Structure Constants and Integrable Bootstrap in Planar N=4 SYM Theory

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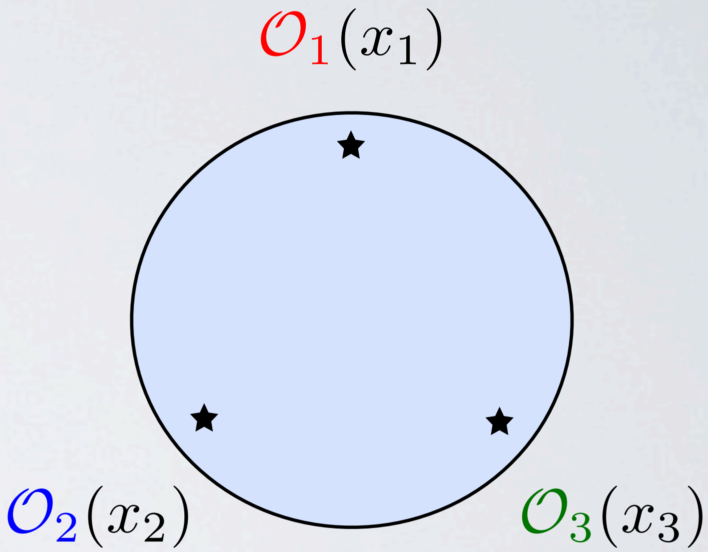
based on work with Shota Komatsu and Pedro Vieira

Plan / Goal

Compute Structure Constants of Single Trace Operators in Planar N=4 SYM theory

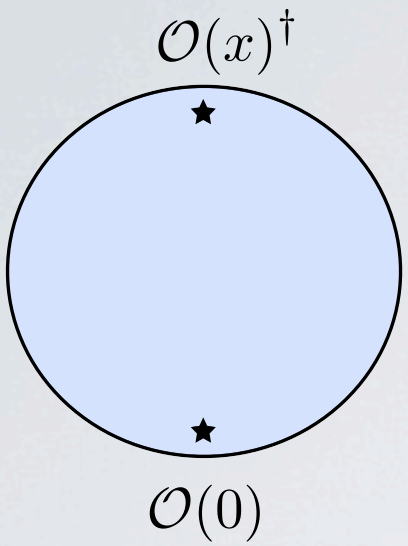
$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{x_{12}^{\Delta_{12}}x_{23}^{\Delta_{23}}x_{13}^{\Delta_{13}}}$$



Question : How Do We Compute That At Finite Coupling?

Answer : Use Integrability and Hexagon Bootstrap Program

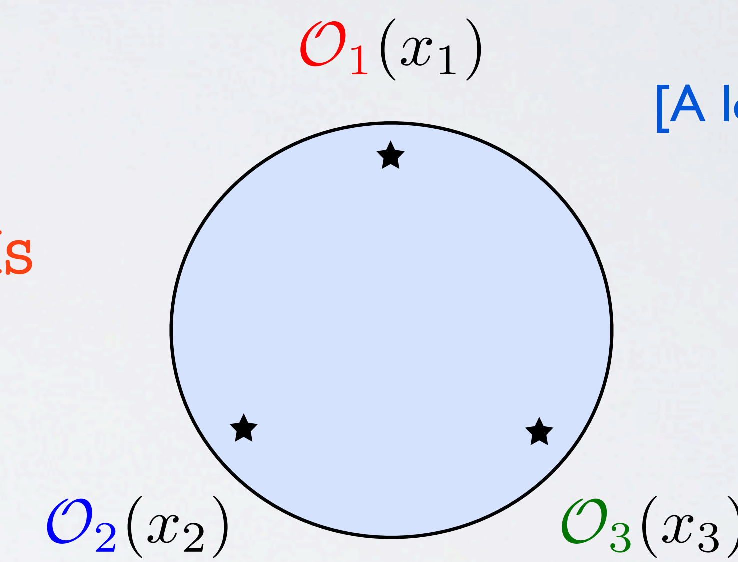
Part of a Bigger Program = Solving Strings in AdS



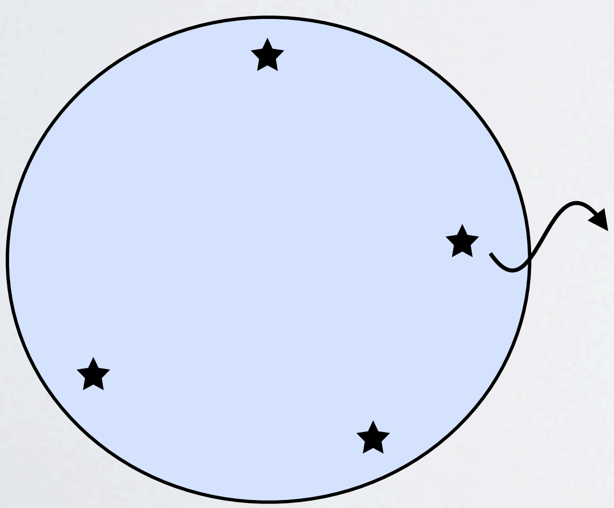
Solved

[Many people here]

What This Talk Is About :



[A lot of recent works]



Two Cross Ratios
Use OPE or
Better?

This is all planar but hopefully
can also describe handles = string loops

Spectrum Success Story

2009

Thermodynamic Bethe Ansatz

2005

Symmetries
Beisert S-matrix

Beisert-Staudacher
Asymptotic Bethe Ansatz

Quantum Spectral Curve

BMN Vacuum
Spin Chain Picture

Two Years Ago
(See Gromov's
Talk@Strings14)

2002

Can we follow a similar path for structure constants?

QCD Story
Perturbative
Integrability

1995-1998

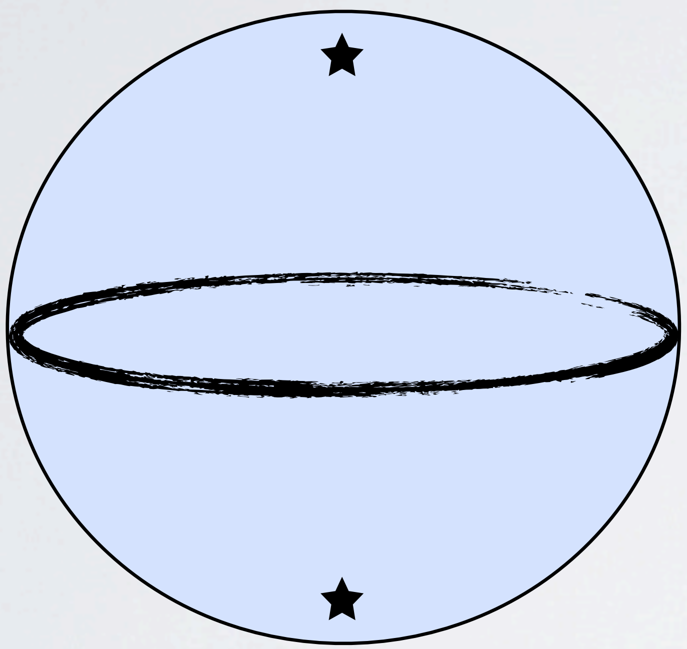
Lessons From The Cylinder

2pt Function

$$\langle \mathcal{O}(x)^\dagger \mathcal{O}(0) \rangle = \frac{1}{x^{2\Delta}}$$

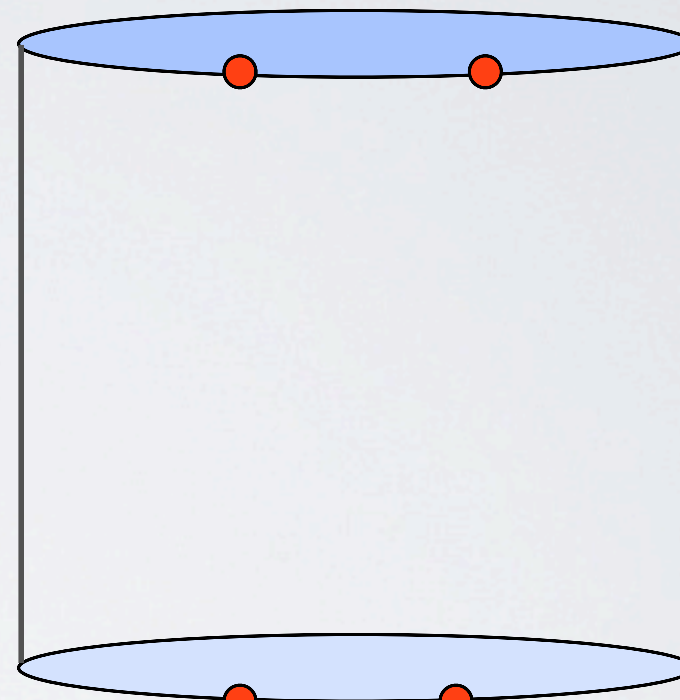
Δ = Energy Eigenvalue
In Finite Volume L

$\mathcal{O}(x)^\dagger$



$\mathcal{O}(0)$

Future



Past

Magnon = Excitation

$$\mathrm{tr}\,[\ldots ZYZ\ldots ZYZ\ldots](0)$$

Pick BMN Vacuum

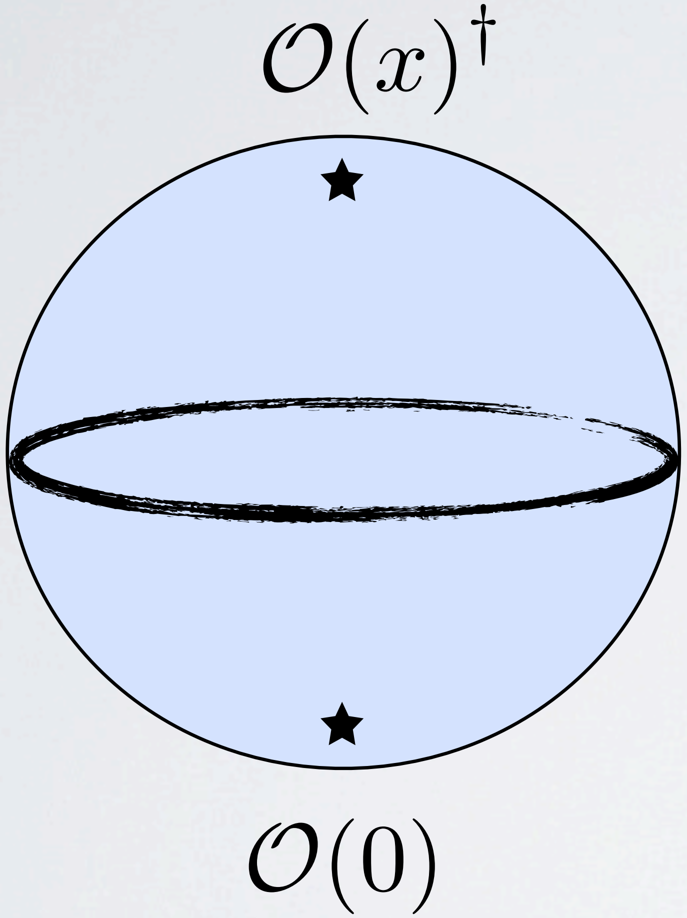
$$\mathrm{tr}\, Z^L$$

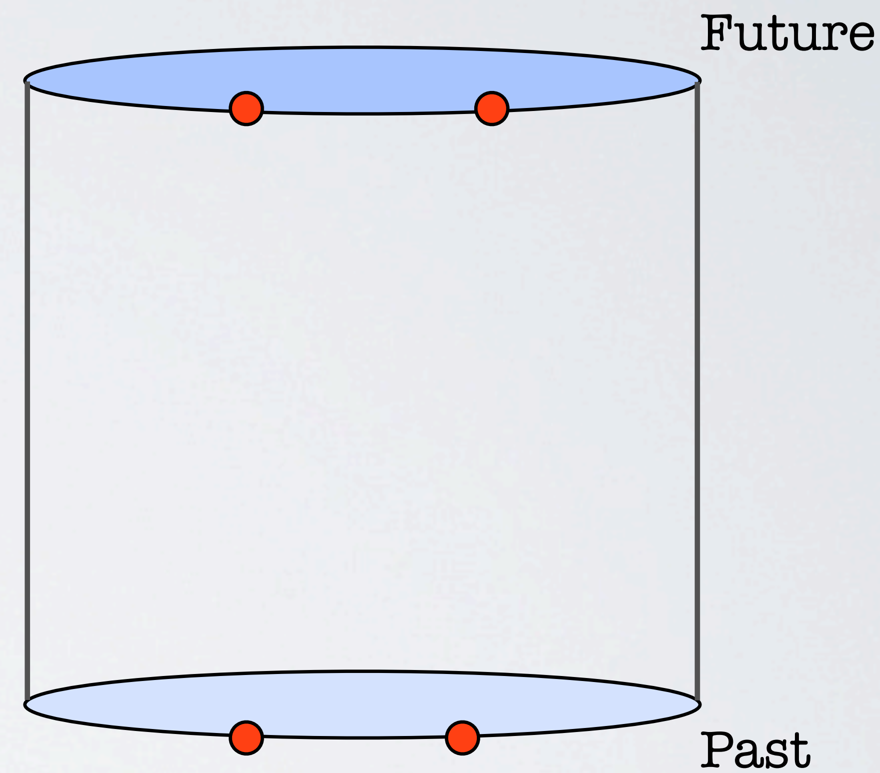
Length = Number of Adjoint
Fields = L

2pt Function

$$\langle \mathcal{O}(x)^\dagger \mathcal{O}(0) \rangle = \frac{1}{x^{2\Delta}}$$

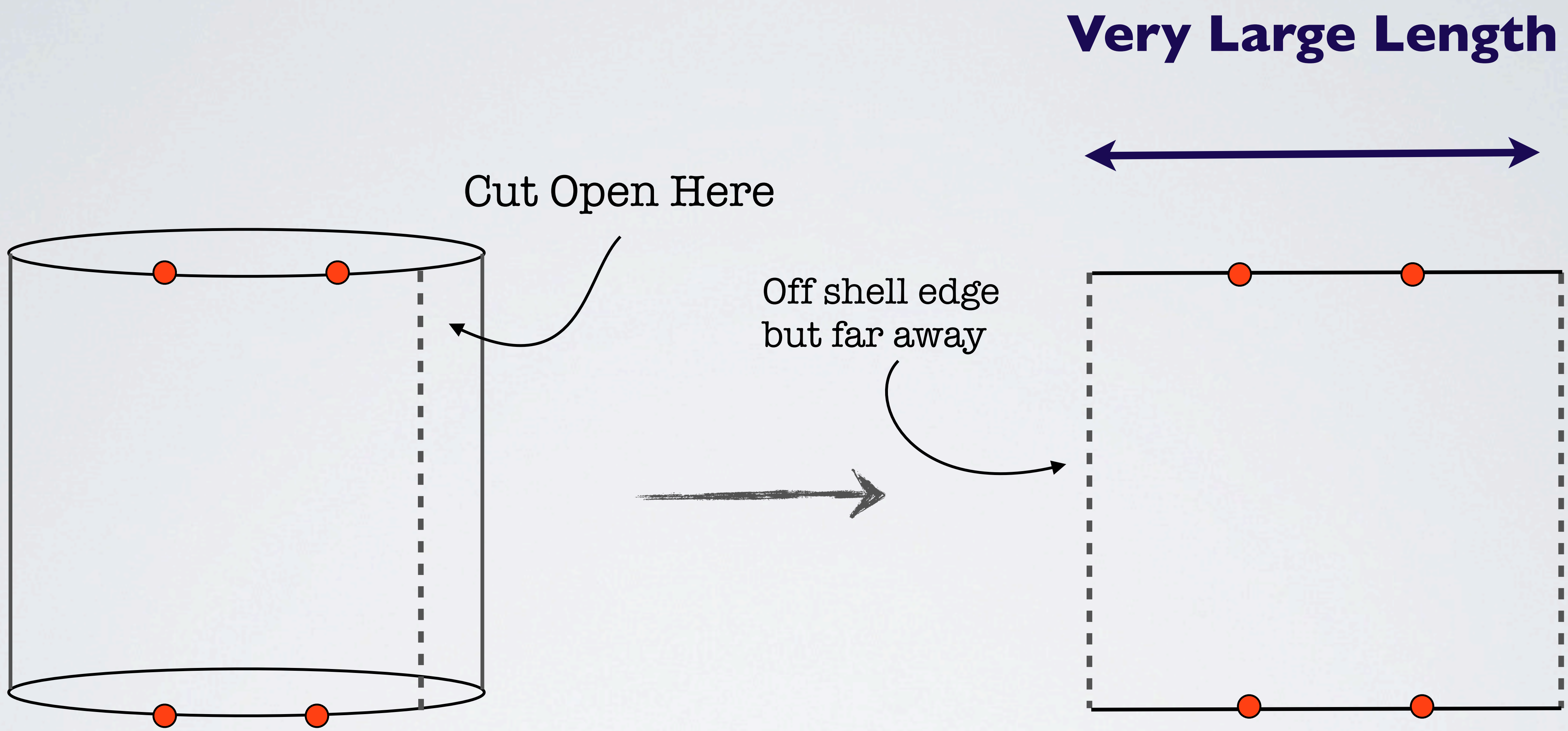
Δ = Energy Eigenvalue
In Finite Volume L





Finite-Volume Problem Are Difficult ...
... Start with Large Volume First

Decompactification = Cutting Procedure

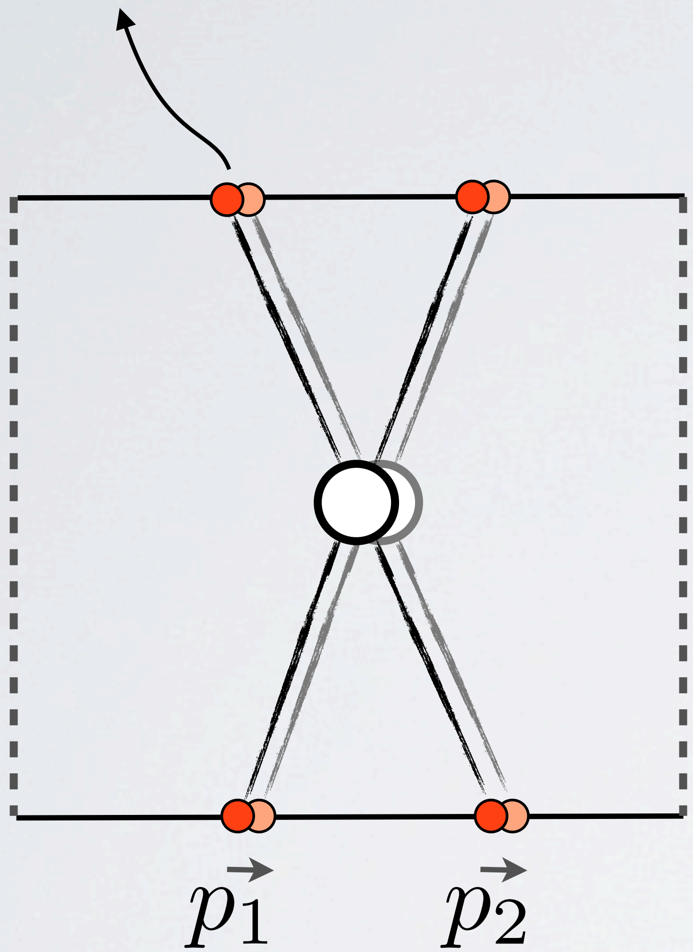


Asymptotic or Infinite Volume L Description

Power of Symmetry

Magnon in bi-fund irrep of residual symmetry group of BMN vacuum :

[Beisert'05]



$$\underbrace{PSU(2|2)}_{\text{Left}} \times \underbrace{PSU(2|2)}_{\text{Right}} \ltimes \mathbb{R}^3$$

Central extensions : contain BMN energy

Beisert S-Matrix

$$\mathbb{S}_{12} \sim S^0_{12}\, \mathcal{S}_{12} \times \dot{\mathcal{S}}_{12}$$

Left / Right S-Matrix

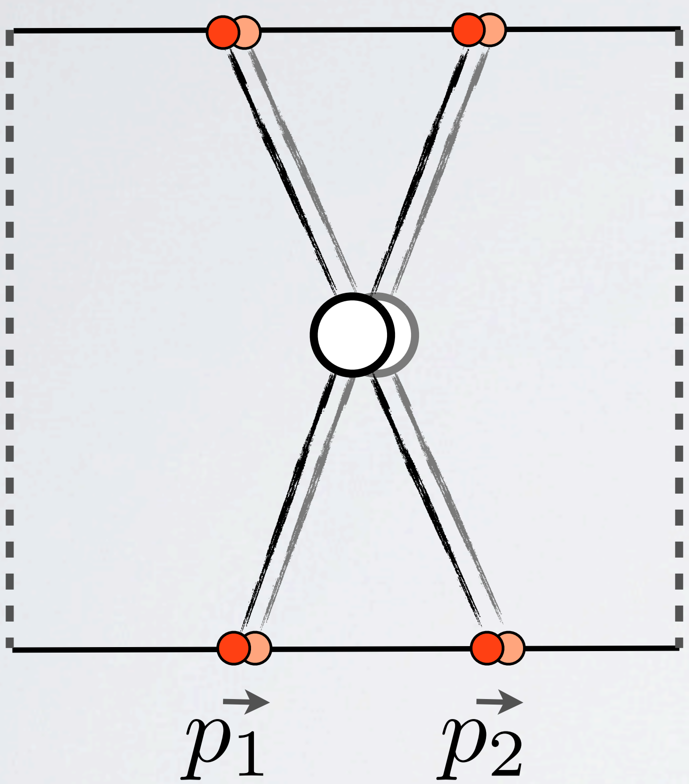
Fixed by Symmetry and Crossing

[Janik'05]

Power of Symmetry

Asymptotic = IR Solution

[Beisert'05]



Energy

$$E = 1 + 2g\left(\frac{i}{x^+} - \frac{i}{x^-}\right)$$

Momentum

$$p = \frac{1}{i}\log\frac{x^+}{x^-}$$

Rapidity

$$x^\pm + \frac{1}{x^\pm} = \frac{u \pm i/2}{g}$$

Scattering Phase

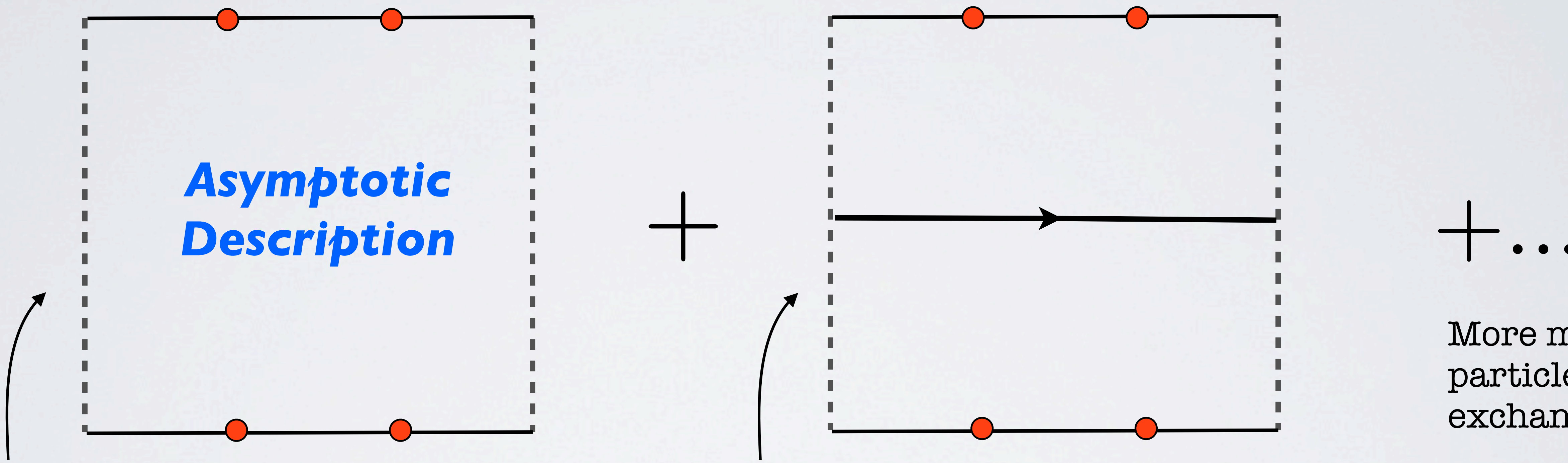
[Beisert,Eden,Staudacher'06]

$$S^0_{12} = \frac{x_1^+ - x_2^-}{x_1^- - x_2^+}\,\frac{1 - 1/x_1^- x_2^+}{1 - 1/x_1^+ x_2^-}\,\frac{1}{\sigma_{12}^2}$$

Systematic Improvement

Include Finite Size Effects = so-called Wrapping Effects

[Ambjorn,Janik,Kristjansen'05]
[Bajnok,Janik'08]



Vacuum (in Mirror = double Wick rotated Theory)

Virtual Effect : Exchange of 1-particle in Mirror Channel

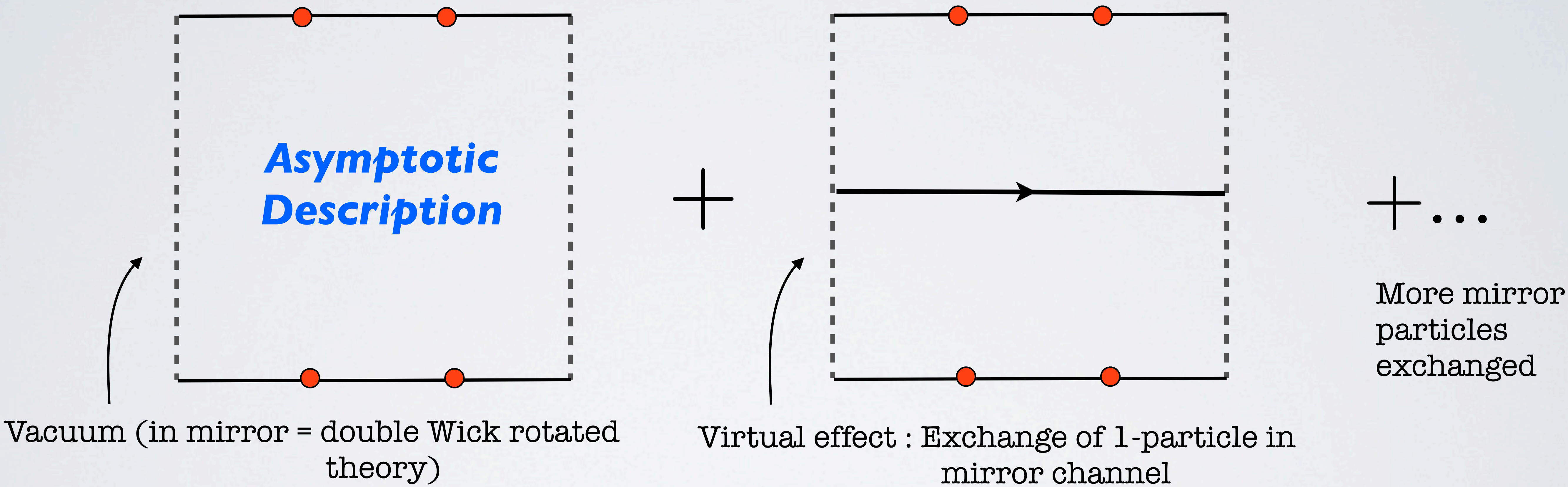
More mirror particles exchanged

Wrapping Corrections = $e^{-L\times E} \sim O(g^{2L})$

Systematic Improvement

Include Finite Size Effects = so-called Wrapping Effects

[Ambjorn,Janik,Kristjansen'05]
[Bajnok,Janik'08]

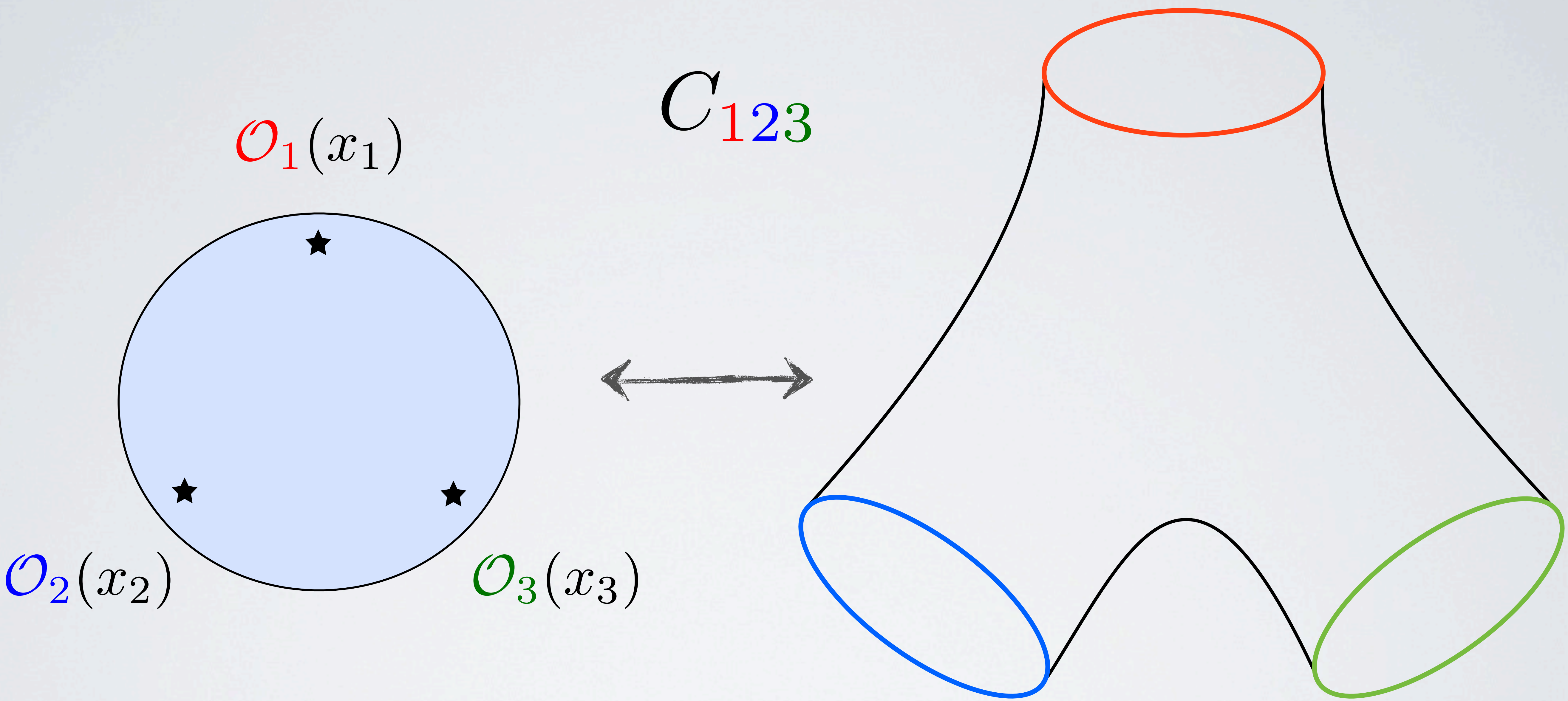


Resummation of All Finite Size Corrections = TBA Eqs

[Kazakov et al.'09]
[Arutyunov et al.'09]
[Bombardelli et al.'09]

3-pt Function

3-pt Function / Pair of Pants

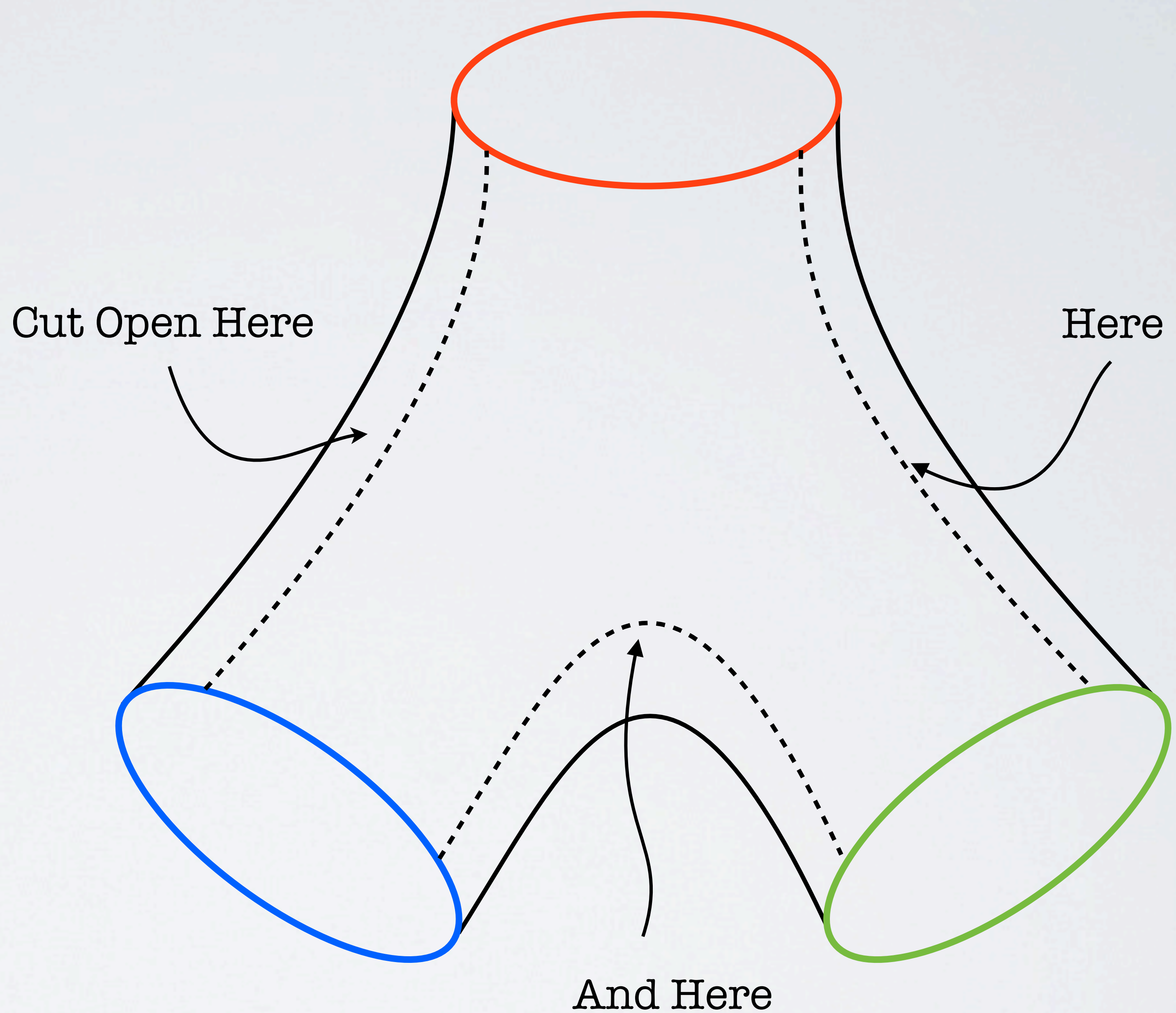


Three Punctures
Sphere

Pair of Pants

Toward Asymptotic Description

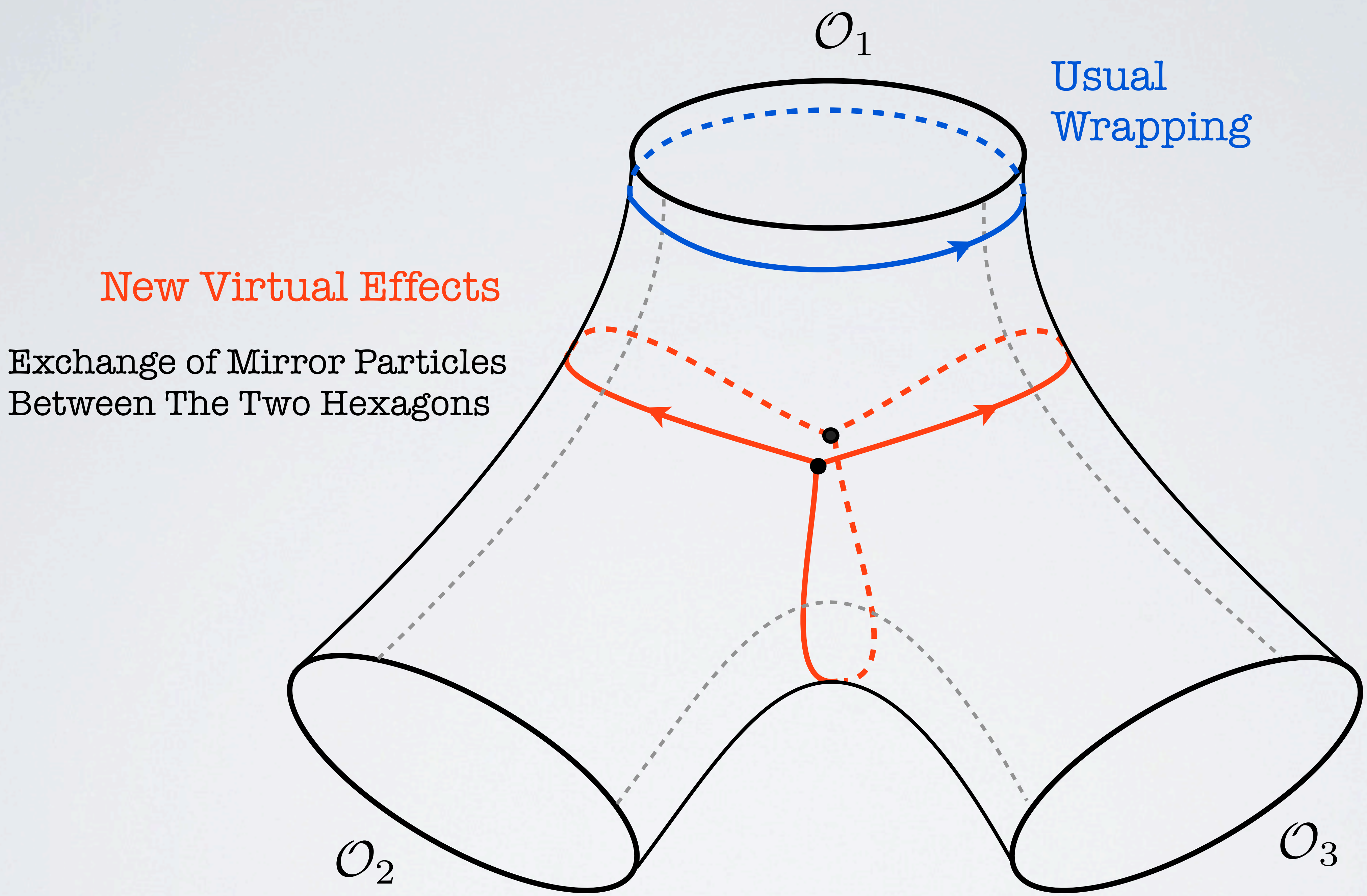
Cutting Procedure



Philosophy :
Closed String
=
(Open String)^2

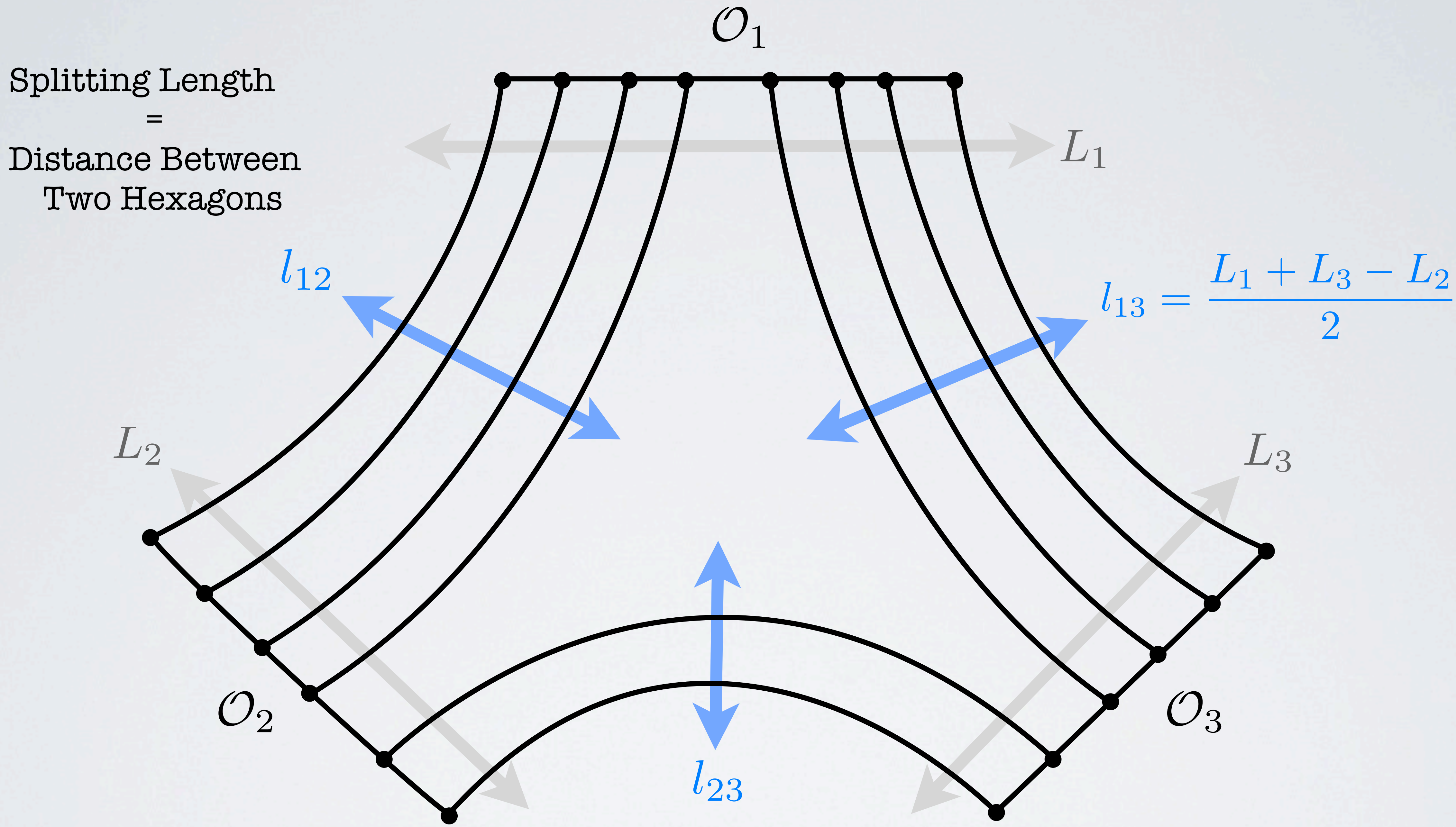
Slogan : Pair of Pants = (Hexagon)^2

New Finite Size Effects



Can Get a Good Asymptotic Description When These New Effects Are Small

Dual Spin Chain Picture

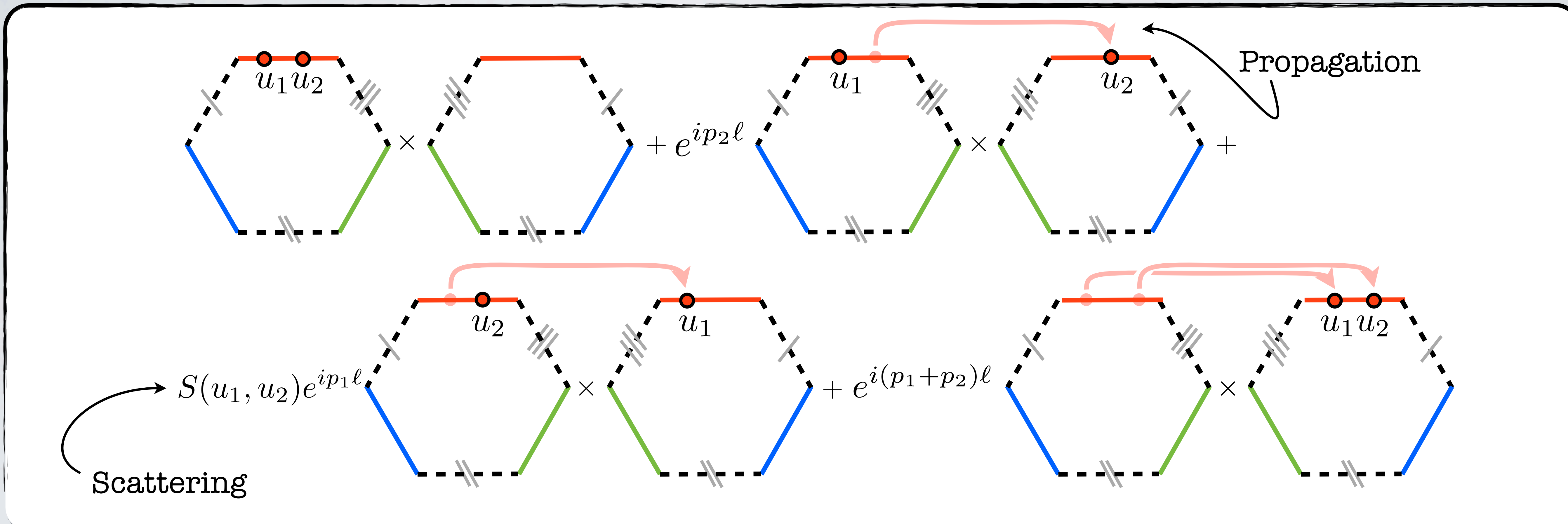


Three Bridges In Total = Three-Distance Problem

New Wrapping Effects = $O(g^{l_{ij}})$ at Weak Coupling

How to Combine Hexagons Into Asymptotic = IR Description

Split The Wave Funtion :

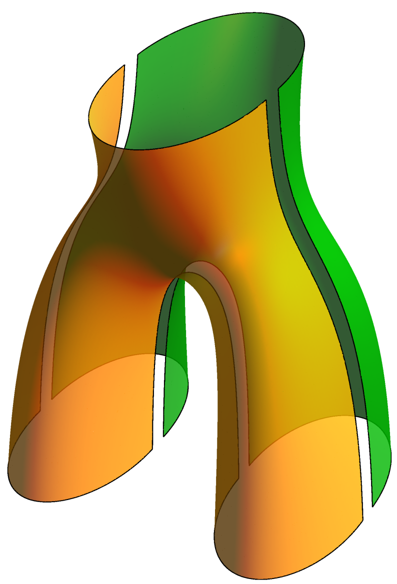


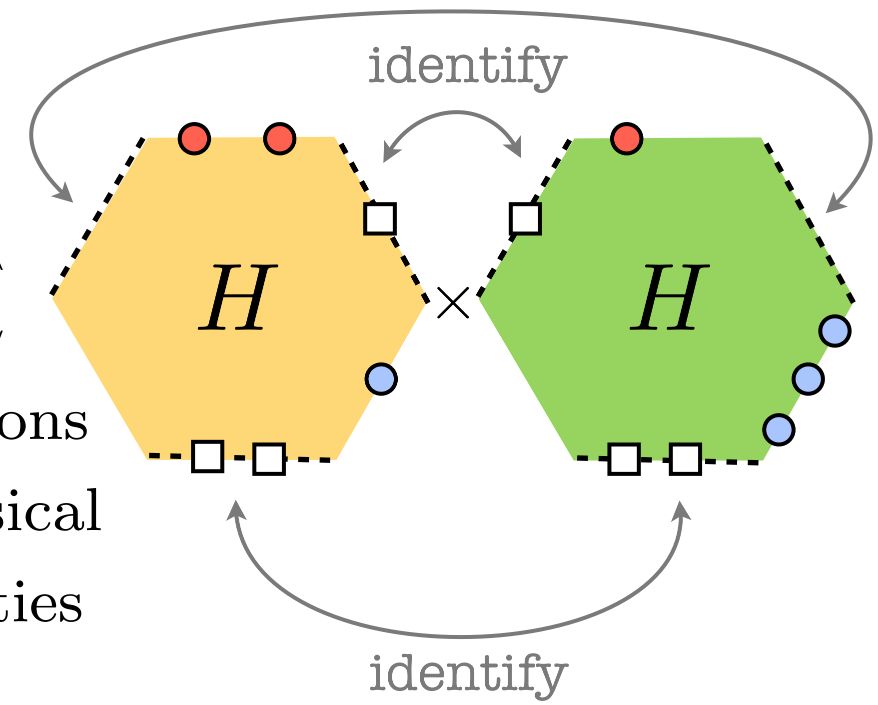
Leftover Information About Spin Chain State :

Sum Over Bipartite Partition of Bethe Roots

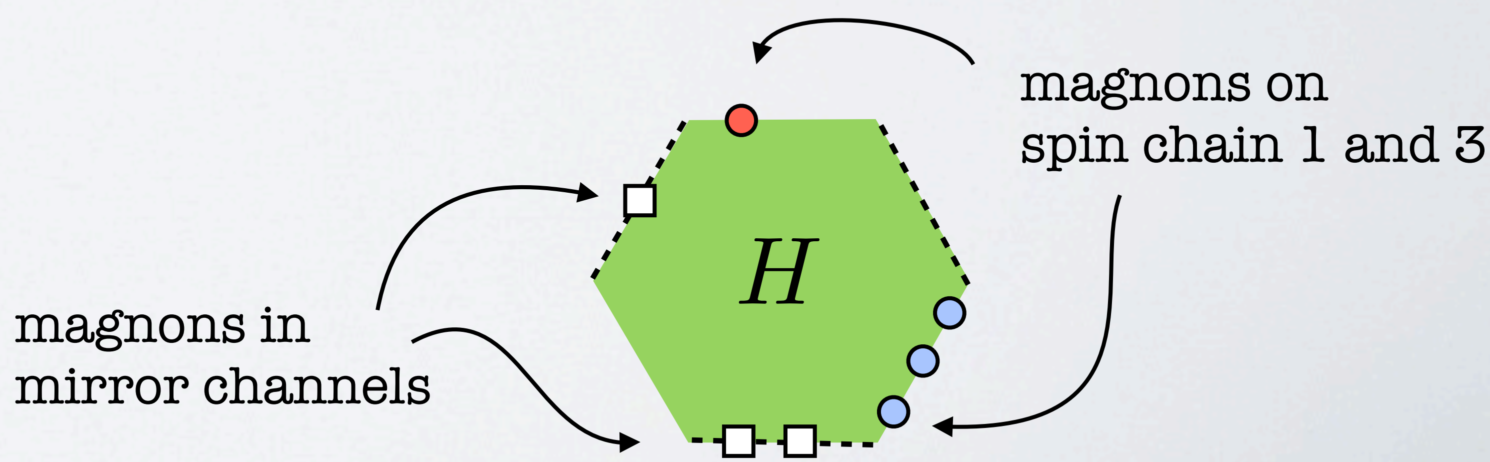
Summary Hexagon Picture

3-pt Function = Finite Volume Correlator of Two Hexagons

$C_{123} =$  $\sim \int$ (momentum of) mirror particles where we glue $\square$ $\sum$ partitions of physical rapidities $\circ$

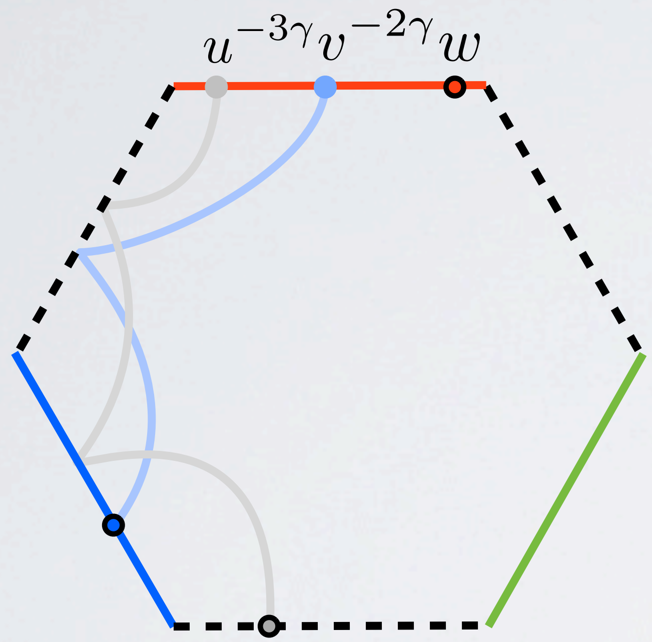


Elementary Patch :
Hexagon Form Factor



Building Blocks = Hexagon Form Factors

Mirror/Crossing Moves



All-On-Top = Creation Form Factor

Spin Chain State = String of bifundamentals

$$\mathfrak{h}^{A_1\dot{A}_1,\ldots,A_N\dot{A}_N}(u_1,\ldots,u_N) = \langle\mathfrak{h}|\big(|\chi_1^{A_1\dot{A}_1}\cdots\chi_N^{A_N\dot{A}_N}\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3\big)$$

Hexagon Vertex

How To Fix The Hexagon Form Factors?

Use Super-Symmetry

3pt-Function = (BMN)^3

$$\mathcal{O}_2 = \mathrm{tr}\, \bar{Z}(\infty)^{L_2}$$

$$\mathcal{O}_3 = \mathrm{tr}\, \tilde{Z}(1)^{L_3}$$

$$\mathcal{O}_1 = \mathrm{tr}\, Z(0)^{L_1}$$

Two BMN vacua

+

One Twisted BMN Vacuum

$$\tilde{Z} = Z + \bar{Z} + Y - \bar{Y}$$

Needed for overlap with ops 1 and 2

Needed for BPS condition

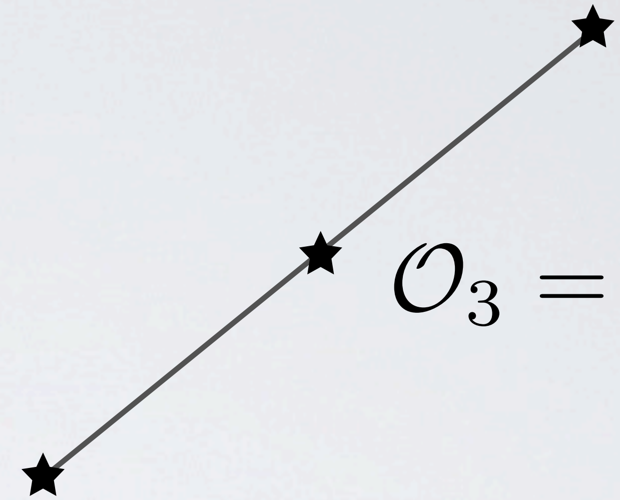
Part of Family of Twisted Correlators

see [Drukker,Plefka'09]

Use Super-Symmetry

$$\mathcal{O}_2 = \mathrm{tr}\,\bar{Z}(\infty)^{L_2}$$

3pt-Function = (BMN)^3



$$\mathcal{O}_3 = \mathrm{tr}\,\tilde{Z}(1)^{L_3}$$

Bosonic Subgroup :

$$O(3) \times O(3)$$

Fix a Line
in Spacetime

Fix Three
(real) Scalars
Out of Six

$$\mathcal{O}_1 = \mathrm{tr}\,Z(0)^{L_1}$$

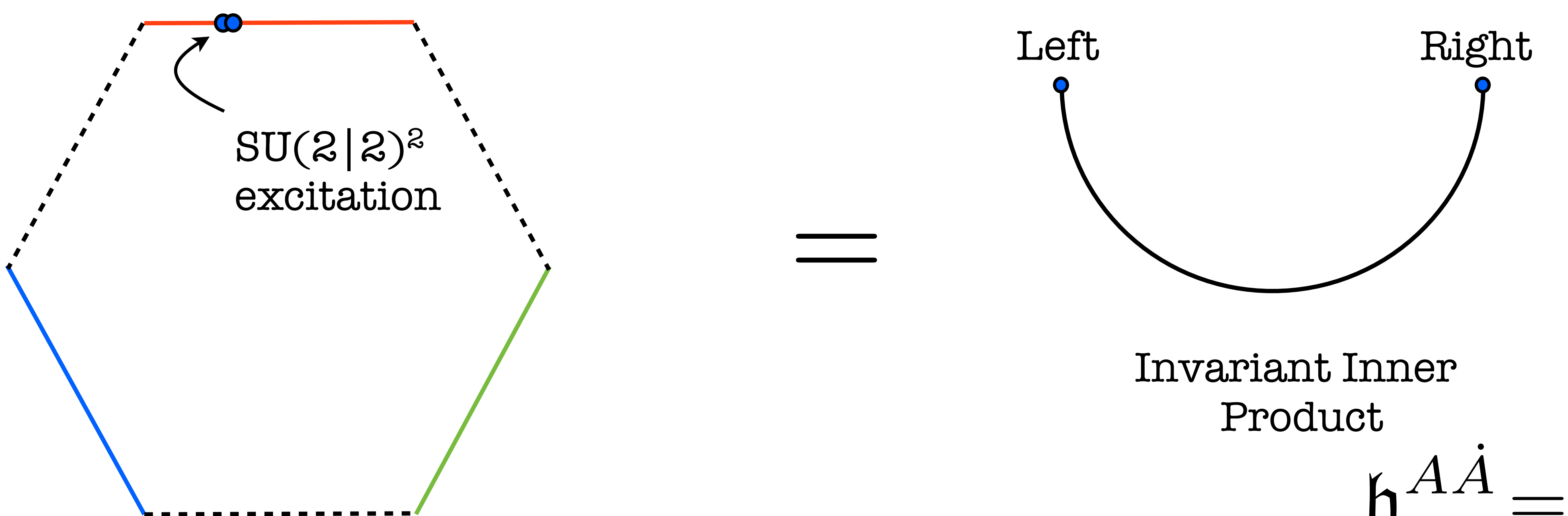
+ 8 Supercharges : $\mathcal{Q}^a{}_\alpha + \epsilon^{ab}\epsilon_{\alpha\beta}\dot{\mathcal{S}}^\beta{}_b$

Total : $PSU(2|2)$ i.e. Diagonal Subgroup of BMN Group

$$PSU(2|2)_{\text{Left}} \times PSU(2|2)_{\text{Right}}$$

Power of Symmetry

Fix 1-pt Hex Form Factor = Invariant Left-Right Inner Product

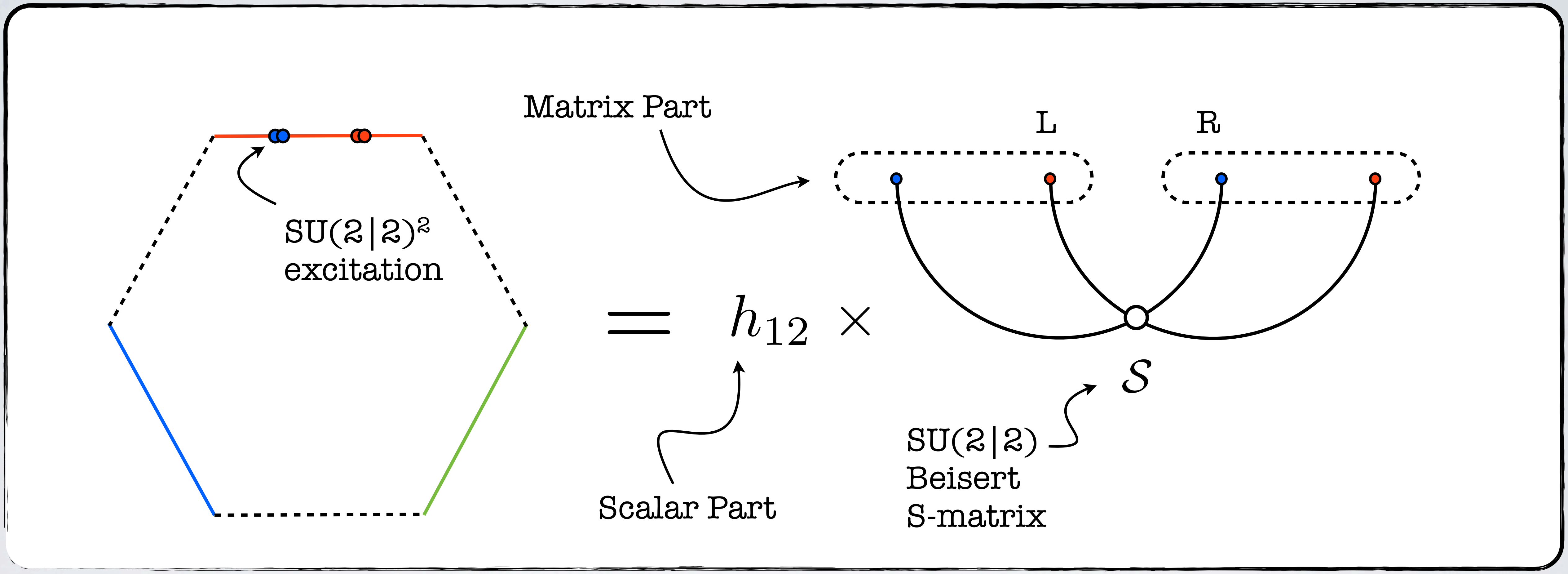


$$\mathfrak{h}^{A\dot{A}} = \langle \chi^{\dot{A}} | \chi^{A} \rangle$$

$$\mathfrak{h}^{a\dot{a}} = \langle \mathfrak{h} | \Phi^{a\dot{a}} \rangle = \epsilon^{a\dot{a}} \quad \mathfrak{h}^{\alpha\dot{\alpha}} = \langle \mathfrak{h} | \mathcal{D}^{\alpha\dot{\alpha}} \rangle = N\epsilon^{\alpha\dot{\alpha}}$$

Power of Symmetry

Fix 2-pt Hex Form Factor

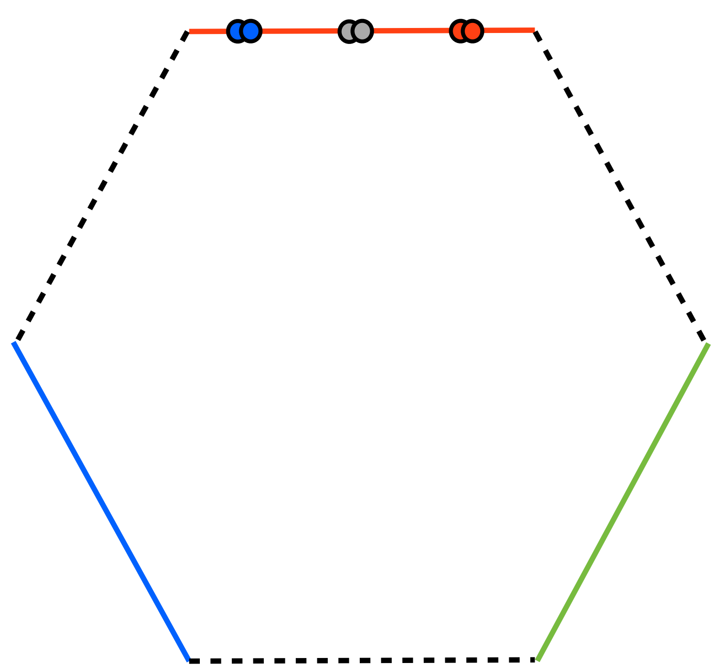


Fixed by Symmetry up to a Scalar Factor

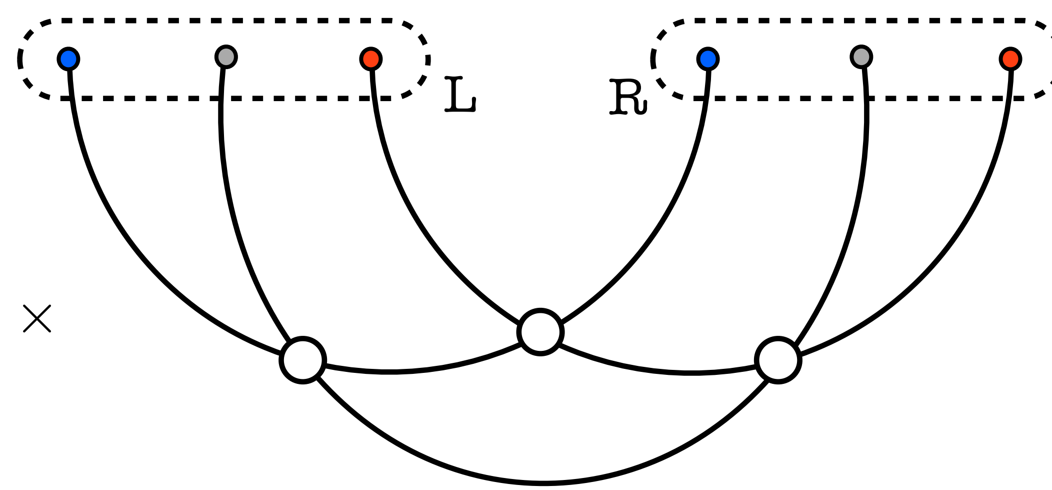
$$\mathfrak{h}^{A_1\dot{A}_1,A_2\dot{A}_2} = (-1)^{\dot{f}_1 f_2} \times h_{12} \times \langle \chi_2^{\dot{A}_2}\chi_1^{\dot{A}_1} | \mathcal{S}_{12} | \chi_1^{A_1}\chi_2^{A_2} \rangle$$

Generalization

N-pt Hexagon Form Factor (conjecture)



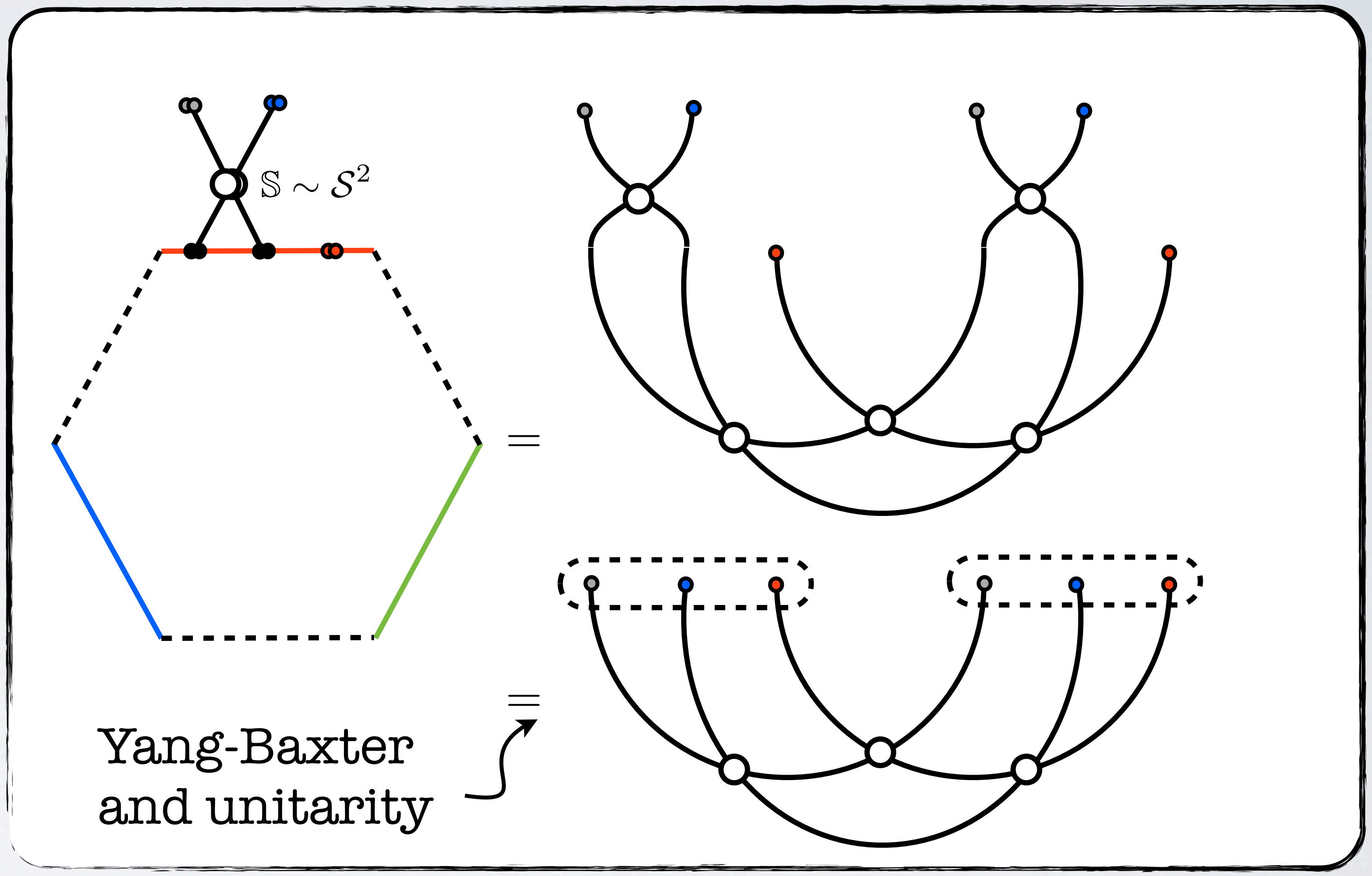
$$= \prod_{i<j}^{N} h(u_i, u_j) \times$$



$$\mathfrak{h}^{A_1 \dot{A}_1 \cdots A_N \dot{A}_N} = (-1)^{\mathfrak{f}} \prod_{i<j}^{N} h_{ij} \langle \chi_N^{\dot{A}_N} \cdots \chi_1^{\dot{A}_1} | \mathcal{S} | \chi_1^{A_1} \cdots \chi_N^{A_N} \rangle$$

Watson Equation

Hex preserved by S-matrix $\langle \mathfrak{h}|(\mathbb{S}_{ii+1}-\mathbb{I})|\ldots\chi_i^{A_i\dot{A}_i}\chi_{i+1}^{A_{i+1}\dot{A}_{i+1}}\ldots\rangle = 0$

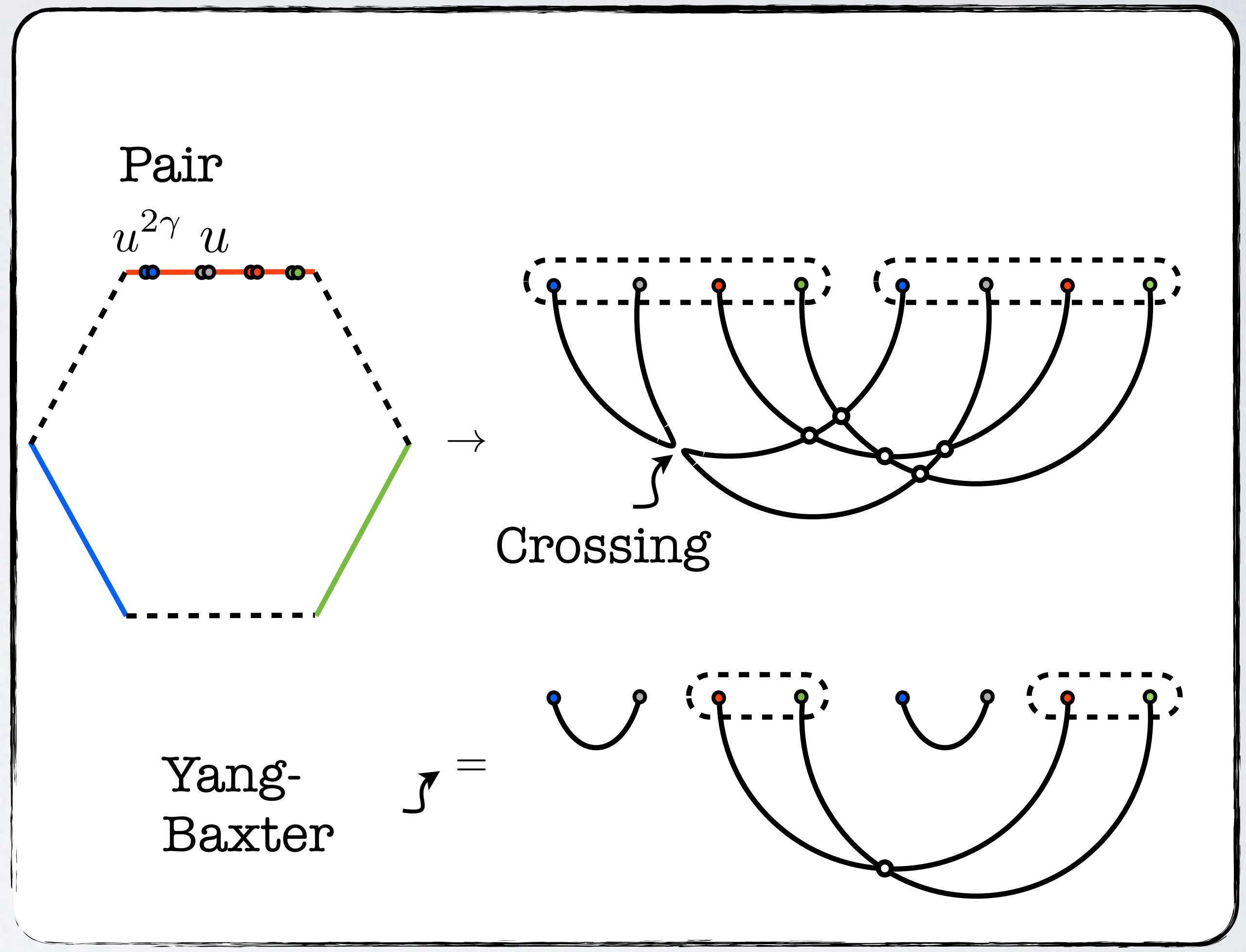


This is Automatic if

$$h_{12}/h_{21} = S^0_{12} = \frac{x_1^+ - x_2^-}{x_1^- - x_2^+}\frac{1-1/x_1^- x_2^+}{1-1/x_1^+ x_2^-}\frac{1}{\sigma_{12}^2}$$

Decoupling Condition

Pair particle-antiparticle with zero energy must decouple



This is Automatic if

$$h(u_1^{2\gamma}, u_2)h(u_1, u_2) = \frac{x_1^- - x_2^-}{x_1^- - x_2^+}\frac{1 - 1/x_1^+ x_2^-}{1 - 1/x_1^+ x_2^+}$$

Solution to Watson and Crossing

(Not unique but conjectured to be the right one :)

$$h_{12} = \frac{x_1^- - x_2^-}{x_1^- - x_2^+} \frac{1 - 1/x_1^- x_2^+}{1 - 1/x_1^+ x_2^+} \frac{1}{\sigma_{12}}$$

Our Hexagons Are Now Fully Determined

All-Loop Asymptotic Formula

Consider 2 BPS Operators and 1 non-BPS Operator

$$\left(\frac{C_{123}^{\bullet\circ\circ}}{C_{123}^{\circ\circ\circ}}\right)^2 = \frac{\prod_{k=1}^{S}\mu(u_k)}{\det\,\partial_{u_i}\phi_j\prod_{i<j}S(u_i,u_j)}\times\mathcal{A}^2$$

Normalization of Spin Chain State

i.e.

BPS

$$\langle\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\rangle = C_{123}^{\bullet\circ\circ}\times\frac{\text{tensor}}{x_{12}^{\Delta_{12}}x_{23}^{\Delta_{23}}x_{31}^{\Delta_{31}}}$$

e.g. $\mathcal{O}_1 = \mathrm{tr}D^S Z^{L_1}$

All-Loop Asymptotic Formula

Consider 2 BPS Operators and 1 non-BPS Operator

$$\left(\frac{C_{123}^{\bullet\circ\circ}}{C_{123}^{\circ\circ\circ}}\right)^2 = \frac{\prod_{k=1}^{S}\mu(u_k)}{\det\,\partial_{u_i}\phi_j \prod_{i<j} S(u_i,u_j)} \times \mathcal{A}^2$$

Hexagon Part

Sum Over Partition of Bethe Roots

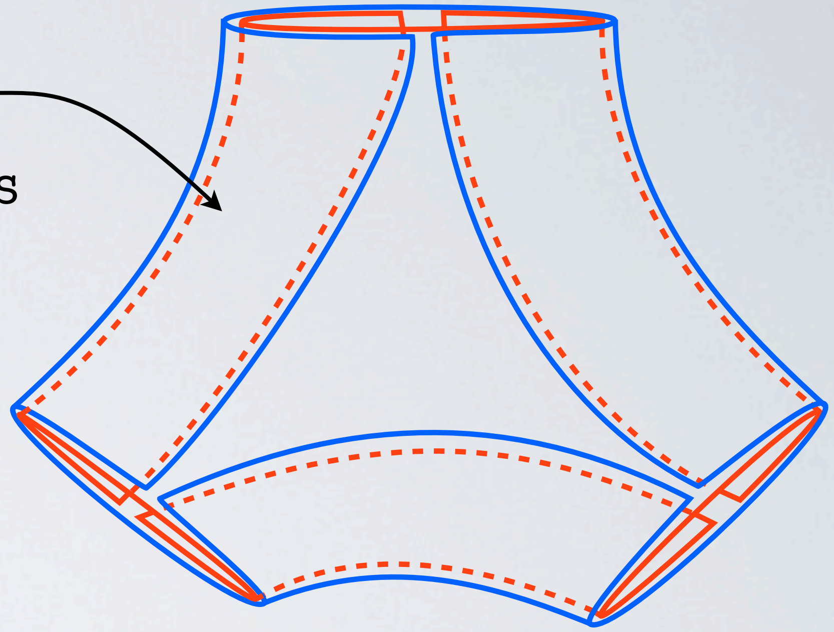
$$\mathcal{A} = \prod_{i<j} h(u_i,u_j) \sum_{\alpha\cup\bar{\alpha}=\mathbf{u}} (-1)^{|\bar{\alpha}|} \prod_{j\in\bar{\alpha}} e^{ip_j\ell} \prod_{i\in\alpha, j\in\bar{\alpha}} \frac{1}{h(u_i,u_j)}$$

Valid to All Loops Up to Finite Size Effects

Include Leading Finite Size Corrections

First Finite Size Effect

Corrections coming from exchange of a single particle in the three mirror channels



$$\mathcal{A} \to \mathcal{A} + \delta\mathcal{A}_{12} + \delta\mathcal{A}_{23} + \delta\mathcal{A}_{31}$$

Asymptotic = vacuum contribution

Integral over Momenta of Exchanged Particle

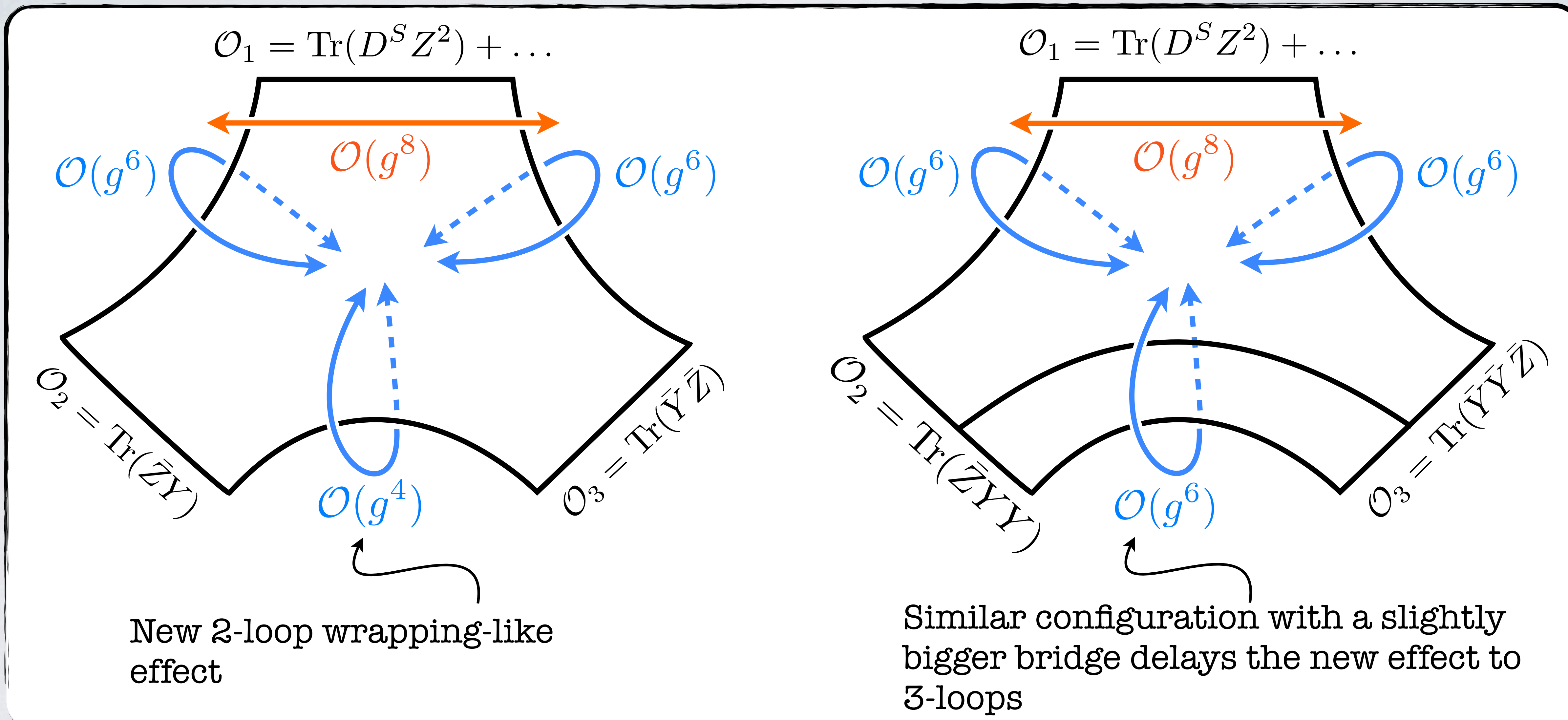
Splitting length

$$\delta\mathcal{A} = \sum_{a\geqslant 1} \int \frac{du}{2\pi} \mu_a^{\gamma}(u) \times \left(\frac{1}{x^{[+a]}x^{[-a]}}\right)^{\ell} \times \mathrm{int}_a(u|\{u_i\})$$

Include hexagon interaction between exchanged mirror particle and magnons on spin chain

Applications / Tests At Weak Coupling

Shortest Probes With Ops of Length 2 or 3



Asymptotic Result the Same for Both :

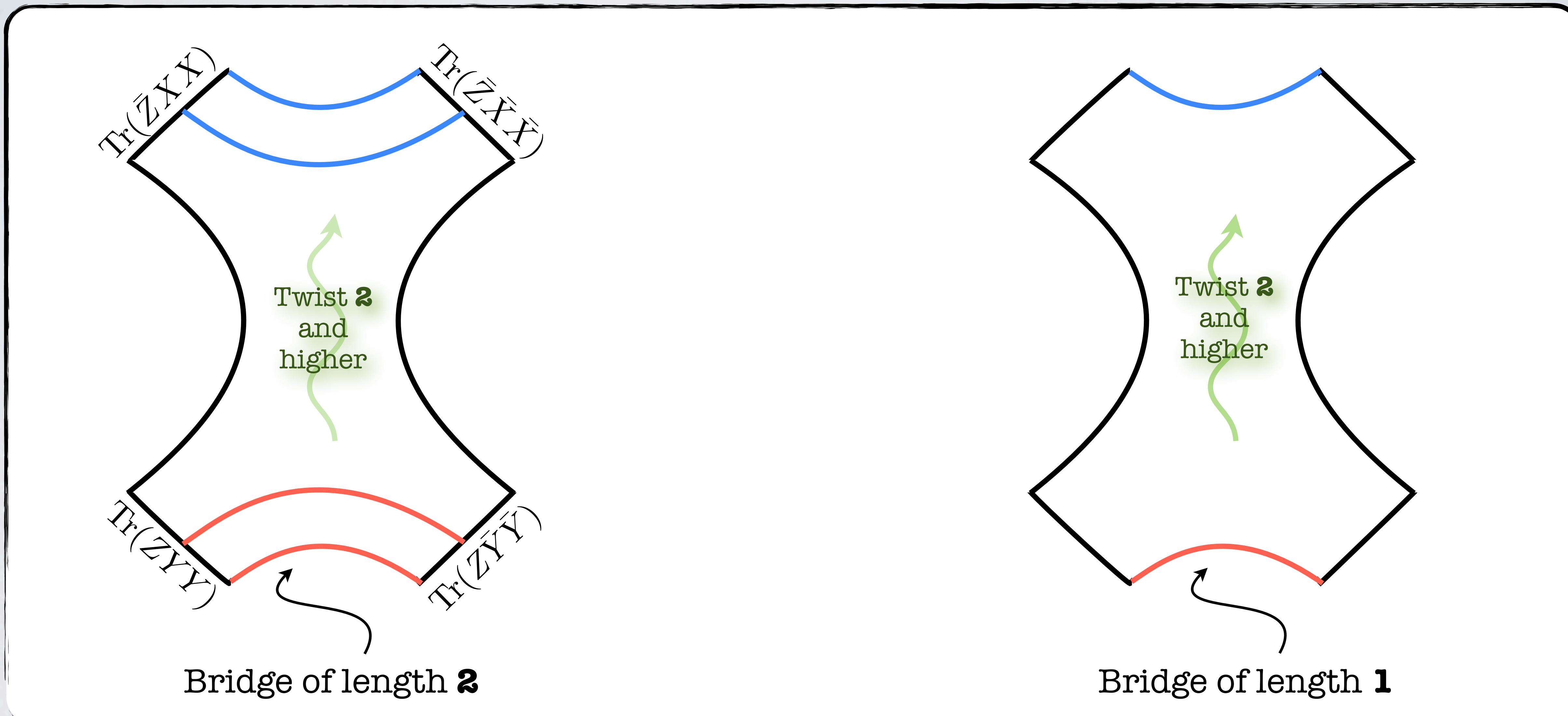
- Valid up to 1-loop on the left

- Valid up to 2-loop on the right

Getting Data From OPE of BPS Correlators

A lot of recent high-loop results

e.g.
[Eden,Heslop,Korchemsky,Sokatchev'11]
[Eden'12],[Chicherin,Sokatchev'14]



We can get what we want playing with external states

Comparison with Data

$$\left(\frac{C_{123}^{\bullet\circ\circ}}{C_{123}^{\circ\circ\circ}}\right)^2$$

Spin	"*Long*" *Bridge i.e. length* $\ell = 2$
2	$\frac{1}{6} - 2g^2 + 28g^4 + \dots$
4	$\frac{1}{70} - \frac{205}{882}g^2 + \frac{36653}{9261}g^4 + \dots$
6	$\frac{1}{924} - \frac{553}{27225}g^2 + \frac{826643623}{2156220000}g^4 + \dots$
8	$\frac{1}{12870} - \frac{14380057}{9018009000}g^2 + \frac{2748342985341731}{8530540523505000}g^4 + \dots$
10	$\frac{1}{184756} - \frac{3313402433}{27991929747600}g^2 + \frac{156422034186391633909}{622011694049832340800000}g^4 + \dots$

Spin	"*Short*" *Bridge i.e. length* $\ell = 1$
2	$\frac{1}{6} - 2g^2 + (28 + 12\zeta_3)g^4 + \dots$
4	$\frac{1}{70} - \frac{205}{882}g^2 + \left(\frac{76393}{18522} + \frac{10}{7}\zeta_3\right)g^4 + \dots$
6	$\frac{1}{924} - \frac{553}{27225}g^2 + \left(\frac{880821373}{2156220000} + \frac{7}{55}\zeta_3\right)g^4 + \dots$
8	$\frac{1}{12870} - \frac{14380057}{9018009000}g^2 + \left(\frac{5944825782678337}{17061081047010000} + \frac{761}{75075}\zeta_3\right)g^4 + \dots$
10	$\frac{1}{184756} - \frac{3313402433}{27991929747600}g^2 + \left(\frac{171050793565932326659}{622011694049832340800000} + \frac{671}{881790}\zeta_3\right)g^4 + \dots$

New 2-loop wrapping-like effect

Perfect Agreement with Hexagon Prediction Including Zeta's Coming From Wrapping

Conclusions

Strategy for Computing Structure Constants :
- Cut Open Pair of Pants into Hexagons
- Glue Hexagons Back Together in the end

Integrable Bootstrap Leads to All-Loop Conjecture for the Hexagons

Outlook

Is the Gluing Prescription Complete?
Can We Resum / Control All Finite Size Effects?

Strong Coupling Resummation Test

String Theory Predicts :

Area Minimal Surface
(classical)

$$\log\ C_{123}^{\bullet\circ\circ} = \frac{\sqrt{\lambda}}{2\pi}\text{Area} + \text{one-loop determinant}$$

Semiclassical Result :

Comes from asymptotic part

$$\frac{\sqrt{\lambda}}{2\pi}\text{Area} = \oint \frac{du}{2\pi}\left[\text{Li}_2(e^{ip_1+ip_2-ip_3}) - \frac{1}{2}\text{Li}_2(e^{2ip_1})\right]$$

$$+\text{finite size contributions}$$

Can we reproduce the infinite sequence of finite size effects predicted by the classical string theory?

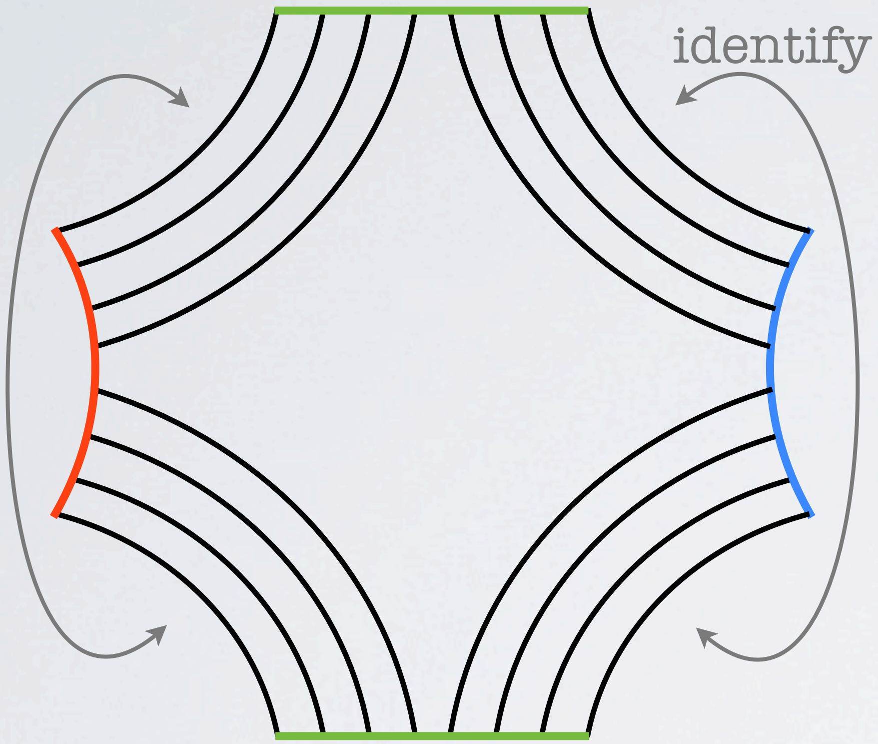
Outlook

Is the Gluing Prescription Complete?
Can We Resum / Control All Finite Size Effects?

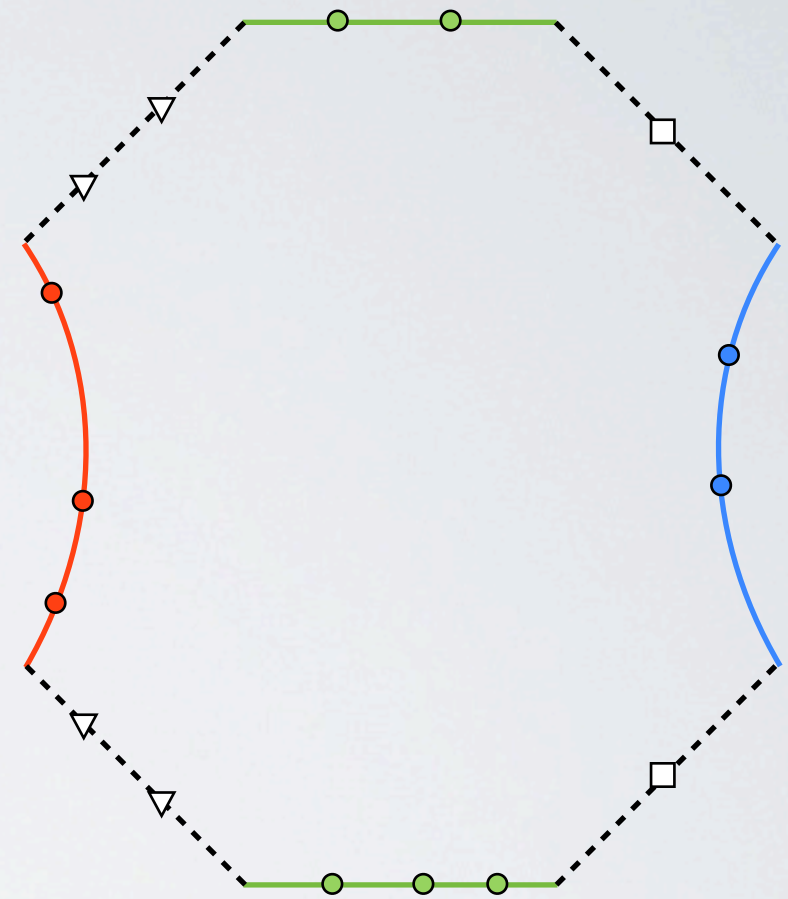
What About Extremal Correlators?

Extremal = Octagon

Should be related to SFT vertex (see Janik's talk)



$$\Leftrightarrow \quad \underset{\substack{\text{(momentum of)}\\ \text{mirror particles}\\ \text{where we glue}\\ \triangledown,\,\square}}{\int} \quad \underset{\substack{\text{partitions}\\ \text{of physical}\\ \text{rapidities of}\\ \text{large operator}\\ \bullet}}{\sum}$$



Can we bootstrap it?

What is the group of residual symmetries?

Further Outlook

Is the Gluing Prescription Complete?
Can We Resum / Control Finite Size Effects?

What About Extremal Correlators?

Embedding Inside Quantum Spectral Curve? (Finite Volume Bootstrap)

Add Boundaries and Describe Open Strings or Mixed Correlators?

Etc.

THANK YOU!