# String field theory vertex from integrability

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and work in progress...

## Outline

**Motivation for the physical problem**

**Motivation for our approach**

- Lessons from the spectral problem
- Lessons from form factors

**Light-cone String Field Theory Vertex – the pp-wave**

**Functional equations for the string vertex**

**What happens in $AdS_5 \times S^5$?**

**Conclusions**

## Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$

  $\equiv$ quantized energy levels of a string for any value of $\lambda \equiv g_{YM}^2 N_c$

  $\equiv$ equivalently (anomalous) dimensions of operators in $\mathcal{N} = 4$ SYM

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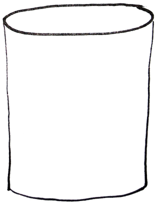

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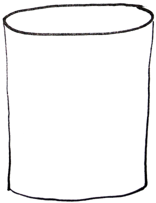

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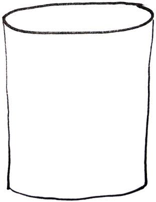

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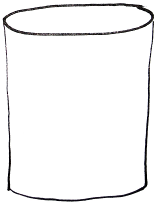

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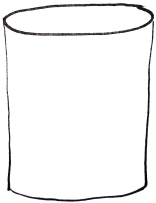

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**Key question:**

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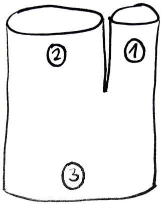


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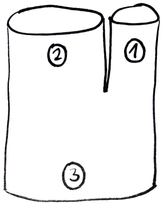


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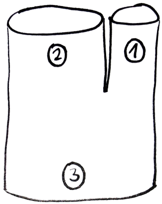


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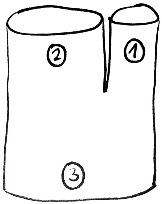


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- Previously we knew how to proceed only for a **free** worldsheet theory
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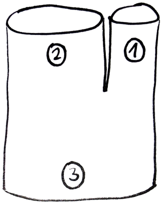

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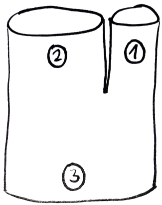

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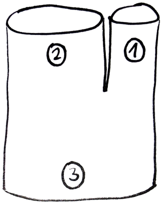

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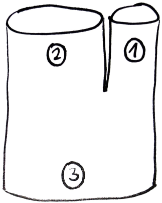

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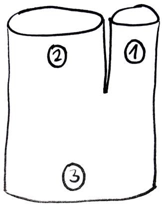

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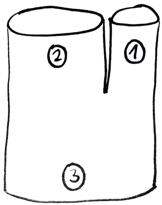

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**I) solve the worldsheet theory on an infinite plane**

1. Particle momenta/rapidities can be analytically continued into the complex plane...
2. We get **crossing** equation + unitarity
3. ... and symmetry + Yang-Baxter equation
4. determines analytically the **S-matrix**

**Works equally for relativistic and $AdS_5 \times S^5$ case...**

**Key role** of the infinite plane $\longrightarrow$ only there do we have crossing+analyticity which allows for solving for the S-matrix (functional equations for the S-matrix)

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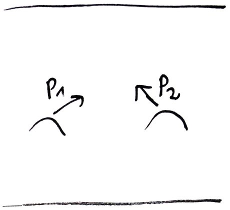


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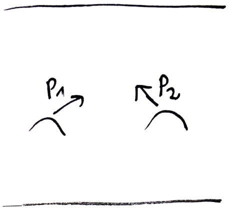


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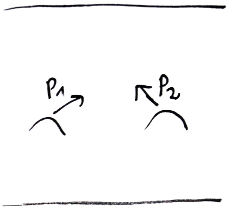


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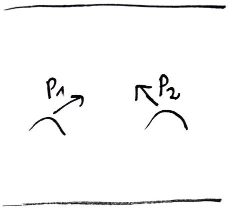


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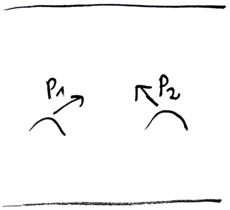


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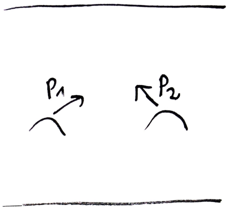


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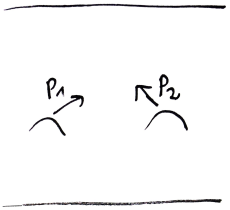


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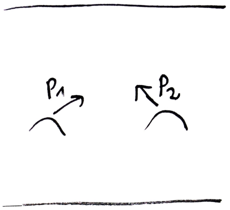


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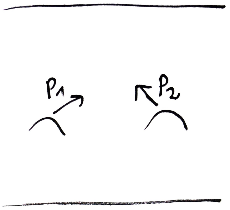


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**II) solve the theory on a (large!) cylinder**

1. Bethe Ansatz Quantization

$$e^{ip_k \mathbf{L}} \prod_{l \neq k} S(p_k, p_l) = 1$$

2. Get the energies from

$$E = \sum_k E(p_k) = \sum_k \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p_k}{2}}$$

**This gives the spectrum up to wrapping corrections...**

relativistic $\sim e^{-mL}$ weak coupling $\sim \lambda^{L}$ strong coupling $\sim e^{-\frac{2\pi L}{\sqrt{\lambda}}}$

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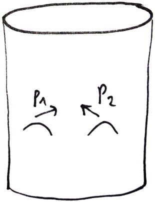


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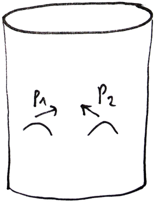


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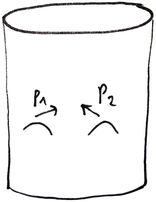


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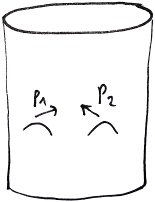


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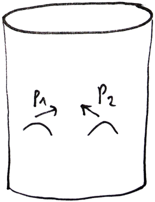


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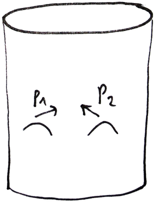


**II) solve the theory on a (large!) cylinder**

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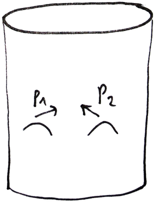


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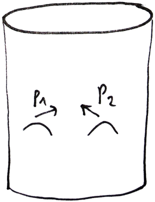


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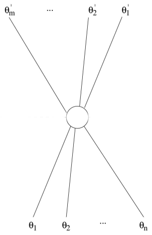
**Proceed to form factors...**

# Form factors

- Form factors are expectation values of a local operator sandwiched between specific multiparticle *in* and *out* states $p_k = m \sinh \theta$




- Form factors in infinite volume (on an infinite plane) satisfy a definite set of **functional equations**

$$f(\theta_1,\theta_2) = S(\theta_1,\theta_2)\, f(\theta_2,\theta_1)$$
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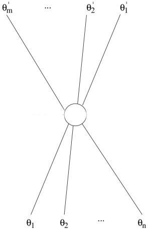


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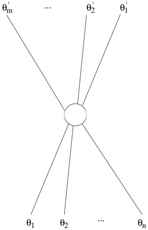


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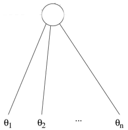


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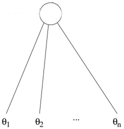


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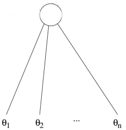


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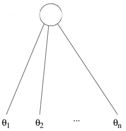


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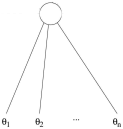


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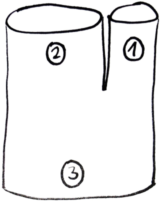


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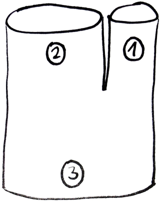


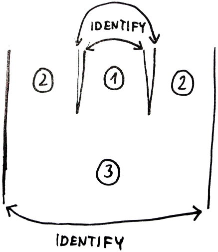


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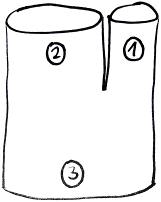


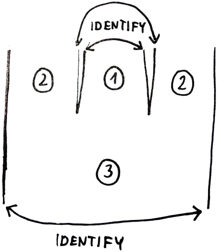


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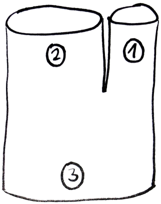


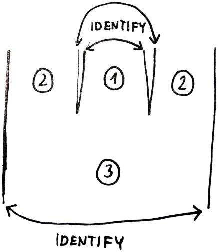


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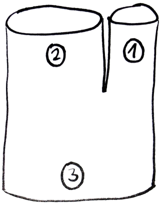


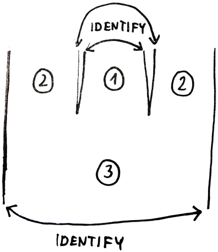


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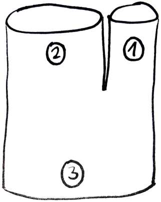


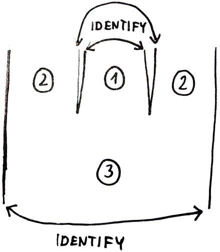


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He, Schwarz, Spradlin, Volovich
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He, Schwarz, Spradlin, Volovich
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- Exact expressions involve novel special functions $\Gamma_\mu(z)$

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  (these are exactly wrapping terms $e^{-ML_r}$!!)
- Going to an exponential basis (BMN basis) one got in this limit

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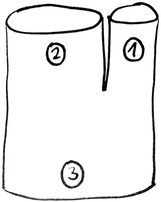


- The emission of string #1 can be understood as an insertion of some macroscopic (not completely local) operator...
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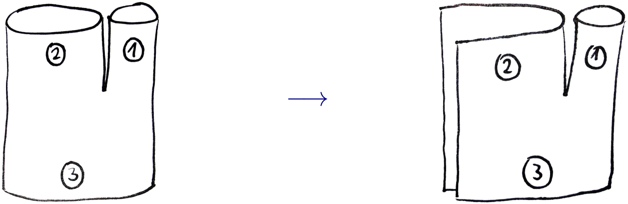


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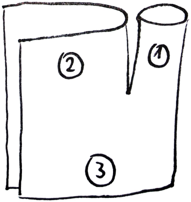


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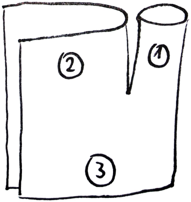


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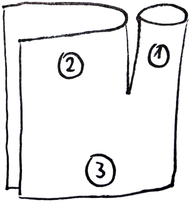


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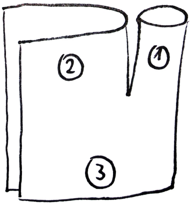


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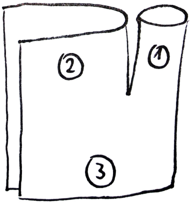


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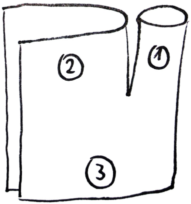


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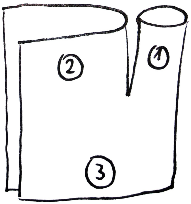


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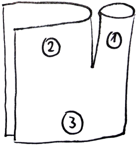
### Functional equations for the (decompactified) string vertex

written here for two incoming particles and, for the moment, free theory




- The exact pp-wave solution, involving the $\Gamma_\mu(\theta)$ special function solves these equations and can be reconstructed from them!

$$n(\theta)n(\theta + i\pi) = -\frac{1}{2\pi^2} ML \sinh\theta \sin\frac{p(\theta)L}{2}$$

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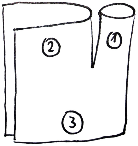


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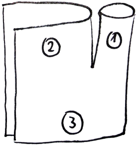


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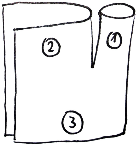


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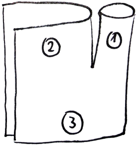
- The exact pp-wave solution, involving the $\Gamma_\mu(\theta)$ special function solves these equations and can be reconstructed from them!

$$n(\theta) n(\theta + i\pi) = -\frac{1}{2\pi^2} ML \sinh\theta \sin\frac{p(\theta)L}{2}$$

- This includes **all exponential wrapping corrections** $e^{-\mu\alpha_1} = e^{-ML}$ for the $\#1$ string
- Straightforward generalization of the axioms to an **interacting** integrable QFT

## Functional equations for the (decompactified) string vertex

written here for two incoming particles and, for the moment, free theory

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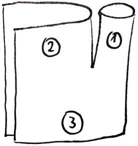
**Functional equations for the (decompactified) string vertex**

written here for two incoming particles and interacting worldsheet theory

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## Step II) The string vertex — back to finite volume

We considered so far the ‘decompactified string vertex’...

but ultimately we are interested in the finite volume one...

**Use the same prescription as for form factors...**

We neglect exponential corrections for strings #2 and #3 but **keep** all size dependence of string #1...

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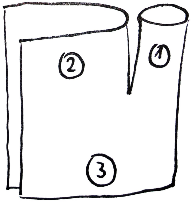


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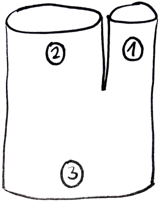


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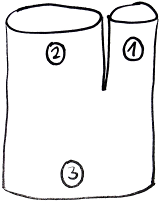


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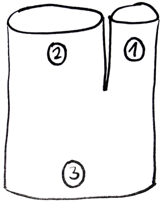


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## What happens in $AdS_5 \times S^5$?

**Novel kinematics**

- Complex rapidities are defined on a covering of an elliptic curve
- Only $e^{ip}$ (and not the momentum $p$ itself) is a well defined elliptic function
- The phase factors $e^{ip\mathbf{L}}$ make sense directly only for integer $\mathbf{L}$ which is nice from the point of view of $\mathcal{N}=4$ SYM...

**Complicated dynamics**

- The S-matrix does not depend on the difference of rapidities
- The S-matrix is nondiagonal which drastically complicates solving form factor axioms (which are a special case of our SFT axioms)

**We would like to separate the two problems...**

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## The $AdS_5 \times S^5$ Neumann coefficient

Factorize the $AdS_5 \times S^5$ Neumann coefficient as

$$\mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1 i_2} = \underbrace{\mathbf{F}(z_1, z_2)_{i_1, i_2}}_{\textit{form factor}} \cdot \underbrace{N^{33}_{\mathbf{L}}(z_2, z_1)}_{\textit{includes all } \mathbf{L} \textit{ dependence}}$$

- The $N^{33}_{\mathbf{L}}(z_2, z_1)$ includes **all** dependence on the size of the string $\mathbf{L}$ at any coupling ($\equiv$ infinite set of wrapping corrections)
- Currently we have a solution of the functional equations for any *even* $\mathbf{L}$ at arbitrary coupling

$$N^{33}_{\mathbf{L}}(z_2, z_1) \sim \text{conventional elliptic functions} \times \text{Elliptic Gamma}$$

- This goes over to the pp-wave answer up to an overall normalization

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## Conclusions

- We propose a framework for formulating functional equations for string interactions (light cone string field theory vertex) when the worldsheet theory is **integrable**
- This approach should work in particular for strings in the full $AdS_5 \times S^5$ geometry
- A key step is the existence of an infinite volume setup, which allows for formulating functional equations incorporating e.g. crossing
- Second step involves reduction to (large) finite size of two of the three strings

- We reproduced pp-wave string field theory formulas for the Neumann coefficients
- We solved for the 'kinematical' part of the $AdS_5 \times S^5$ Neumann coefficient describing exact volume dependence (currently for even $L$) at any coupling – may describe all order wrapping w.r.t. one string

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