

# Conformal Bootstrap Review

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Strings 2015

# Outline

① Bootstrap Review

② Bootstrap Bounds

③ Bootstrap Future

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# Conformal Bootstrap

- ▶ The conformal bootstrap aims to use basic consistency conditions to map out and solve the space of CFTs
  - ▶ Conformal symmetry:  $SO(D, 2)$
  - ▶ Associativity of the OPE (crossing symmetry)
  - ▶ Unitarity (reflection positivity)
- ▶ Beautiful success story in 2D  
[Ferrara, Gatto, Grillo '73; Polyakov '74; Belavin, Polyakov, Zamolodchikov '83]
- ▶ Great progress in  $D > 2$  starting in 2008  
[Rattazzi, Rychkov, Tonni, Vichi '08; ...]

# Motivations

Many motivations to learn about CFTs in  $D > 2$ :

- ▶ 3D: Condensed Matter and Statistical Systems at Phase Transitions
- ▶ 4D: Scenarios for Physics Beyond the Standard Model
- ▶ Structure of QFT and space of CFTs
- ▶ AdS/CFT Correspondence (precise way to study quantum gravity)

# Operator Product Expansion

Basic tool: Operator Product Expansion

$$\sigma(x)\sigma(0) = \sum_{\mathcal{O} \in \sigma \times \sigma} \lambda_{\mathcal{O}} C_I(x, \partial) \mathcal{O}^I(0)$$

- ▶  $C_I(x, \partial)$  fixed by conformal symmetry and sums up descendants
  - ▶ E.g., for scalars  $C(x, \partial) \sim x^{\Delta - 2\Delta_\sigma} [1 + \frac{1}{2}x^a \partial_a + \dots]$
- ▶ Converges when this expresses a change of basis in radial quantization
- ▶ Lets us expand correlation functions in terms of CFT data:  $\{\Delta, \ell, \lambda\}$

# Single Correlator Bootstrap

Simplest bootstrap involves evaluating scalar 4-point functions with OPE:

$$\begin{aligned}
 & \langle \overline{\sigma(x_1)\sigma(x_2)}\overline{\sigma(x_3)\sigma(x_4)} \rangle \\
 &= \sum_{\mathcal{O} \in \sigma \times \sigma} \lambda_{\mathcal{O}}^2 C_I(x_{12}, \partial_2) C_J(x_{34}, \partial_4) \langle \mathcal{O}^I(x_2) \mathcal{O}^J(x_4) \rangle \\
 &\equiv \frac{1}{x_{12}^{2\Delta_\sigma} x_{34}^{2\Delta_\sigma}} \sum_{\mathcal{O} \in \sigma \times \sigma} \lambda_{\mathcal{O}}^2 g_{\Delta,\ell}(u, v)
 \end{aligned}$$

- ▶  $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$ ,  $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$  conformally-invariant cross ratios
- ▶  $g_{\Delta,\ell}(u, v)$  conformal block, labeled by  $\Delta = \dim(\mathcal{O})$  and  $\ell = \text{spin}(\mathcal{O})$

# Scalar Conformal Blocks

Explicit formulas in even  $D$  [Dolan, Osborn '00; '03]:

$$\begin{aligned} g_{\Delta,\ell}^{2D}(u,v) &= k_{\Delta+\ell}(z)k_{\Delta-\ell}(\bar{z}) + z \leftrightarrow \bar{z} \\ g_{\Delta,\ell}^{4D}(u,v) &= \frac{z\bar{z}}{z-\bar{z}}[k_{\Delta+\ell}(z)k_{\Delta-\ell-2}(\bar{z}) - z \leftrightarrow \bar{z}] \\ k_\beta(x) &= x^{\beta/2} {}_2F_1(\beta/2, \beta/2, \beta; x) \end{aligned}$$

where  $u = z\bar{z}$  and  $v = (1-z)(1-\bar{z})$

- ▶ Conformal blocks are eigenfunctions of  $SO(D, 2)$  Casimir (like  $Y_\ell^m$ 's)
- ▶ Outside of even  $D$ , can be computed recursively to arbitrary precision  
[El-Showk, Paulos, DP, Rychkov, Simmons-Duffin, Vichi; Kos, DP, Simmons-Duffin '13; '14]

# Crossing Symmetry

$\langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4) \rangle$  is symmetric under permutations of  $x_i$ :

- ▶ Switching  $x_1 \leftrightarrow x_3$  gives the crossing relation ( $S = T$ )

$$\sum \begin{array}{c} 1 \\ \diagup \quad \diagdown \\ \text{\O} \\ \diagdown \quad \diagup \\ 2 \qquad 3 \end{array} = \sum \begin{array}{c} 1 \\ \diagup \quad \diagdown \\ \text{\O} \\ \diagup \quad \diagdown \\ 2 \qquad 3 \end{array}$$

$$v^{\Delta_\phi} \sum_{\mathcal{O} \in \sigma \times \sigma} \lambda_{\mathcal{O}}^2 g_{\Delta, \ell}(u, v) = u^{\Delta_\phi} \sum_{\mathcal{O} \in \sigma \times \sigma} \lambda_{\mathcal{O}}^2 g_{\Delta, \ell}(v, u)$$

- ▶ Obtain similar sum rules from every 4-point function  $\langle \mathcal{O}^I \mathcal{O}^J \mathcal{O}^K \mathcal{O}^L \rangle$

# Conformal Bootstrap Steps

The conformal bootstrap has three steps:

1. Write down the crossing symmetry conditions
2. Study how they constrain the CFT data:  $\{\Delta, \ell, \lambda\}$ 
  - ▶ Analytical Approaches
  - ▶ Numerical Approaches
3. Interpret the constraints

# Setting up the Bootstrap

In general, to formulate the sum rules for a given correlator, one must:

- ▶ Classify all operators and tensor structures appearing in  $\mathcal{O}^I \times \mathcal{O}^J$  OPE  
→ Embedding Formalism [Dirac '36; ...; Weinberg '10; Giombi, Prakash, Yin '11;  
Costa, Penedones, DP, Rychkov '11; Simmons-Duffin '12; Costa, Hansen '14; Elkhidir,  
Karateev, Serone '14; Iliesiu, Kos, DP, Pufu, Simmons-Duffin, Yacoby, to appear]

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- ▶ Compute conformal blocks for each combination of tensor structures  
→ Spinning Blocks [Costa, Penedones, DP, Rychkov '11; Echeverri, Elkhidir, Karateev, Serone '15; Iliesiu, Kos, DP, Pufu, Simmons-Duffin, Yacoby, to appear]

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- ▶ Impose constraints from additional global (super)symmetries  
→ Superconformal Blocks [Dolan, Osborn '01; '04; Dolan, Gallot, Sokatchev '04; DP, Simmons-Duffin '10; Fortin, Intriligator, Stergiou '11; Berkooz, Yacoby, Zait '14; Fitzpatrick, Kaplan, Khandker, Li, DP, Simmons-Duffin '14; Khandker, Li, DP, Simmons-Duffin '14; Chester, Lee, Pufu, Yacoby '14; Bobev, El-Showk, Mazac, Paulos '15]

# Studying the Bootstrap

Analytical Approaches:

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- ▶ Large  $N \rightarrow$  Perturb around known  $N = \infty$  solutions  
[Vasiliev, Pismak, Khonkonen '81; ...; Heemskerk, Penedones, Polchinski, Sully '09;  
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- ▶ Lightcone limit ( $u, v \rightarrow 0$ ) → Large spin asymptotics  
[Fitzpatrick, Kaplan, DP, Simmons-Duffin '12; Komargodski, Zhiboedov '12; Alday, Bissi  
'13; Fitzpatrick, Kaplan, Walters '14; Vos '14; Kaviraj, Sen, Sinha '15; '15; Alday, Bissi,  
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- ▶ Find solvable sectors of highly supersymmetric theories  
[Beem, Lemos, Peelaers, Rastelli, van Rees '13; Beem Rastelli, van Rees '14; Beem,  
Peelaers, Rastelli, van Rees '14; Lemos, Peelaers '14; Chester, Lee, Pufu, Yacoby '14]

# Studying the Bootstrap

## Numerical Approaches:

- ▶ Rigorously exclude assumptions about spectrum by applying functionals  
[Rattazzi, Rychkov, Tonni, Vichi '08; Rychkov, Vichi '09; Caracciolo, Rychkov '09; Poland, Simmons-Duffin '10; Rattazzi, Rychkov, Vichi '10; '10; Vichi '11; Poland, Simmons-Duffin, Vichi '11; Rychkov '11; Liendo, Rastelli, van Rees '12; El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi '12; '13; Beem, Rastelli, van Rees '13; Kos, Poland, Simmons-Duffin '13; '14; Alday, Bissi '13; '14; Gaiotto, Mazac, Paulos '13; Bashkirov '13; Berkooz, Yacoby, Zait '14; Nakayama, Ohtsuki '14; '14; '14; Chester, Lee, Pufu, Yacoby '14; '14; Caracciolo, Echeverri, von Harling, Serone '14; Paulos '14; Bae, Rey '14; Beem, Lemos, Liendo, Rastelli, van Rees '14; Chester, Pufu, Yacoby '14; Bobev, El-Showk, Mazac, Paulos '15; '15; Kos, Poland, Simmons-Duffin, Vichi '15; Iliesiu, Kos, Poland, Pufu, Simmons-Duffin, Yacoby, '15]

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- ▶ Reconstruct approximate numerical solutions to sum rule  
[El-Showk, Paulos '12; Gliozzi '13; El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi '14; Gliozzi, Rago '14; Gliozzi, Liendo, Meineri, Rago '15]

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# How Does Crossing Symmetry Lead to CFT Bounds?

Crossing relation for  $\langle\sigma\sigma\sigma\sigma\rangle$ :

$$0 = F_{0,0}(u, v) + \underbrace{\sum \lambda_{\mathcal{O}}^2 F_{\Delta,\ell}(u, v)}_{\text{unit op.}} + \underbrace{\sum}_{\text{everything else}}$$

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$$0 = F_{0,0}(u, v) + \underbrace{\lambda_{\mathcal{O}}^2 F_{\Delta,\ell}(u, v)}_{\text{unit op.}} + \underbrace{\sum_{\ell' > 0} \lambda_{\mathcal{O},\ell'}^2 F_{\Delta,\ell'}(u, v)}_{\text{everything else}}$$

- ▶ Make an assumption: e.g., all scalars have dimension  $\Delta > \Delta_{\min}$
- ▶ Search for a linear functional  $\alpha = \sum a_{mn} \partial_z^m \partial_{\bar{z}}^n \Big|_{z=\bar{z}=1/2}$  such that

$$\alpha(F_{0,0}) = 1, \quad \text{and}$$

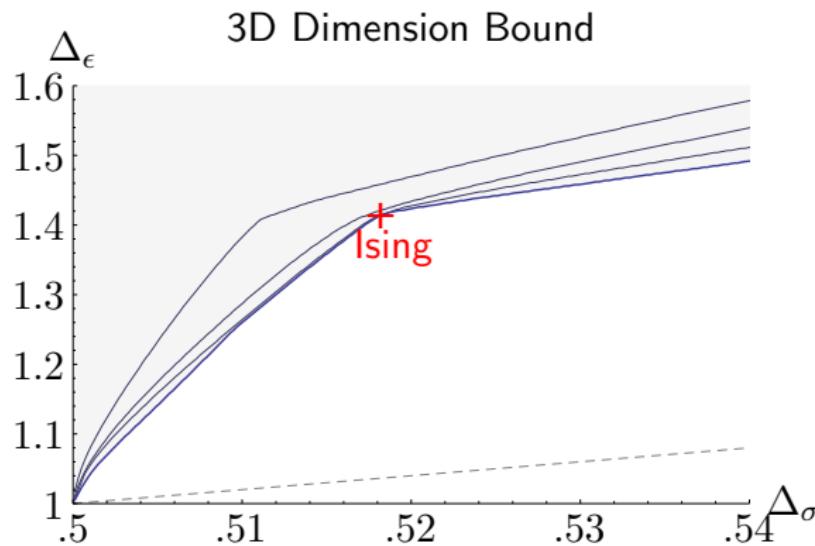
$$\alpha(F_{\Delta,\ell}) \geq 0, \quad \text{for all other } \mathcal{O} \in \sigma \times \sigma.$$

- ▶ If you find one, the assumption is ruled out!

# CFT Bounds

- ▶ Solved with linear or (more generally) semidefinite programming  
[Rattazzi, Rychkov, Tonni, Vichi '08; DP, Simmons-Duffin, Vichi '11]
  - ▶ State of the art: SDPB [Simmons-Duffin '15]
- ▶ Many nice results between 2008-2015 following this approach in (2-6)D, as well as generalizations to SUSY and other global symmetries
- ▶ Here I will start with the 3D story and show you some recent results...

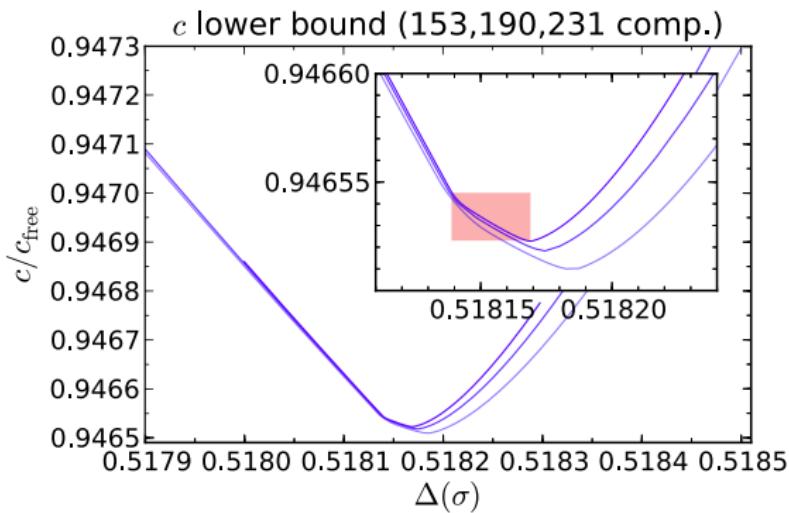
# 3D Dimension Bounds



[El-Showk, Paulos, DP, Rychkov, Simmons-Duffin, Vichi, '12]

- ▶ Bound on leading scalar in  $\sigma \times \sigma \sim 1 + \epsilon + \dots$
- ▶ 3D Ising dimensions from Monte Carlo:  
 $\Delta_\sigma \simeq 0.51813(5)$ ,  $\Delta_\epsilon \simeq 1.41275(25)$  [Hasenbusch '10]

# $c$ -minimization and Spectrum Extraction

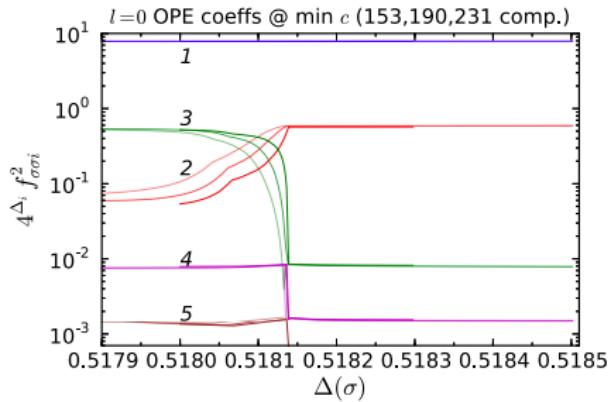
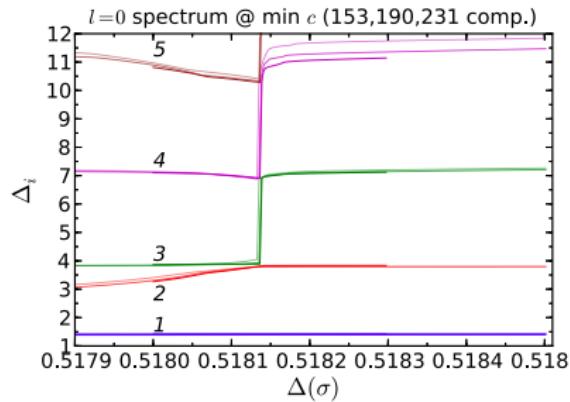


[El-Showk, Paulos, DP, Rychkov, Simmons-Duffin, Vichi, '14]

- ▶ Under the conjecture that the central charge  $\langle TT \rangle \propto c$  is minimized, a precise spectrum in  $\sigma \times \sigma \sim 1 + \epsilon + \epsilon' + \dots$  can be extracted:

$$\Delta_\sigma \simeq 0.518154(15), \Delta_\epsilon \simeq 1.41267(13), \Delta_{\epsilon'} = 3.8303(18), \dots$$

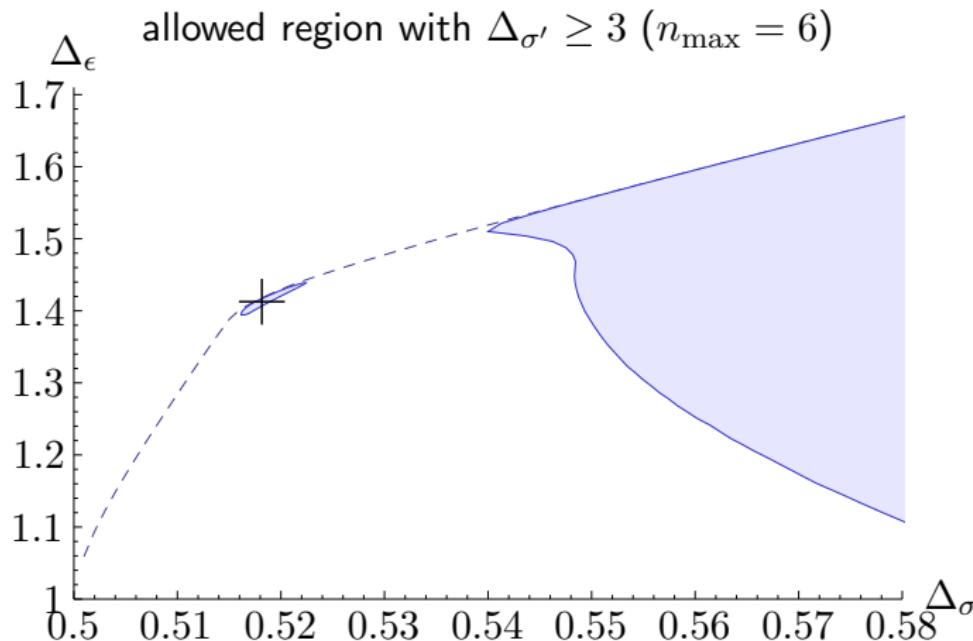
# $c$ -minimization and Spectrum Extraction



[El-Showk, Paulos, DP, Rychkov, Simmons-Duffin, Vichi, '14]

- ▶ “Kink”  $\leftrightarrow$  operators merge and disappear from spectrum!
- ▶ Reminiscent of null states in 2D or equations of motion in  $(4 - \epsilon)D$   
 $\rightarrow$  Non-perturbative equation of motion?
- ▶ E.g., in  $\phi^4$  theory, expect  $\partial^2\phi \sim \phi^3 \rightarrow$  gap in  $\mathbb{Z}_2$ -odd spectrum...

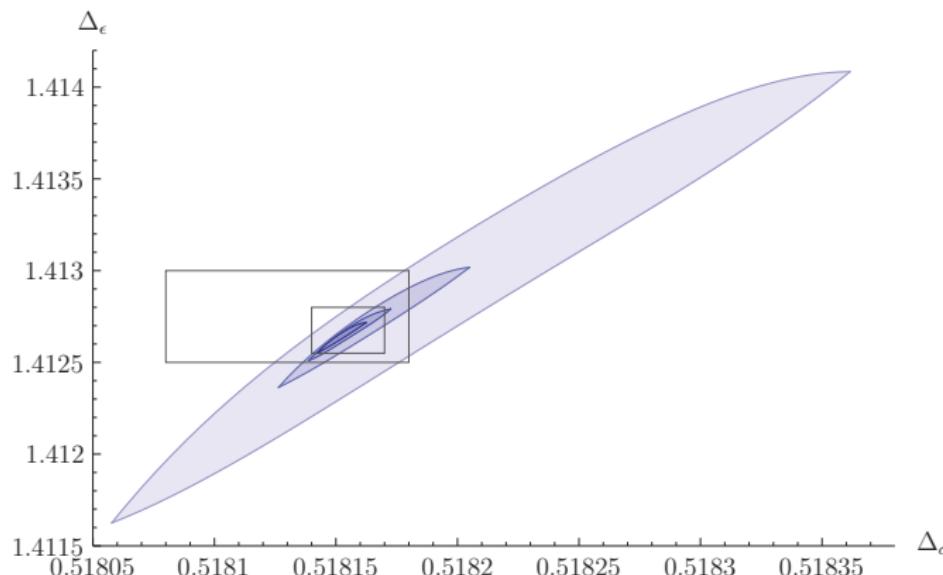
# Mixed Correlator Bounds



[Kos, DP, Simmons-Duffin '14]

- ▶ Combining constraints from  $\langle \sigma\sigma\sigma\sigma \rangle$ ,  $\langle \sigma\sigma\epsilon\epsilon \rangle$ ,  $\langle \epsilon\epsilon\epsilon\epsilon \rangle$ , we can impose that  $\sigma$  is only relevant  $\mathbb{Z}_2$ -odd scalar in  $\sigma \times \epsilon$ , yielding a closed island!

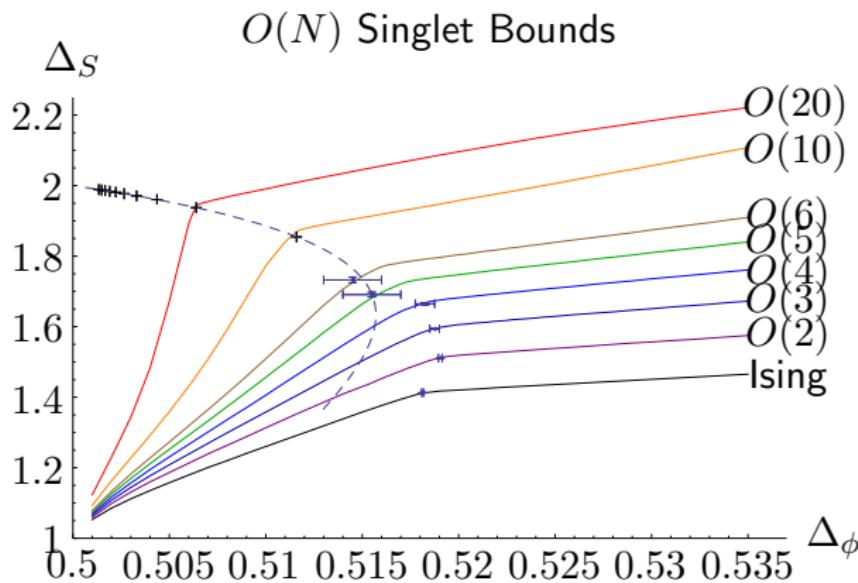
# Mixed Correlator Bounds



[Kos, DP, Simmons-Duffin '14; Simmons-Duffin '15]

- ▶ Pushing farther (with help of SDPB), region keeps shrinking!  
 $\{\Delta_\sigma, \Delta_\epsilon\} = \{0.518151(6), 1.41264(6)\}$

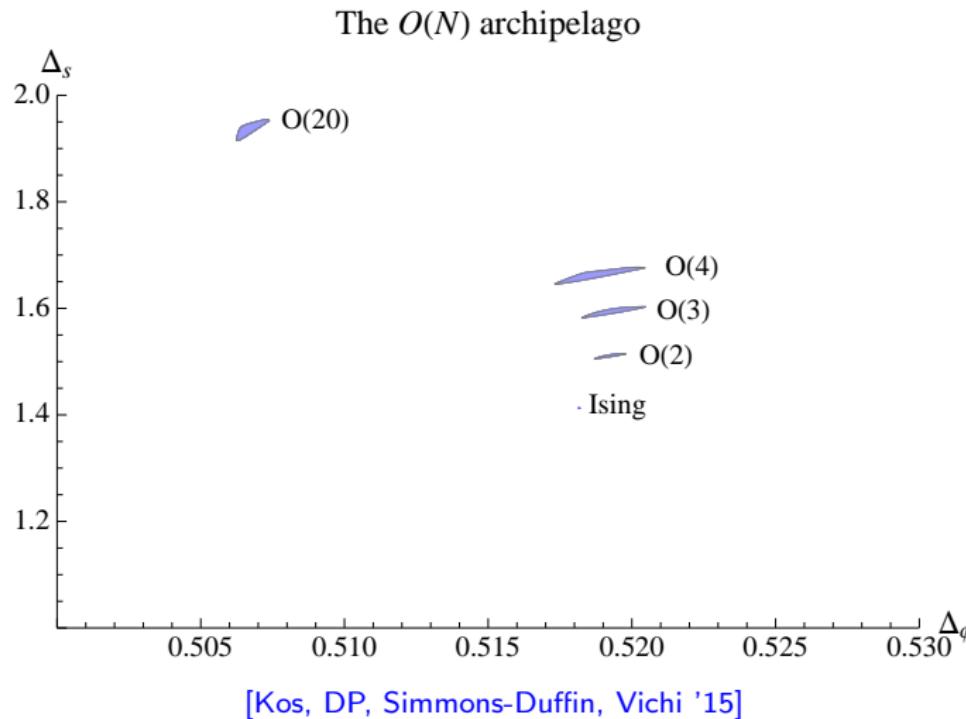
# 3D $O(N)$ Bounds



[Kos, DP, Simmons-Duffin '13]

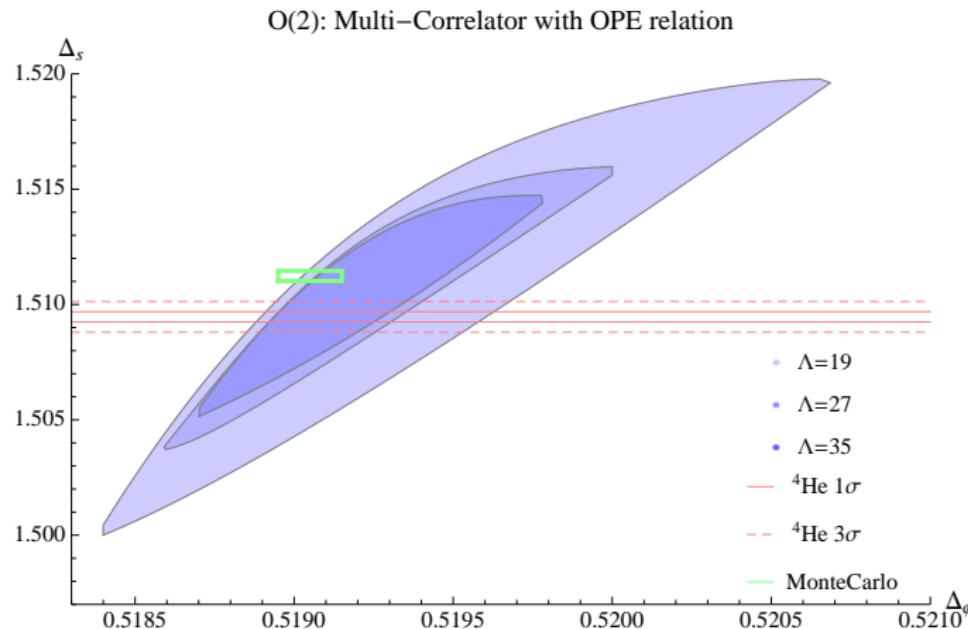
- Extension to  $\langle \phi_i \phi_j \phi_k \phi_l \rangle$ , where  $\phi_i$  is  $O(N)$  vector
- OPE  $\phi_i \times \phi_j \sim 1 + S + T_{ij} + \dots$  contains singlets and two-index tensors

# Mixed $O(N)$ Correlators



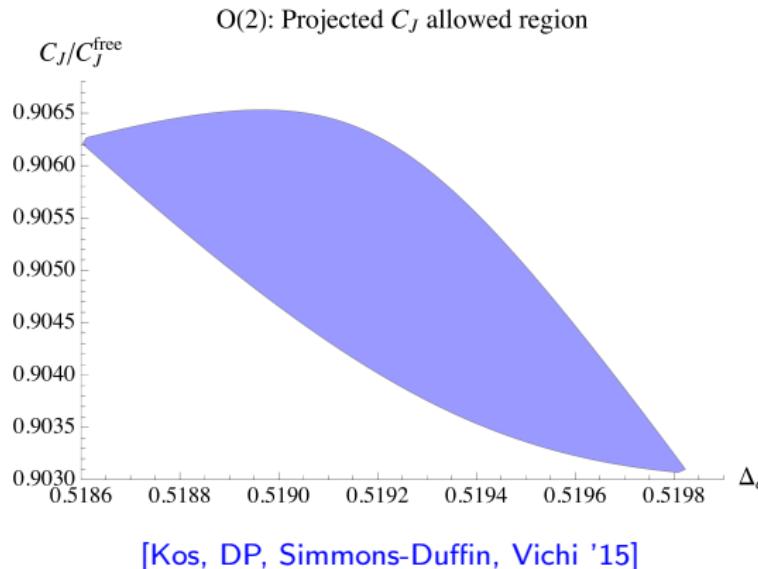
- ▶ Assumes a single relevant  $O(N)$  fundamental  $\phi_i$  and singlet  $s$

# $O(2)$ Zoom



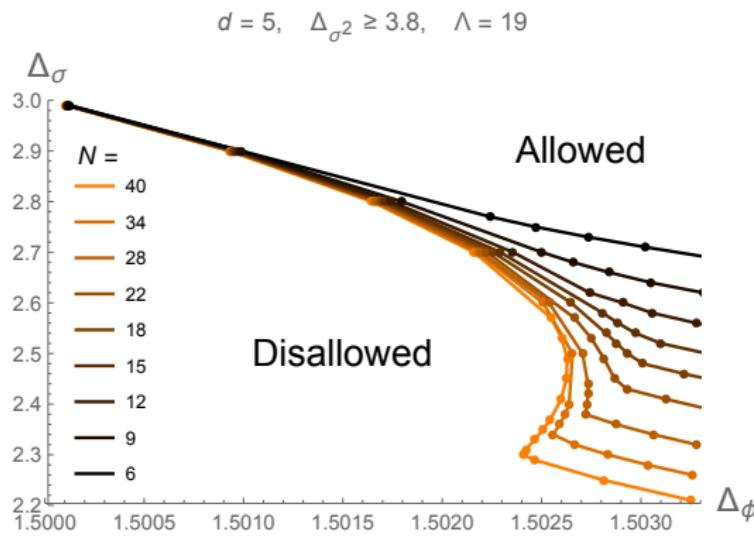
[Kos, DP, Simmons-Duffin, Vichi '15]

# $O(2)$ Current Central Charge



- ▶ Rigorous determination of  $\langle JJ \rangle \propto C_J$ , giving high-frequency conductivity  $2\pi\sigma_\infty = C_J/32 = 0.3554(6)$  in  $O(2)$  superconductors
- ▶ Quantum Monte Carlo:  $0.355(5)$  [Gazit, Podolsky, Auerbach '14]  
(statistical errors only)  $0.3605(3)$  [Katz, Sachdev, Sørensen, Witczak-Krempa '14]

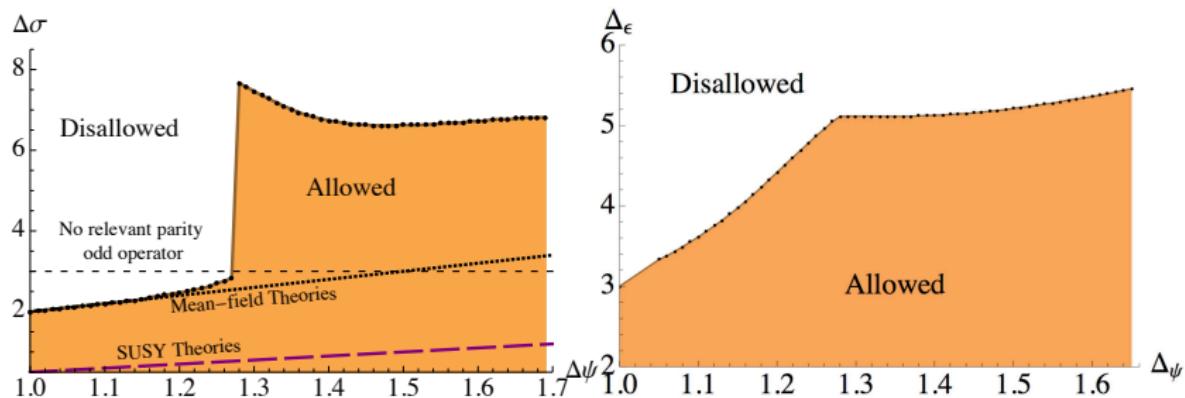
# 5D $O(N)$ Bounds



[Chester, Pufu, Yacoby '14]

- ▶ Explorations of 5D  $O(N)$  models (also [Nakayama, Ohtsuki '14; Bae, Rey '14])
- ▶ Disappearance of kink  $\leftrightarrow$  bottom of conformal window?

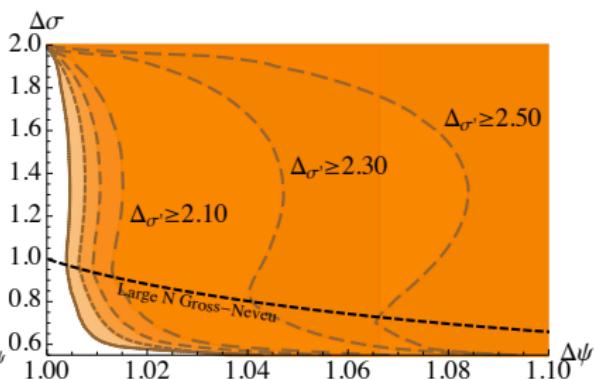
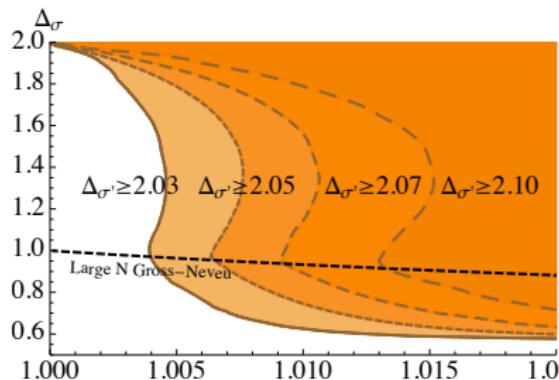
# 3D Fermion Bootstrap



[Iliesiu, Kos, DP, Pufu, Simmons-Duffin, Yacoby, to appear]

- ▶ Bootstrap for  $\langle \psi\psi\psi\psi \rangle$ , where  $\psi$  is a Majorana fermion in a 3D CFT
- ▶ Bounds on leading parity-odd/even scalars in  $\psi \times \psi \sim \mathbb{1} + \sigma + \epsilon$
- ▶ Features at  $\Delta_\psi \sim 1.28$  may describe 3D CFT with no relevant scalars!

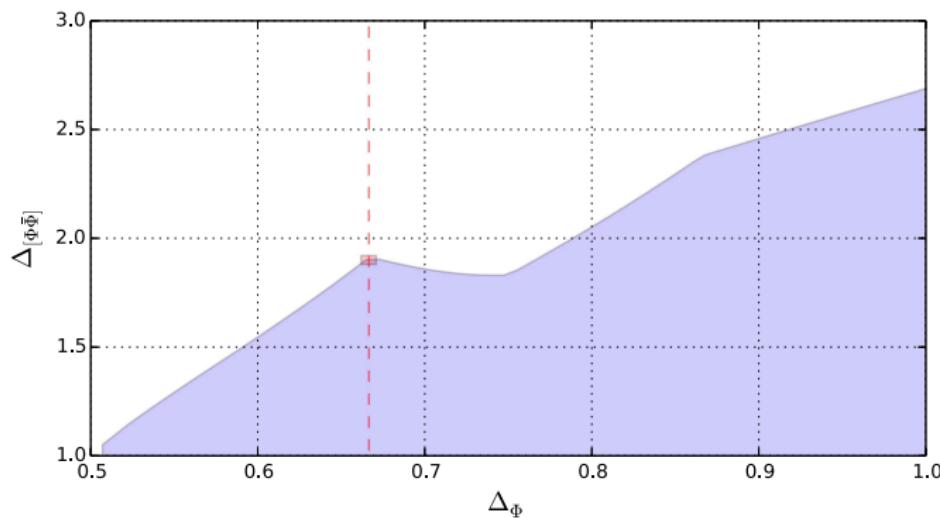
# 3D Fermion Bootstrap



[Iliesiu, Kos, DP, Pufu, Simmons-Duffin, Yacoby, to appear]

- ▶ Scanning over  $\Delta\sigma'$  reveals features that seem to coincide with Gross-Neveu(-Yukawa) models ( $\mathcal{L} \sim \sigma \bar{\psi}_i \psi_i + \sigma^4$ ) with  $N$  fermions
- ▶ Taking  $N \rightarrow 1$  should reveal  $\mathcal{N} = 1$  supersymmetric Ising model

# 3D $\mathcal{N} = 2$ Supersymmetry

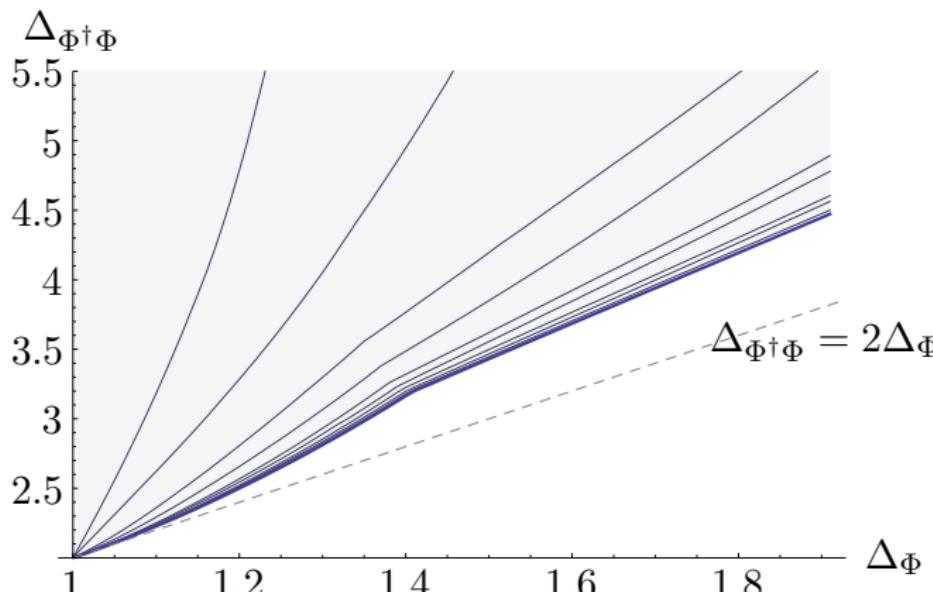


[Bobev, El-Showk, Mazac, Paulos '15]

Bound on leading unprotected scalar in  $\Phi^\dagger \times \Phi$  reveals 3 kinks:

- ▶  $\Delta_\Phi = 2/3$ :  $\mathcal{N} = 2$  Ising,  $W = \Phi^3$  ( $\Phi^2$  disappears)
- ▶  $\Delta_\Phi = .75/.86$ : Interpretation not yet clear ( $\Phi^2$  reappears/disappears)

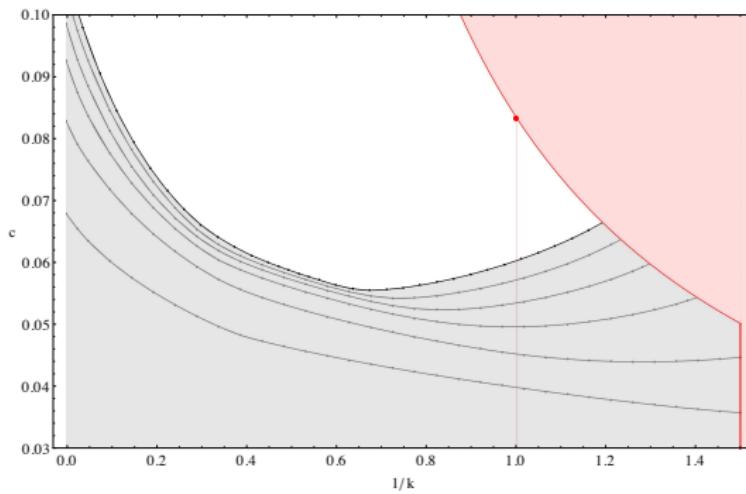
# 4D $\mathcal{N} = 1$ Supersymmetry



[DP, Simmons-Duffin, Vichi '11; DP, Stergiou, in progress]

- ▶ 3rd kink interpolates to older mysterious 4D  $\mathcal{N} = 1$  kink
- ▶ Is there a 4D  $\mathcal{N} = 1$  SCFT with  $\Phi^2$  absent,  $\Delta_\Phi \sim 1.41$ , and small central charge ( $c \sim 2c_{\text{free chiral}}$ )?

# 4D $\mathcal{N} = 2$ Supersymmetry

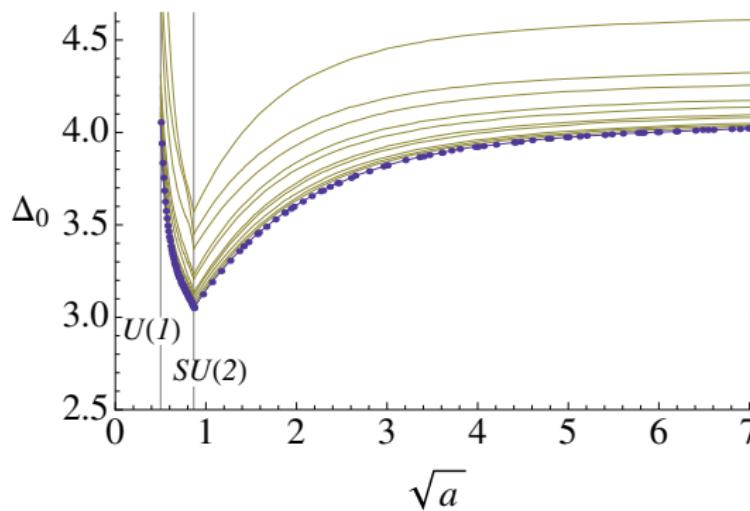


[Beem, Lemos, Liendo, Rastelli, van Rees '14]

Central charge bounds  $\{c, 1/k\}$  in  $SU(2)$  theories from flavor-current 4pf

- ▶ Upper bound  $k \geq \frac{16c}{1+4c}$  from unitarity + mapping to 2D chiral algebra
- ▶ Lower bound consistent with theory of free hypermultiplets  
(other known CFTs have  $c \geq 1/2$ )

# 4D $\mathcal{N} = 4$ Supersymmetry

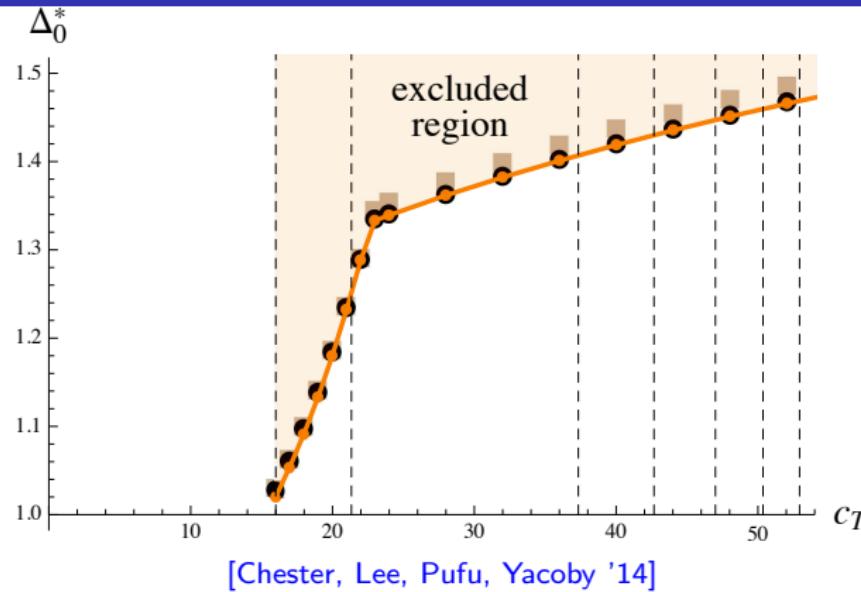


[Beem, Rastelli, van Rees '13]

Bound on dimension of long multiplet  $\Delta_0$  appearing in  $\mathcal{T}_{20'} \times \mathcal{T}_{20'}$

- ▶ Conjectured to be saturated by one of self-dual points  $\tau = i, e^{i\pi/3}$
- ▶ Similar bounds in 6D (2,0) theories [Beem, Lemos, Rastelli, van Rees, to appear]

# 3D $\mathcal{N} = 8$ Supersymmetry



Bound on dimension of long multiplet  $\Delta_0^*$  appearing in  $\mathcal{T}_{35_c} \times \mathcal{T}_{35_c}$

- ▶  $c_T = 16$ :  $\Delta_0^* \rightarrow 1$  [U(1) SYM  $\rightarrow$   $U(1)_1 \times U(1)_{-1}$  ABJM]
- ▶  $c_T = 64/3$ : Near kink [O(3) SYM  $\rightarrow$   $U(2)_2 \times U(1)_{-2}$  ABJ]
- ▶  $c_T \rightarrow \infty$ :  $\Delta_0^* \rightarrow 2$  [U( $\infty$ ) SYM  $\rightarrow$   $U(\infty)_1 \times U(\infty)_{-1}$  ABJM]

# Outline

① Bootstrap Review

② Bootstrap Bounds

③ Bootstrap Future

# Bootstrap Future

- ▶ Pursue mixed correlator analyses → find more islands!
- ▶ Map out landscape of 3D CFTs with fermions + scalars
- ▶ Understand how to isolate conformal gauge theories
  - ▶ 3D QED/QCD/Chern-Simons + matter
  - ▶ 4D QCD/SQCD in conformal window
- ▶ Clarify space of CFTs in  $D = 5, 6$ , and  $D > 6$
- ▶ Conformal manifolds in the bootstrap

# Bootstrap Future

- ▶ Compute missing conformal/superconformal blocks
  - ▶ General Lorentz representations (e.g.,  $\Psi \times \Psi \sim F^{\mu\nu}$ )
  - ▶ General non-chiral/mixed 4-point functions in SCFTs
  - ▶ 4D  $\mathcal{N} = 2$  stress-tensor 4-point functions
  - ▶ ...
- ▶ Study bootstrap for current and stress-tensor 4-point functions
- ▶ Improve algorithms for high-precision semidefinite programming  
(or find an even better approach!)
- ▶ Better understand gaps  $\leftrightarrow$  non-perturbative equations of motion

# Bootstrap Future



- ▶ I'm hopeful that we'll create a full map of the space of CFTs with a small number of relevant operators...we may even discover a new world!