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Some analytic results from conformal bootstrap

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Strings 2015, ICTS, Bangalore

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- Since the seminal work of Rattazzi, Rychkov, Tonni and Vichi in 2008, many interesting results have emerged using conformal bootstrap.
- Most of these impressive results rely heavily on (clever) numerics.
- I will talk about some analytic results that follow from conformal bootstrap.

- Work done with Apratim Kaviraj, Kallol Sen arXiv:1502.01437, 1504.00772 and in progress.

See Apratim's poster and cf Kallol's gong show talk

- Builds on work by Fitzpatrick, Kaplan, Poland, Simmons-Duffin; Komargodski, Zhiboedov.

- Related work by Alday, Bissi, Lukowski for N=4 SYM.

Summary of main results

- Given a (4d for most part) CFT with a scalar operator of dimension Δ_ϕ and a spin-2 (minimal) twist-2 operator there is an infinite sequence of large spin operators of dimension

$$\Delta = 2\Delta_\phi + 2n + \ell + \gamma(n,\ell)$$ Anomalous dim.

$\ell \gg n \gg 1$ $$\gamma(n,\ell) = -\frac{160}{c_T}\frac{n^4}{\ell^2}$$ UNIVERSAL

$$\langle T_{ab}(x) T_{cd}(x') \rangle = \frac{c_T}{|x-x'|^{2d}} \mathcal{I}_{ab,cd}(x-x')$$

$n \gg \ell \gg 1$ $$\gamma(n,\ell) = -\frac{80}{c_T}\frac{n^3}{\ell}$$?UNIVERSAL?
assume large N

- For large N, we can think of these operators as double trace operators of the form

$$O_1 \partial_{\mu_1} \cdots \partial_{\mu_\ell} (\partial^2)^n O_1$$

Heemskerk, Penedones, Polchinski, Sully; El-Showk, Papadodimas

- However the CFT bootstrap analysis of course only yields conformal dimension, spin and the OPE coefficients and not the precise form of these operators.

Why is this interesting?

- Result is universal. Does not depend on lagrangian or the dimension of the seed operator. Just assumes twist gap of these operators from other operators in the spectrum.

- Anomalous dimension of double trace operators is related to bulk Shapiro time delay. Sign of anomalous dimension is related to causality. Interplay between unitarity of CFT and causality of bulk.

  Camanho, Edelstein, Maldacena, Zhiboedov

- Can be extended to arbitrary (eg. 3d) dimensions. May be relevant for 3d Ising model at criticality.

  El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi

- Can compare with AdS/CFT. Two different ways to calculate the anomalous dimensions
  a) Eikonal approximation of 2-2 scattering. Cornalba, Costa, Penedones, Schiappa
  b) Energy shift in a black hole background. Fitzpatrick, Kaplan, Walters; Kaviraj, Sen, AS
- Turns out that the result matches exactly with the AdS/CFT prediction. Kaviraj, Sen, AS

Quick review of bootstrap

"s-channel" "t-channel"

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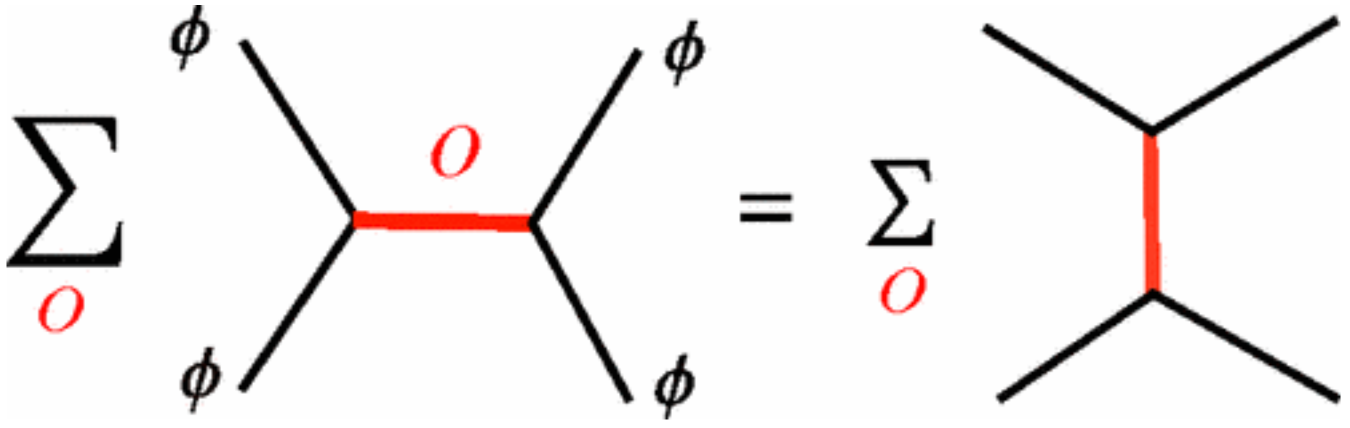

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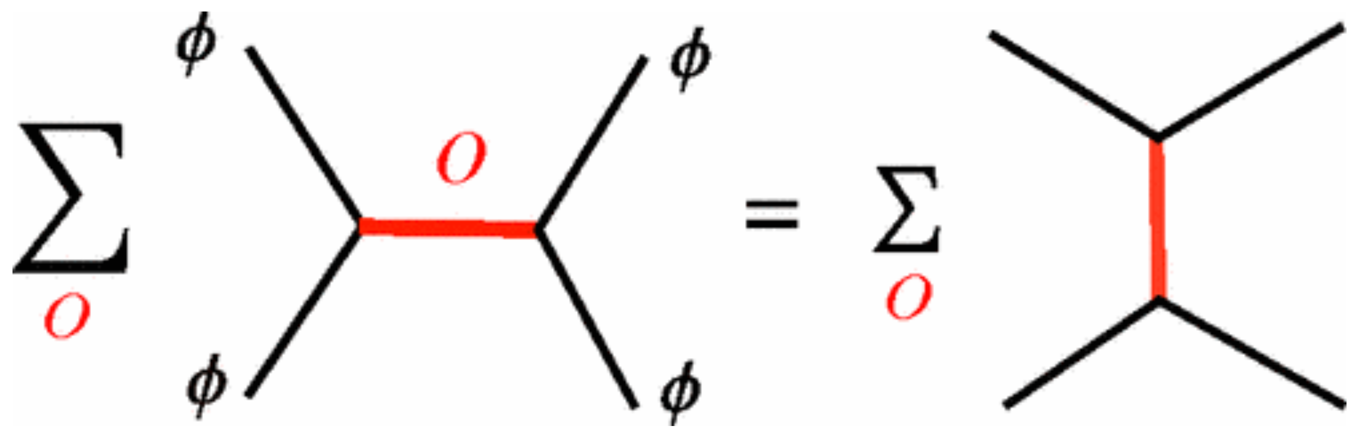

"s-channel" "t-channel" even spin

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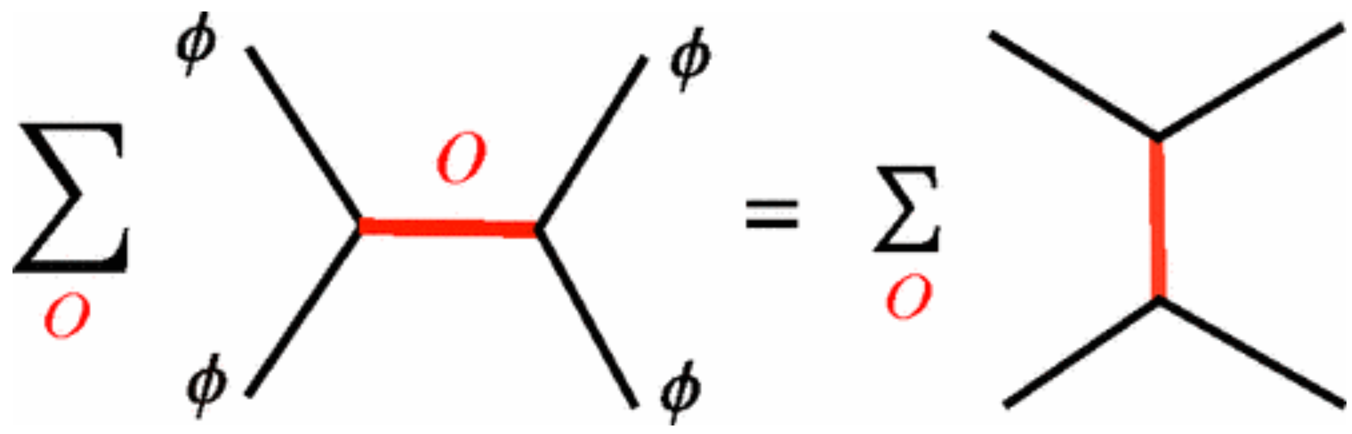

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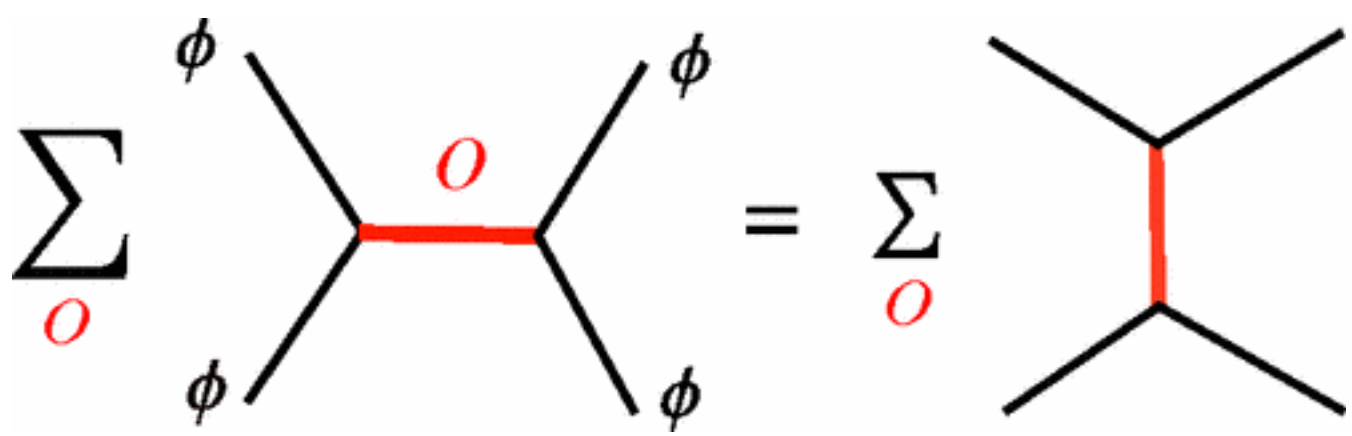

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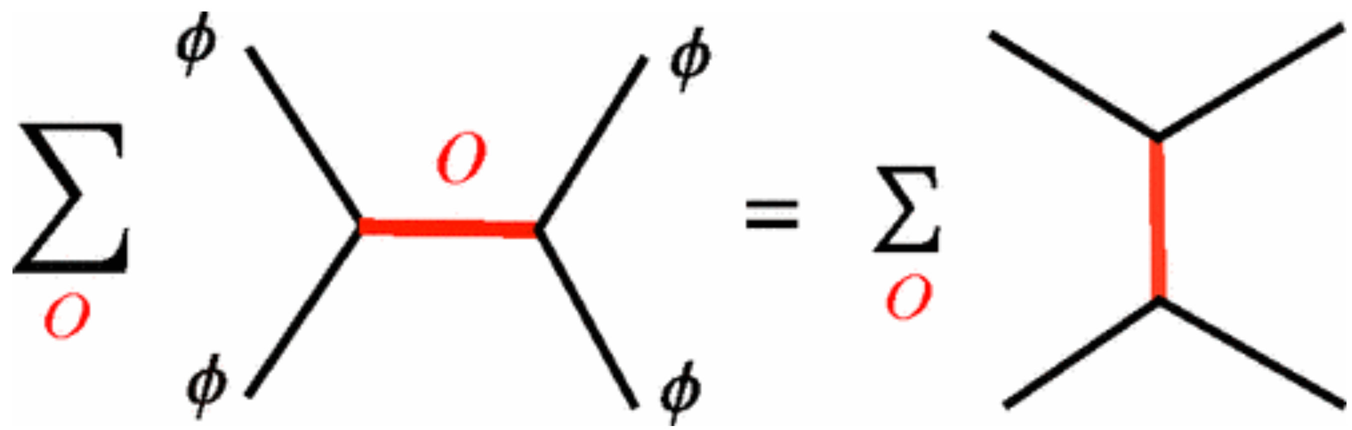

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Can only be reproduced upon considering large spin operators on the RHS

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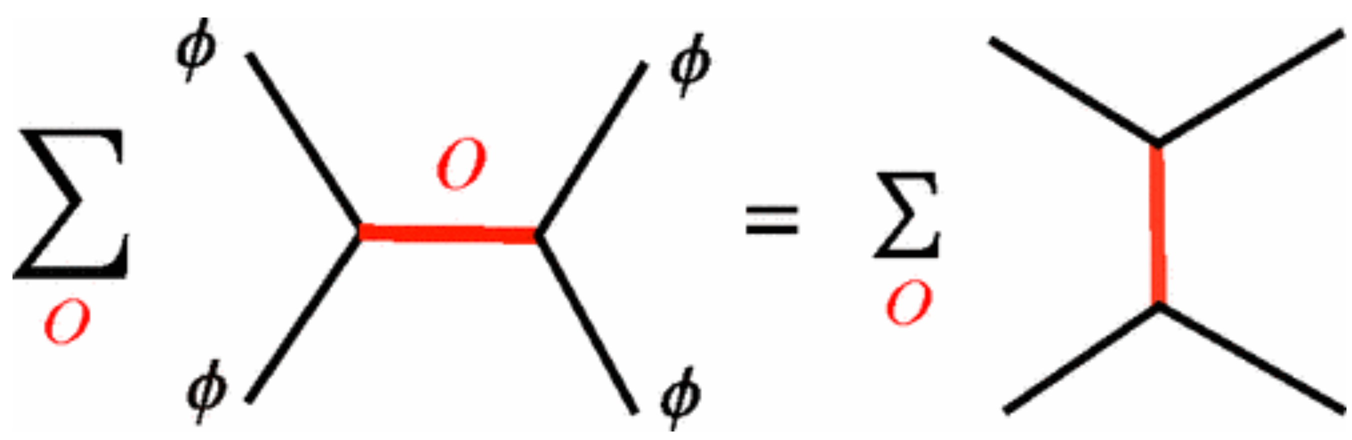

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$$u = \frac{x_{12}^2 x_{34}^2}{x_{24}^2 x_{13}^2}\,, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{24}^2 x_{13}^2}$$

Conformal cross ratios

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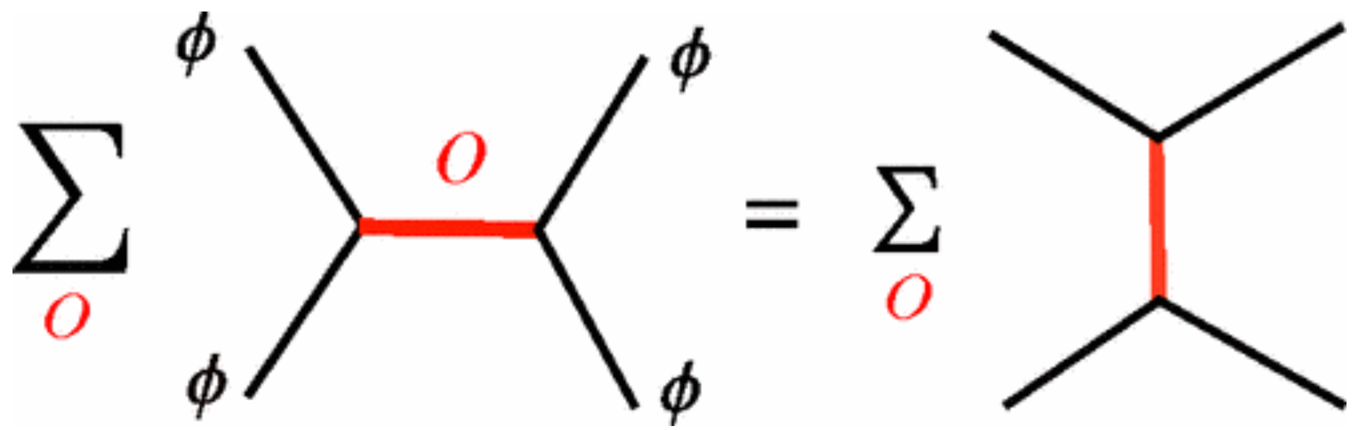

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$u \leftrightarrow v$

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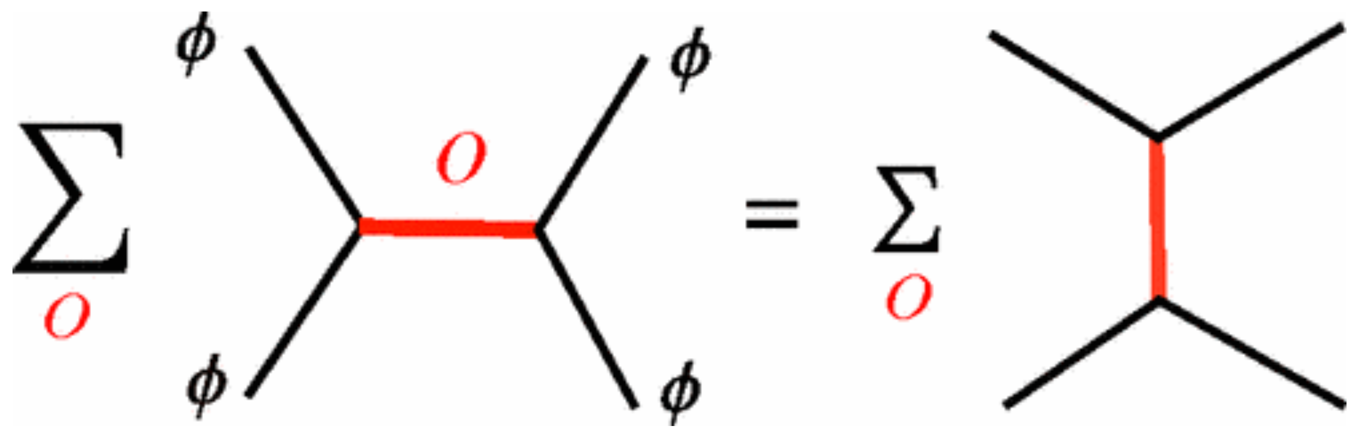

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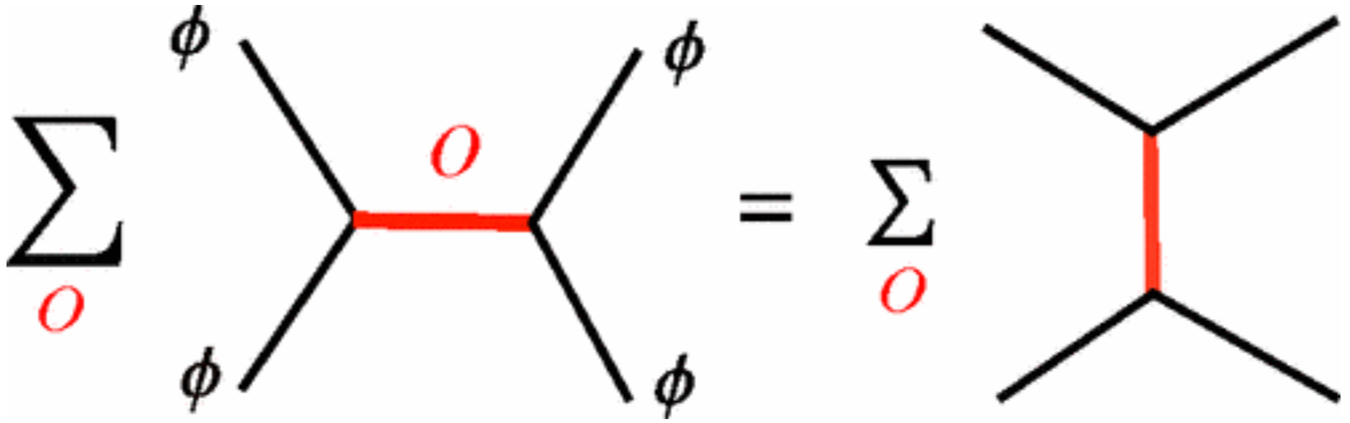

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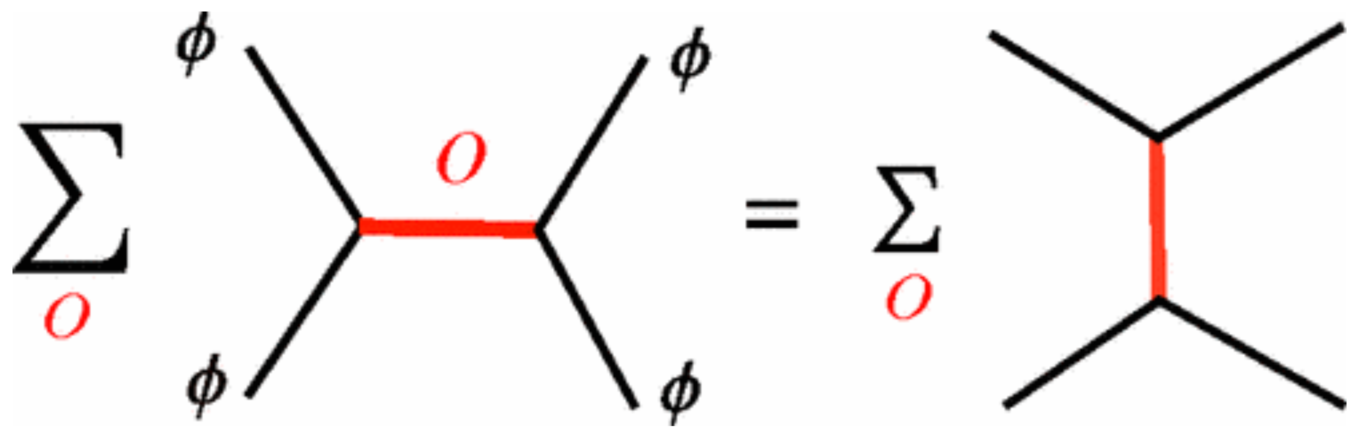

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$P_{\tau,\ell}$ OPE x OPE

$g_{\tau,\ell}(u,v)$ Dolan, Osborn; Blocks

Closed form expressions for conformal blocks are known only in even dimensions.

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"factorizes" (twist,spin,v) x (twist, u, d)

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Recursion relations for blocks in any dimension

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$$- \frac{4(d-\Delta-3)(d-\Delta-2)}{d-2\Delta-2)(d-2\Delta)} \left[\frac{(\Delta+\ell)^2}{16(\Delta+\ell-1)(\Delta+\ell+1)} g^{(d-2)}_{\Delta,\ell+2}(v,u)\right.$$
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$$(1-v)^2 F^{(d)}(\tau,v) = 16F^{(d-2)}(\tau-4,v) - 2vF^{(d-2)}(\tau-2,v) + \frac{(d-\tau-2)^2}{16(d-\tau-3)(d-\tau-1)} v^2 F^{(d-2)}(\tau,v)\,.$$

New results from bootstrap

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Gauss Hypergeometric

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$$F^{(d)}(\tau, v) = \frac{2^{\tau}}{(1-v)^{\frac{d-2}{2}}}\, {}_2F_1\left(\frac{1}{2}(\tau - d + 2), \frac{1}{2}(\tau - d + 2), (\tau - d + 2), v\right)$$

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Bootstrap equation demands at leading order

$$1 \approx (\text{function of u}) \times v^{\tau/2-\Delta_\phi}(1-v)^{\Delta_\phi} F^{(d)}(\tau, v)$$

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To match powers of v, we must have

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Fitzpatrick et al; Komargodski, Zhiboedov

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Fitzpatrick et al; Komargodski, Zhiboedov

Same as what appears in MFT. OPE's known.

$$\Delta = 2\Delta_\phi + 2n + \ell + \gamma(n,\ell)$$

We can go to subleading order

$$P_m = P_{MFT} + c(n,\ell)$$

It can be shown that the anomalous dimension at large spin goes like an inverse power of the spin for spacetime dimension>2.

This means that we can treat the inverse spin as an expansion parameter and this result is true even for theories which do not have a "large N".

Our objective is to determine the n-dependence for the anomalous dimension.

After some clever detective work we find

τ_m Twist of exchanged operator

$$\gamma(n,\ell)\ell^{\tau_m} = \sum_{m=0}^{n} C^{(d)}_{n,m} B^{(d)}_m$$

nested sums

$$C^{(d)}_{n,m} = \frac{(-1)^{m+n}}{8}\left(\frac{\Gamma[\Delta_\phi]}{(\Delta_\phi - d/2+1)_m}\right)^2 \frac{n!}{m!(n-m)!}(2\Delta_\phi + n + 1 - d)_m$$

$$B^{(d)}_k = -\frac{16 P_m \Gamma[\tau_m + 2\ell_m]\Gamma[\tau_m/2 + \ell_m + k]^2}{\Gamma[1+k]^2\Gamma[\tau_m/2+\ell_m]^4}$$

$$\times {}_3F_2\left(-k,-k,-\frac{\tau_m}{2} - \ell_m - \frac{d-2}{2} + \Delta_\phi; 1-\ell_m - \frac{\tau_m}{2} - k, 1 - \ell_m - \frac{\tau_m}{2} - k; 1\right)$$

${}_3F_2$: Since k is a positive integer, this is a polynomial.

Progress is possible in 4d (and similar techniques apply in even d)

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$$ {}_3F_2(-m, -m-4+\Delta_\phi; -2-m, -2-m; 1) = \sum_{k=0}^{m} \frac{(m+1-k)^2(m+2-k)^2}{(m+1)^2(m+2)^2} \frac{\Gamma[\Delta_\phi - 4 + k]}{\Gamma[k+1]\Gamma[\Delta_\phi - 4]} $$

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$$

Use basic defn

$$
=\frac{4[6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)]}{(m+1)(m+2)\Gamma[m+3]}\frac{\Gamma[m+\Delta_\phi-1]}{\Gamma[\Delta_\phi+1]}
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To reduce to one sum

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To reduce to one sum

$$ \gamma(n,\ell)\ell^2=-(-1)^n\frac{80}{3c_T}\sum_{m=0}^{n}(-1)^m[6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)] \times\frac{\Gamma[\Delta_\phi]\Gamma[n+1]\Gamma[2\Delta_\phi+m+n-3]}{\Gamma[m+1]\Gamma[n-m+1]\Gamma[\Delta_\phi+m-1]\Gamma[2\Delta_\phi+n-3]} $$

Progress is possible in 4d (and similar techniques apply in even d)

$$ {}_3F_2(-m,-m-4+\Delta_\phi;-2-m,-2-m;1) = \sum_{k=0}^{m} \frac{(m+1-k)^2(m+2-k)^2}{(m+1)^2(m+2)^2} \frac{\Gamma[\Delta_\phi-4+k]}{\Gamma[k+1]\Gamma[\Delta_\phi-4]} $$

Use basic defn

$$ = \frac{4[6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)]}{(m+1)(m+2)\Gamma[m+3]} \frac{\Gamma[m+\Delta_\phi-1]}{\Gamma[\Delta_\phi+1]} $$

To reduce to one sum

$$ \gamma(n,\ell)\ell^2 = -(-1)^n \frac{80}{3c_T} \sum_{m=0}^{n} (-1)^m [6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)] $$

$$ \times \frac{\Gamma[\Delta_\phi]\Gamma[n+1]\Gamma[2\Delta_\phi+m+n-3]}{\Gamma[m+1]\Gamma[n-m+1]\Gamma[\Delta_\phi+m-1]\Gamma[2\Delta_\phi+n-3]} $$

Using the following:

Progress is possible in 4d (and similar techniques apply in even d)

$$ {}_3F_2(-m,-m-4+\Delta_\phi;-2-m,-2-m;1)=\sum_{k=0}^{m}\frac{(m+1-k)^2(m+2-k)^2}{(m+1)^2(m+2)^2}\frac{\Gamma[\Delta_\phi-4+k]}{\Gamma[k+1]\Gamma[\Delta_\phi-4]} $$

Use basic defn

$$ =\frac{4[6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)]}{(m+1)(m+2)\Gamma[m+3]}\frac{\Gamma[m+\Delta_\phi-1]}{\Gamma[\Delta_\phi+1]} $$

To reduce to one sum

$$ \gamma(n,\ell)\ell^2=-(-1)^n\frac{80}{3c_T}\sum_{m=0}^{n}(-1)^m[6m^2+6m(\Delta_\phi-1)+\Delta_\phi(\Delta_\phi-1)] $$

$$ \times\frac{\Gamma[\Delta_\phi]\Gamma[n+1]\Gamma[2\Delta_\phi+m+n-3]}{\Gamma[m+1]\Gamma[n-m+1]\Gamma[\Delta_\phi+m-1]\Gamma[2\Delta_\phi+n-3]} $$

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$$ \Gamma[x]\Gamma[1-x]=\frac{\pi}{\sin\pi x} $$

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Using the following:

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$$ \Gamma[x]=\int_0^\infty t^{x-1}e^{-t}dt $$

So effectively we just need to do the integral

$$\int_0^\infty \int_0^\infty dx dy\ b(n,x,y) e^{-(x+y)} x^a y^b$$

which can be easily done by going to polar coordinates.

d=4

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d=4

$$\boxed{\begin{aligned}\gamma(n,\ell)\ell^2 = -\frac{80}{3c_T}\bigg(&6n^4 + 12n^3(2\Delta_\phi - 3) + 6n^2(11 - 14\Delta_\phi + 5\Delta_\phi^2) + 6n(2\Delta_\phi - 3)(\Delta_\phi^2 - 2\Delta_\phi + 2)\\ &+\Delta_\phi^2(\Delta_\phi - 1)^2\bigg)\end{aligned}}$$

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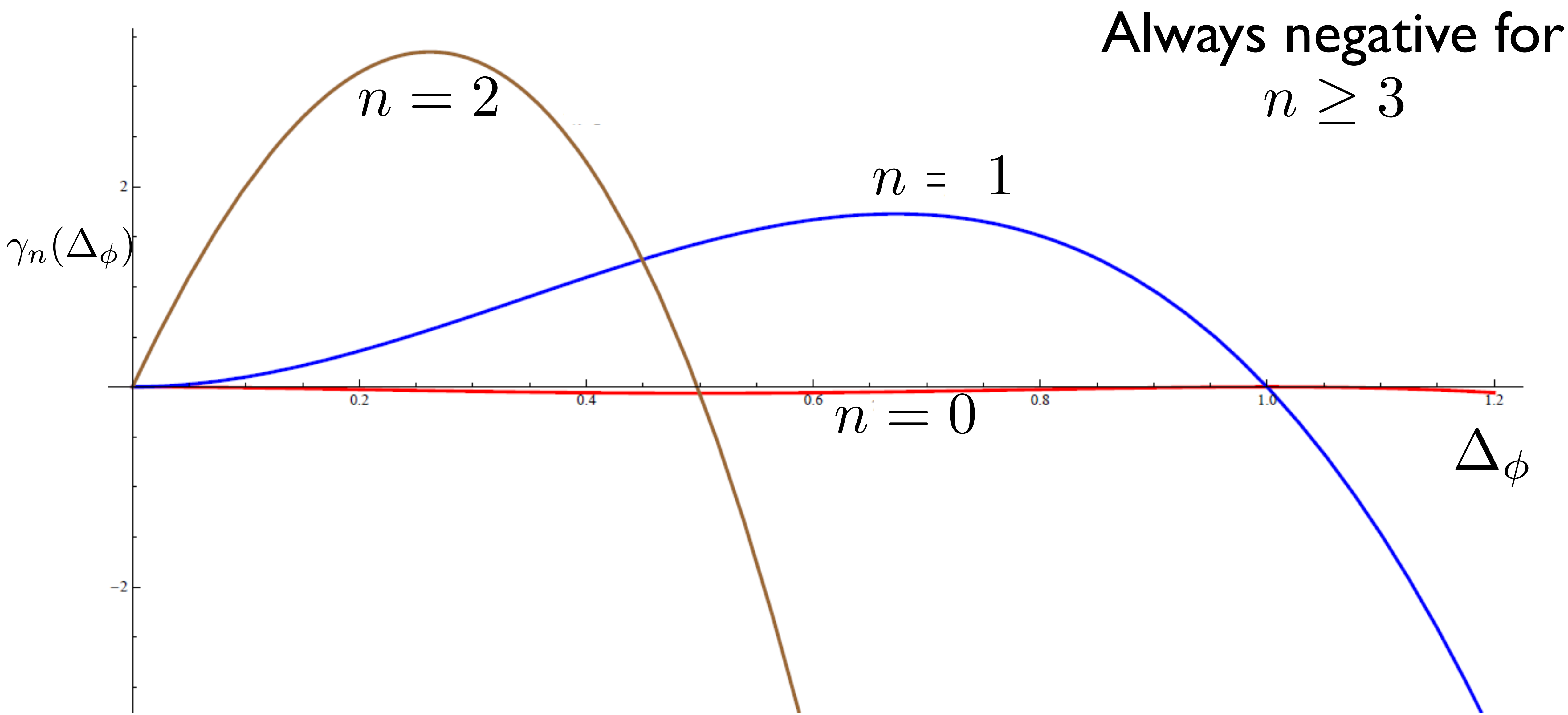
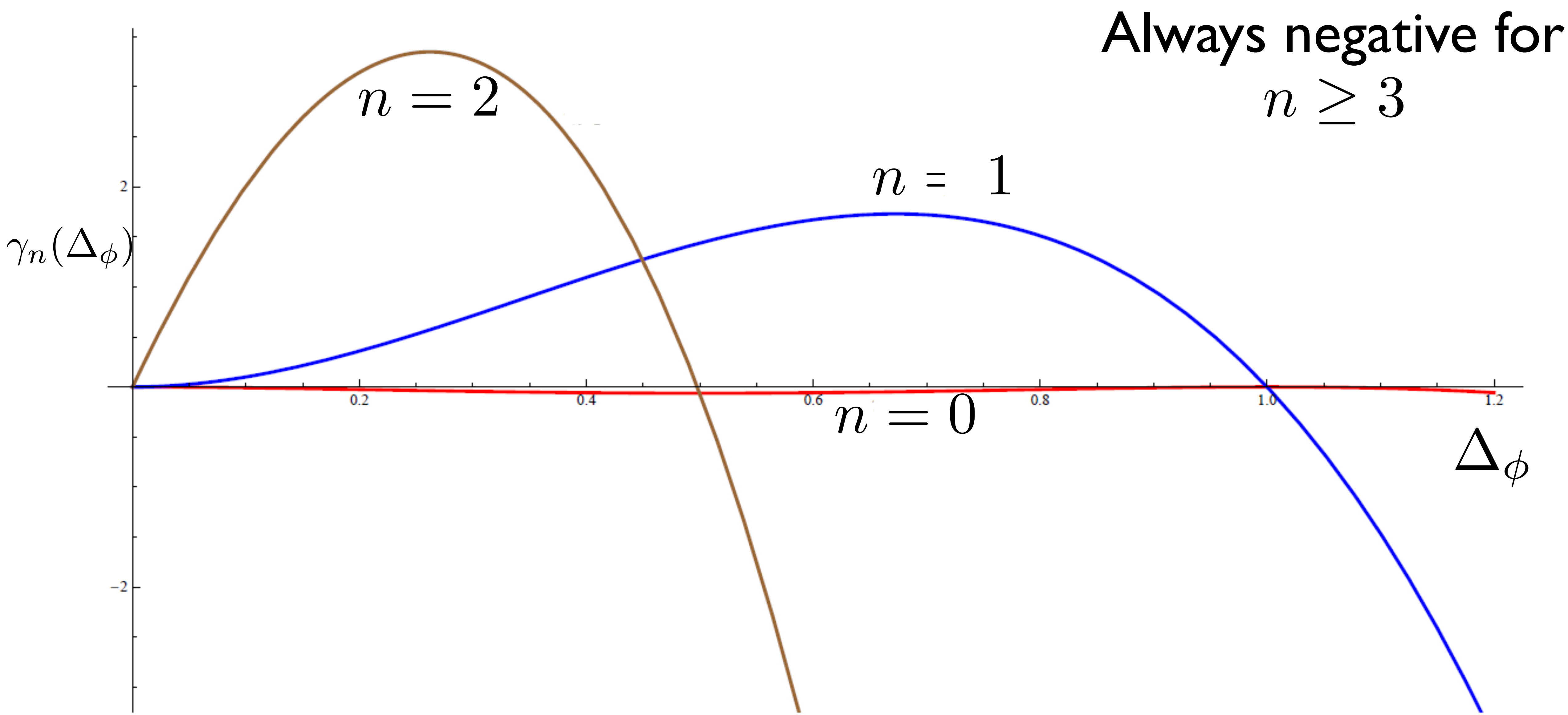
This is negative and monotonically decreasing with n for any conformal dimension satisfying the unitarity bound



Always negative for
$n \geq 3$
$n = 2$
$n = 1$
$n = 0$
$\gamma_n(\Delta_\phi)$
Δ_ϕ
2
-2
0.2
0.4
0.6
0.8
1.0
1.2



<u>If unitarity bound is violated</u> anomalous dimensions can be positive.

Comments on general dimensions

Assume minimal twist for stress tensor exchange d-2

$$\ell \gg n \gg 1$$

$$\gamma(n,\ell)\ell^{d-2} = -P_m \frac{\Gamma[d+1]\Gamma[d+2]}{2\Gamma[1+\frac{d}{2}]\Delta_\phi^2} n^d$$

With some effort this can be derived analytically in all d. In terms of c_T:

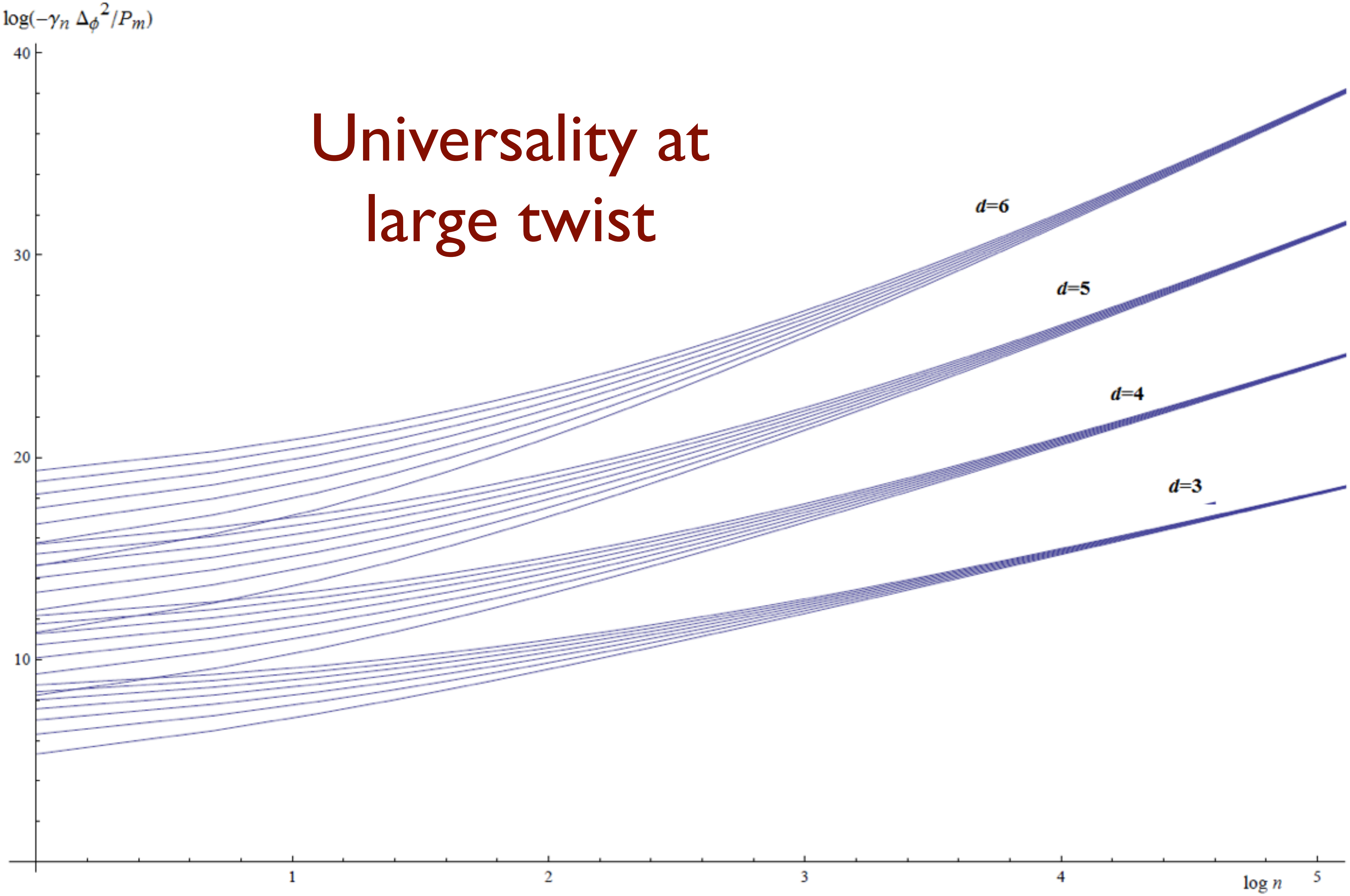
$$\gamma(n,\ell) = -\frac{d^2}{2(d-1)^2 c_T} \frac{\Gamma[d+1]\Gamma[d+2]}{\Gamma[1+\frac{d}{2}]^4} \frac{n^d}{\ell^{d-2}}$$

For this to match with the AdS Eikonal calculation, we need

$$P_m = \frac{16 G_N}{\pi} \frac{\Gamma(d-1)\Gamma(1+\frac{d}{2})^3}{\Gamma(d+1)\Gamma(d+2)} \Delta_\phi^2$$

Exactly expected from AdS/CFT

Universality at large twist



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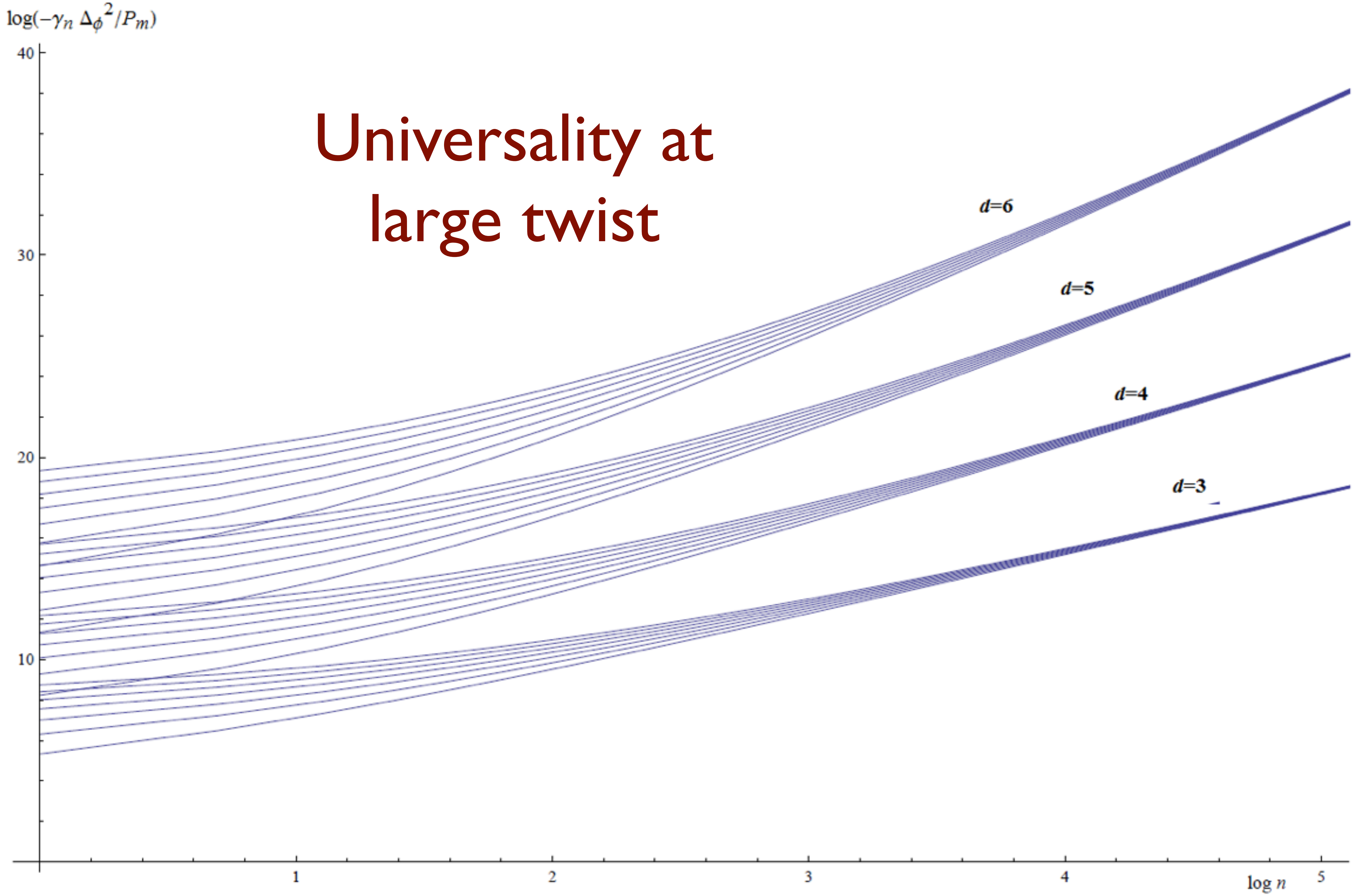
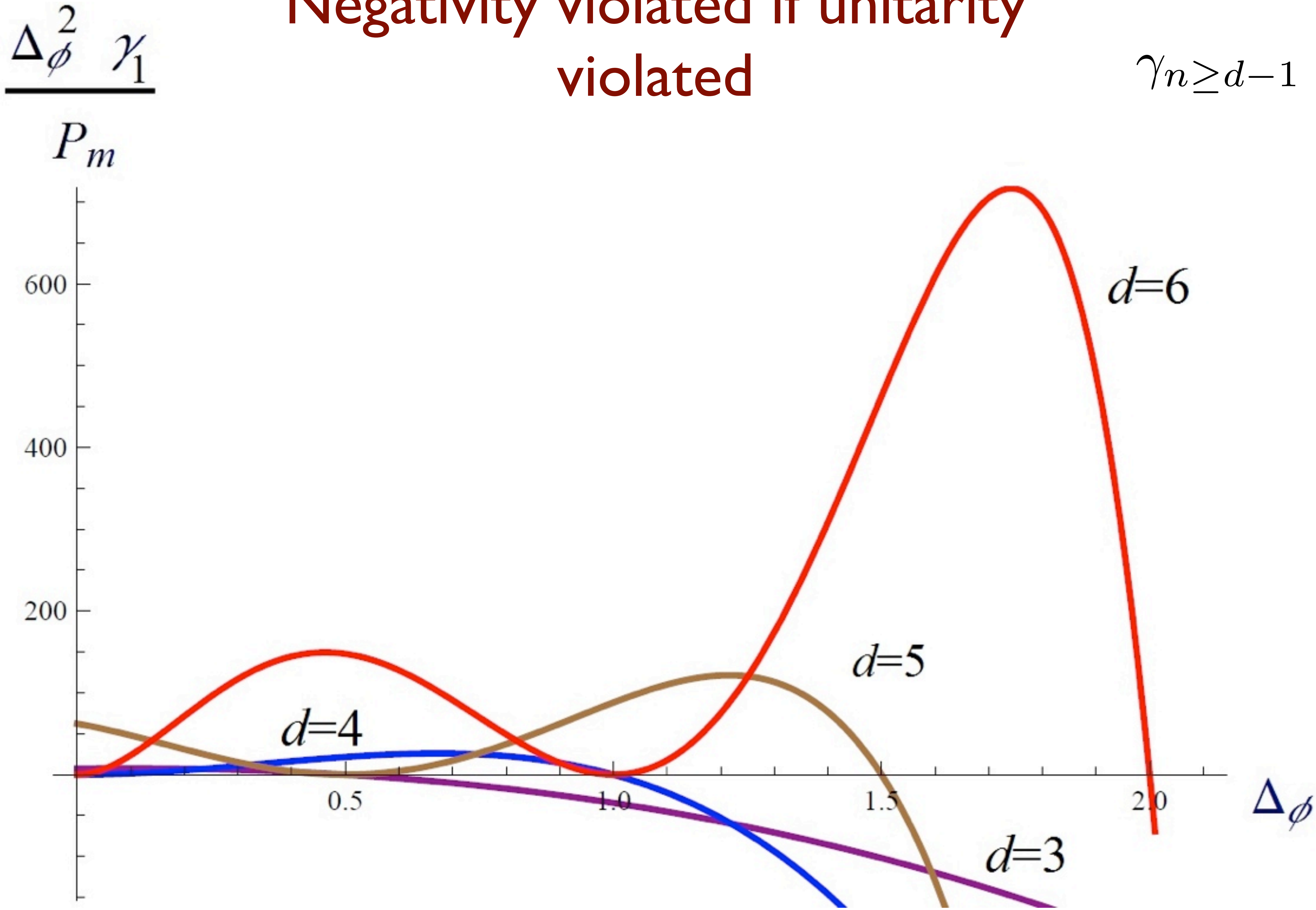


Plots in diverse spacetime dimensions for various conformal dimensions. Asymptotes indicate same intercept independent of conformal dimension.

Negativity violated if unitarity violated

$$\gamma_{n \geq d-1} < 0$$



Subleading terms (d=4)

Starting with the differential equation one can also get the subleading terms in $1/\ell$

$$\gamma(n,\ell) = \frac{\gamma_n^0}{\ell^{\tau_m}} + \frac{\gamma_n^1}{\ell^{\tau_m+1}} + \frac{\gamma_n^2}{\ell^{\tau_m+2}} + \dots .$$

t-channel

$$z^{(\tau_m+1)/2} \sum_n [(1-2\Delta_\phi + (2n+2\Delta_\phi)\tau_m)\gamma_n^0 + 2\gamma_n^1] \frac{(\Delta_\phi - 1)_n{}^2 \Gamma(\Delta_\phi - \frac{1}{2} - \frac{\tau_m}{2})^2}{2\Gamma(n+1)\Gamma(\Delta_\phi)^2\Gamma(2\Delta_\phi + n - 3)_n}$$
$$\times v^n \log v \, F^{(d)}[2\Delta_\phi + 2n, v] .$$

not in *s-channel*

$$\gamma(n,\ell) = -\frac{160}{c_T}\frac{n^4}{\ell^2}(1-\tau_m\frac{n}{\ell})$$

universal

In principle we can extract order by order.

Role of higher spin exchange from CFT

$$n \gg \ell \gg 1$$

Introduce $h = \Delta_\phi + n + \ell, \bar{h} = \Delta_\phi + n$

Using saddle point methods and the other limit we find

$$\gamma(n,\ell) \propto -\frac{n^{2\ell_m+\tau_m-2}(\ell+n)^{2-\tau_m}}{\ell(\ell+2n)}$$

Agrees exactly with AdS/CFT results of Cornalba, Costa, Penedones, Schiappa '07

Valid for large spin, twist

For $n \gg \ell \gg 1$

$$\gamma(n,\ell) \propto -\frac{n^{2\ell_m-1}}{\ell}$$

$\ell_m = 0$

allows small anom. dim

Depends only on spin!

$$\ell_m = 2$$
$$n \gg \ell \gg 1 \qquad \gamma(n,\ell) \sim -\frac{n^3}{\ell}\left(\frac{1}{N^2} + \frac{\#}{\Delta^2} + \#\frac{n^4}{\Delta^4} + \cdots\right)$$

From massive spin=2

Comes from OPE coefficient of higher spin=4 exchange

This means that adding a finite set of higher spin modes will not change the sign of the anomalous dimension.

Anom. dims. will not be small for some very large twist.

To allow for perturbative unitarity we may need to add an infinite set of higher spin modes.

cf Camanho, Edelstein, Maldacena, Zhiboedov

- It will be very interesting to see what a consistent CFT spectrum can be which leads to perturbatively small anomalous dimensions in a theory with large N and a gap.

Holography

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- The double trace operators can be thought of as two massive particles in AdS rotating around each other. The anomalous dimension arises due to the interacting energy of these particles.
- Essential idea is to do perturbation theory in inverse distance corresponding to a Newtonian approximation in AdS.

- It has been shown that for n=0, the result of the calculation agrees with the bootstrap prediction. (Unlike Eikonal where both spin and n needed to be large)
- Non-zero n is quite hard. However, we have been able to make progress (barring overall constants) at large n, i.e., $\ell \gg n \gg 1$
- It turns out to give exactly the same universal behaviour predicted by bootstrap!

Non-renormalization from holography

$\ell \gg n \gg 1$

Higher derivative correction

$$\delta E^{d}_{n,\ell_{orb}} = -\frac{\mu}{2}\int r(1+\alpha'^{h} r^{-2h})dr \left[\sum_{k,\alpha=0}^{n}\left(\frac{E^2_{n,\ell}}{(1+r^2)^2}\psi_k(r)\psi_\alpha(r)+\partial_r\psi_k(r)\partial_r\psi_\alpha(r)\right)\right] = \mathcal{I}_1+\mathcal{I}_2\,,$$

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$$\delta E^{4}_{n,\ell_{orb}} = -\frac{\mu(\ell+2n)^2\Gamma(\ell+2+n)}{4\Gamma(\ell+2)\Gamma(n+\Delta-1)}\sum_{k=0}^{n}(-1)^k\frac{\Gamma(k+\ell+n+\Delta)}{\Gamma(\ell+2+k)\Gamma(n+1-k)\Gamma(2+k+\ell+\Delta)\Gamma(k+1)}$$
$$\times\left[\Gamma(1+\ell+k)\Gamma(1+\Delta)\,{}_3F_2\Big(-n,k+\ell+1,\ell+n+\Delta;\ell+2,2+k+\ell+\Delta;1\Big)\right.$$
$$\left.+\alpha'^{h}\Gamma(1+\ell+k-h)\Gamma(1+\Delta+h)\,{}_3F_2\Big(-n,k+\ell+1-h,\ell+n+\Delta;\ell+2,2+k+\ell+\Delta;1\Big)\right]$$

The spin dependence for the Einstein term can be shown to be $\frac{1}{\ell^2}$ while the higher derivative term gives $\frac{1}{\ell^{2h+2}}$. Thus no 't Hooft coupling dependence!

Prediction for susy bootstrap: N=4 't Hooft coupling shows up at ℓ^{-8}

- The $\ell \gg n \gg 1$ result exactly agrees with the CFT calculation.
- It will be interesting to do the other limit to check non-universality due to higher derivative corrections and compare with causality constraints.

Summary

- We have derived certain interesting universal results using conformal bootstrap.
- We should understand the large twist limit better both from the CFT side (what is a consistent spectrum) and from the gravity side (role of higher derivative corrections).

Thank you for listening