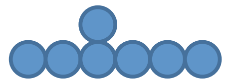
All the known examples of (2,0) and (1,0) CFT's can be realized in IIB/F-theory.

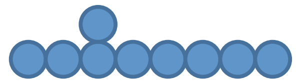
(2,0) : ADE singularities of IIB

A_N 

D_N 

E_6 

E_7 

E_8 

O(-1) curve, unlike the O(-2) curve, when shrunk leaves no imprint of singularity in the base. 122222 corresponds to n blow ups of the base.

Basic classification strategy: Classify all the possible bases that can appear in F-theory, up to blow ups and adding 7-branes wrapping the cycles (i.e. classify the maximally higgsed phase because in going to Higgs phase we first shrink cycles).

Surprising result: All allowed endpoints correspond to orbifold singularities

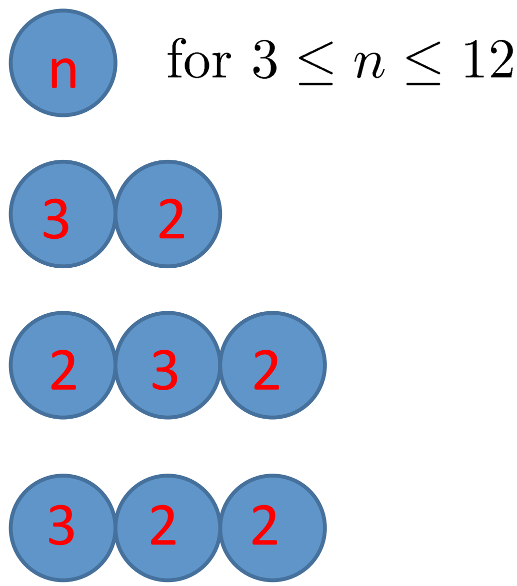
$$\mathbf{C}^2/\Gamma, \qquad \Gamma \subset U(2)$$

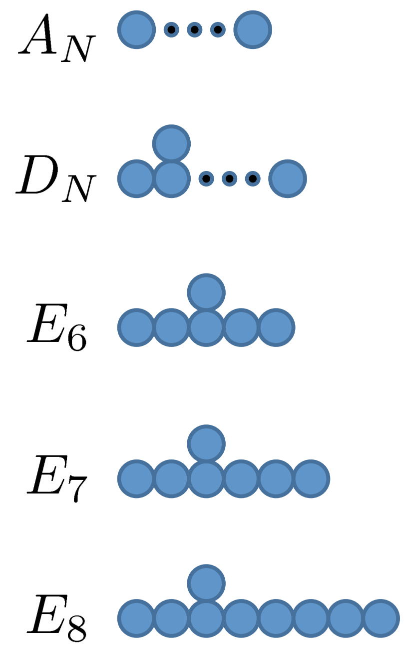
for special subgroups.

For a single curve:

$-n$ curve: $\mathcal{O}(-n) \to \mathbb{P}^1$	gauge symmetry on the $\mathbb{P}^1$
3	$\mathfrak{su}_3$
4	$\mathfrak{so}_8$
5	$\mathfrak{f}_4$
6	$\mathfrak{e}_6$
7	$\mathfrak{e}_7 + (1/2)$ hyper
8	$\mathfrak{e}_7$
12	$\mathfrak{e}_8$

Building Blocks





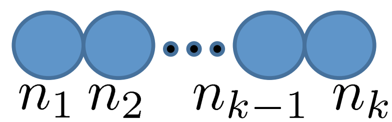
Non-Higgsable Clusters (Morrison-Taylor)

In addition to ADE endpoints, there are two additional series of endpoints:

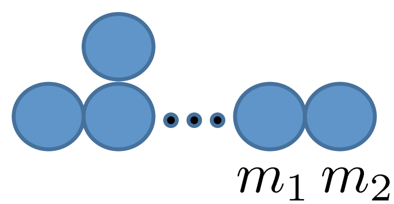
A-type, D-type similar to the usual ADE case except the cyclic element does not have determinant 1.

Once the base singularity is resolved we get:

Generalized A



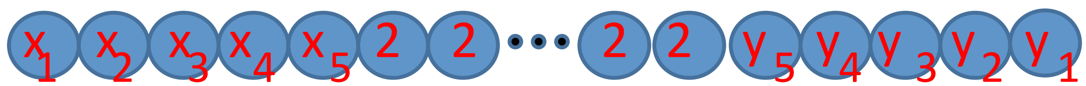
Generalized D



A-type Endpoint List

(c.f. paper for outliers)

Generic case at 11 or more curves



$2 \leq x_i \leq x_i^{max}$

A_n

$2 \leq y_i \leq y_i^{max}$

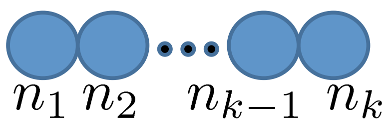
$$x_1^{max} x_2^{max} x_3^{max} x_4^{max} x_5^{max} \in \{7, 24, 223, 2223, 22223\}$$

$$y_5^{max} y_4^{max} y_3^{max} y_2^{max} y_1^{max} \in \{7, 42, 322, 3222, 32222\}$$

U(2) subgroup e.g.
A-case, generated by:

$$(z_1, z_2) \mapsto (\omega z_1, \omega^q z_2)$$

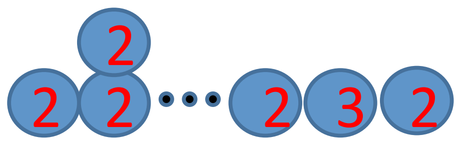
$$\omega = \exp(2\pi i/p)$$

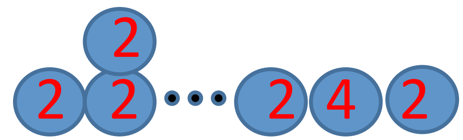


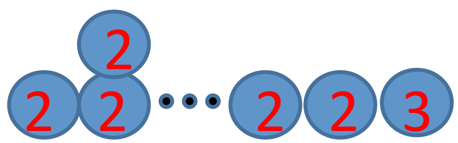
$$\frac{p}{q} = n_1 - \frac{1}{n_2 - \dots \frac{1}{n_k}}$$

D-type Endpoint List

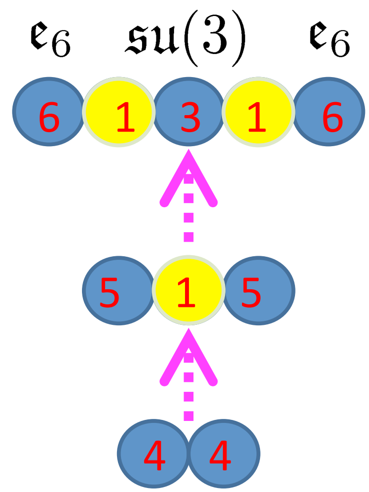
(no outliers in this case)



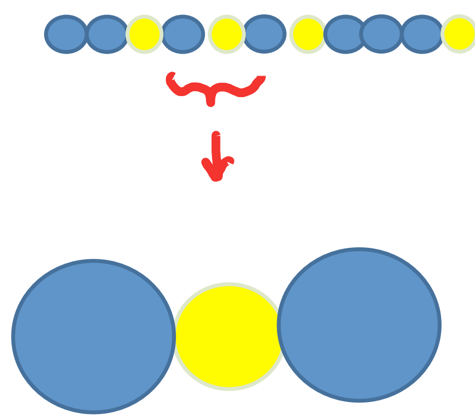


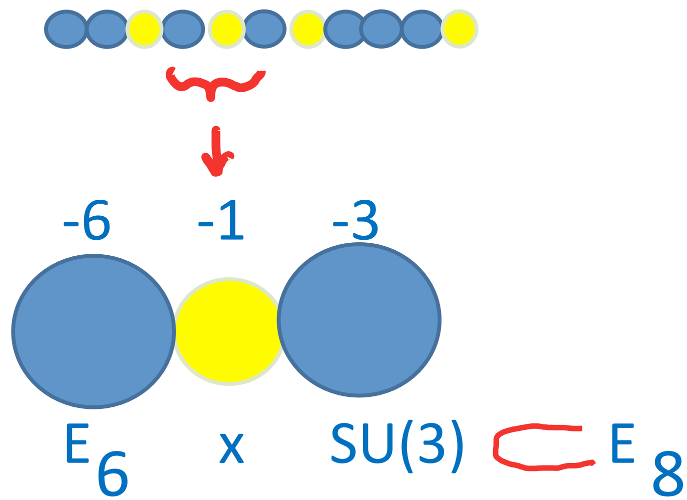


Each of these end points leads to a canonical tensor branch (which require additional blow ups for the elliptic 3-fold singularities) leading to a final geometry of blow ups



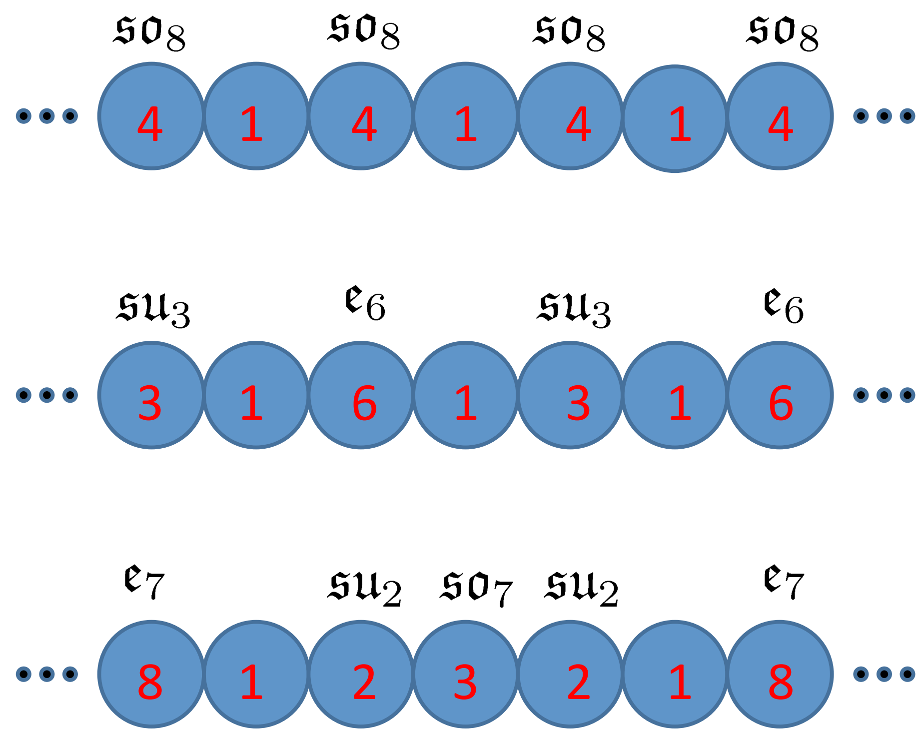
Typically we have a geometry of the form:




-6
-1
-3
E 6
x
SU(3)
E 8

Emergence of repeating patterns

for example:



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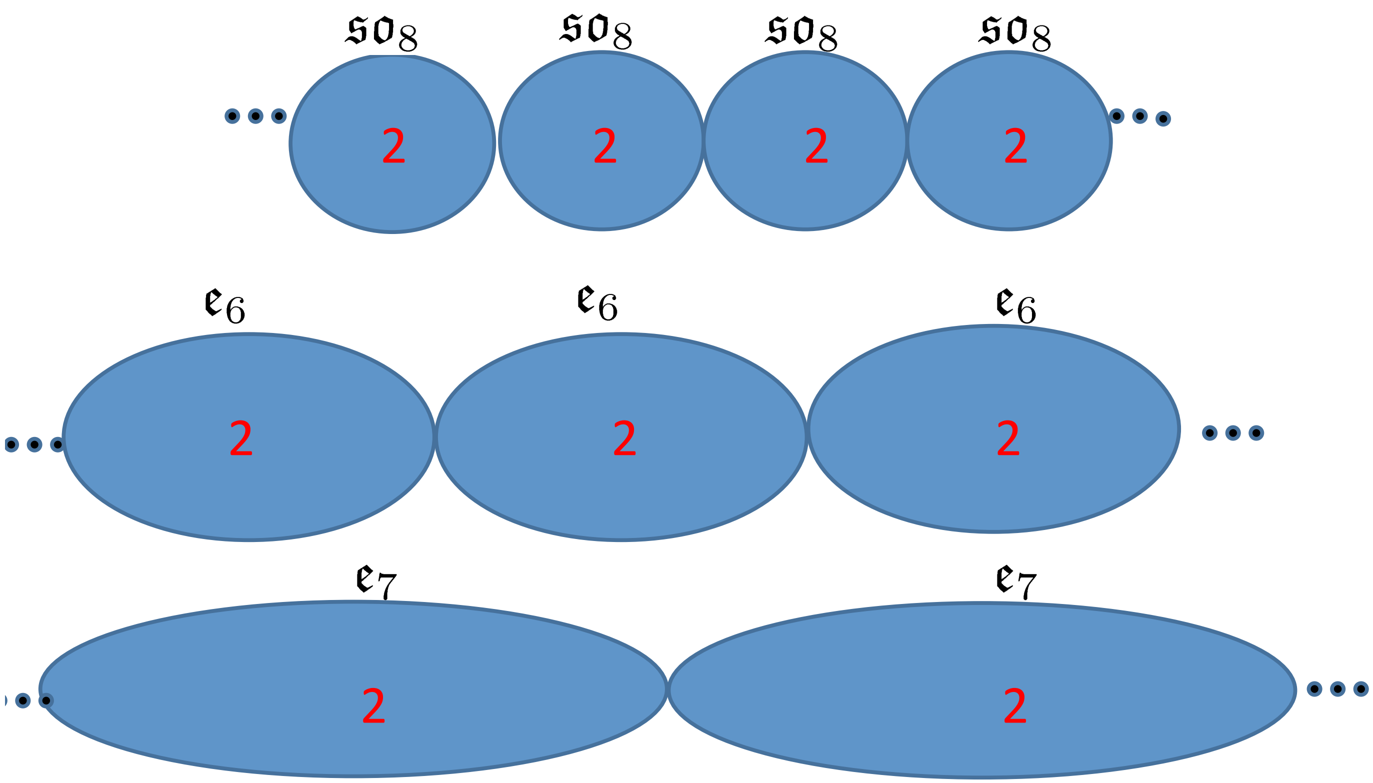
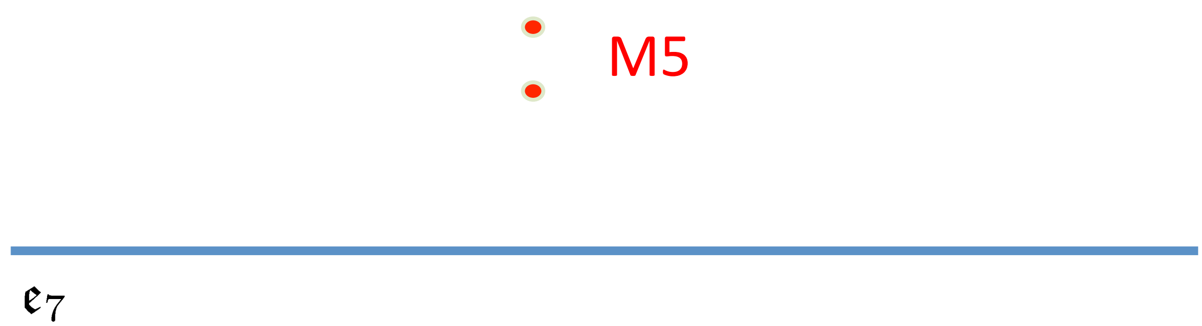


M-theory interpretation: M5 branes probing D,E singularities. For example:



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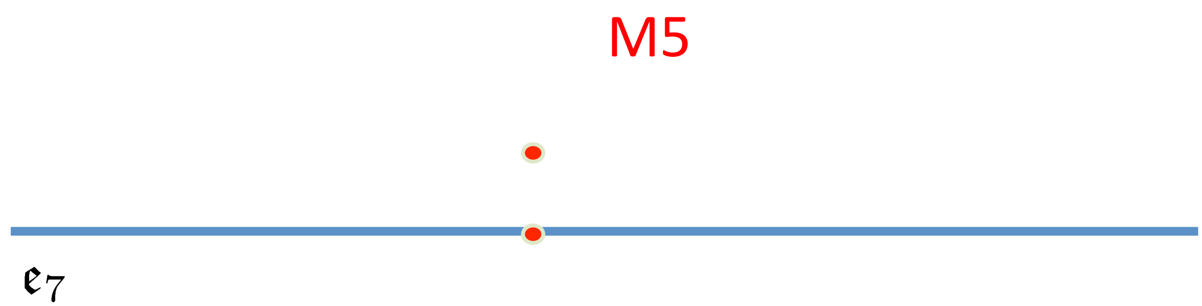

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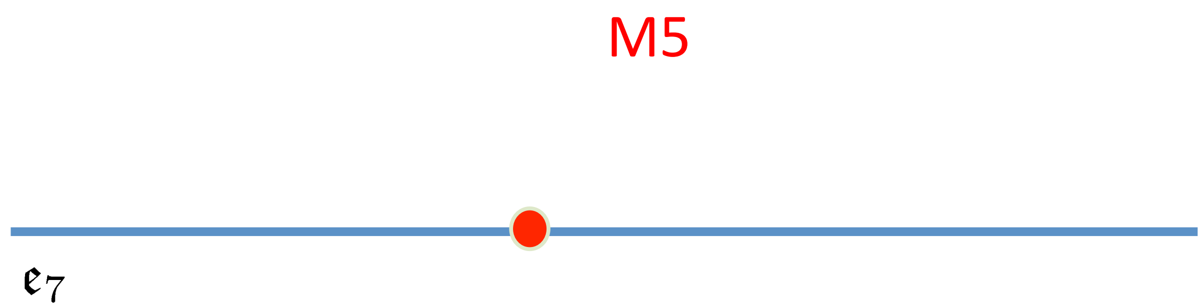

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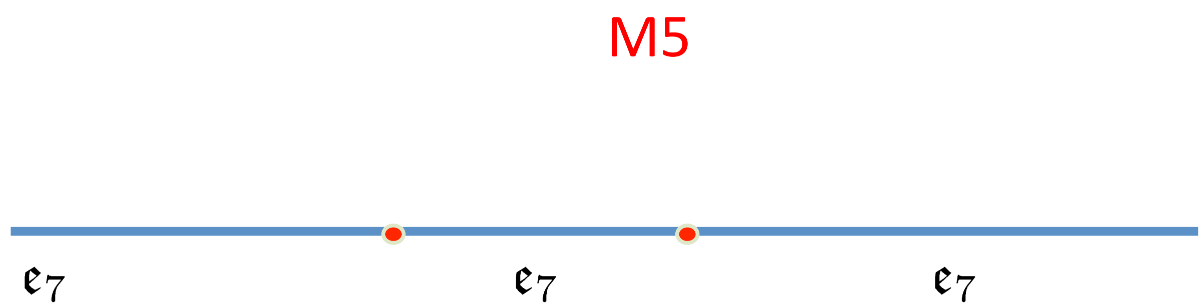

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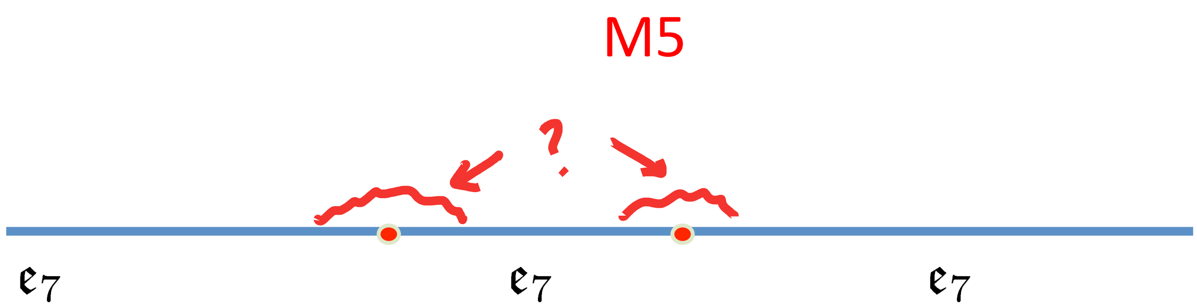

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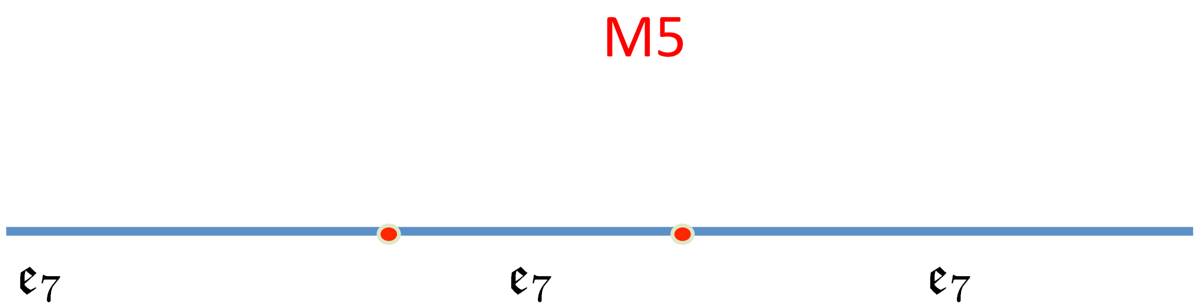

M-theory interpretation: M5 branes probing D,E singularities. For example:

M5

New fractional M5 branaes

$\mathfrak{e}_7$ $\mathfrak{e}_7$ $\mathfrak{e}_7$

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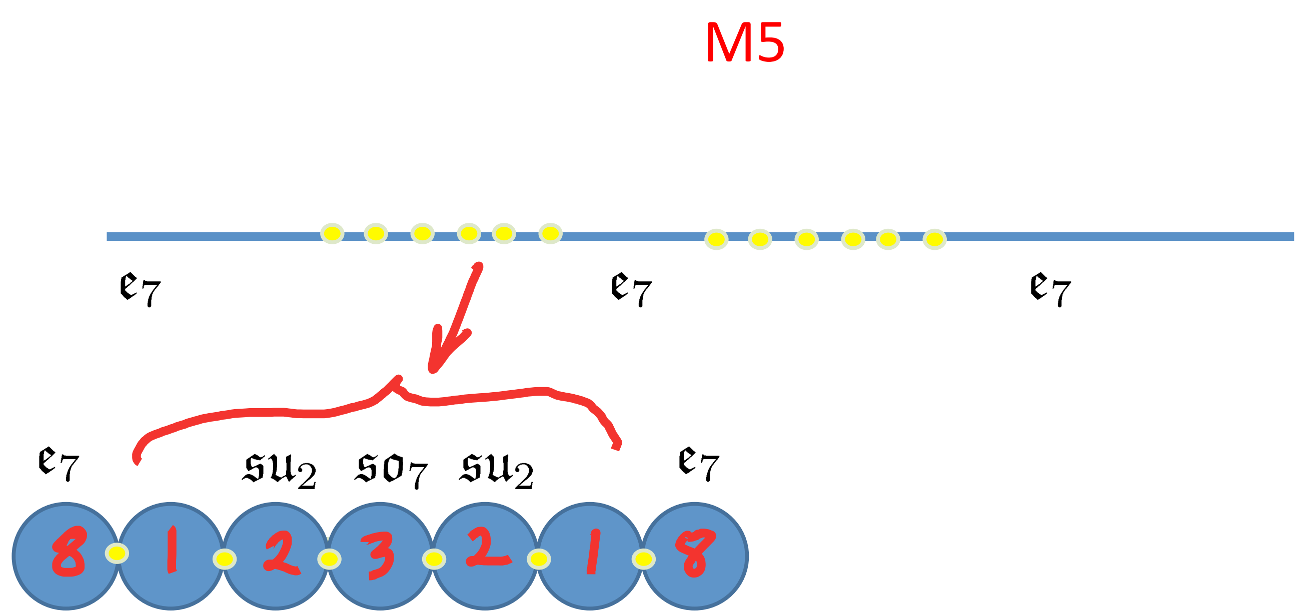

bifundamental `E7 matter' =SCFT

M-theory Large N Duals

Some of these theories admit simple large N duals:

$$AdS^7 \times S^4/\Gamma_{ADE}$$

$$AdS^7 \times S^4/\mathrm{Z}_2$$

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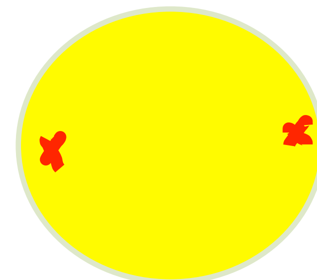
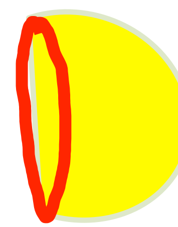


$AdS^7 \pounds\ S^4 = Z_2$



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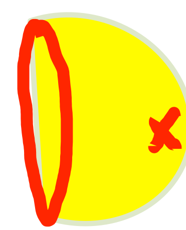

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e.g. E7: 52222....22225

$$AdS^7 \times S^4/\mathrm{Z}_2$$

1222222222

$$AdS^7 \times S^4/\Gamma_{ADE} \times \mathrm{Z}_2$$

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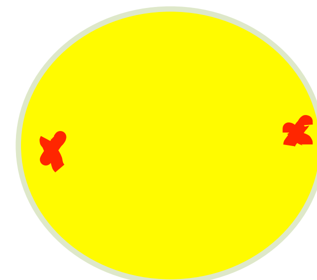
$A\,dS^{7}\ £\ \ S^{4}{=}\text{i}_{\ A\ D\ E}$



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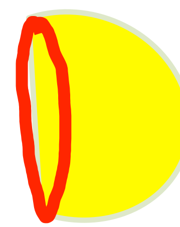

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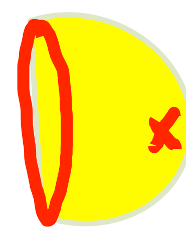


Computation of the Superconformal Index

$$Z_{S^5\times S^1} = \int_{\mathcal{M}_C} Z_{\mathbf{R}^4\ltimes T^2}\; Z'_{\mathbf{R}^4\ltimes T^2}\; Z''_{\mathbf{R}^4\ltimes T^2}$$

[Lockhart,V; Kim^3]

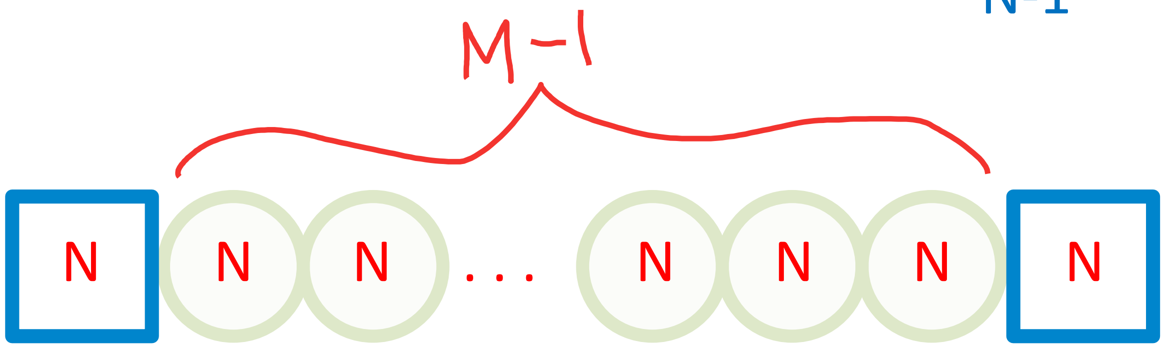
$Z_{\mathbf{R}^4\ltimes T^2}$ can be computed using string instantons:

$$Z_{\mathbf{R}^4\ltimes T^2} = \sum_n e^{-n\cdot t}\; Z^n_{T^2}$$

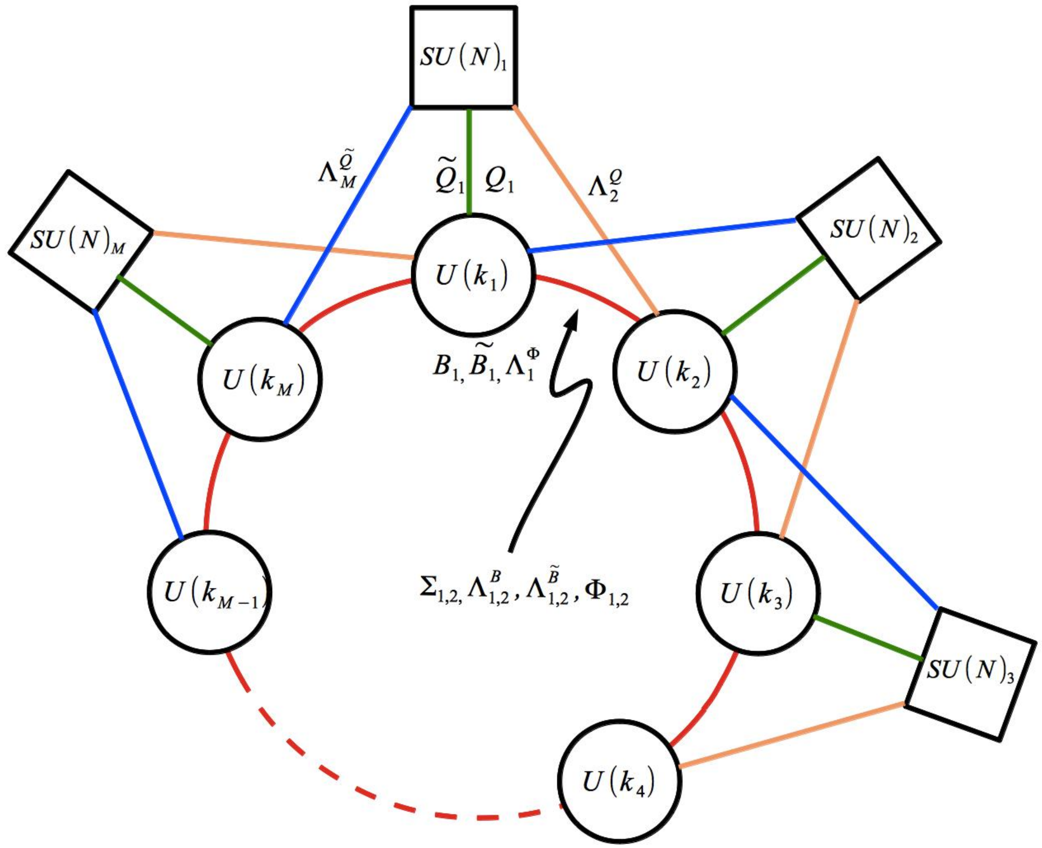
$Z^n_{T^2}$: The elliptic genus of n strings twisted by rotations of $\mathbf{R}^4$ and global symmetries of CFT.

Alternatively $Z_{\mathbf{R}^4\times T^2}$ can be computed using topological string/Nekrasov partition function.

For the theory of M5 branes probing A_{N-1}:



There are M-1 different strings. They form a (4,0) supersymmetric theory in 2d. If we denote their numbers by $k_1, \ldots, k_{M-1}$, the corresponding theory is a quiver theory:



The elliptic genus can be computed

[Gadde,Gukov; Benini,Eager,Hori,Tachikawa]

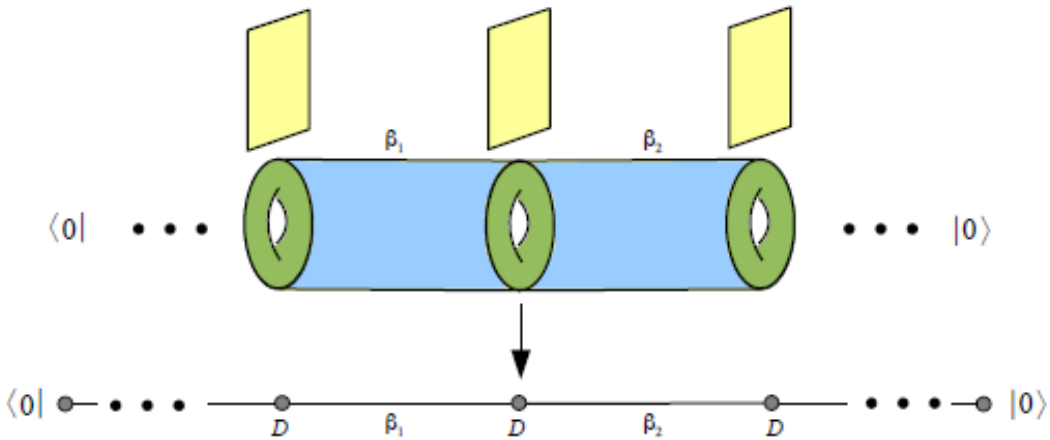
One learns unlike fundamental strings, tensionless strings do form bound states which is reflected in:

$$Z_2(\tau) \neq \frac{1}{2}[(Z_1(\tau)^2 + Z_1(2\tau) + Z_1(\frac{\tau}{2}) + Z_1(\frac{\tau+1}{2})]$$

A Dual Description: QM

In M-theory, strings arise as M2 branes suspended between M5 branes. Leads to a dual perspective:

3=2+1; i.e. string, versus QM



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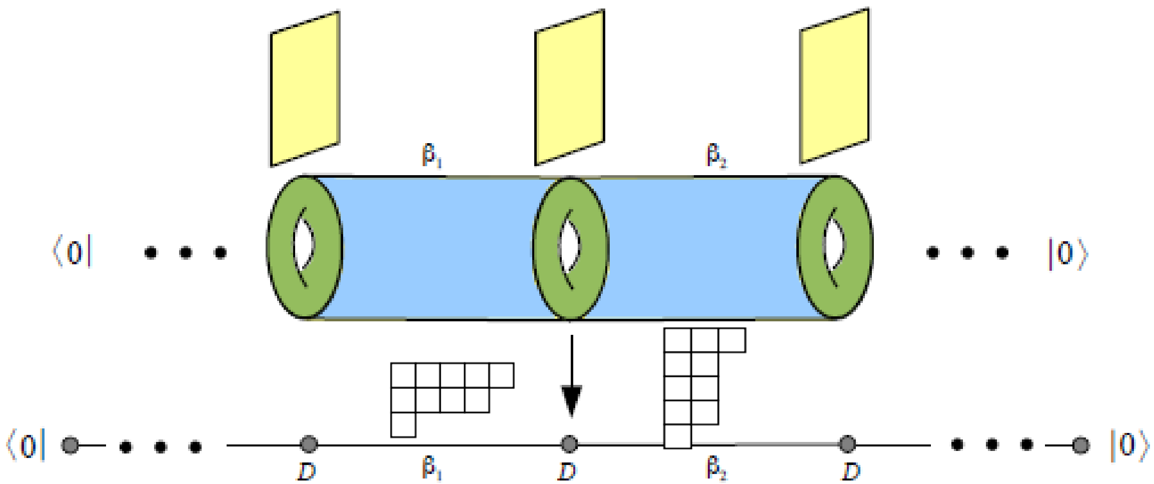
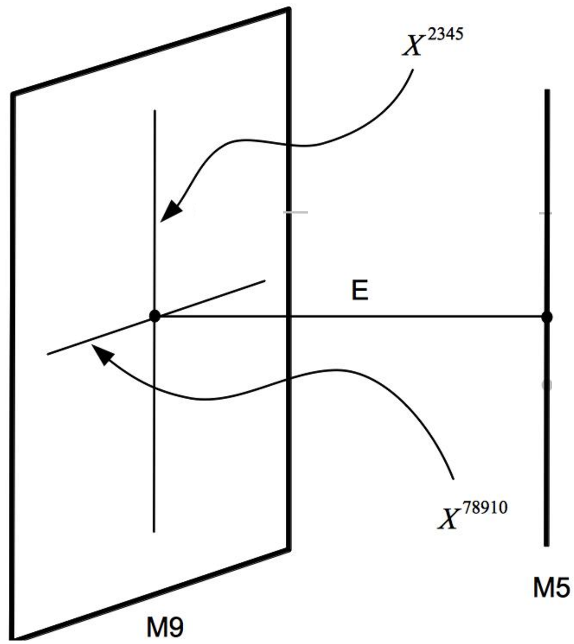


$$\langle 0|...D\ \exp(-\beta_1 H)D\ \exp(-\beta_2 H)D\ ...|0\rangle$$

E-string:

1

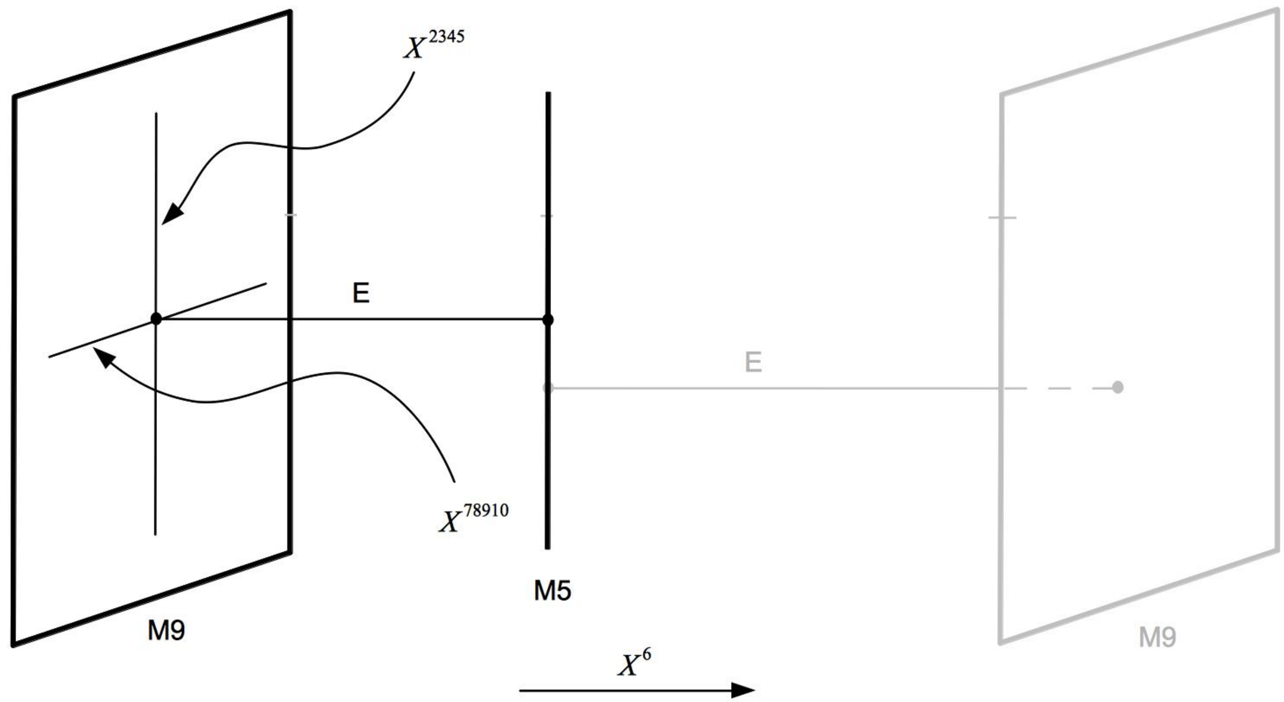


$$\langle M9|\exp(-\beta_1 H)D|0\rangle$$

$$Z_2^{\text{E-str}} = D^{M9,L}_{\square\square} D^{M5}_{\square\square\,\emptyset} + D^{M9,L}_{\begin{smallmatrix}\square\\ \square\end{smallmatrix}} D^{M5}_{\begin{smallmatrix}\square\\ \square\end{smallmatrix}\,\emptyset}$$

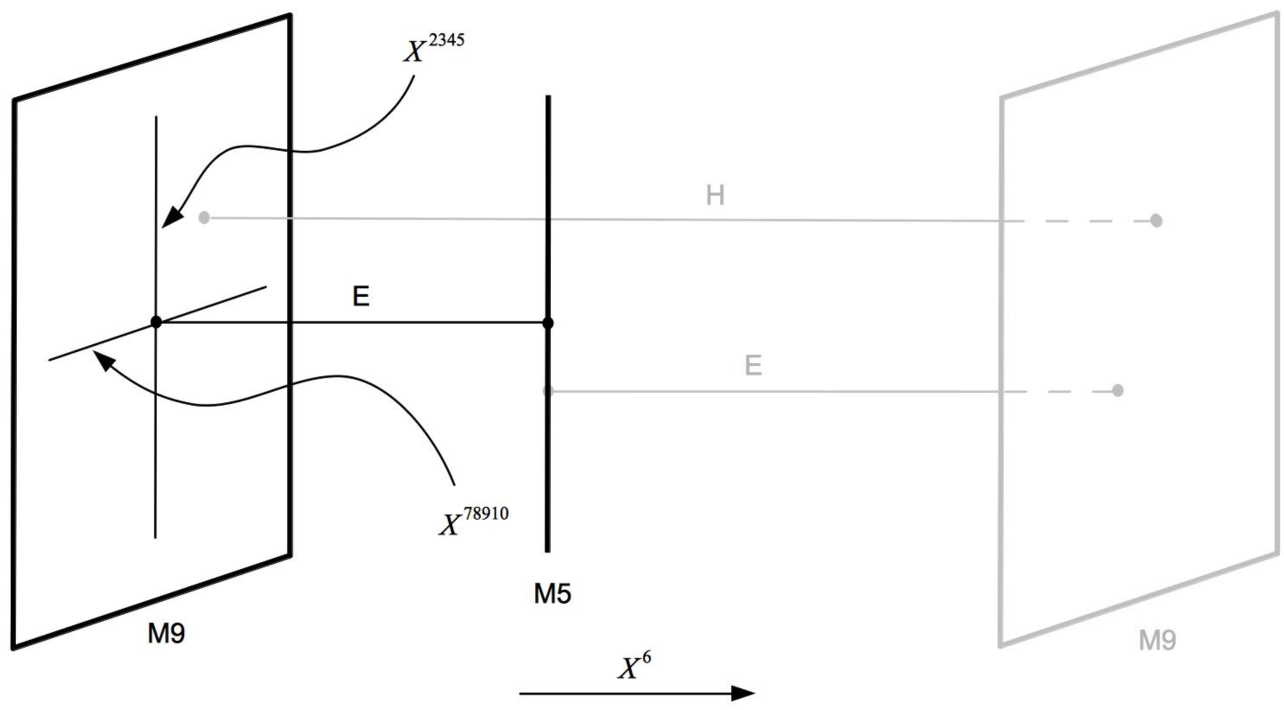
Using recent results for topological string on K3/2

[Huang,Klemm,Poretschkin] extract $Z_2 \to \langle M9^{(2)}|$



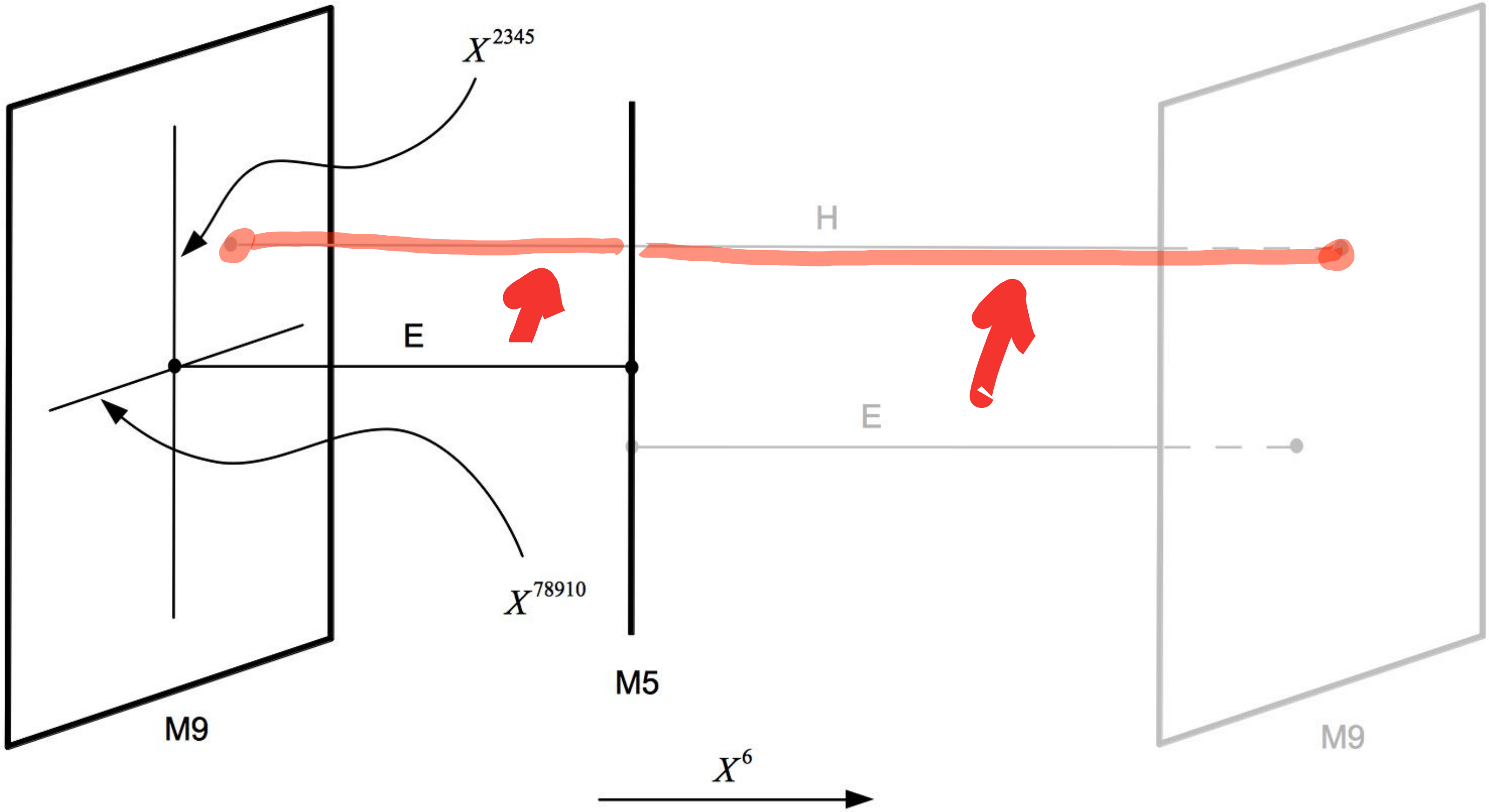
$$\langle M9 | \exp(-\beta_1 H) D \, \exp(-\beta_2 H) | M9 \rangle$$

E+E → H



$$\langle M9|\exp(-\beta_1 H)\ \exp(-\beta_2 H)|M9\rangle$$
$$= Z_{Heterotic}$$

E+E → H



$$\langle M9|\exp(i\beta_1 H)\ \exp(i\beta_2 H)|M9\rangle$$
$$= Z_{Heterotic}$$

$$Z_2^{\rm het}(\tau,\epsilon,\vec m_{E_8,L},\vec m_{E_8,R}) = 2\,\frac{N(\tau,\vec m_{E_8,L},\epsilon)N(\tau,\vec m_{E_8,R},\epsilon)}{\eta(\tau)^{24}\theta_1(\tau;-3\epsilon)\theta_1(\tau;-2\epsilon)^2\theta_1(\tau;-\epsilon)\theta_1(\tau;\epsilon)^2\theta_1(\tau;2\epsilon)^2} + 2\,\frac{N(\tau,\vec m_{E_8,L},\epsilon)N(\tau,\vec m_{E_8,R},\epsilon)}{\eta(\tau)^{24}\theta_1(\tau;-\epsilon)\theta_1(\tau;\epsilon)^2\theta_1(\tau;2\epsilon)^2\theta_1(\tau;3\epsilon)\theta_1(\tau;4\epsilon)^2}.$$

Comparing to the usual expression

$$Z_2^{\rm het}(\tau,\vec\epsilon,\vec m_{E_8\times E_8}) = \frac{1}{2}\Big[\Big(Z_1^{\rm het}(\tau,\vec\epsilon,\vec m_{E_8\times E_8})\Big)^2 + Z_1^{\rm het}(2\tau,2\vec\epsilon,2\vec m_{E_8\times E_8}) + Z_1^{\rm het}(\frac{\tau}{2},\vec\epsilon,\vec m_{E_8\times E_8}) + Z_1^{\rm het}(\frac{\tau+1}{2},\vec\epsilon,\vec m_{E_8\times E_8})\Big].$$

$$Z_1^{\rm het} = \left(\frac{\vartheta_{E_8}(\vec m_{E_8,L})\times\vartheta_{E_8}(\vec m_{E_8,R})}{\eta^{16}}\right)\frac{\eta^4}{\theta_1(\epsilon_1)\theta_1(\epsilon_2)\theta_1(\epsilon_3)\theta_1(\epsilon_4)},$$

leads to highly non-trivial identities, that we have checked to high order.

Conclusions

1-Classification of maximally Higgsed branches of (1,0) theories completed (within F-theory) leading to a generalized ADE classification.

2-Strings in the Coulomb branch of some of these theories are being understood and used to compute supersymmetry protected quantities for the (1,0) theory.