Aspects of 2d (0,2) theories

Abhijit Gadde
IAS
arXiv:1506.xxxx

AG, Sergei Gukov, Pavel Putrov
arXiv: 1404.5314, 1310.0818

Exact non-perturbative RG flow in (0,2) theories

Abhijit Gadde
IAS
arXiv:1506.xxxx

AG, Sergei Gukov, Pavel Putrov
arXiv: 1404.5314, 1310.0818

What makes them special?

What makes them special?

- 2d (0,2) theories have two supercharges of the same spin

What makes them special?

- 2d (0,2) theories have two supercharges of the same spin

- Naturally, the algebra leads to chiral theories.

What makes them special?

- 2d (0,2) theories have two supercharges of the same spin
- Naturally, the algebra leads to chiral theories.
- Global symmetries have non-vanishing 't Hooft anomaly, which implies existence of gapless modes

What makes them special?

- 2d (0,2) theories have two supercharges of the same spin

- Naturally, the algebra leads to chiral theories.

- Global symmetries have non-vanishing 't Hooft anomaly, which implies existence of gapless modes

- Autonomous theory of the gapless modes is a conformal field theory.

What makes them special?

- 2d (0,2) theories have two supercharges of the same spin
- Naturally, the algebra leads to chiral theories.
- Global symmetries have non-vanishing 't Hooft anomaly, which implies existence of gapless modes
- Autonomous theory of the gapless modes is a conformal field theory.
- Generically, (0,2) theories flow to non-trivial conformal fixed points.

Behavior of the microscopic gauge theory

Behavior of the microscopic gauge theory

- In 2 dimensions, gauge coupling is dimensionful. It is classically relevant.

Behavior of the microscopic gauge theory

- In 2 dimensions, gauge coupling is dimensionful. It is classically relevant.

- In addition, there could be other classically relevant interactions (Superpotential terms that lead to Yukawa type interactions)

Behavior of the microscopic gauge theory

- In 2 dimensions, gauge coupling is dimensionful. It is classically relevant.
- In addition, there could be other classically relevant interactions (Superpotential terms that lead to Yukawa type interactions)
- Microscopic gauge theory undergoes non-trivial RG flow.

Behavior of the microscopic gauge theory

- In 2 dimensions, gauge coupling is dimensionful. It is classically relevant.
- In addition, there could be other classically relevant interactions (Superpotential terms that lead to Yukawa type interactions)
- Microscopic gauge theory undergoes non-trivial RG flow.
- In fact, RG flow naturally splits into two stages.

Stages of the RG flow

Fast Flow

Stages of the RG flow

Fast Flow

- During this stage, the dimensionful parameters, including gauge coupling, flow rapidly to infinity.
- A good description of the theory at the end of this stage is in terms of a non-linear sigma model

Stages of the RG flow

Fast Flow

- During this stage, the dimensionful parameters, including gauge coupling, flow rapidly to infinity.
- A good description of the theory at the end of this stage is in terms of a non-linear sigma model

Slow Flow

- The sigma model has Kahler and complex structure moduli that are classically dimensionless.
- The next stage of RG flow is in this space. At one loop it is logarithmic. Eventually takes the theory to the fixed point.

A picture of the RG flow

Gauge theory

$$\Big\downarrow \quad \text{Fast} \quad g_{\mathrm{YM}} \to \infty, \quad m \to \infty$$

NLSM

$E \to M$

$$\Big\downarrow \quad \text{Slow} \quad t_{\mathrm{Kahler}}, \quad t_{\mathrm{Complex}}$$

CFT

A picture of the RG flow

$$
\begin{tikzcd}
\text{Gauge theory} \arrow[d, "\text{Fast}"', "{\begin{matrix} g_{\mathrm{YM}} \to \infty \\ m \to \infty \end{matrix}}"] \\
\begin{matrix} \text{NLSM} \\ E \to M \end{matrix} \arrow[d, "\text{Slow}"', "{\begin{matrix} t_{\mathrm{Kahler}} \\ t_{\mathrm{Complex}} \end{matrix}}"] \\
\text{CFT}
\end{tikzcd}
$$

- Normalizable vacuum gives rise to state operator correspondence for the CFT

A picture of the RG flow

Gauge theory

Fast $\Big\downarrow$ $g_{\mathrm{YM}} \to \infty$, $m \to \infty$

NLSM
$E \to M$

Slow $\Big\downarrow$ t_{Kahler}, t_{Complex}

CFT

- Normalizable vacuum gives rise to state operator correspondence for the CFT
- Global symmetries are promoted to affine Kac-Moody symmetries

A picture of the RG flow

$$\begin{tikzcd}
\text{Gauge theory} \arrow[d, "\text{Fast}"', "{g_{\mathrm{YM}} \to \infty,\ m \to \infty}"] \\
\text{NLSM} \\[-1em]
E \to M \arrow[d, "\text{Slow}"', "{t_{\mathrm{Kahler}},\ t_{\mathrm{Complex}}}"] \\
\text{CFT}
\end{tikzcd}$$

- Normalizable vacuum gives rise to state operator correspondence for the CFT
- Global symmetries are promoted to affine Kac-Moody symmetries
- One way to ensure it is to make sure that the sigma model target is compact

(0,2) Multiplets

(0,2) Multiplets

- Chiral multiplet: $\bar{D}_+ \Phi = 0$

$$\Phi = \phi + \sqrt{2}\theta^+ \psi_+ - i\theta^+ \bar{\theta}^+ \partial_+ \phi$$

(0,2) Multiplets

- Chiral multiplet: $\bar{D}_+ \Phi = 0$

$$\Phi = \phi + \sqrt{2}\theta^+ \psi_+ - i\theta^+ \bar{\theta}^+ \partial_+ \phi$$

  - Complex scalar
  - Complex right-moving fermion

(0,2) Multiplets

- Chiral multiplet: $\bar{D}_+\Phi = 0$

$$\Phi = \phi + \sqrt{2}\theta^+\psi_+ - i\theta^+\bar{\theta}^+\partial_+\phi$$

    - Complex scalar
    - Complex right-moving fermion

- Fermi multiplet: $\bar{D}_+\Psi = 0$

$$\Psi = \psi_- - \sqrt{2}\theta^+ G - i\theta^+\bar{\theta}^+\partial_+\psi_-$$

(0,2) Multiplets

- Chiral multiplet: $\bar{D}_+ \Phi = 0$

$$\Phi = \phi + \sqrt{2}\theta^+ \psi_+ - i\theta^+ \bar{\theta}^+ \partial_+ \phi$$

  - Complex scalar
  - Complex right-moving fermion

- Fermi multiplet: $\bar{D}_+ \Psi = 0$

$$\Psi = \psi_- - \sqrt{2}\theta^+ G - i\theta^+ \bar{\theta}^+ \partial_+ \psi_-$$

  - Complex left-moving fermion

(0,2) Multiplets

- Chiral multiplet: $\bar{D}_+ \Phi = 0$

  $$\Phi = \phi + \sqrt{2}\theta^+ \psi_+ - i\theta^+ \bar{\theta}^+ \partial_+ \phi$$

  - Complex scalar
  - Complex right-moving fermion

- Fermi multiplet: $\bar{D}_+ \Psi = 0$

  $$\Psi = \psi_- - \sqrt{2}\theta^+ G - i\theta^+ \bar{\theta}^+ \partial_+ \psi_-$$

  - Complex left-moving fermion

- Vector multiplet:

(0,2) Multiplets

- Chiral multiplet: $\bar{D}_+ \Phi = 0$

  $$\Phi = \phi + \sqrt{2}\theta^+ \psi_+ - i\theta^+ \bar{\theta}^+ \partial_+ \phi$$

  - Complex scalar
  - Complex right-moving fermion

- Fermi multiplet: $\bar{D}_+ \Psi = 0$

  $$\Psi = \psi_- - \sqrt{2}\theta^+ G - i\theta^+ \bar{\theta}^+ \partial_+ \psi_-$$

  - Complex left-moving fermion

- Vector multiplet:

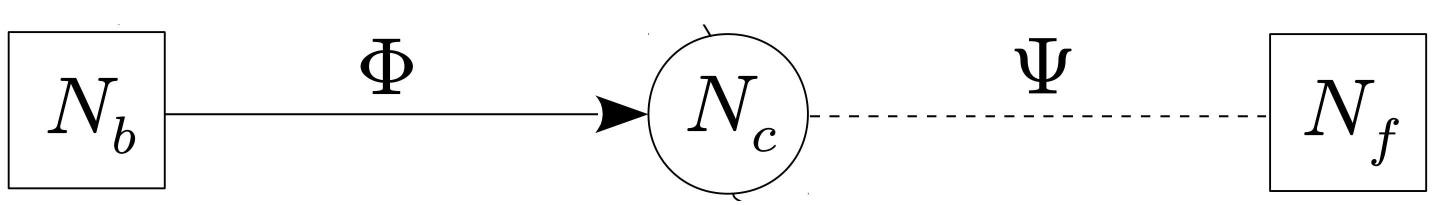
  - Gauge invariant d.o.f.: Fermi multiplet Λ

(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter

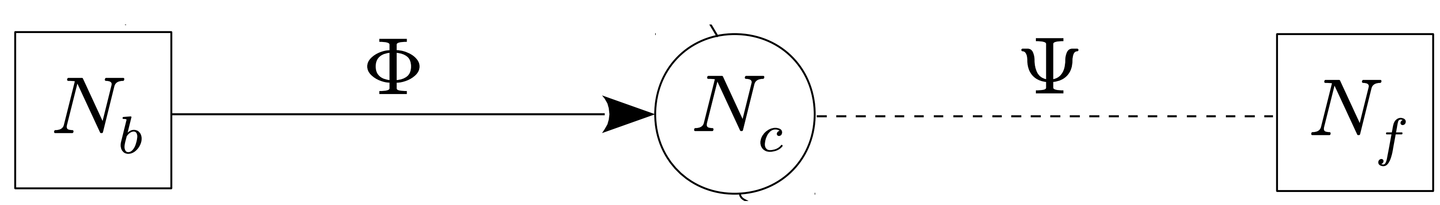
(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi



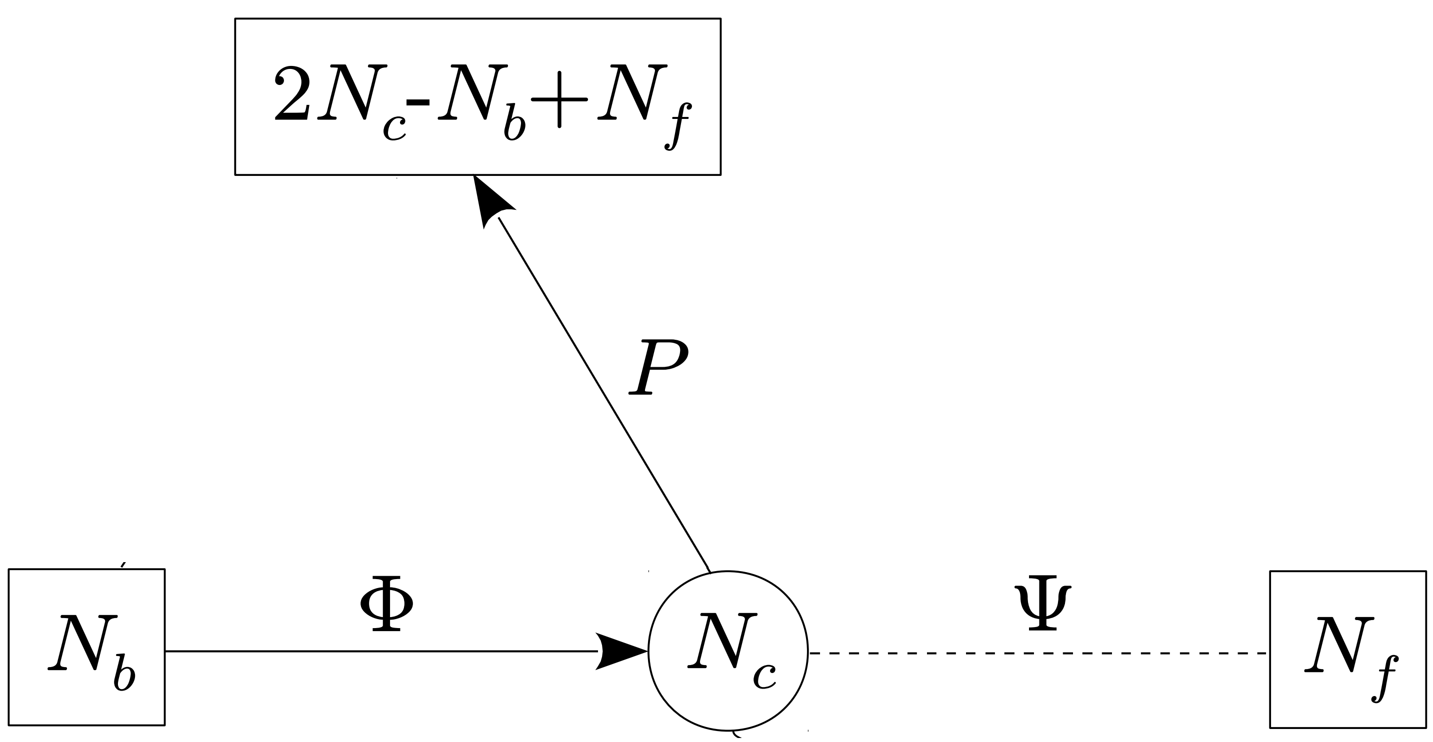
(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation



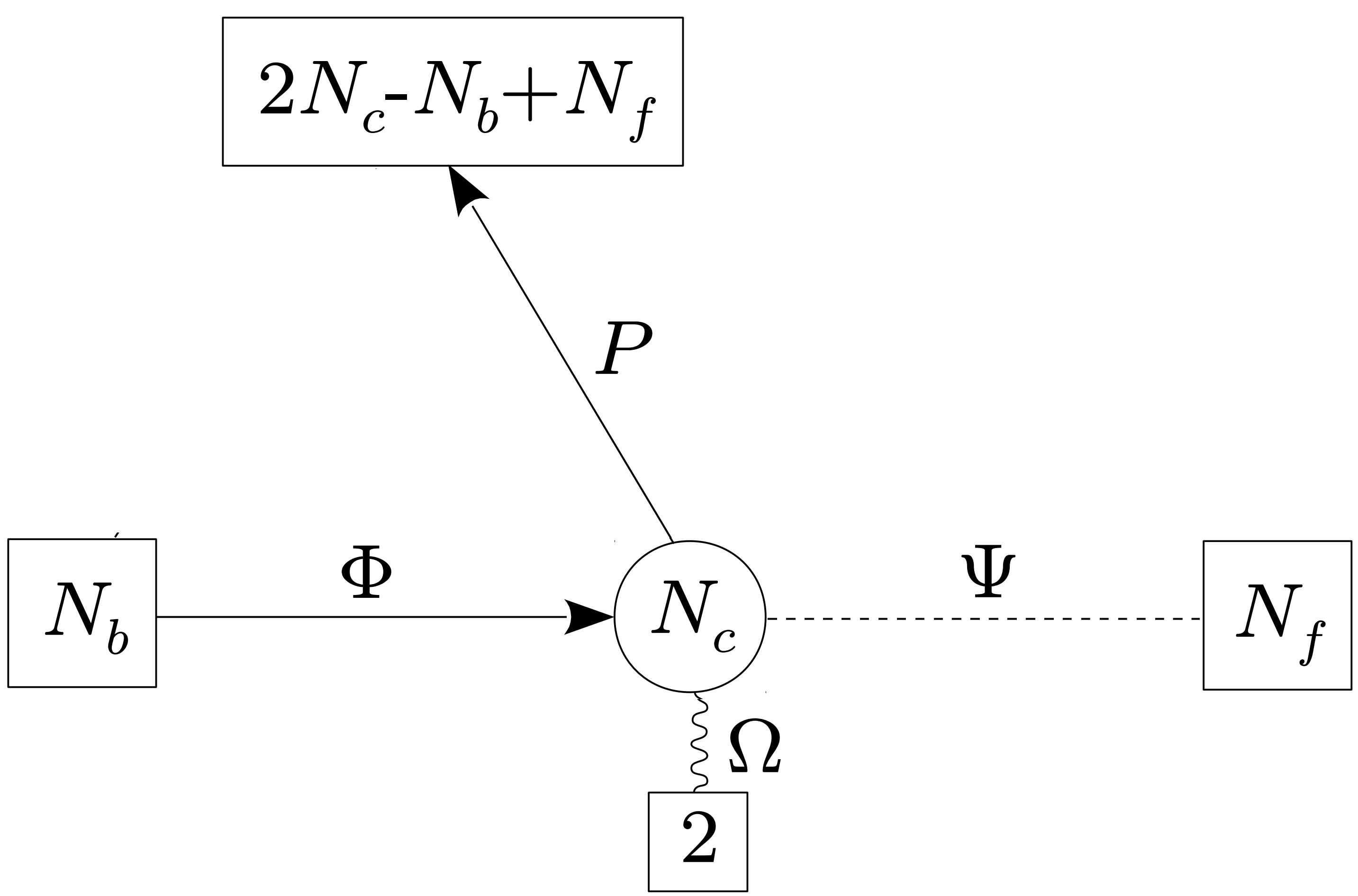
(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation



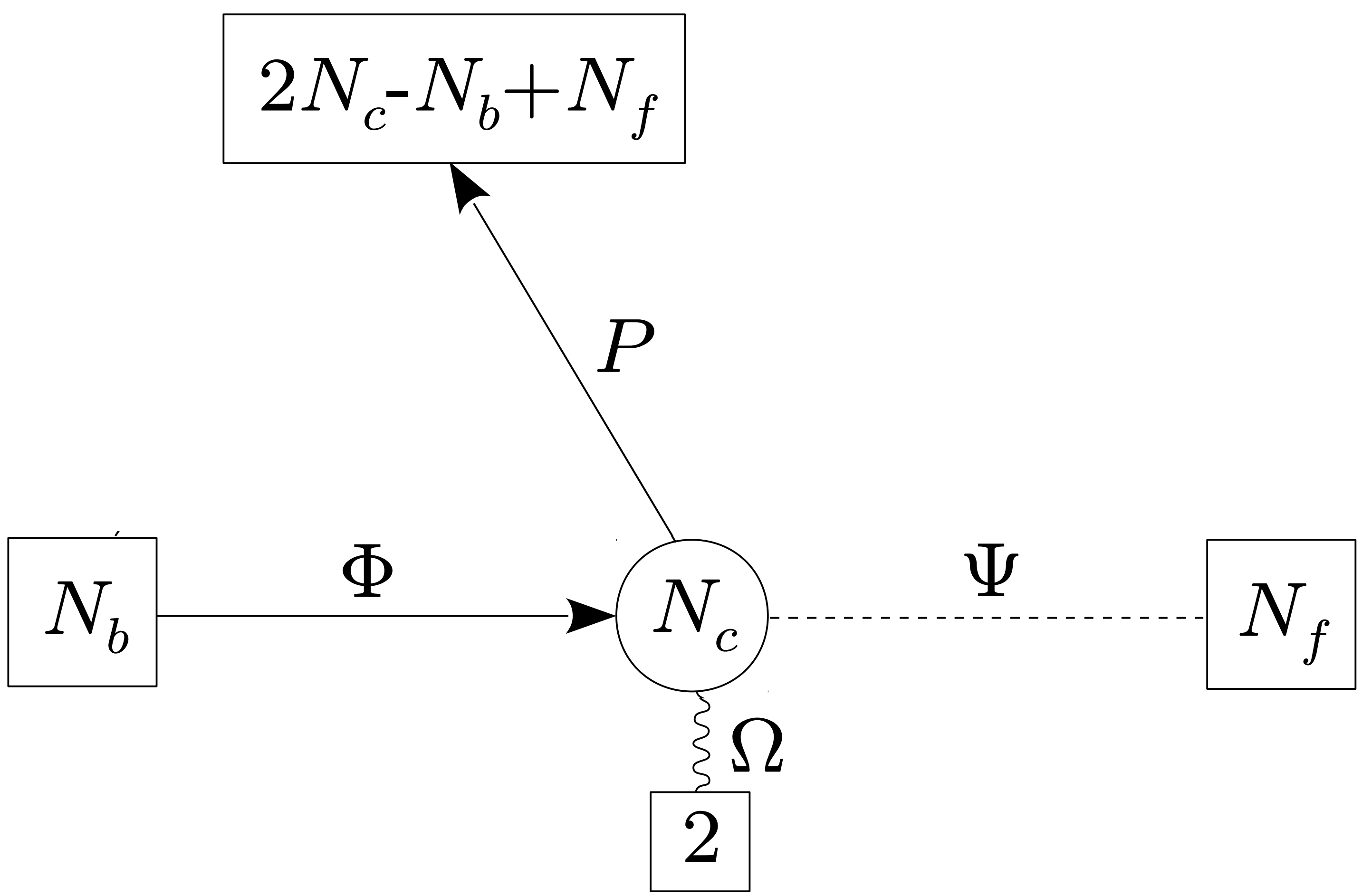
(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation



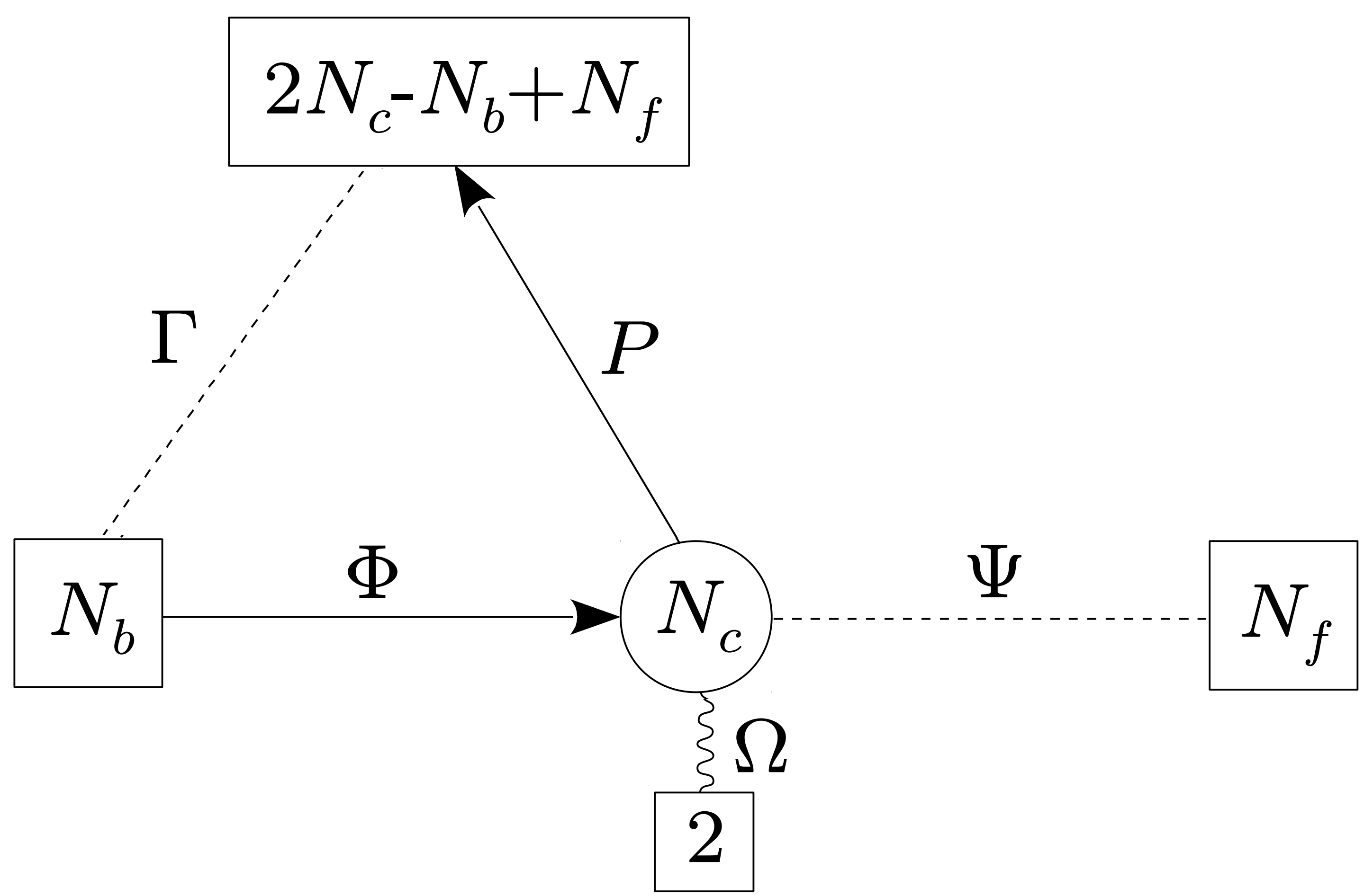
(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation
- + Normalizable vacuum



(0,2) SQCD

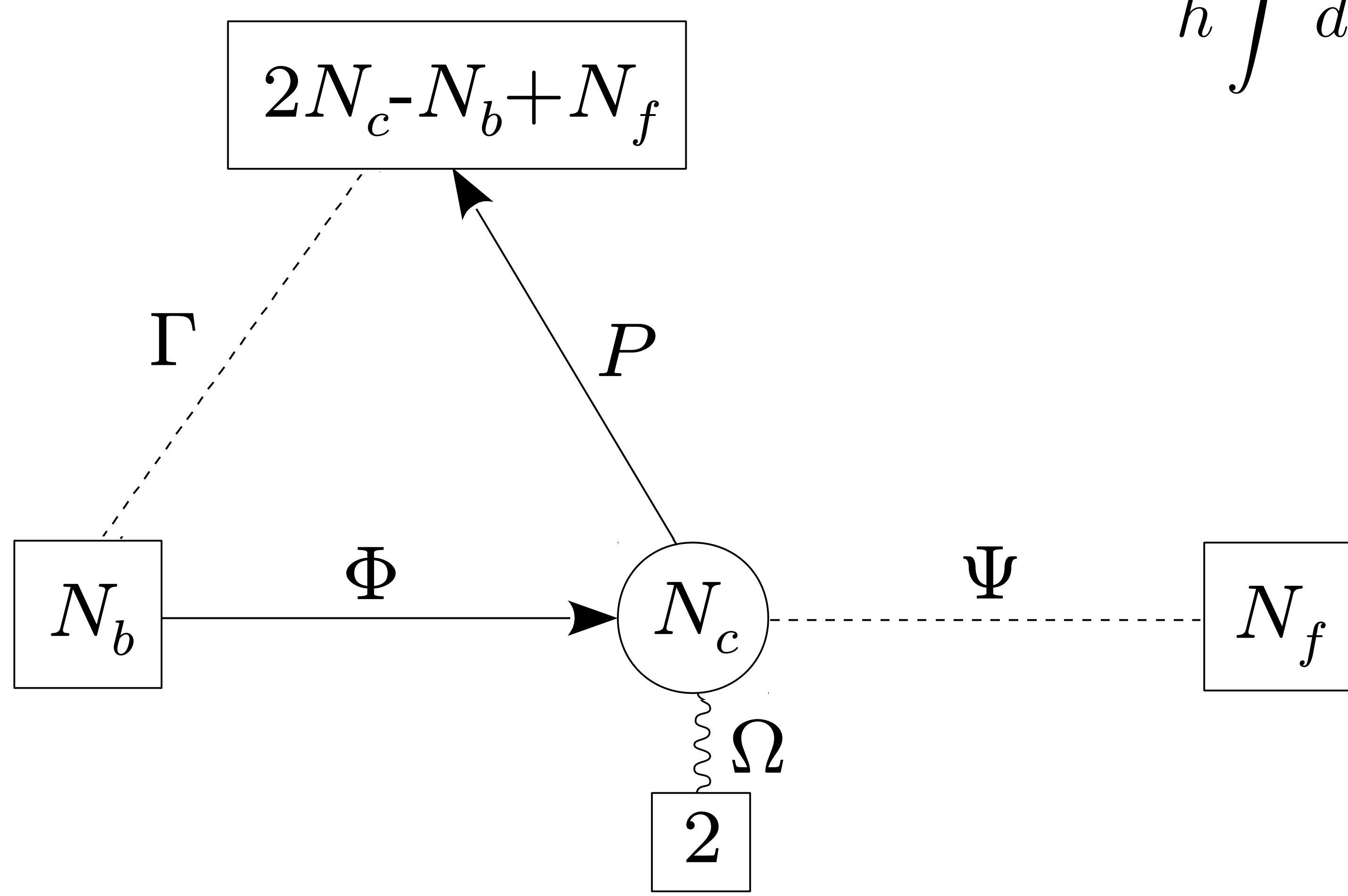
- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation
- + Normalizable vacuum



(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation
- + Normalizable vacuum

$$h \int d\theta \, \Gamma P \Phi + t \int d\theta \, \Lambda$$



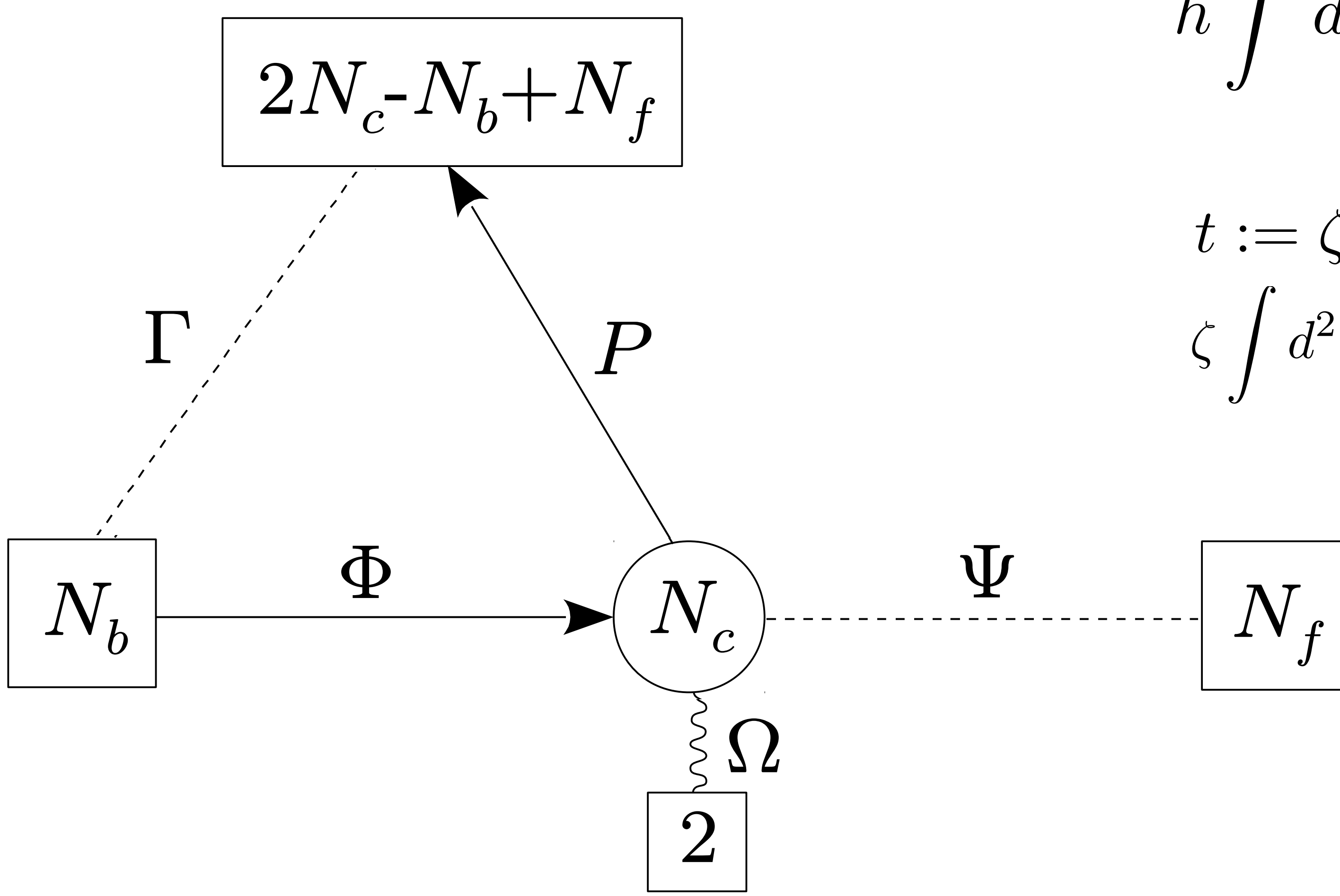
(0,2) SQCD

- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation
- + Normalizable vacuum

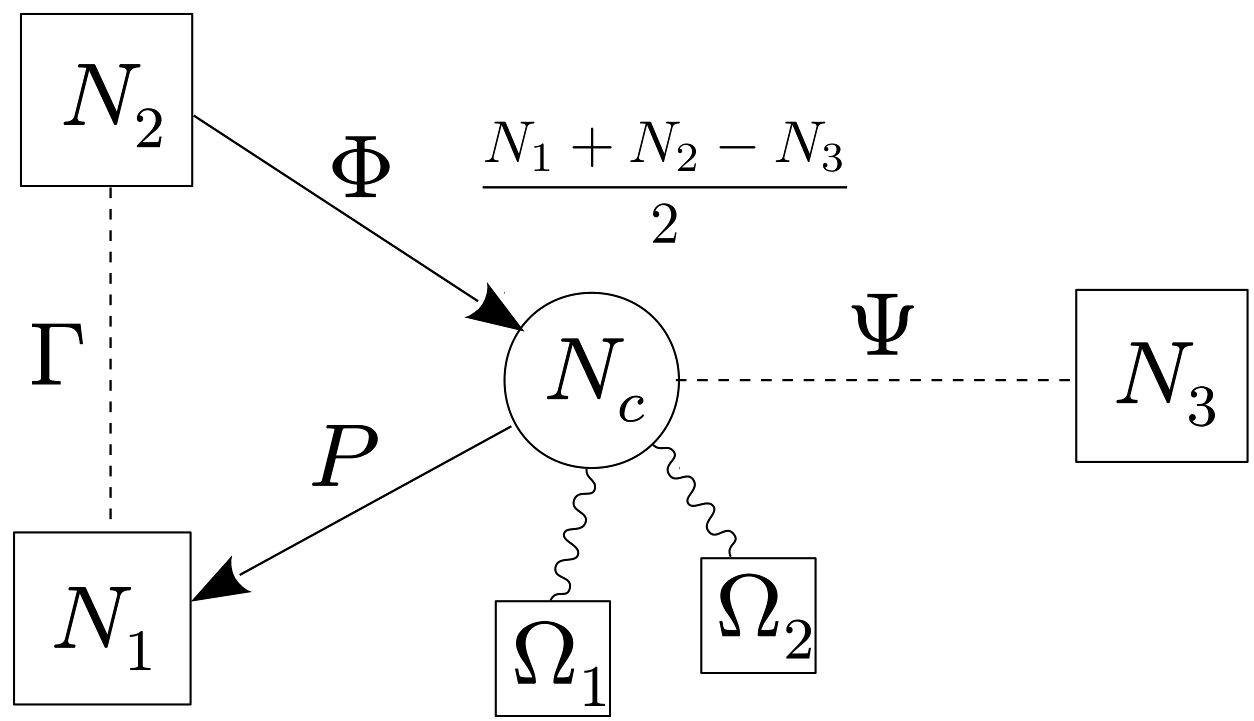
$$h \int d\theta\, \Gamma P \Phi + t \int d\theta\, \Lambda$$

$$t := \zeta + i\theta$$

$$\zeta \int d^2x\, \mathrm{tr}\, D + \theta \int \mathrm{tr}\, F$$

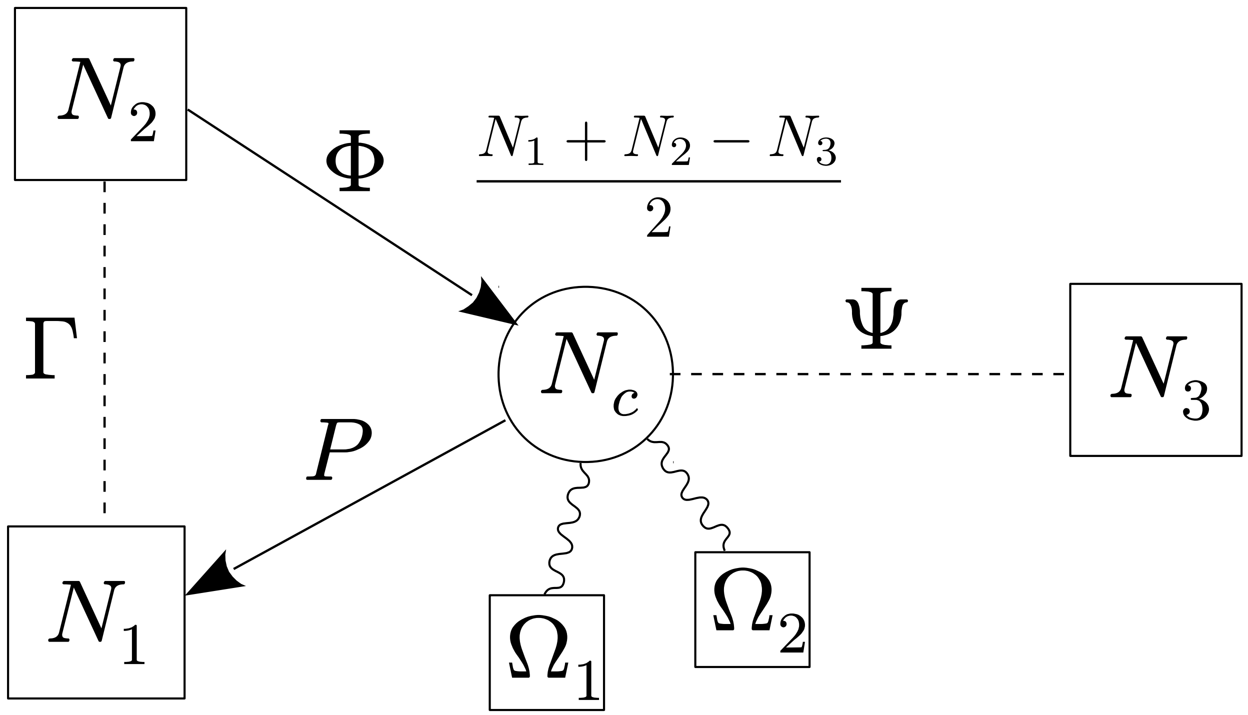


(0,2) SQCD



$=: \mathcal{T}_{N_1,N_2,N_3}$

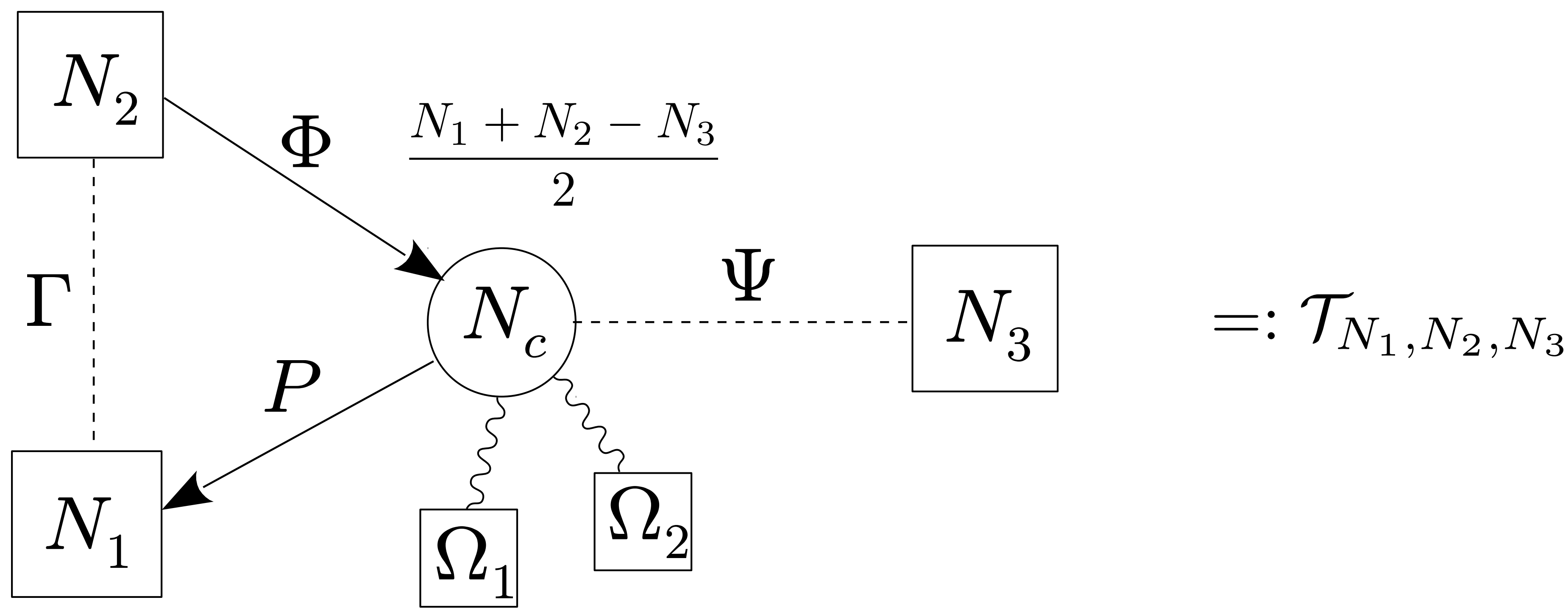
(0,2) SQCD



$$=: \mathcal{T}_{N_1, N_2, N_3}$$

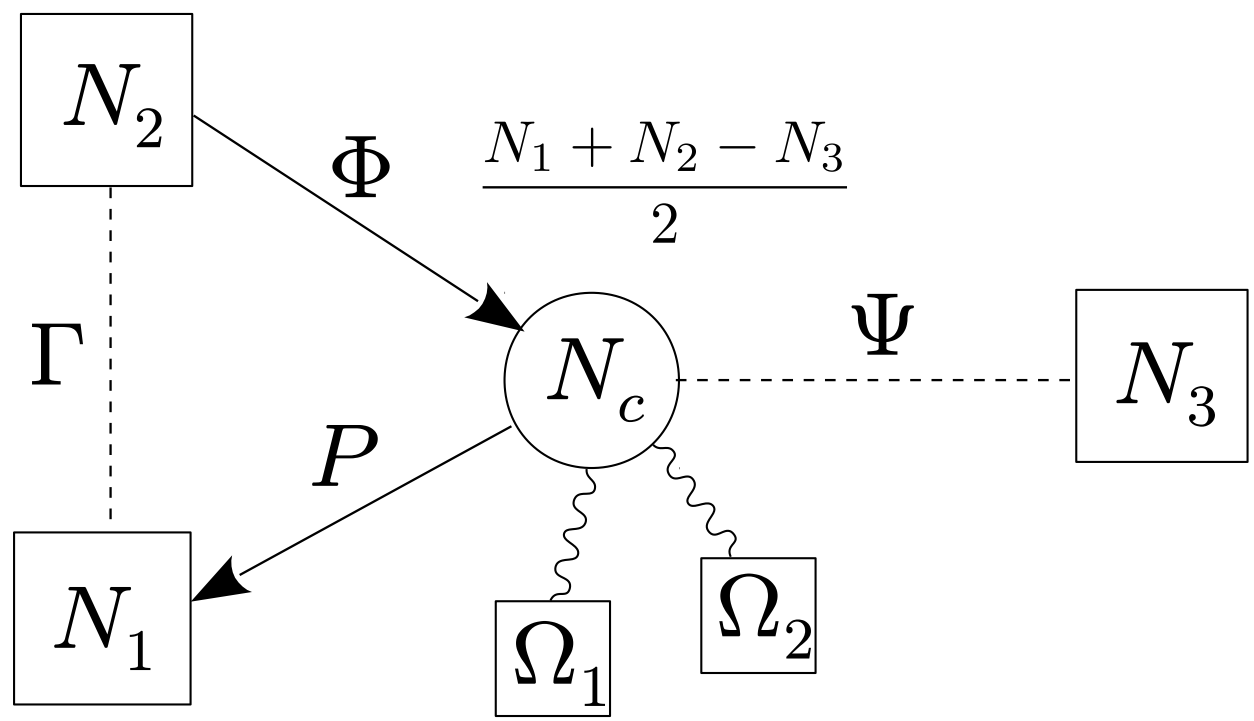
- Triality is invariance of the fixed point under permutations of N's

(0,2) SQCD



- Triality is invariance of the fixed point under permutations of N's
- The support is obtained by computing the elliptic genus

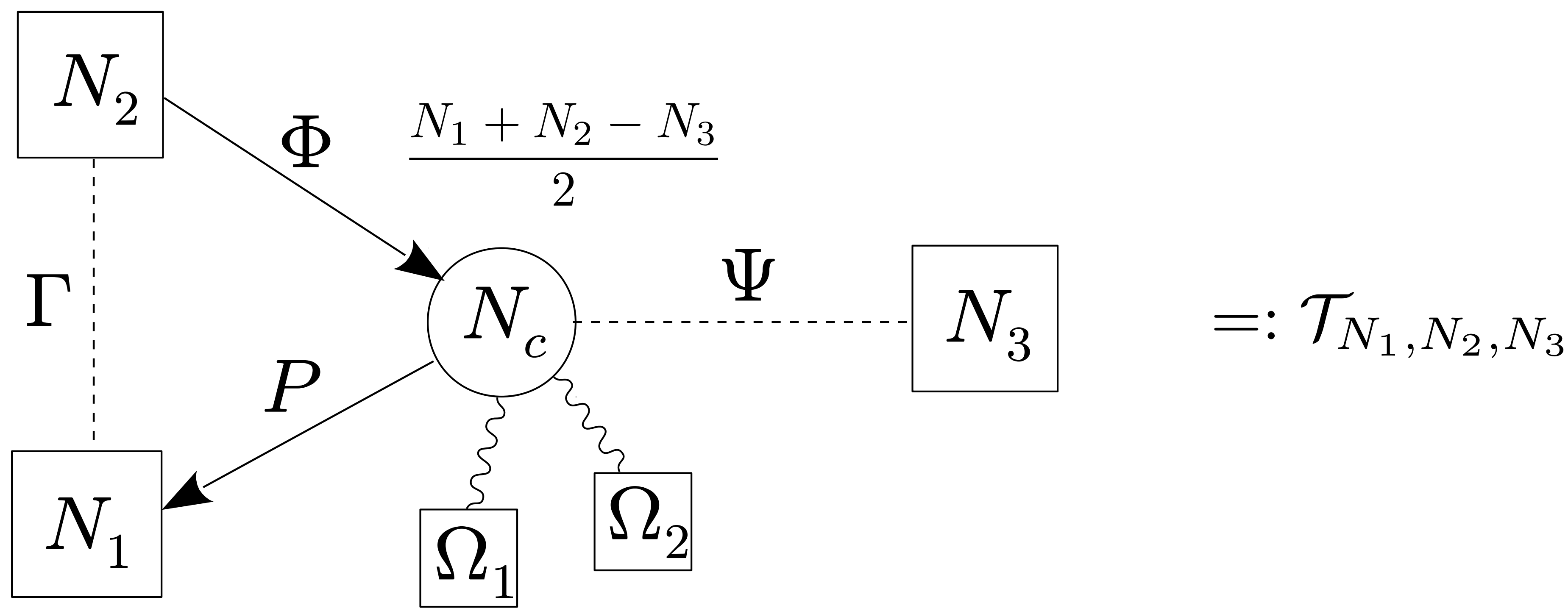
(0,2) SQCD



$=: \mathcal{T}_{N_1,N_2,N_3}$

- Triality is invariance of the fixed point under permutations of N's
- The support is obtained by computing the elliptic genus
- The IR fixed point can be solved explicitly to prove triality!

(0,2) SQCD



- Triality is invariance of the fixed point under permutations of N's
- The support is obtained by computing the elliptic genus
- The IR fixed point can be solved explicitly to prove triality!
- Two possible IR CFTs related to each other by charge conjugation

RG flow to sigma model

RG flow to sigma model

- Target space of NLSM

RG flow to sigma model

- Target space of NLSM
  - F term: $\Phi P = 0$
  - D term: $(|\Phi|^2 - |P|^2 = \zeta)/U(N_c)$

RG flow to sigma model

- Target space of NLSM
  - F term: $\Phi P = 0$
  - D term: $(|\Phi|^2 - |P|^2 = \zeta)/U(N_c)$
- Ψ engineers the tautological bundle S
- Γ engineers the "orthogonal" bundle Q

RG flow to sigma model

- Target space of NLSM
  - F term: $\Phi P = 0$
  - D term: $(|\Phi|^2 - |P|^2 = \zeta)/U(N_c)$
- Ψ engineers the tautological bundle S
- Γ engineers the "orthogonal" bundle Q

- Target space for $\zeta > 0$ $\quad S^{\oplus N_3} \oplus Q^{\oplus N_2} \to Gr(N_c, N_1)$

RG flow to sigma model

- Target space of NLSM
  - F term: $\Phi P = 0$
  - D term: $(|\Phi|^2 - |P|^2 = \zeta)/U(N_c)$
- Ψ engineers the tautological bundle S
- Γ engineers the "orthogonal" bundle Q

- Target space for $\zeta > 0$ $\quad S^{\oplus N_3} \oplus Q^{\oplus N_2} \to Gr(N_c, N_1)$
- Target space for $\zeta < 0$ $\quad S^{\oplus N_3} \oplus Q^{\oplus N_1} \to Gr(N_c, N_2)$

RG flow to sigma model

- Target space of NLSM
  - F term: $\Phi P = 0$
  - D term: $(|\Phi|^2 - |P|^2 = \zeta)/U(N_c)$
- Ψ engineers the tautological bundle S
- Γ engineers the "orthogonal" bundle Q

- Target space for $\zeta > 0$ $S^{\oplus N_3} \oplus Q^{\oplus N_2} \to Gr(N_c, N_1)$
- Target space for $\zeta < 0$ $S^{\oplus N_3} \oplus Q^{\oplus N_1} \to Gr(N_c, N_2)$

$Gr(k, N) \sim Gr(N-k, N)$

RG flow to sigma model

- Target space of NLSM
  - F term: $\Phi P = 0$
  - D term: $(|\Phi|^2 - |P|^2 = \zeta)/U(N_c)$
- Ψ engineers the tautological bundle S
- Γ engineers the "orthogonal" bundle Q

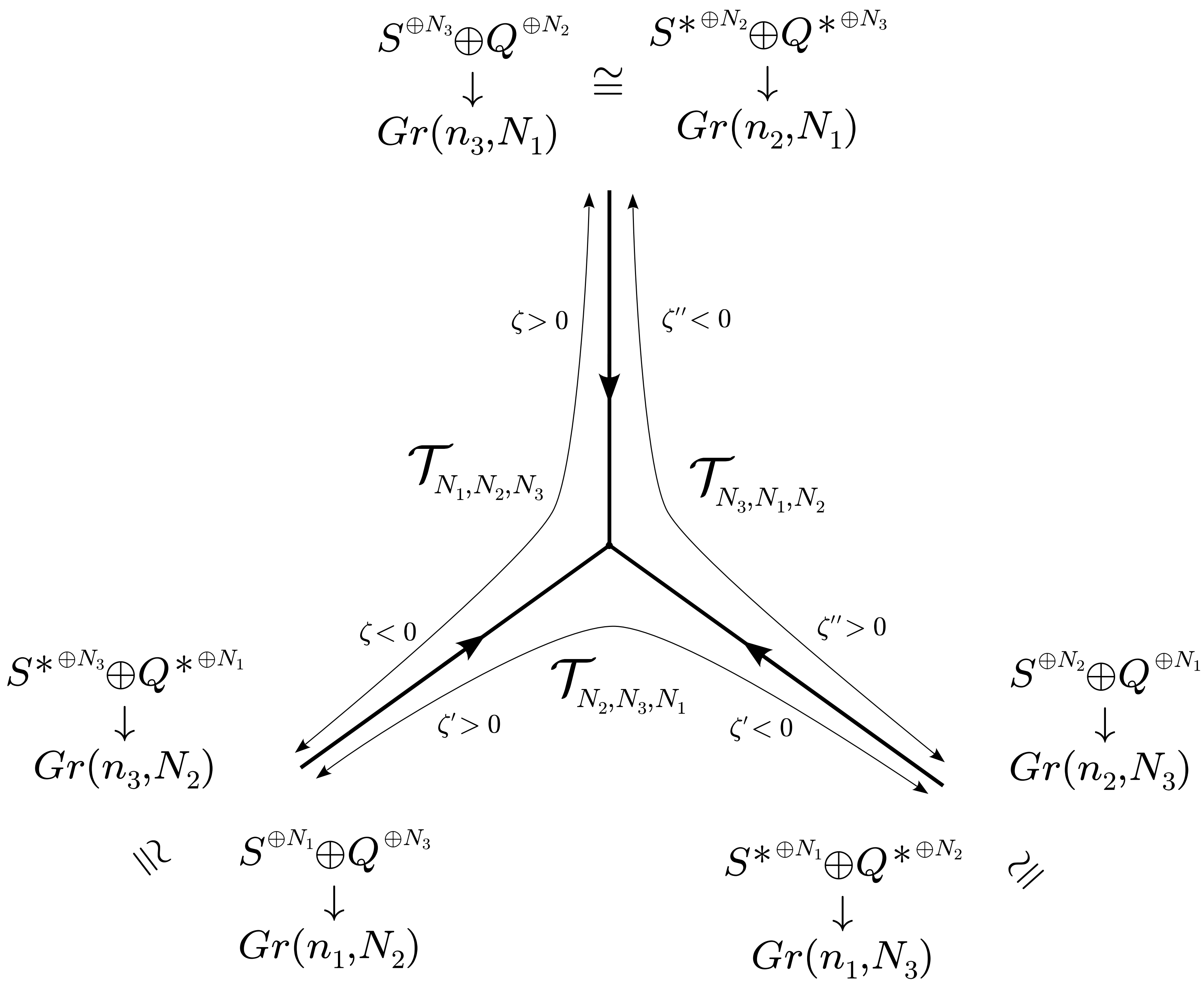
- Target space for $\zeta > 0$ $\quad S^{\oplus N_3} \oplus Q^{\oplus N_2} \to Gr(N_c, N_1)$
- Target space for $\zeta < 0$ $\quad S^{\oplus N_3} \oplus Q^{\oplus N_1} \to Gr(N_c, N_2)$

$Gr(k, N) \sim Gr(N-k, N)$

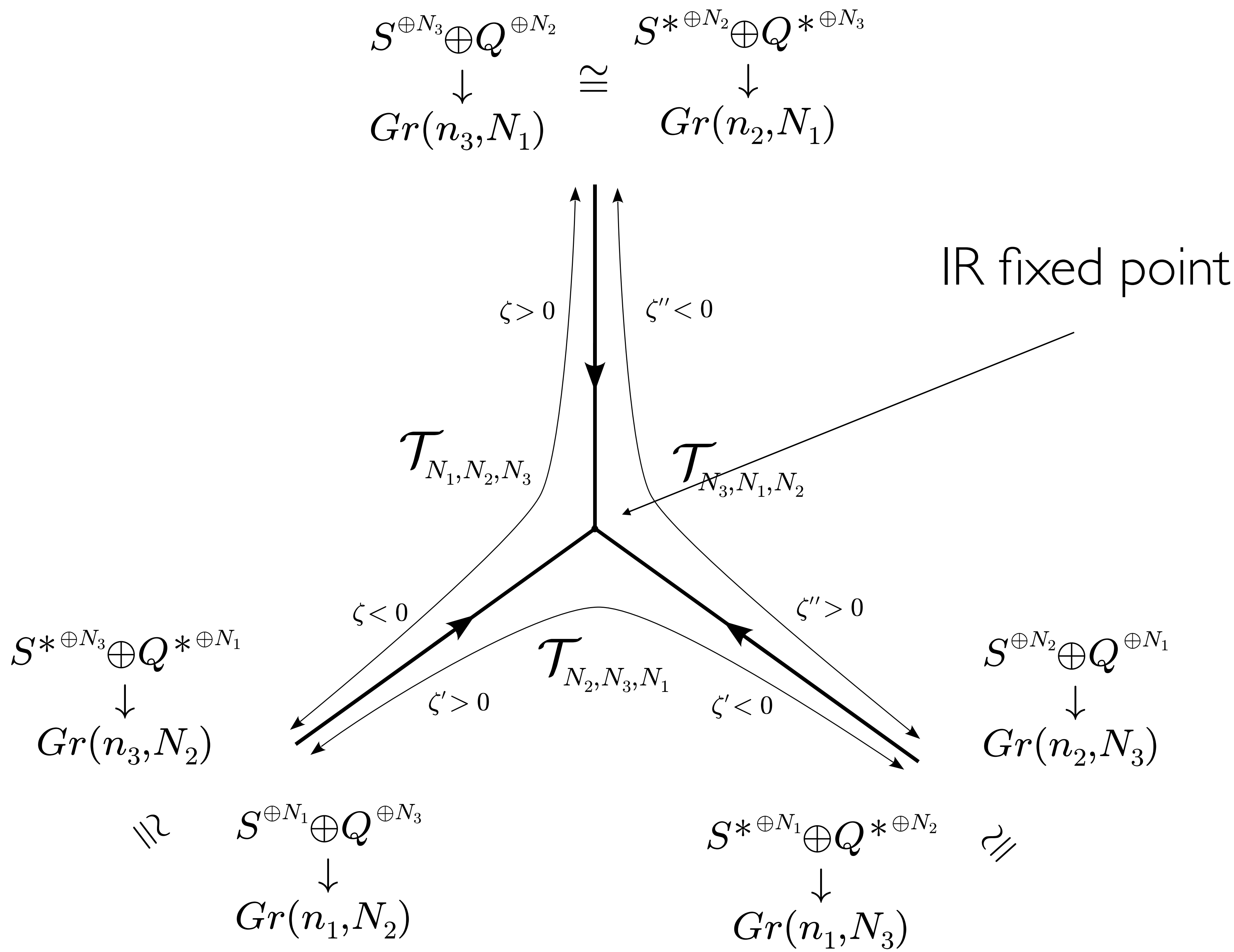
$$\wr$$

$$Q^{\oplus N_3} \oplus S^{\oplus N_2} \to Gr(N_2 - N_c, N_2)$$

Triality of sigma models

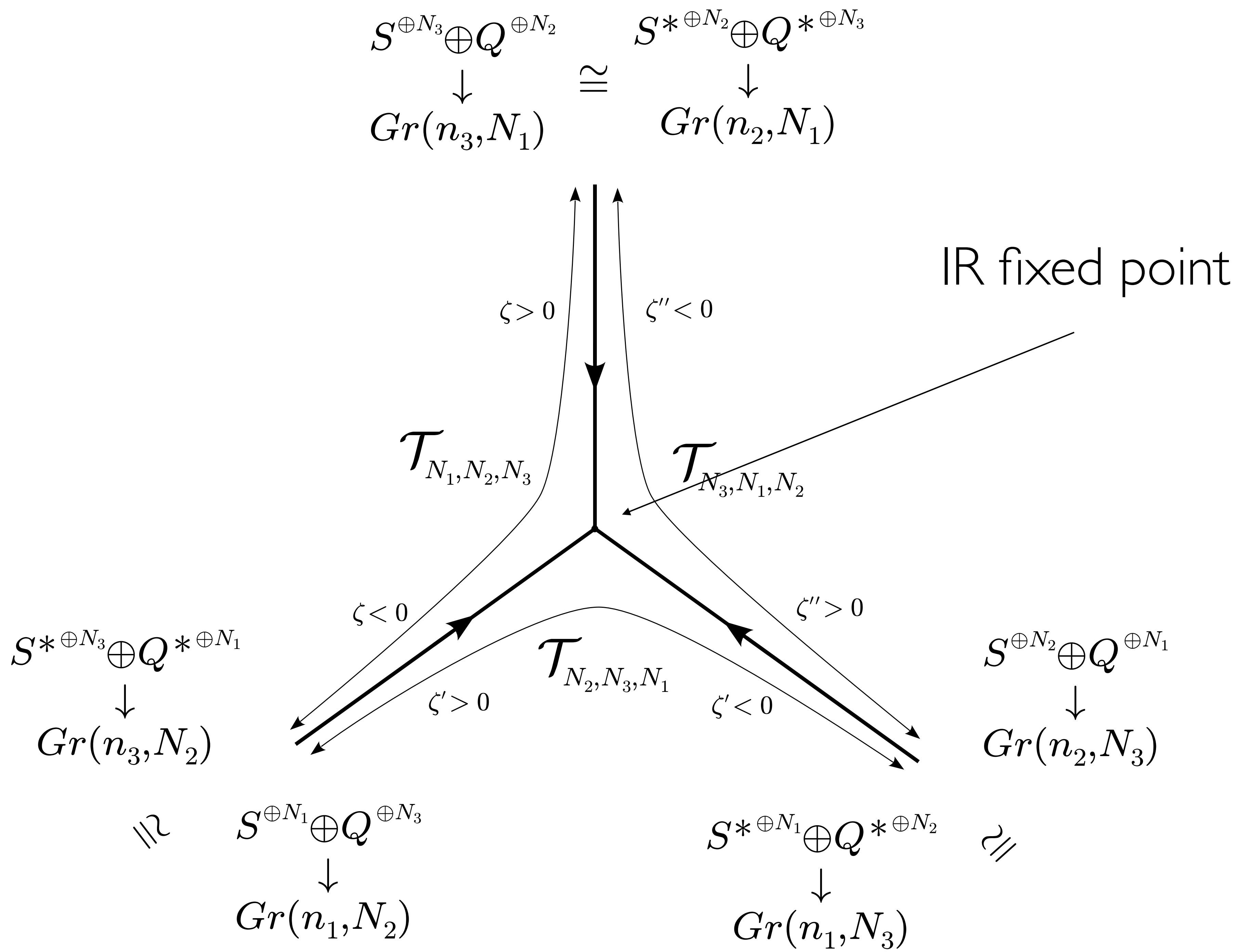


Triality of sigma models



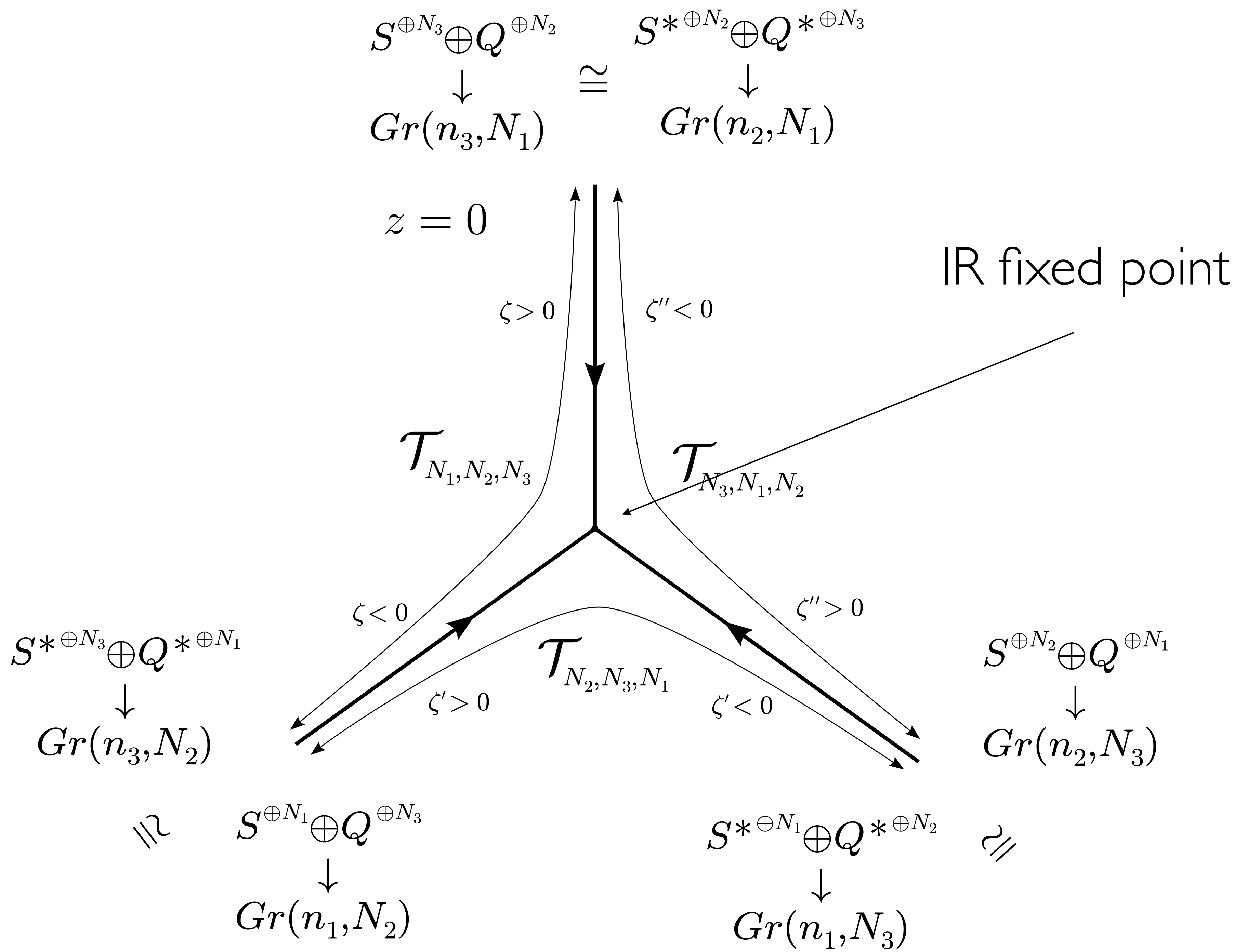
Triality of sigma models

- Let $z := e^{-t}$



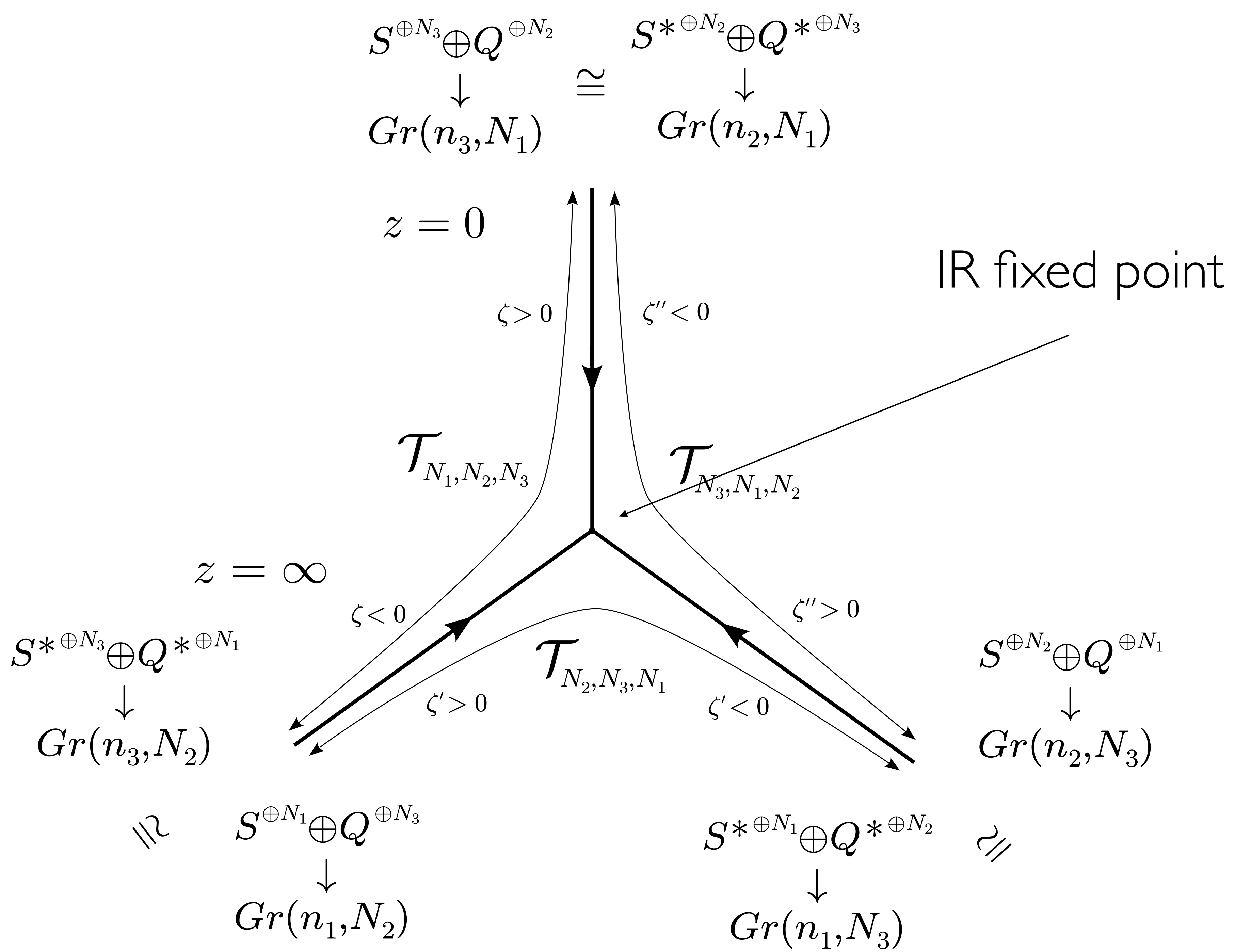
Triality of sigma models

- Let $z := e^{-t}$



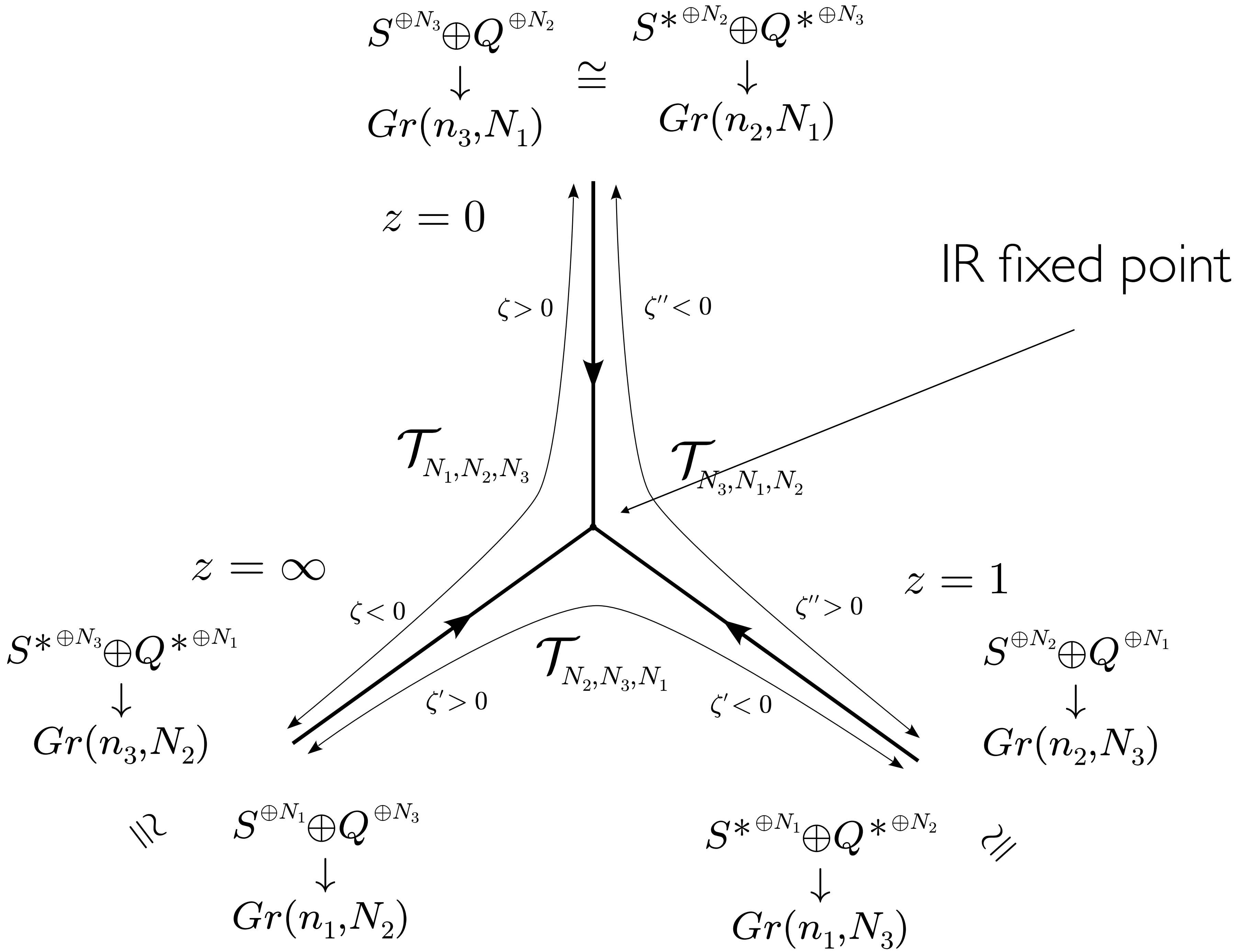
Triality of sigma models

- Let $z := e^{-t}$



Triality of sigma models

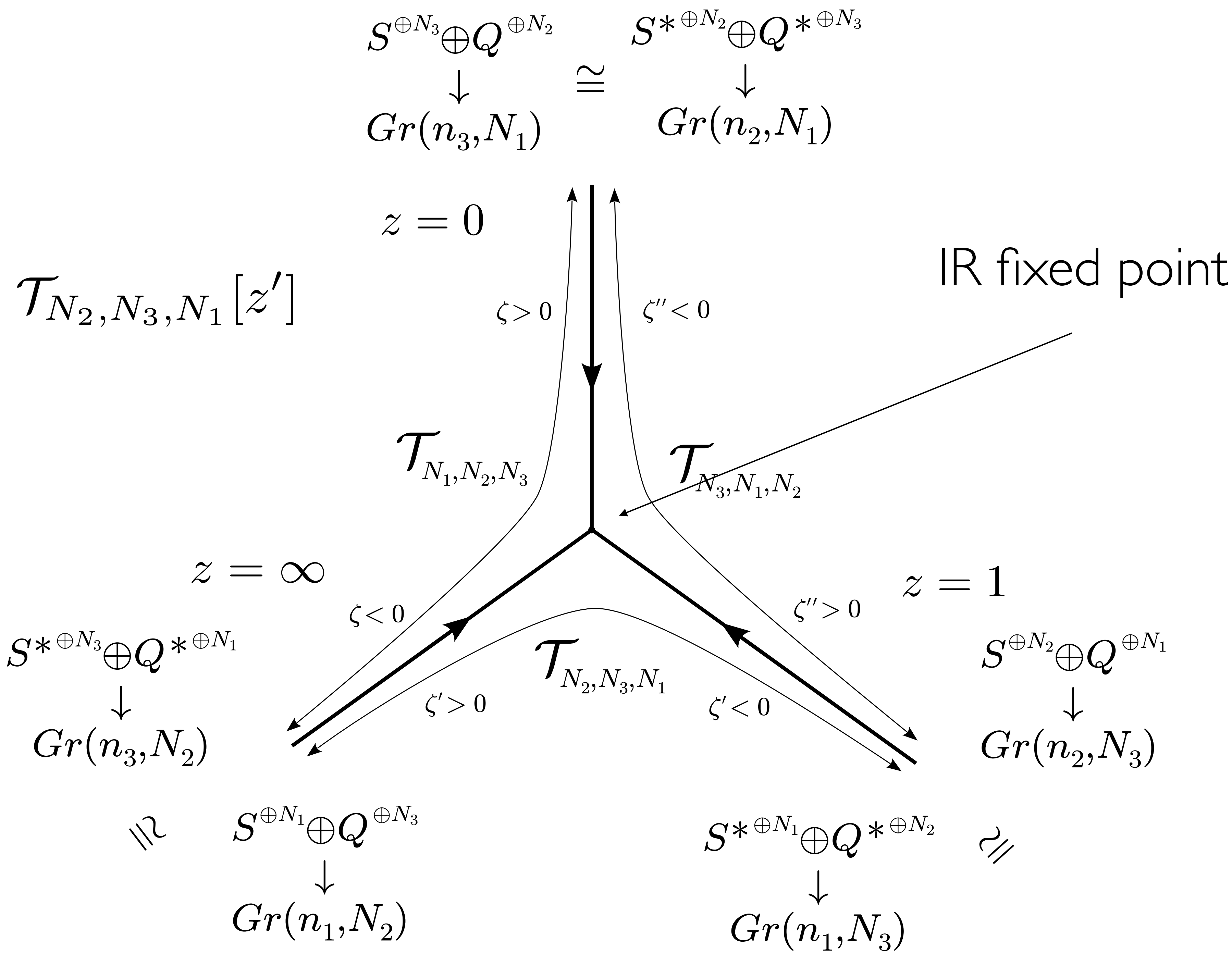
- Let $z := e^{-t}$



Triality of sigma models

- Let $z := e^{-t}$

  $\mathcal{T}_{N_1,N_2,N_3}[z] \sim \mathcal{T}_{N_2,N_3,N_1}[z']$

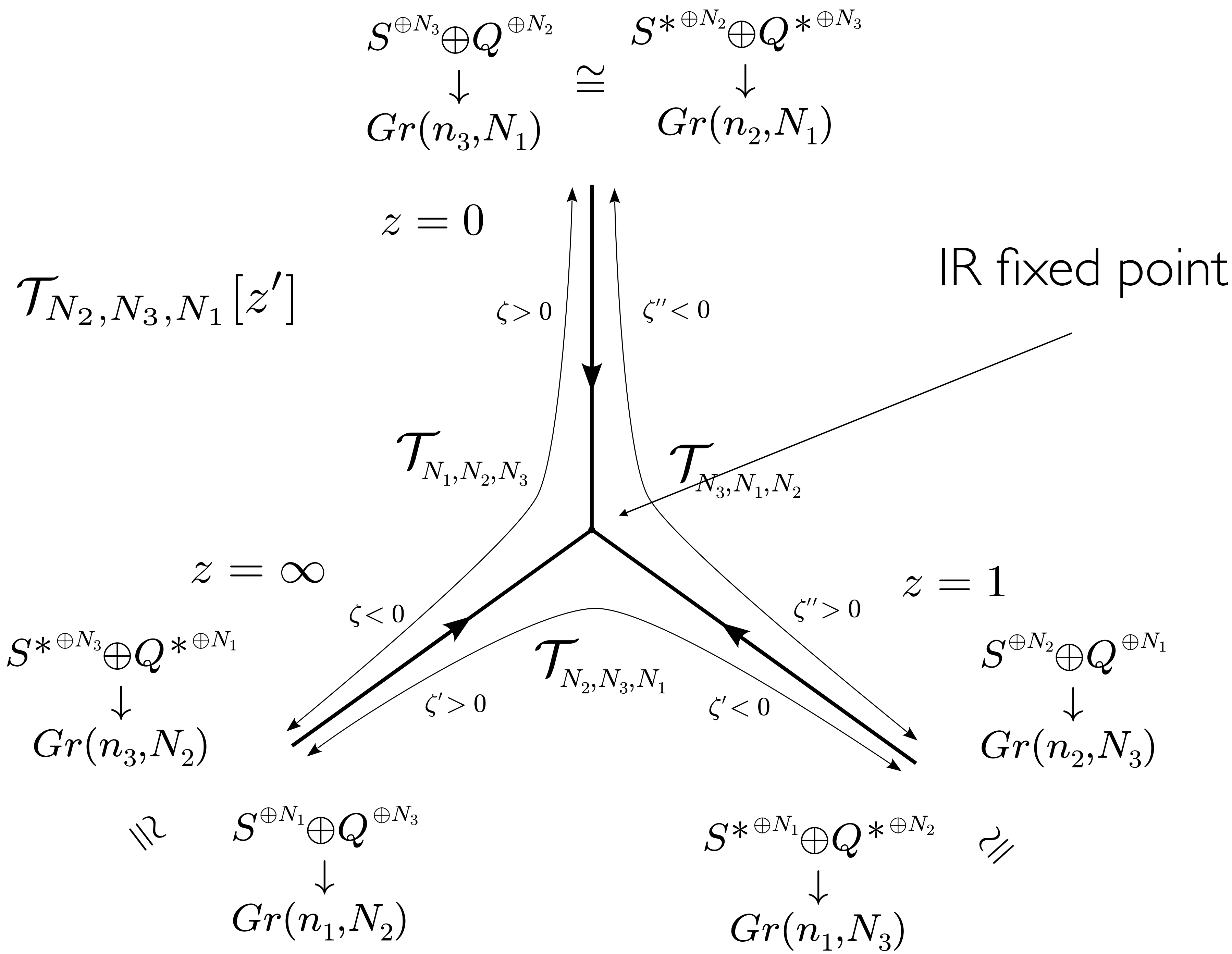


Triality of sigma models

- Let $z := e^{-t}$

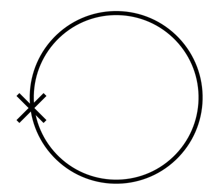
  $$\mathcal{T}_{N_1,N_2,N_3}[z] \sim \mathcal{T}_{N_2,N_3,N_1}[z']$$

  $$z' = \frac{1}{1-z}$$



One loop beta function

- The FI parameter runs due to the coupling $D|\phi|^2 - D|P|^2$
- Feynman diagram contributing to its one loop beta function is

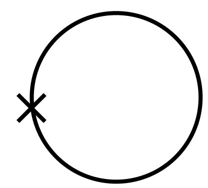


$$\sim \int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2}$$

- Only scalars run in the loop but when (say) Φ gets a vev P is massive and vice versa

One loop beta function

- The FI parameter runs due to the coupling $D|\phi|^2 - D|P|^2$
- Feynman diagram contributing to its one loop beta function is



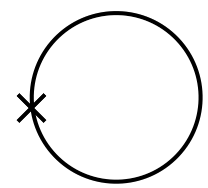
$$\sim \int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2}$$

- Only scalars run in the loop but when (say) Φ gets a vev P is massive and vice versa

- For $\zeta \to \infty$ $$\frac{d\zeta}{d\log\Lambda} = \frac{N_1}{2\pi}$$

One loop beta function

- The FI parameter runs due to the coupling $D|\phi|^2 - D|P|^2$
- Feynman diagram contributing to its one loop beta function is



$$\sim \int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2}$$

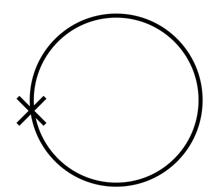
- Only scalars run in the loop but when (say) Φ gets a vev P is massive and vice versa

- For $\zeta \to \infty$ $$\frac{d\zeta}{d\log\Lambda} = \frac{N_1}{2\pi}$$

- For $\zeta \to -\infty$ $$\frac{d\zeta}{d\log\Lambda} = -\frac{N_2}{2\pi}$$

One loop beta function

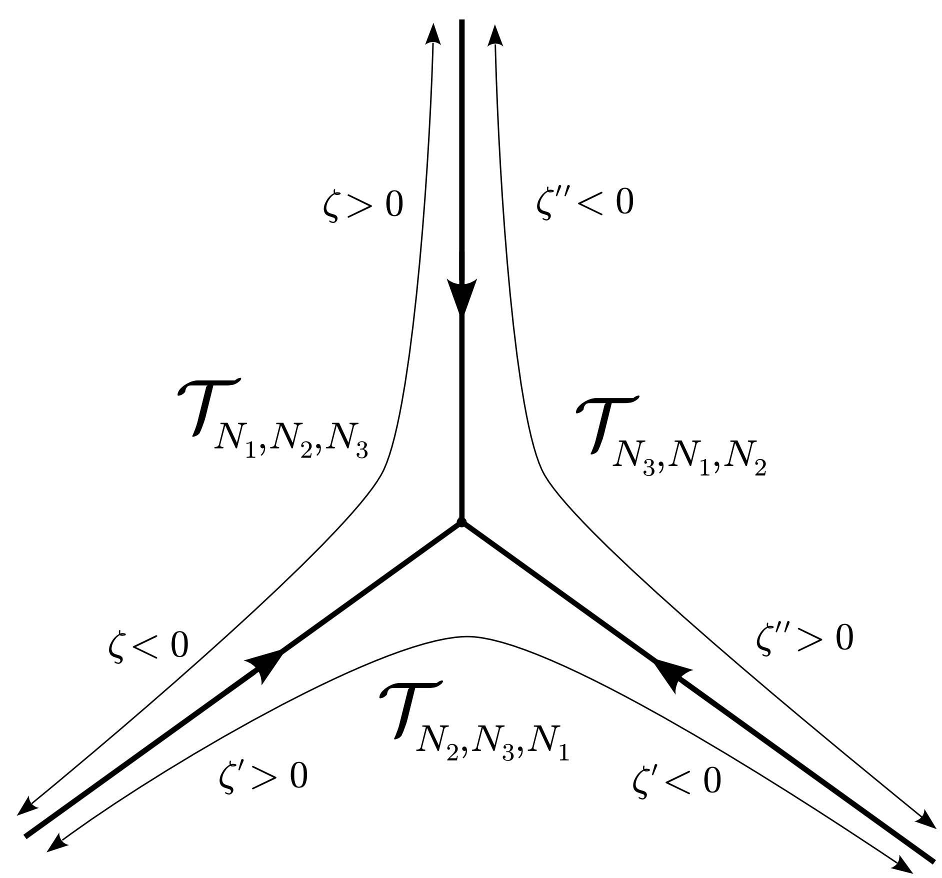
- The FI parameter runs due to the coupling $D|\phi|^2 - D|P|^2$
- Feynman diagram contributing to its one loop beta function is

 $$\sim \int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2}$$

- Only scalars run in the loop but when (say) Φ gets a vev P is massive and vice versa

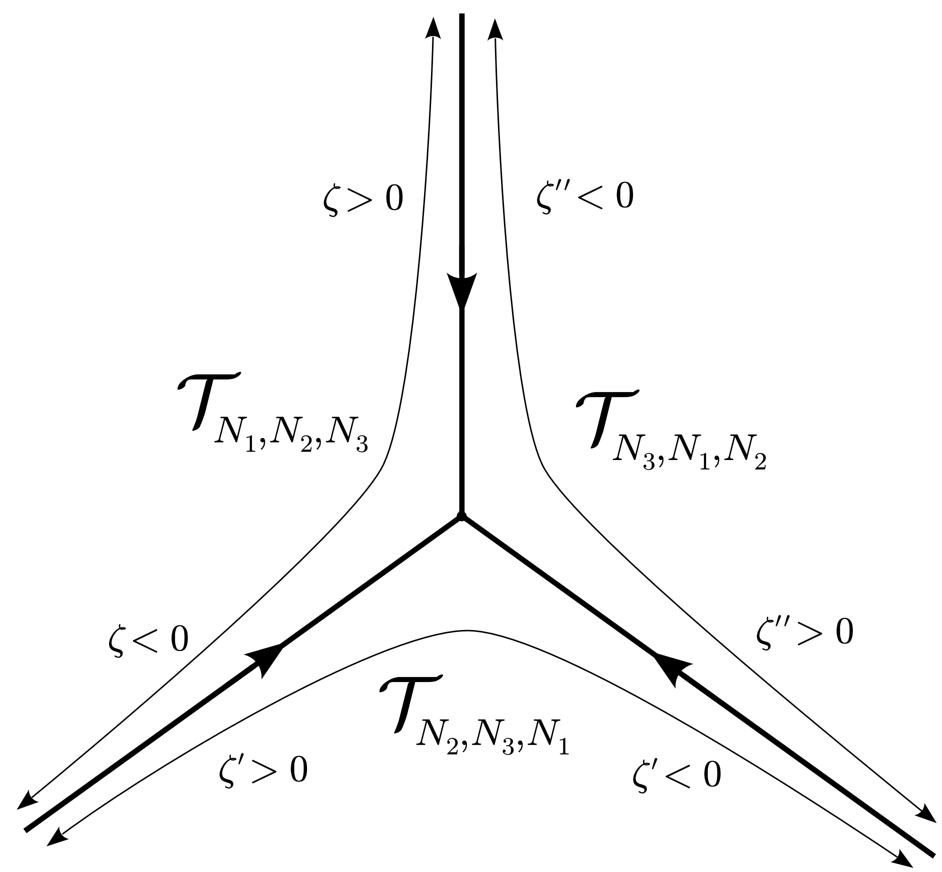
- For $\zeta \to \infty$ $$\frac{d\zeta}{d\log\Lambda} = \frac{N_1}{2\pi} \qquad d\log\Lambda = \frac{1}{N_1}\frac{dz}{z}$$
- For $\zeta \to -\infty$ $$\frac{d\zeta}{d\log\Lambda} = -\frac{N_2}{2\pi} \qquad d\log\Lambda = -\frac{1}{N_2}\frac{dz}{z}$$

Limits of beta function



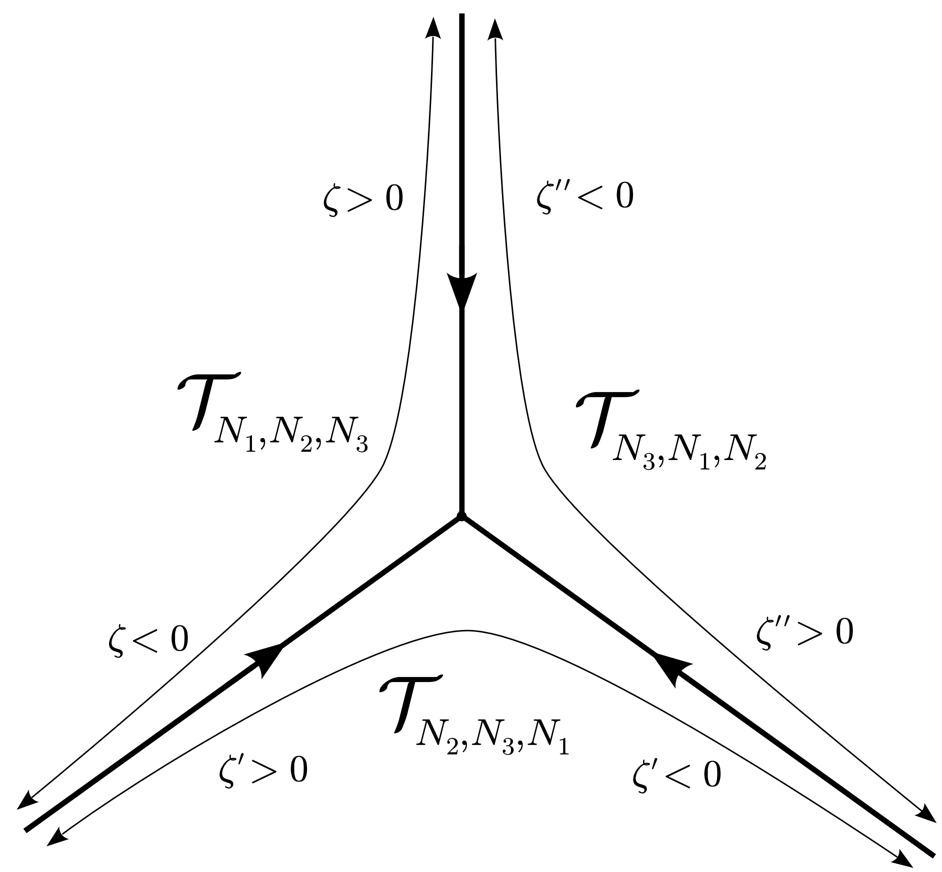
Limits of beta function

$$d \log \Lambda = \frac{1}{N_1} \frac{dz}{z}$$



Limits of beta function

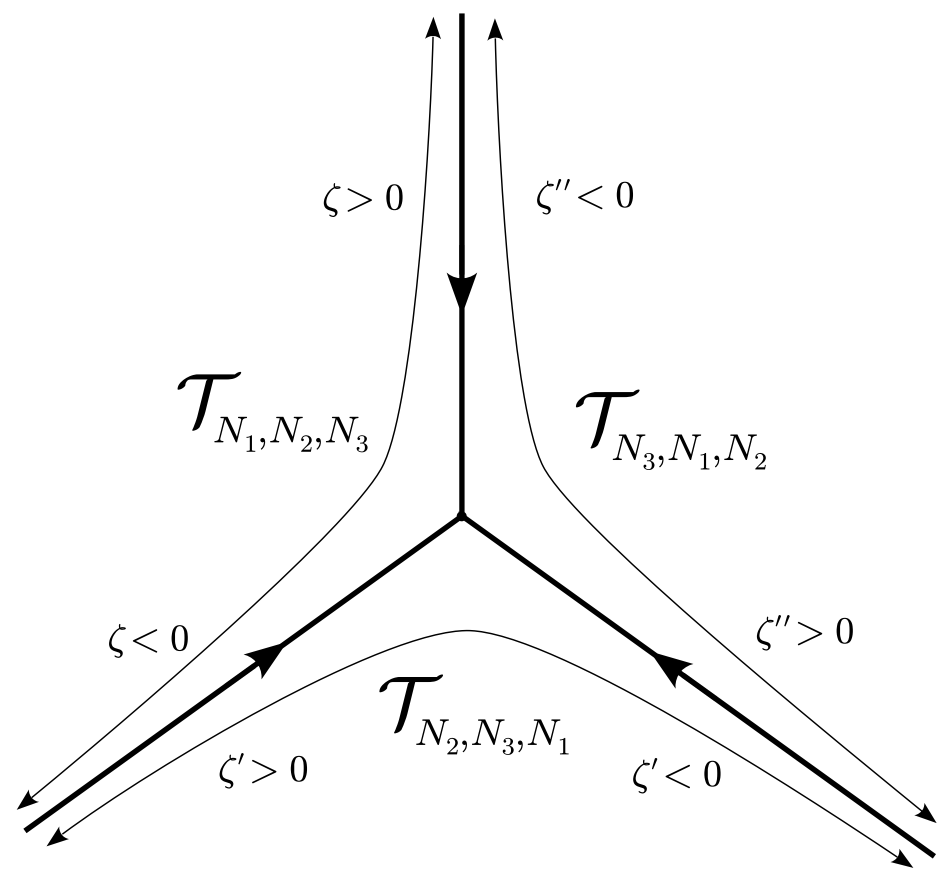
$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z}$$



$$d\log\Lambda = -\frac{1}{N_2}\frac{dz}{z}$$

Limits of beta function

$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z}$$



$$d\log\Lambda = -\frac{1}{N_2}\frac{dz}{z}$$

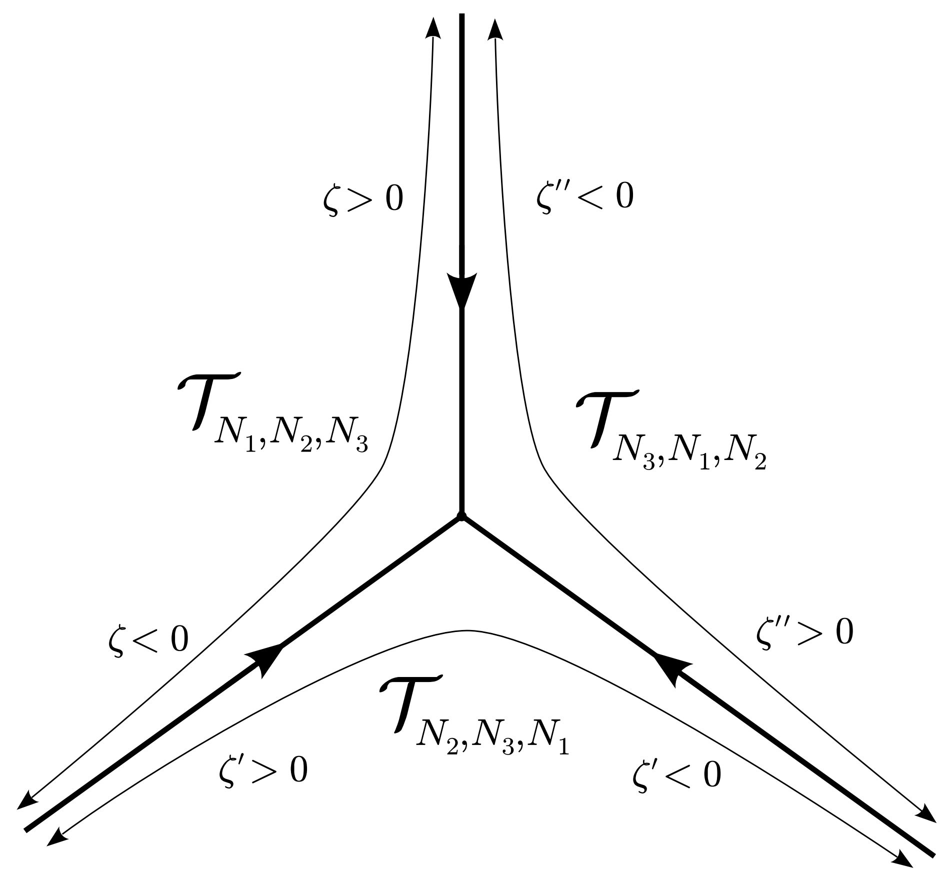
$$d\log\Lambda = \frac{1}{N_3}\frac{dz}{z-1}$$

Limits of beta function

- Three UV fixed points
- Poles are at

$$0, \infty, 1$$

$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z}$$



$$d\log\Lambda = -\frac{1}{N_2}\frac{dz}{z}$$

$$d\log\Lambda = \frac{1}{N_3}\frac{dz}{z-1}$$

Limits of beta function

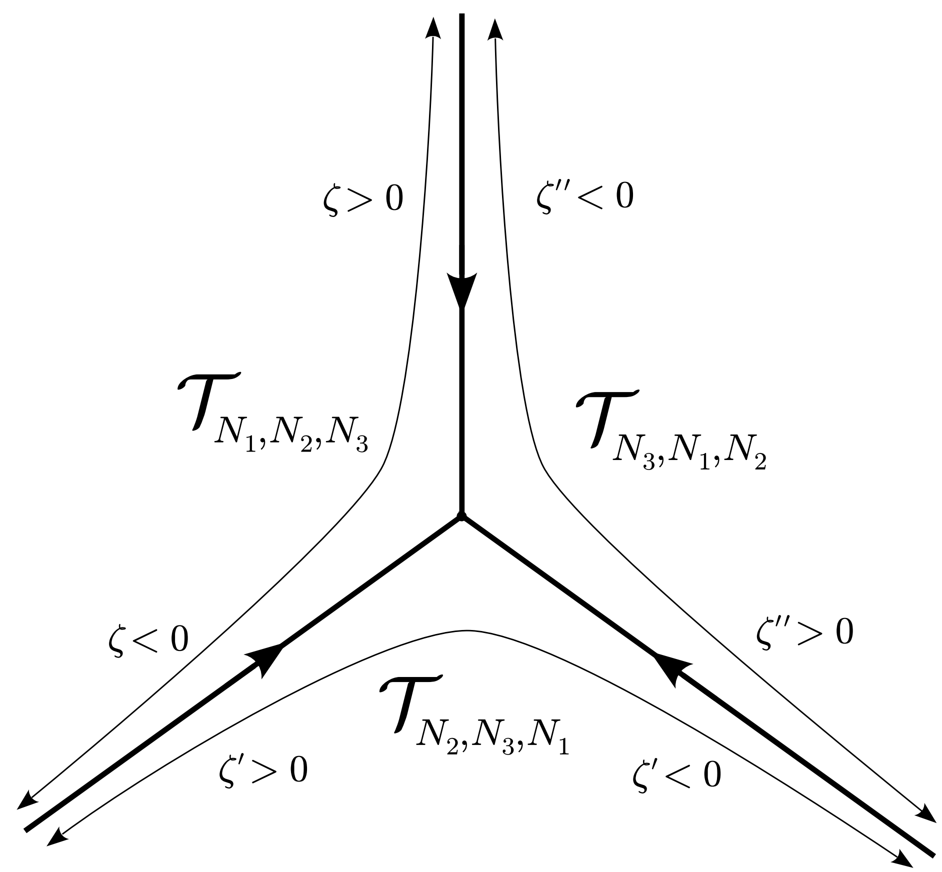
- Three UV fixed points
- Poles are at

  $$0, \infty, 1$$

- With residues

  $$\frac{1}{N_1}, \frac{1}{N_2}, \frac{1}{N_3}$$

$$d \log \Lambda = \frac{1}{N_1} \frac{dz}{z}$$



$$d \log \Lambda = -\frac{1}{N_2} \frac{dz}{z}$$

$$d \log \Lambda = \frac{1}{N_3} \frac{dz}{z-1}$$

Limits of beta function

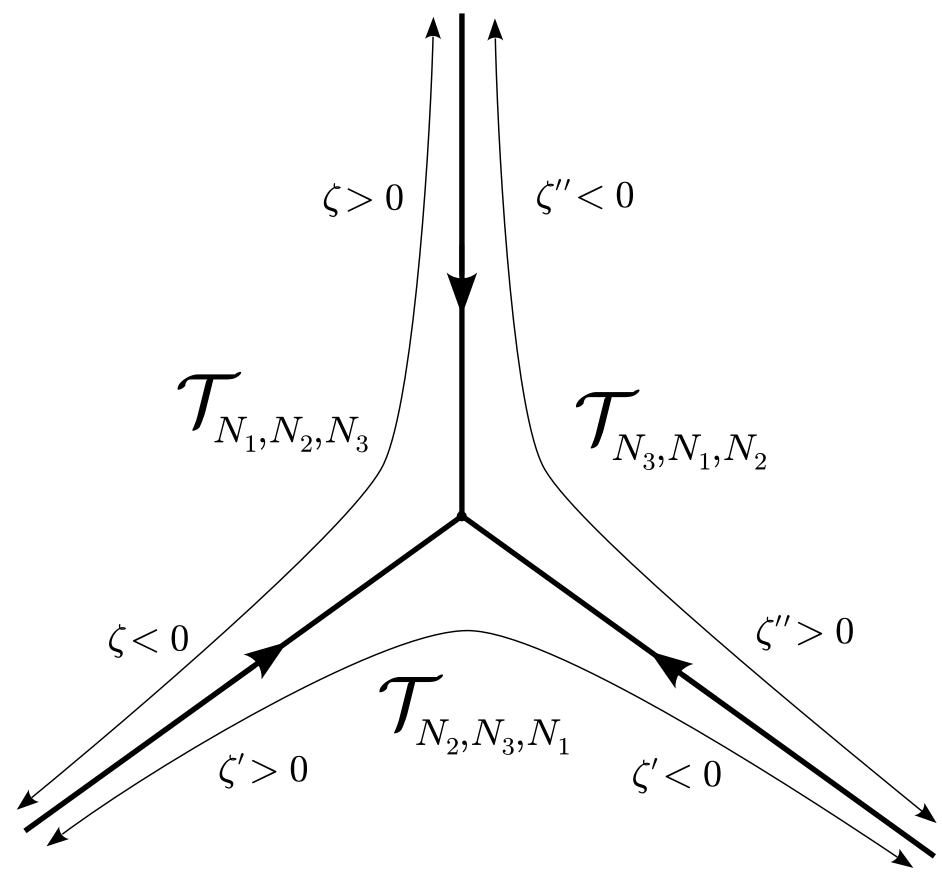
- Three UV fixed points
- Poles are at

  $0, \infty, 1$

- With residues

  $\frac{1}{N_1}, \frac{1}{N_2}, \frac{1}{N_3}$

$$d \log \Lambda = \frac{1}{N_1} \frac{dz}{z}$$



$$d \log \Lambda = -\frac{1}{N_2} \frac{dz}{z}$$

$$d \log \Lambda = \frac{1}{N_3} \frac{dz}{z-1}$$

- Two IR fixed points
- Total residue must be

  $-\left(\frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3}\right)$

The exact beta function

$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z} + \frac{1}{N_3}\frac{dz}{z-1}$$
$$-\frac{1}{2}\Big(\frac{1}{N}_1 + \frac{1}{N}_2 + \frac{1}{N}_3\Big)\Big(\frac{(1+x)dz}{z-a} + \frac{(1-x)dz}{z-b}\Big)$$

The exact beta function

$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z} + \frac{1}{N_3}\frac{dz}{z-1}$$

$$-\frac{1}{2}\Big(\frac{1}{N}_1 + \frac{1}{N}_2 + \frac{1}{N}_3\Big)\Big(\frac{(1+x)dz}{z-a} + \frac{(1-x)dz}{z-b}\Big)$$

- Require consistency with triality
- ie Invariance under $z \to \dfrac{1}{1-z}$ and cyclic permutation of N's
- x=1 and a, b are fixed points of equation $z = \dfrac{1}{1-z}$

The exact beta function

$$d\log\Lambda = \frac{1}{N_1}\frac{dz}{z} + \frac{1}{N_3}\frac{dz}{z-1}$$
$$-\frac{1}{2}\Big(\frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3}\Big)\Big(\frac{(1+x)dz}{z-a} + \frac{(1-x)dz}{z-b}\Big)$$

- Require consistency with triality
- ie Invariance under $z \to \frac{1}{1-z}$ and cyclic permutation of N's
- x=1 and a, b are fixed points of equation $z = \frac{1}{1-z}$

$$\boxed{\begin{aligned} d\log\Lambda &= \frac{1}{N_1}\frac{dz}{z} + \frac{1}{N_3}\frac{dz}{z-1} \\ &-\frac{1}{2}\Big(\frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3}\Big)\Big(\frac{dz}{z-e^{\frac{i\pi}{3}}} + \frac{dz}{z-e^{-\frac{i\pi}{3}}}\Big)\end{aligned}}$$

Renormalization group flow lines