

# String spreading and S-matrix `data'

with M. Dodelson:

arXiv:1504.05536-7 and work in progress

\*light cone spreading calculations  
and Black Hole dynamics

\*4,5, and 6 point string amplitudes  
in flat spacetime

+ extensive discussions with S. Giddings,  
D. Marolf, G. Veneziano, T. Bachlechner &  
L. McAllister, D. Stanford & S. Shenker; S.  
Caron-Huot; ...

'14 (D-brane production) E.S. (+J.  
Polchinski); Puhm, Rojas, Ugajin

Question: What is the leading breakdown of effective field theory at a horizon, in string theory?

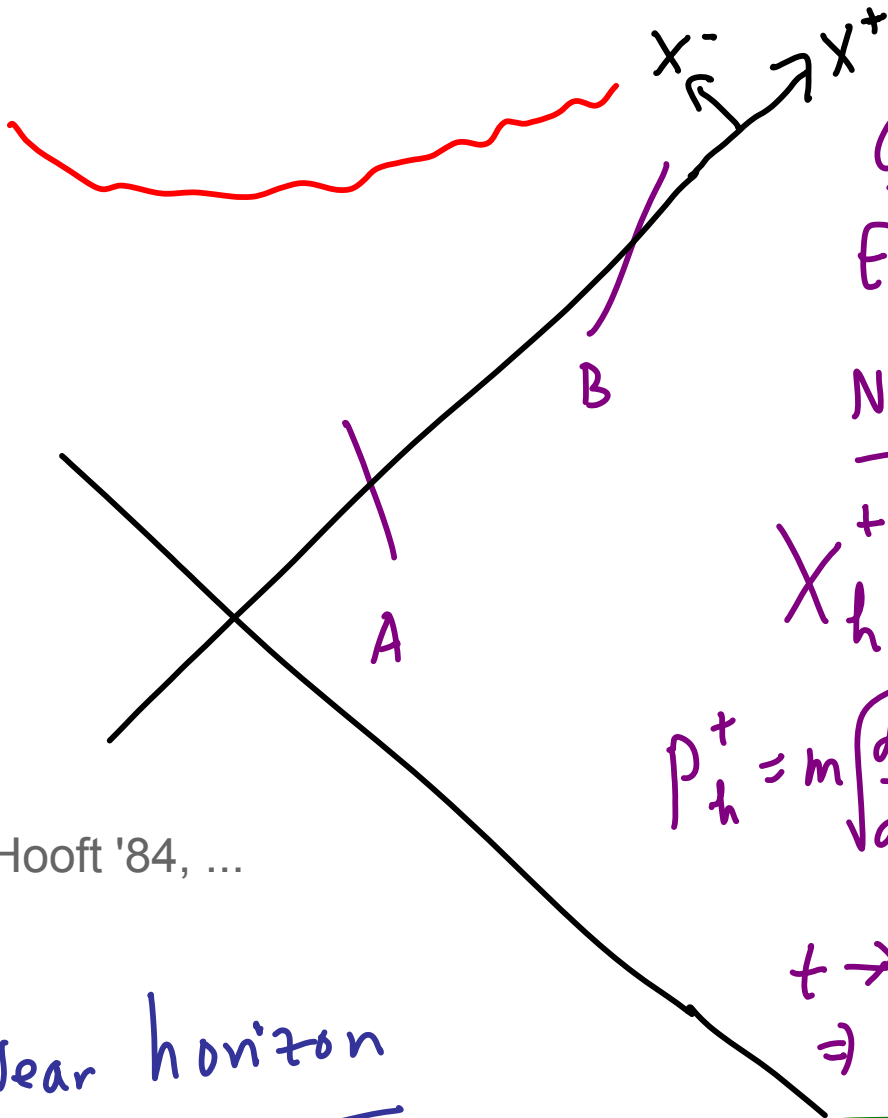
Naive estimate: EFT valid for small  $\alpha'R$  (and small tidal forces).

Despite weak curvature, over long times a large energy can develop.

As we will see: this, combined with concrete string spreading dynamics, calls into question the above estimate. The string-

theoretic modification of GR this suggests is consistent with observations, and potentially important for black hole physics, real and thought-experimental.

$$ds^2 = -\frac{2r_s}{r} e^{1-\frac{r}{r_s}} dX^+ dX^- + r^2 d\Omega^2$$



Outside :  
E, m fixed

Near Horizon :

$$X_h^+ = 2r_s \sqrt{e} \frac{E}{m} e^\eta$$

$$p_h^+ = m \left| \frac{dX^+}{dX^-} \right|_h = m e^\eta$$

$$t \rightarrow t + \Delta t \Rightarrow \eta \rightarrow \eta + \frac{\Delta t}{2r_s}$$

Near horizon

$$\cdot S \sim 2 p_{B,h}^+ p_{A,h}^- \sim e^{\frac{\Delta t}{2r_s}} m^2$$

$$\cdot X_B^+ - X_A^+ \propto p_B^+ \propto e^{\frac{\Delta t}{2r_s}}$$

'tHooft '84, ...

Near horizon: huge Energy, but  
separated along  $X^+$ .

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## String Spreading

- Susskind '94

- Brown Polchinski

Strassler Tan '06

Light Cone gauge  $X^- \sim p^- \tau$ ,

Constraint determines  $X^+$  in terms of  $X^\perp$

$$\langle 4 | (X_\perp - x_\perp)^2 | \psi \rangle = \sum_n^{n_{\max}} \frac{1}{n} = \log \frac{n_{\max}}{n_0} + O\left(\frac{1}{n_{\max}}\right)$$

$$\langle 4 | (X^+ - x^+)^2 | \psi \rangle \approx \frac{1}{(p^-)^2} \sum_n^{n_{\max}} n \approx \frac{n_{\max}^2}{(p^-)^2}$$

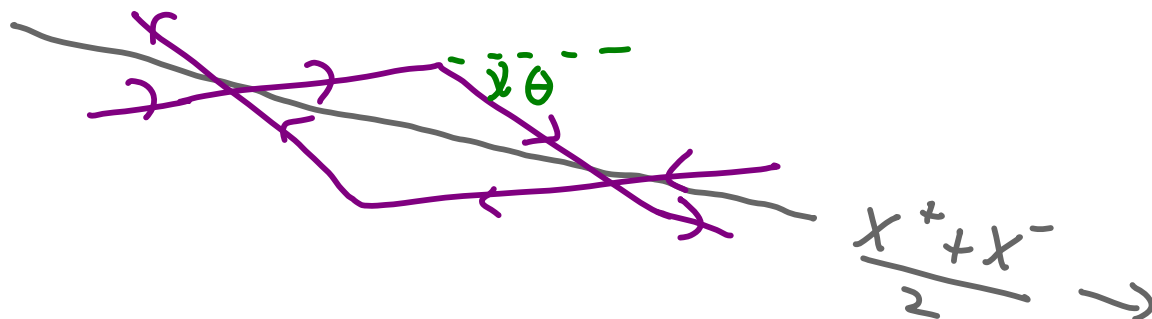
$n_{\max} \leftrightarrow$  light cone time resolution

- Apparent asymmetry between  $X_{\perp}$  and  $X^{\pm}$  directions?

No: the RMS longitudinal spreading is detectable for

$X^{\pm} \approx$  direction of relative motion

More precisely: Brick wall frame



respects time-reversal symmetry

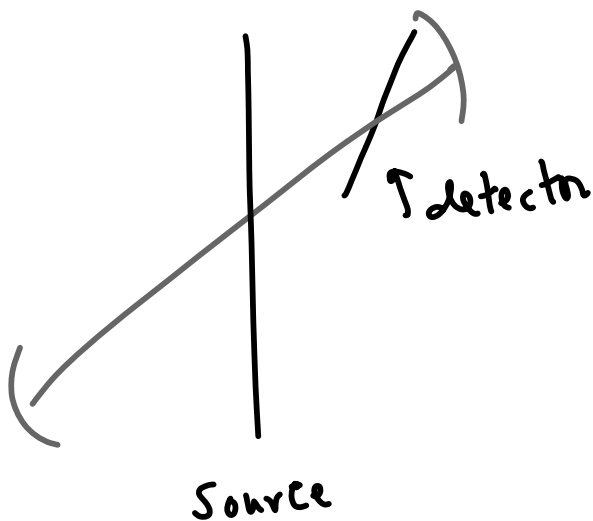
# Light cone time resolution:

$$\Delta X^- \sim \frac{1}{P_{\text{detector}}^+}$$

$$N_{\text{max}} \leq \frac{q'}{\Delta T}$$

$$Y \sim \frac{X^-}{P_s^-}$$

$$\Rightarrow N_{\text{max}} \leq \underbrace{P_s^- P_d^+}_{S} q'$$



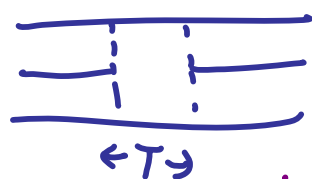
Dependence on detector trajectory?

Measurement degrades as  $\uparrow P_{\perp}$ :

conservatively  $N_{\text{max}} \sim \frac{P_s^- P_d^+}{P_{\perp}^2 + m^2} \xrightarrow{(m \approx 0)} \frac{S}{-t}$

(detector moves  $\sim \Delta X^+$  in time  $\sim \Delta X^-$ )

★ This physical idea is confirmed explicitly in BPST '06 calculation of 4-point Regge amplitude in light-cone gauge.



in brick wall frame:  $p_{\perp r} \sim \frac{k_{\perp}}{z}$ ,  $m=0$

$$\sum_{n=1}^{\infty} \frac{1}{n + \frac{n^2 T}{2s' p_s^-}} = \sum_{n=1}^{n_{\max}} \frac{1}{n} = \log \frac{n_{\max}}{n_0} + O\left(\frac{1}{n_{\max}}\right)$$

$\underbrace{\hspace{10em}} \rightarrow n_{\max} \sim \frac{S}{-t}$

Appears in  
BPST Calculation  
with saddle point  
(cf Gross-Mende)

$T \sim \frac{k_{\perp}^2}{p_d^+ + p_s^+} \approx \frac{k_{\perp}^2}{p_d^+} \quad (m=0)$

To sum up light cone gauge calculations:

Vacuum fluctuations of string embedding coordinates can interact with a sufficiently sensitive detector: up to mode number  $n_{\text{max}} \sim |s/t|$  for  $|t| > 1/\alpha'$ .

Note that detectability of vacuum fluctuations is familiar in other circumstances (Unruh detectors, density perturbations,...).



Longitudinal spreading is related by a constraint to transverse.

Such constrained variables are familiar and physical: perhaps the most basic is the expansion of the universe (related to matter by Hamiltonian constraint). Similarly the oscillation of a string along its direction (expansion of the worldsheet universe).

Nonetheless, it is worthwhile to check for this effect in gauge-invariant S-matrix amplitudes.

## S-matrix 'data' analysis:

Convolve amplitudes with wavepackets.

Phase of amplitude determines **peak** central trajectories -- including impact parameters and time shifts -- of external states <sup>(Giddings)</sup> (i.e. **most probable among the wavepackets indexed by their central trajectories**).

$$\int d\tilde{k} e^{i\tilde{k}\Delta Y} e^{-\frac{(k-\tilde{k})^2}{\sigma^2}} e^{i\delta(\tilde{k})} \hat{A}(\tilde{k})$$

Work in Regge regime (details below) where incoming states bend slightly into outgoing states, exchanging transverse momentum.

(Wavepackets narrow enough to justify focus on leading term in Regge limit.) **Trace the trajectories back into the middle of the process to attempt a geometric**

**interpretation.** Find nontrivial check of naive geometry: meeting of trajectories + 'yo yo' solutions + Bremsstrahlung radiation at higher orders. **At five (and now six) points this geometry exhibits longitudinal nonlocality.**

Full Disclosure: the wavepackets have a momentum space width

$$\sigma \ll 1/\alpha' \log|s/t|$$

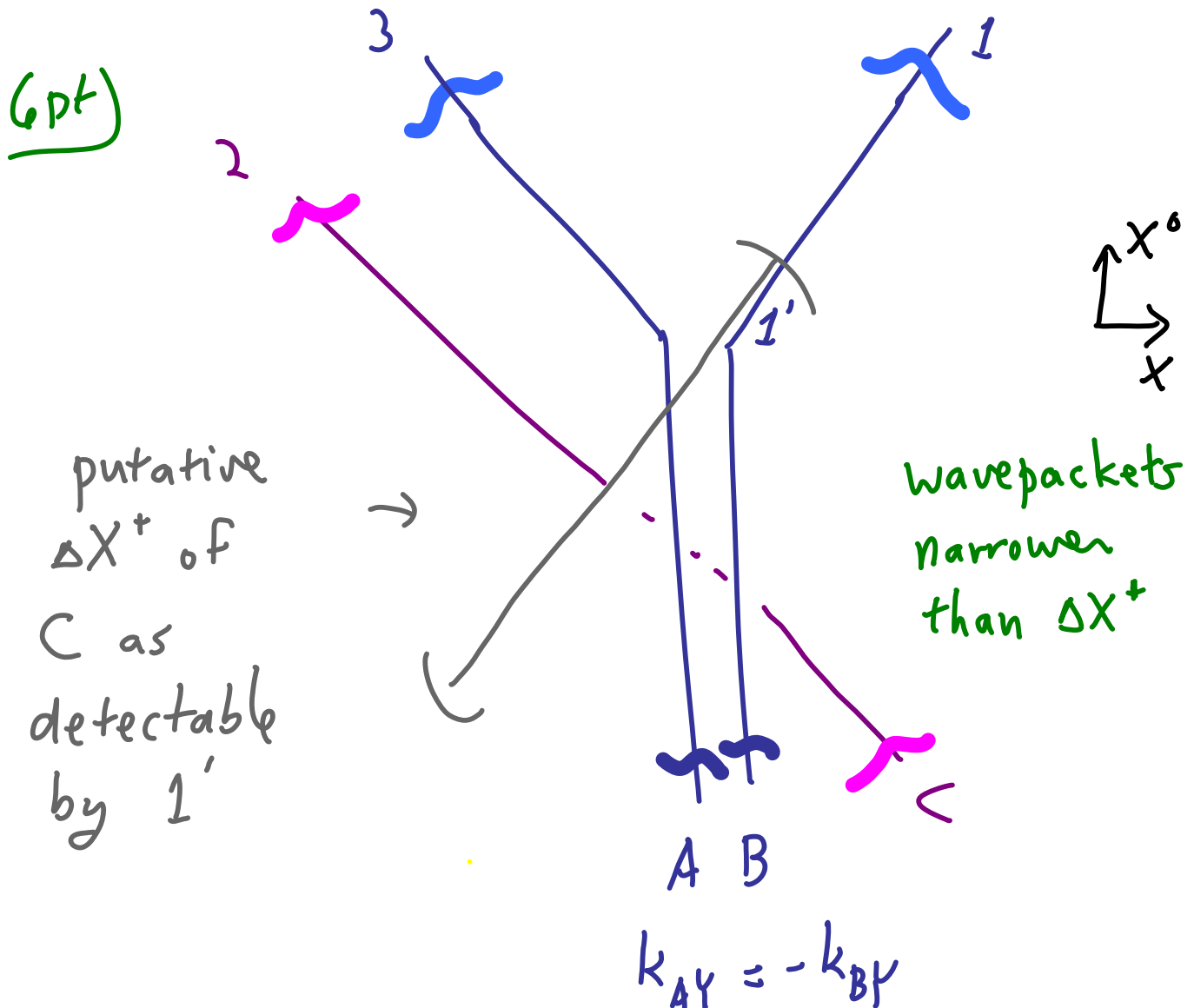
Although this allows us to localize the trajectories well within the scales of interest, in position space they are broad enough that many other impact parameters contribute to the amplitude, beyond the peak one.

cf S. Giddings, S. Caron Huot

The peak trajectories are the most probable among these wavepackets, but are not position eigenstates.

A similar setup to the black hole appears at six points in tree level flat space string S matrix:

cf D. Marolf



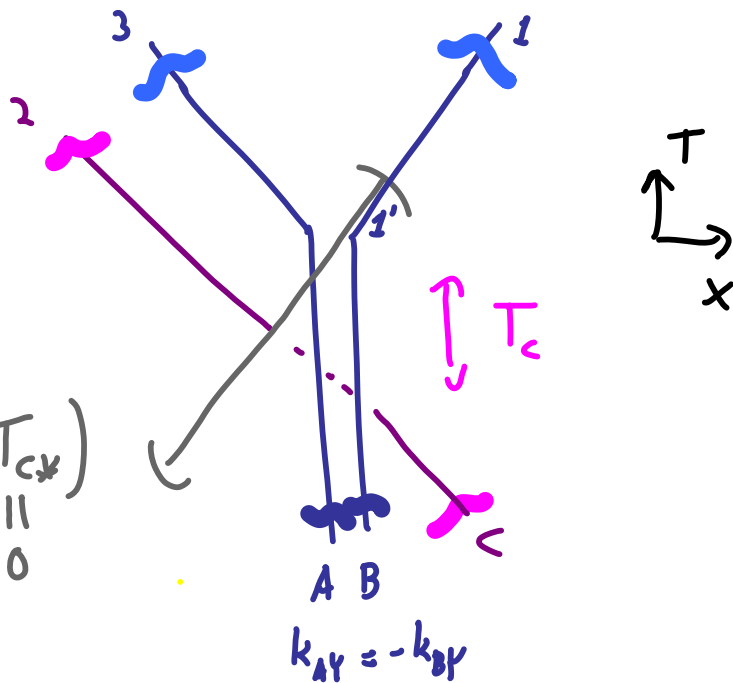
Does C interact w/  $1'$ ?

6pt)

QFT:

$$\int d\tilde{\omega} \frac{e^{-i\tilde{\omega} T_{c*}}}{2\pi\tilde{\omega}} = \theta(T_c - T_{c*})$$

11  
0



step function in  $T_c$  near  $1'$  pole  
(the process happens for  $T_c > 0$ , not otherwise). Can also work off-pole.

String Theory:

Explicit calculation:

$$A \sim A_{\text{hard}} e^{i\sigma} \hat{A}_{\text{int}} A_{c1'13 \text{ Regge}}$$

$\uparrow$   
 $e^{i\tilde{\omega}(\pi f_9')}$

$$T_c = -2\pi f_9'$$

6pt)

As compared to QFT, in <sup>open & closed</sup> ST  
the feature at  $T_c = 0$  is  
shifted earlier by  $2\pi f_g'$ ,  
the scale expected from longitudinal  
spreading. It is also deformed  
via the  $s^+$  type factors.

+ necessary condition for  $\Delta X^+$  satisfied

- hard scattering factor involves  
~  $f_g'$  scales which might complicate  
the interpretation

Back to 4, 5 points)

- Let us take as given the transverse spreading

$$\rho(x_{\perp}) \sim e^{-\frac{(x_{\perp}(\pi) - x_{\perp}(0))^2}{g' \log \frac{s}{-t}}}$$

Note this density is real, consistent with  $|\Psi|^2$ .

- Can be seen from impact parameter transform in forward scattering
- well-established in BPST

Assuming that, we find features of tree level string amplitudes that indicate longitudinal nonlocality.

# Warmup:



$$g_0^2 \frac{e^{i'(-\frac{s}{t})} g' t}{(-1 - g' t)(g' s)}$$

$1 + g' t$   
 $e$   
phase

$$\frac{\sin(\pi g' (s+t))}{\sin(\pi g' s)}$$

$$+ \dots$$

$$\sum_n e^{-i\pi(s + i(\epsilon_H F))/n}$$

series of add'l  
oscillations  
before splitting  
Seiberg-Susskind-  
Toumbas

$$V_k \rightarrow \int d\tilde{k} e^{-\frac{(\tilde{k}-k)^2}{\alpha^2}} V_k e^{i(\tau, b, \dots) \cdot \tilde{k}}$$

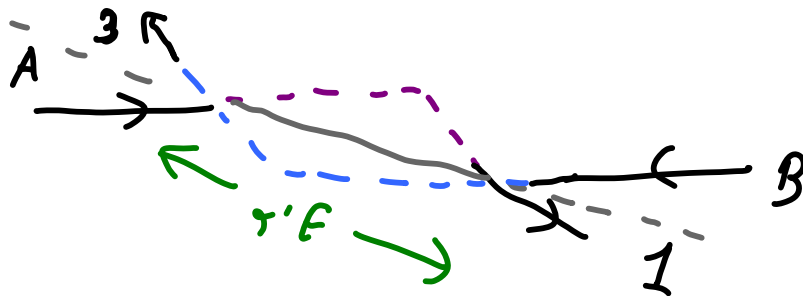
→ Peak trajectories

$$b_{AB*} = -2\pi\alpha' E \sin\theta_1 \neq 0!$$

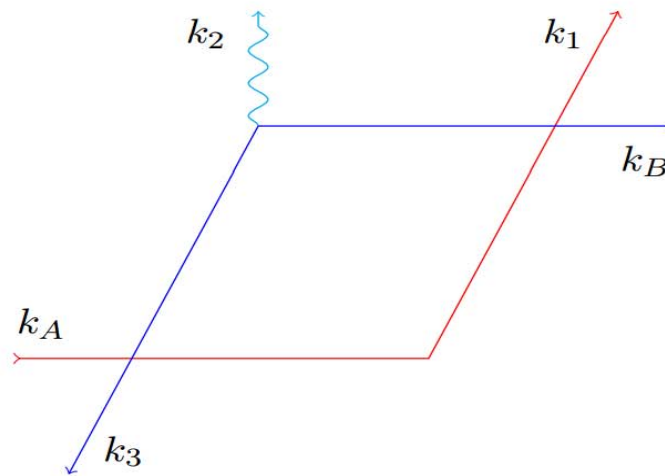
$$T_{1*} = 2\pi\alpha' E (1 - \cos\theta_1) \approx \pi E \theta_1^2$$

time delay  
(cf attractive  
potential)

distinguish  
via  
localized  
wavepackets







Naive to trace back, but:

- \*Trajectories meet (all dimensions)

- \*`yo yo' string solution for created string fits geometry nontrivially

- \*5-point upgrade: radiation leg (2) emerges as expected for Bremsstrahlung

- \*When join? Instantaneous, purely transverse joining+splitting hard to make sense of, but subtle (bonus slides)

Purely transverse effect? (bonus slide)

(see appendix D)

Fourier transform

$$\int dq_{\perp} s^{-\alpha' q_{\perp}^2} e^{i\pi\alpha' q_{\perp}^2} e^{iq_{\perp} b} = \frac{\exp\left(-\frac{b^2}{4\alpha'(\log s - i\pi)}\right)}{\sqrt{\frac{\log s}{\pi} - i}},$$

• Not a real density

cf  $A(\vec{q}) = A_{pt}(\vec{q}) F(\vec{q})$

source  $\rho(\vec{r}) = |\psi_s(\vec{r})|^2$

$$F(\vec{q}) = \int d\vec{r} e^{i\vec{q} \cdot \vec{r}} \rho(\vec{r})$$

real



all directions

Interpret Fourier transform as wave function? (bonus slide)

- Known ground state wavefunction

$$\psi_0(\Delta y) = N_0 e^{\frac{-(\Delta y)^2}{\sigma' \log \frac{n_{\max}}{n_0}}}$$

- Alternate hypothesis:

$$\psi_{\pi}(\Delta y) = N_{\pi} e^{\frac{-(\Delta y)^2}{\sigma'(\log n_{\max} - i\pi)}}$$

Joining interaction could produce A

But  $\langle \psi_{\pi} | (\Delta y)^2 | \psi_{\pi} \rangle \sim \log n_{\max} - \frac{\pi^2}{\log n_{\max}}$

Whereas  $\langle \Delta X_{\perp}^2 \rangle = \sum_{n=1}^{n_{\max}} \frac{1}{n} = \log \frac{n_{\max}}{n_0} + o\left(\frac{1}{n_{\max}}\right)$

Exactly as in BPST

no  $\frac{1}{\log n_{\max}}$  term

(bonus slide)

The interaction timescale<sup>\*</sup> from  
BPST/GM is  $\Delta X_{\text{int}}^- \sim -\frac{g't}{P_A^+}$

During this time, A travels  
along  $X^+$  a distance

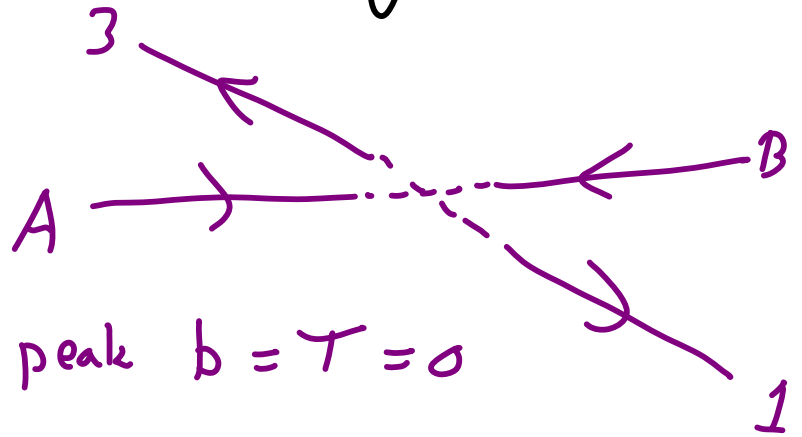
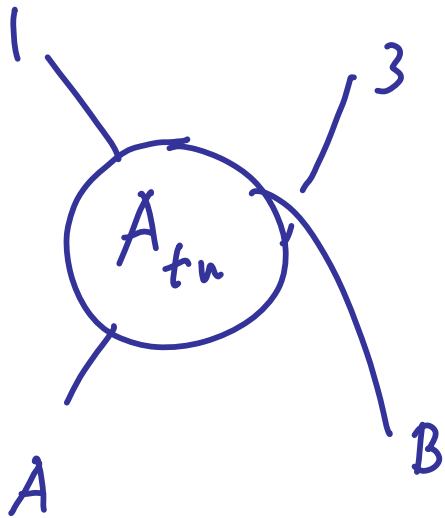
$$\Delta X_{\text{int}}^+ \sim \Delta X^- \frac{P_A^{+2}}{P_\perp^2} \sim \frac{\Delta X^- P_A^{+2}}{-t} \sim g' \dot{P}_A^+ \\ \sim F_{g'}$$

Consistent with early joining as above

\* defined by analytic continuation

5pt)

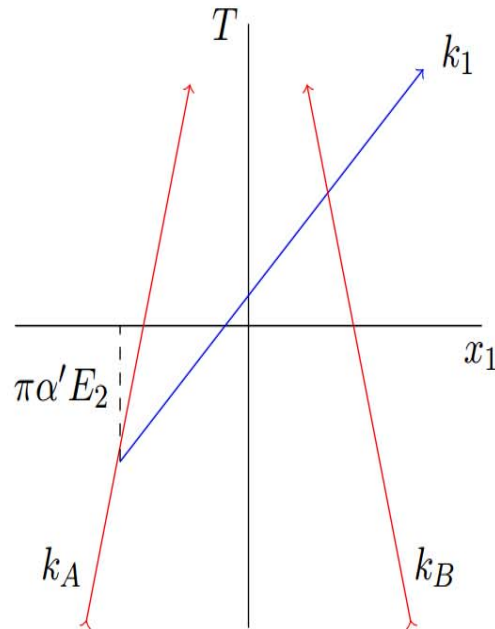
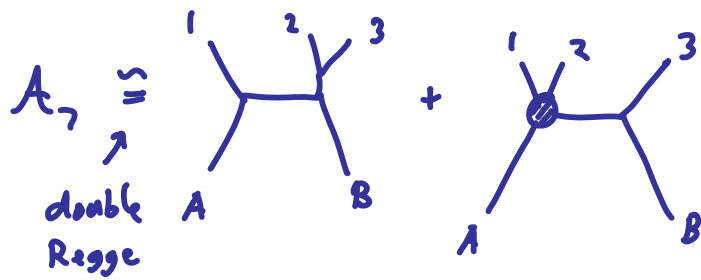
One of the 4 pt diagrams has zero time delay/advance



In this one, when we upgrade to 5 points we find time advance.

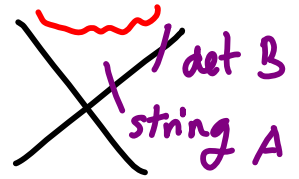
$$A_2 = \frac{g_0^2}{g'} \Gamma(1+k_{A1}) \Gamma(1+k_{B3}) e^{i\pi(k_{B3}-k_{A1})} \frac{K_{A1}-K_{B3}}{|K_{23}|} \times K_{A3}^{-1-k_{A1}} \times U(1+k_{A1}, 1+k_{A1}-k_{B3}, K-i\epsilon)$$

$$K_{IJ} \equiv 2g' k_I \cdot k_J \quad K = \frac{K_{23} K_{A2}}{K_{A3}}$$



Trace back. Assumption here is that A turns directly into 1, a hypothesis tested by nontrivial meeting, and as occurred in Regge 4 pts where also tested by Brem. and yo-yo sol'n. Given that, the interaction is early, indicating longitudinal non-locality.

## Back to horizons:



\*Applying detectable spreading estimate from above:

$$\Delta X^+ \sim \frac{n_{\max}}{P_{\text{source}}} \sim \frac{P_{\text{detector}}^+}{P_{\perp}^2 + m^2}$$

requires B decay to B'  $m_{B'} \ll m_B$

$\Rightarrow$  Late system can detect early  
infalling string for

$$m_B > \frac{r_s}{\alpha'} \quad (m \sim E_s)$$

$$m_B > \frac{r_s^{\frac{1}{2}}}{\alpha'^{\frac{3}{4}}} \quad (\text{dropped from R s.t. } \sqrt{\alpha'} \ll R - r_s \ll r_s)$$

Satisfies conditions for breakdown of EFT

## Remarks:

\*This is causal, just nonlocal. Weakly curved BH accelerates trajectories to large center of mass energy.

\*AdS/CFT constrains late time behavior (via OPEs), consistent because of  $n_{\text{max}} \sim s/(-t)$

Shenker-Stanford; Camanho, Edelstein, Maldacena, Zhiboedov, ...

\*Relative boost sets in outside the horizon.

--May apply to AMPS paradox (late infaller)

--Consistent with observational constraints in cosmology (see below) and BH physics.

--May apply to real black holes:

modification of GR due to string theory.

Could it affect 'ringdown' (quasinormal mode) physics in GW detection ??

Applications to Event horizon telescope??



# Cosmological horizons

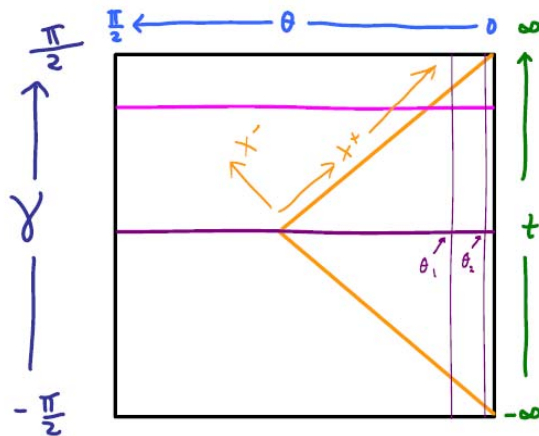


Figure 6: Trajectories 1 and 2 in the late de Sitter universe, as described in the text. For small values of the global spatial coordinate  $\theta$ , the trajectories fall across the indicated observer horizon at a late global time, so that the spatial slices are nearly flat as in our observed universe. Within that regime, the hierarchy  $\frac{\theta_2}{\theta_1} \ll 1$  leads to a large relative boost at the horizon, generated by the cosmological background.

- Late universe :
  - $\Delta t$  of order  $L_{\text{ds}}$
  - $m_{\text{det}}$  huge
- $\Rightarrow$  not ruled out ✓
- Early U :  $\forall$  data consistent with vacuum initial conditions during inflation
- $\Rightarrow$  no strings + detectors involved ✓

## Final remarks/outlook:

\*This effect is subtle, more theoretical `data' tests in progress  
--six points --background fields (linear dilaton, AdS, ...) --radiation profile(beyond peak Brem.)  
--relation to Gross-Mende --(...)

\*Substantial evidence+physical motivation for longitudinal spreading => breakdown of EFT at weakly-curved horizons in string theory. At the very least this represents a large theoretical uncertainty on a basic question. Tantalizing hint of physics beyond GR intrinsic to string theory, with potential for real and thought-experimental application.