Generalized F-Theorem and the Epsilon-Expansion

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Based mainly on SG, Klebanov, arXiv:1409.1937

Fei, SG, Klebanov, Tarnopolsky, to appear

The c-theorem

- A deep problem in QFT is how to define a "good" measure of the number of degrees of freedom which decreases along RG flows
- In *d*=2, this was solved by Alexander Zamolodchikov, who constructed a *c-function* which monotonically decreases under RG flow and is stationary at fixed points. At RG fixed points, this c-function is equal to the CFT central charge, which is also the Weyl anomaly

$$\langle T^{\mu}_{\mu} \rangle = -\frac{c}{12}\mathcal{R}$$

- The central charge can also be found from the Euclidean path integral on a two-sphere of radius R

$$F = -\log Z_{S^2} = -c/3\ \log R + \ldots$$

The a-theorem

- In d=4 there are two Weyl anomaly coefficients

$$\langle T^\mu_\mu \rangle = -\frac{a}{16\pi^2}\left(\mathcal{R}^2_{\mu\nu\lambda\rho} - 4\mathcal{R}^2_{\mu\nu} + \mathcal{R}^2\right) + \frac{c}{16\pi^2} W^2_{\mu\nu\rho\sigma}$$

- The *a*-coefficient, which multiplies the 4d Euler density, can be extracted from the Euclidean path integral on the 4d sphere:

$$F = -\log Z_{S^4} = a\ \log R + \ldots$$

- Cardy conjectured that *a* decreases along any RG flow

$$a_{UV} > a_{IR}$$

- A proof was provided a few years ago (*Komargodski, Schwimmer*)
- Natural to propose a generalization to all even *d* . In *d*=6, no general proof, but evidence from supersymmetric CFT's (*Cordova, Dumitrescu, Yin; Cordova, Dumitrescu, Intriligator*)

The F-theorem

- How do we extend these successes to odd dimensions where there are no anomalies? This is physically interesting, especially in *d*=3 where there are many CFTs, some of them describing critical points in statistical mechanics and condensed matter physics
- Consider the free energy on the 3-sphere

$$F = -\log Z_{S^3}$$

- In a CFT, after removing power-like divergences (e.g. by zeta or dimensional regulator), it is a well-defined, finite and radius independent number

The F-theorem

- Guided by evidence from *N*=2 susy models, perturbative fixed points and holography, it was proposed that any RG flow between unitary CFT's satisfies the *F-theorem* (*Myers, Sinha; Jafferis, Klebanov, Pufu, Safdi*)

$$F_{UV} > F_{IR}$$

- F is also related to the universal term in the entanglement entropy across a circle of radius R in any 2+1 dimensional CFT (Casini, Huerta, Myers)
- This connection was used to prove the *d*=3 F-theorem (Casini, Huerta)
- Conjectural extension for all odd *d:* what decreases under RG is $\tilde{F} = (-1)^{\frac{d+1}{2}} F = (-1)^{\frac{d-1}{2}} \log Z_{S^d}$ (Klebanov, Pufu, Safdi)

Sphere free energy in continuous d

- Is there some interpolation between "F-theorems" in odd *d* and "*a*-theorems" in even *d*?
- It is natural to study the dimensional continuation of the sphere free energy: the Euclidean path integral of the CFT on S^d, continued to non-integer *d*
- Consider the quantity (*SG, Klebanov*)

$$\tilde{F} = \sin(\pi d/2) \log Z_{S^d} = -\sin(\pi d/2) F$$

- In even *d*, F has a pole in dimensional regularization whose residue is related to the Weyl *a*-anomaly. The multiplication by the factor $\sin(\pi d/2)$ removes the pole and yields the anomaly coefficient
- In odd *d* , it yields the F-values $\tilde{F} = (-1)^{\frac{d+1}{2}} F = (-1)^{\frac{d-1}{2}} \log Z_{S^d}$

Generalized F-theorem in continuous d?

- Therefore, $\tilde{F}$ smoothly interpolates between *a*-anomaly coefficients in even *d* and "F-values" in odd *d*
- Based on the existing F- and a-theorems, it is natural to ask whether a "generalized F-theorem" holds in arbitrary dimension *d*

$$\tilde{F}_{UV} > \tilde{F}_{IR}$$

- We have calculated $\tilde{F}$ in several examples of CFTs that can be defined in continuous dimension, including double-trace flows at large N, weakly relevant flows, and perturbative fixed points in the ε-expansion
- In all *unitary* examples we considered, we find that $\tilde{F}$ indeed decreases under RG flow. (For non-unitary fixed points, the inequality $\tilde{F}_{UV} > \tilde{F}_{IR}$ typically does *not* hold.)
- Note: this is a statement about the value of $\tilde{F}$ at fixed points. We do not construct a monotonic function defined along the RG trajectory

Free conformal scalar in continuous d

- For a free conformally coupled scalar on S^d, the free energy in general *d* can be computed to be (*SG, Klebanov*)

$$
\begin{aligned}
F_s &= \frac{1}{2}\log\det\left(-\nabla^2+\frac{1}{4}d(d-2)\right) \\
&= -\frac{1}{\sin(\frac{\pi d}{2})\Gamma(1+d)}\int_0^1 du\, u\sin\pi u\,\Gamma\left(\frac{d}{2}+u\right)\Gamma\left(\frac{d}{2}-u\right)
\end{aligned}
$$

- Near even *d*, it has simple poles whose coefficients are the *a*-anomalies.
- For example, in $d=4-\epsilon$

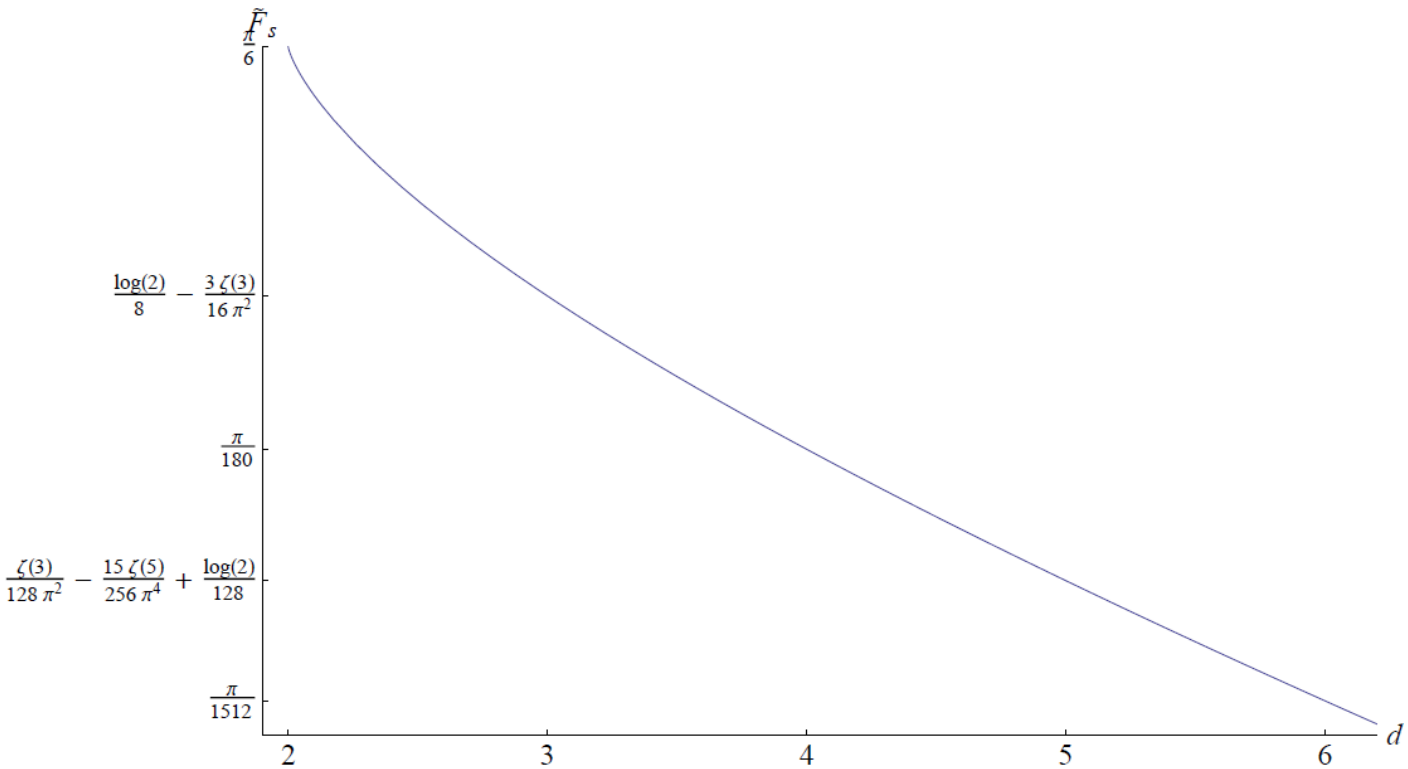
$$F_s=\frac{1}{90\epsilon}+\ldots$$

Free conformal scalar in continuous d

- The value of $\tilde{F} = -\sin(\pi d/2)F$ is then given by

$$\tilde{F}_s = \frac{1}{\Gamma(1+d)} \int_0^1 du\, u \sin \pi u\, \Gamma\left(\frac{d}{2}+u\right)\Gamma\left(\frac{d}{2}-u\right)$$

- This is *positive* for all *d* and smoothly interpolates between *a* and F



Double-Trace Flows

- Consider a large N CFT perturbed by a double-trace operator

$$S_{\mathrm{CFT}_\lambda} = S_{\mathrm{CFT}} + \lambda \int d^d x O_\Delta^2$$

- O_Δ is a single trace scalar primary with dimension Δ in the unperturbed CFT
- When Δ < d/2 the perturbation is relevant, and there is a flow to an IR fixed point where O_Δ has dimension d-Δ+O(1/N) (*Gubser, Klebanov*)
- The change in *F* on *S*^{d} between UV and IR at large N is

$$\delta F_\Delta = F_{IR} - F_{UV} = \frac{1}{2} \log\det \langle O_\Delta O_\Delta \rangle_0 + \mathcal{O}(1/N)$$

Double-Trace Flows

- The 2-point function on the sphere is fixed in terms of the chordal distance s(x,y)

$$\langle O_\Delta(x) O_\Delta(y)\rangle_0 = \frac{1}{s(x,y)^{2\Delta}}$$

- Its determinant can be computed explicitly by decomposing in spherical harmonics. In arbitrary *d* one gets the result (*Gubser, Klebanov; Diaz, Dorn*)

$$\delta F_\Delta = -\frac{1}{\sin(\frac{\pi d}{2})\Gamma(1+d)} \int_0^{\Delta-\frac{d}{2}} du\, u \sin\pi u\, \Gamma\left(\frac{d}{2}+u\right)\Gamma\left(\frac{d}{2}-u\right)$$

Double-Trace Flows

- In terms of $\tilde{F} = -\sin(\pi d/2)F$, we then find

$$\delta\tilde{F}_\Delta = \frac{1}{\Gamma(1+d)} \int_0^{\Delta-\frac{d}{2}} du\, u \sin\pi u\, \Gamma\left(\frac{d}{2}+u\right)\Gamma\left(\frac{d}{2}-u\right)$$

- In a unitary CFT, $\Delta \geq$ d/2-1, and recall $\Delta <$ d/2 for relevant perturbations. Then one can see that $\delta\tilde{F}_\Delta$<0 for all *d*, or:

$$\tilde{F}_{UV} > \tilde{F}_{IR}$$

- This shows that double-trace flows in large N unitary CFT's obey the generalized F-theorem in arbitrary *d*
- An analogous calculation applies to the large N unitary UV fixed points that arise when d/2 < Δ < d/2+1 (e.g. the *d*=3 critical Gross-Neveu model)

Interacting O(N) models in 4<d<6

- Consider the O(N) models with quartic interaction

$$S = \int d^d x \left(\frac{1}{2}(\partial \phi^i)^2 + \frac{\lambda}{4}(\phi^i \phi^i)^2 \right)$$

- In 4<d<6, the model has large N unitary UV fixed points (*Parisi '75*), well defined to all orders in 1/N
- Dual to Vasiliev higher spin gravity in AdS_6 with non-standard boundary conditions (Δ=2) on the bulk scalar (*SG, Klebanov, Safdi*)
- The same 4<d<6 fixed points can be described as IR fixed points of the cubic model (*Fei, SG, Klebanov*)

$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{g_1}{2}\sigma \phi^i \phi^i + \frac{g_2}{6}\sigma^3 \right)$$

Interacting O(N) models in 4<d<6

- In the cubic description, RG flow is from the free theory of N+1 scalars in the UV to the O(N) interacting theory in the IR. Perturbatively unitary in d=6-ε for sufficiently large N (*Fei, SG, Klebanov; Fei, SG, Klebanov, Tarnopolsky*)

- The two descriptions, as either the IR fixed point of the cubic theory or UV fixed point of the quartic theory, imply that F should satisfy the inequalities

$$N\tilde{F}_{\text{free sc.}} < \tilde{F}_{\text{crit.}} < (N+1)\tilde{F}_{\text{free sc.}}$$

A test of the 5d F-theorem

- In d=5, using the large N results for double-trace flows, one finds

$$\tilde{F}_{\rm crit} = N\tilde{F}_{\rm free\ sc.} + \frac{3\zeta(5) + \pi^2\zeta(3)}{96\pi^4} + \mathcal{O}(1/N)$$

- We see that the correction is positive, so that the left side of the inequality $N\tilde{F}_{\rm free\ sc.} < \tilde{F}_{\rm crit.} < (N+1)\tilde{F}_{\rm free\ sc.}$ is satisfied
- The right hand side is also satisfied, because

$$\frac{3\zeta(5) + \pi^2\zeta(3)}{96\pi^4} \simeq 0.001601$$

is smaller than the value of $\tilde{F}$ for a 5d free scalar

$$\tilde{F}_{\rm free\ sc.} = \frac{\log 2}{128} + \frac{\zeta(3)}{128\pi^2} - \frac{15\zeta(5)}{256\pi^4} \simeq 0.00574$$

- Using the continuous d results, one can also show that the same inequalities are satisfied in the full range 4<d<6, supporting the generalized F-theorem

Weakly Relevant Flows

- Another general class of RG flows that one can study are those obtained by perturbing a CFT by a slightly relevant operator O(x) with dimension Δ=*d*-ε (ε<<1)

$$S_g = S_{\mathrm{CFT}_0} + g_b \int d^d x\, O(x)$$

- Working in conformal perturbation theory, the relation between bare coupling g_b and renormalized one, and the corresponding β function, are

$$g_b = \mu^\epsilon \left(g + \frac{C\pi^{d/2}}{\Gamma(d/2)} \frac{g^2}{\epsilon} + O(g^3) \right) \qquad \beta(g) = -\epsilon g + \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} C g^2 + \mathcal{O}(g^3)$$

- Here $C = \mathcal{C}_3/\mathcal{C}_2$, where

$$\langle O(x) O(y) \rangle_0 = \frac{\mathcal{C}_2}{|x-y|^{2\Delta}} \qquad \langle O(x) O(y) O(z) \rangle_0 = \frac{\mathcal{C}_3}{|x-y|^{\Delta} |y-z|^{\Delta} |z-x|^{\Delta}}$$

Weakly Relevant Flows

- There is a perturbative IR fixed point, $\beta(g_*) = 0$, at

$$g_* = \frac{\Gamma\left(\frac{d}{2}\right)\epsilon}{\pi^{\frac{d}{2}} C} + \mathcal{O}(\epsilon^2)$$

- To compute the change in F from UV to IR, we conformally map to the sphere S^d and obtain

$$\delta F = F - F_0 = -\frac{g_b^2}{2}\mathcal{C}_2 I_2(d-\epsilon) + \frac{g_b^3}{6}\mathcal{C}_3 I_3(d-\epsilon) + \mathcal{O}(g_b^4)$$

- I_2 and I_3 are the 2-point and 3-point integrals on S^d (*Cardy*)

$$I_2(\Delta) = \int \frac{d^d x d^d y \sqrt{g_x}\sqrt{g_y}}{s(x,y)^{2\Delta}} = (2R)^{2(d-\Delta)} \frac{2^{1-d}\pi^{d+\frac{1}{2}}\Gamma\left(\frac{d}{2}-\Delta\right)}{\Gamma\left(\frac{d+1}{2}\right)\Gamma(d-\Delta)},$$

$$I_3(\Delta) = \int \frac{d^d x d^d y d^d z \sqrt{g_x}\sqrt{g_y}\sqrt{g_z}}{[s(x,y)s(y,z)s(z,x)]^{\Delta}} = R^{3(d-\Delta)} \frac{8\pi^{\frac{3(1+d)}{2}}\Gamma(d-\frac{3\Delta}{2})}{\Gamma(d)\Gamma(\frac{1+d-\Delta}{2})^3}$$

Weakly Relevant Flows

- In terms of the renormalized coupling *g*, one obtains the result for the change of $\tilde{F} = -\sin(\pi d/2) F$

$$\delta\tilde{F} = \frac{2\pi^{1+d}\mathcal{C}_2}{\Gamma(1+d)}\left[-\frac{1}{2}\epsilon g^2 + \frac{1}{3}\frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} C g^3\right] = \frac{2\pi^{1+d}\mathcal{C}_2}{\Gamma(1+d)}\int_0^g \beta(g)dg$$

- At the fixed point $g=g_*$ we find

$$\delta\tilde{F} = \tilde{F}_{\mathrm{IR}} - \tilde{F}_{\mathrm{UV}} = -\frac{\pi\Gamma\left(\frac{d}{2}\right)^2}{\Gamma(1+d)}\frac{\mathcal{C}_2}{3C^2}\epsilon^3$$

- In a unitary CFT, $\mathcal{C}_2$ is positive and *C* is real, so we find agreement with the generalized F-theorem in all *d:*

$$\tilde{F}_{UV} > \tilde{F}_{IR}$$

- This generalizes to continuous *d* previous computations in odd *d* (*Klebanov, Pufu, Safdi*) and even *d* (*Komargodski*)

Sphere free energy and the ε-expansion

- The fact that $\tilde{F}$ is a smooth function of dimension suggests that, in the spirit of the Wilson-Fisher ε-expansion, it may provide us with a useful tool to estimate the value of F for interacting CFT's for which it is hard to make calculations directly in the physical dimension
- For example, we can consider the 3d Ising model, and more generally the critical O(N) CFT's in d=3
- They are strongly coupled CFT's in d=3, but they have a perturbative description in d=4-ε

The O(N) models in d=4-ε

$$S = \int d^d x \left(\frac{1}{2} \left(\partial_\mu \phi_0^i \right)^2 + \frac{\lambda_0}{4} (\phi_0^i \phi_0^i)^2 \right)$$

$$\beta = -\epsilon\lambda + \frac{N+8}{8\pi^2}\lambda^2 - \frac{3(3N+14)}{64\pi^4}\lambda^3 + \dots$$

$$\lambda_* = \frac{8\pi^2}{N+8}\epsilon + \frac{24(3N+14)\pi^2}{(N+8)^3}\epsilon^2 + \dots$$

- For N=1, ε=1 we get the 3d Ising model. N=1, ε=2 corresponds to the 2d Ising CFT with central charge c=1/2
- The ε-expansion (coupled with resummation techniques) has proved successful for estimating operator dimensions and critical exponents in the d=3 interacting CFT's
- Following a similar approach, we can compute the sphere free energy perturbatively and extrapolate the results to ε=1 to estimate the value of F for 3d Ising and related models

The Wilson-Fisher fixed points in curved space

- To renormalize the theory in curved space in d=4-ε, one starts with the bare action (*Brown-Collins '80; Hathrell '82*)

$$S = \int d^dx\sqrt{g}\left(\frac{1}{2}((\partial_\mu\phi_0^i)^2 + \frac{d-2}{4(d-1)}\mathcal{R}(\phi_0^i)^2) + \frac{\lambda_0}{4}(\phi_0^i\phi_0^i)^2 + \frac{1}{2}\eta_0 H(\phi_0^i)^2 + a_0 W^2 + b_0 E + c_0 H^2\right)$$

$$W^2 = \mathcal{R}_{\mu\nu\lambda\rho}\mathcal{R}^{\mu\nu\lambda\rho} - \frac{4}{d-2}\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \frac{2}{(d-2)(d-1)}\mathcal{R}^2 \qquad H = \frac{\mathcal{R}}{d-1}$$

$$E = \mathcal{R}_{\mu\nu\lambda\rho}\mathcal{R}^{\mu\nu\lambda\rho} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2$$

- Divergences in the free energy are removed by expressing all bare couplings in terms of renormalized ones

$$\lambda_0 = \mu^\epsilon\left(\lambda + \frac{(N+8)}{8\pi^2\epsilon}\lambda^2 + \dots\right),$$

$$a_0 = \mu^{-\epsilon}\left(a + \sum_{i=0}^{\infty}\frac{L_a^{(i)}(\lambda)}{\epsilon^i}\right), \qquad b_0 = \mu^{-\epsilon}\left(b + \sum_{i=0}^{\infty}\frac{L_b^{(i)}(\lambda)}{\epsilon^i}\right), \quad \text{etc.}$$

The Wilson-Fisher fixed points in curved space

- Each renormalized coupling λ, *a*, *b*,… then acquires a non-trivial beta function β_λ, β_a, β_b,…
- The renormalized free energy is a finite function of the renormalized couplings and renormalization scale μ that satisfies the Callan-Symanzik equation

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_\lambda\frac{\partial}{\partial\lambda}+\beta_\eta\frac{\partial}{\partial\eta}+\beta_a\frac{\partial}{\partial a}+\beta_b\frac{\partial}{\partial b}+\beta_c\frac{\partial}{\partial c}\right)F=0$$

- The conformally invariant IR fixed point is obtained by setting to zero *all* beta functions in *d*=4-ε

$$\beta_\lambda=\beta_a=\beta_b=\beta_c=\beta_\eta=0$$

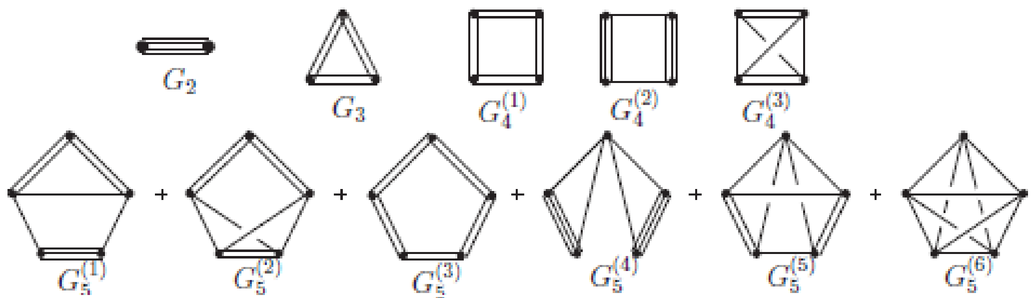
- The sphere free-energy at the IR fixed point in *d*=4-ε

$$F_{\rm IR}(\epsilon)=F(\lambda_*,a_*,b_*,c_*,\eta_*,\mu R)$$

is then a R-independent quantity which is a function of ε only

F for the O(N) scalar theory in d=4-ε

- We performed a perturbative calculation of F to order λ^5 (*Fei, SG, Klebanov, Tarnpolsky, to appear*), i.e. up to 6-loops



- The poles in the above diagrams fix the curvature beta functions to the needed order. At the IR fixed point, we get the final result for $\tilde{F} = -\sin(\pi d/2) F$:

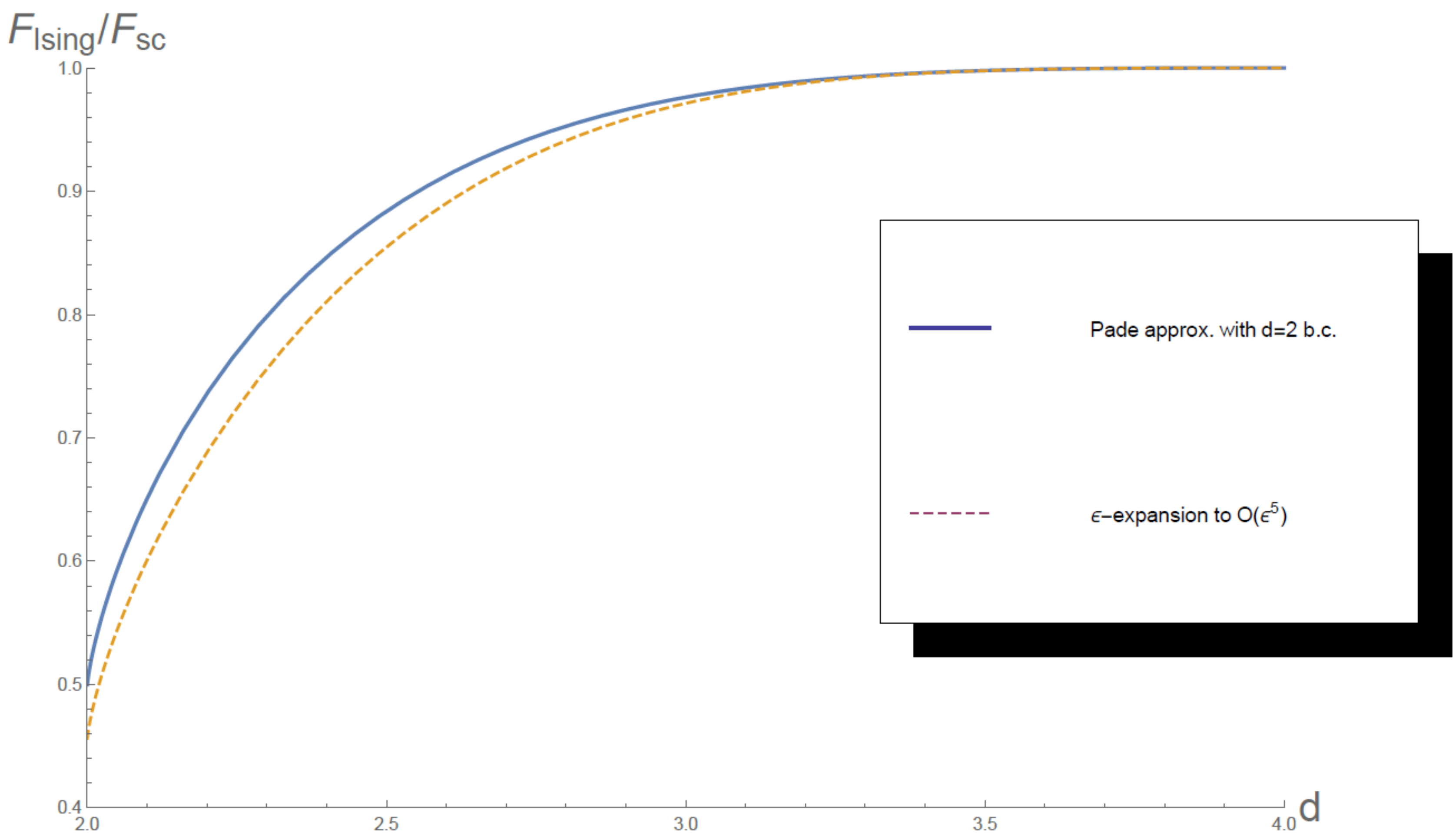
$$\begin{aligned}\tilde{F}_{\rm IR} =& N\tilde{F}_s(\epsilon) - \frac{\pi N(N+2)\epsilon^3}{576(N+8)^2} - \frac{\pi N(N+2)(13N^2+370N+1588)\epsilon^4}{6912(N+8)^4} \\ &+ \frac{\pi N(N+2)}{414720(N+8)^6}\big(10368(N+8)(5N+22)\zeta(3) - 647N^4 - 32152N^3 \\ &\qquad -606576N^2 - 3939520N + 30\pi^2(N+8)^4 - 8451008\big)\,\epsilon^5 + \mathcal{O}(\epsilon^6)\end{aligned}$$

Estimating F for the 3d Ising model

- Extracting precise estimates from the ε–expansion typically requires some resummation technique. A simple approach is to use Pade approximants

$$\text{Padé}_{[m,n]}(\epsilon) = \frac{A_0 + A_1\epsilon + A_2\epsilon^2 + \ldots + A_m\epsilon^m}{1 + B_1\epsilon + B_2\epsilon^2 + \ldots + B_n\epsilon^n}$$

- For the Ising model (N=1), we expect $\tilde{F}$ to be a smooth function of *d*, such that near *d*=4 it reproduces the perturbative ε–expansion, and in *d*=2 it reproduces the exact central charge of the 2d Ising CFT, c=1/2
- The accuracy of the Pade approximants can be greatly improved if we impose the exact value c=1/2 (which in terms of $\tilde{F}$ corresponds to $\tilde{F} = \pi/12$) as a boundary condition at *d*=2



$$\tilde{F} = \tilde{F}_s + \tilde{F}_{\rm int} = \frac{\pi}{180} + 0.0205991\epsilon + 0.0136429\epsilon^2 + 0.00670643\epsilon^3 + 0.00264883\epsilon^4 + 0.000927589\epsilon^5 + O(\epsilon^6)$$

$$\tilde{F}_s = \frac{\pi}{180} + 0.0205991\epsilon + 0.0136429\epsilon^2 + 0.00690843\epsilon^3 + 0.00305846\epsilon^4 + 0.0012722\epsilon^5 + O(\epsilon^6)$$

Estimating F for the 3d Ising model

- Using the constrained Pade approximant method, we get the estimate

$$F_{\text{3d Ising}} \approx 0.0623\,, \qquad \frac{F_{\text{3d Ising}}}{F_{\text{free sc.}}} \approx 0.97$$

- The value of F (and hence of the disk entanglement entropy) for 3d Ising appears to be extremely close to the free field value!
- A qualitatively similar result was found for C_T in the conformal bootstrap approach

$$c_T^{\text{3d Ising}} / c_T^{\text{3d free scalar}} \approx 0.9466$$

(*El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi*)

Conclusion and summary

- We studied dimensional continuation of the sphere free energy and provided evidence for a generalized F-theorem in continuous *d*, interpolating between F-theorems in odd *d* and *a*-theorems in even *d*.

  The quantity that decreases under RG flow is

$$\tilde{F} = \sin(\pi d/2) \log Z_{S^d} = - \sin(\pi d/2) F$$

- The ε–expansion of $\tilde{F}$ can be used to estimate the values of F for interesting CFTs
- For the critical 3d Ising model it is only a few percent lower than for the free conformal scalar
- Can this result be compared with a numerical calculation of the Entanglement Entropy for the 3d Ising model?

Comments on SUSY theories

- Using the dimensional reduction scheme in $d=4-\varepsilon$ (which preserves SUSY), one can smoothly connect theories with 4 supercharges in $d=4,3,2$
- For models with several chiral superfields (no gauge fields), and with $U(1)_R$ symmetry, we proposed a natural version of localization in $2\le d\le 4$ (SG, Klebanov). The exact $\tilde{F}$ is given by

$$\tilde{F} = \sum_{\text{chirals}} \tilde{\mathcal{F}}(\Delta_i) \qquad\qquad \text{trial R-charges: } R_i = 2\Delta_i/(d-1)$$

$$\tilde{\mathcal{F}}(\Delta) = 2(\tilde{F}_s + \tilde{F}_f) + \int_{d/2-1}^{\Delta} dx \frac{\Gamma(d-1-x)\,\Gamma(x)\sin\left(\pi(x-\frac{d}{2})\right)}{\Gamma(d-1)}$$

- If the Δ_i are not all determined by superpotential, they are fixed by extremizing $\tilde{F}$ with respect the Δ_i

Comments on SUSY theories

- This smoothly interpolates between the known results in *d*=4,3,2:

$$d=4: \quad \tilde{\mathcal{F}}(\Delta = 3/2R) = \frac{3\pi}{16}(R-1)(3(R-1)^2-1)$$

$$d=3: \quad \tilde{\mathcal{F}}(\Delta) = -\ell(1-\Delta)$$ (Jafferis' $\ell(z)$ function)

$$d=2: \quad \tilde{\mathcal{F}}(\Delta) = \frac{\pi}{2}(1-2\Delta)\,, \qquad c = 3\sum_i (1-2\Delta_i)$$

- For explicit examples such as $W = X^3\,,\ W = X\sum_{i=1}^{N} Z_i Z_i$, which have non-trivial IR fixed points in *d*=4-ε ("super-Ising", "super O(N)"), we checked that this "interpolating localization" prescription precisely agrees with existing loop calculations of anomalous dimensions in these SCFT's (Ferreira, Jack, Jones), and with direct perturbative calculation of F in the ε-expansion

THANK YOU!