Witten Diagrams Revisited:

Holographic Duals of Conformal Blocks

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Strings 2015

Two pillars of AdS/CFT:

1. Symmetries must match.
2. Bulk Witten diagrams compute boundary correlators.

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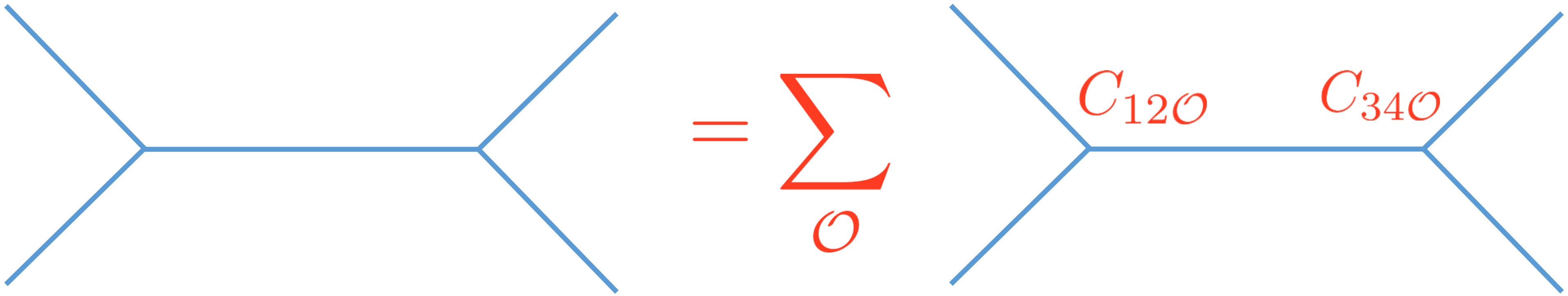
2. Bulk Witten diagrams compute boundary correlators.

CFT correlators admit a conformal block expansion:

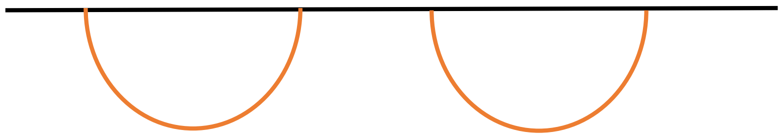


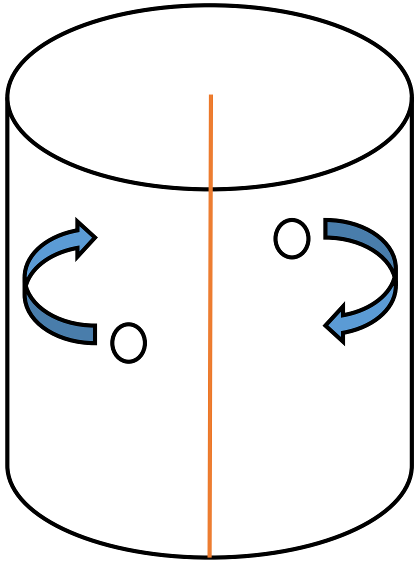
This should be visible in Witten diagrams. Traditionally, however, it is not. More recent methods are complicated. Can't we do better?

- This raises a very natural question:

What is the holographic dual of a conformal block?

- In d=2 CFT, large central charge Virasoro blocks do geometrize.





[Headrick; Hartman; Fitzpatrick, Kaplan, Walters; Roberts, Stanford]

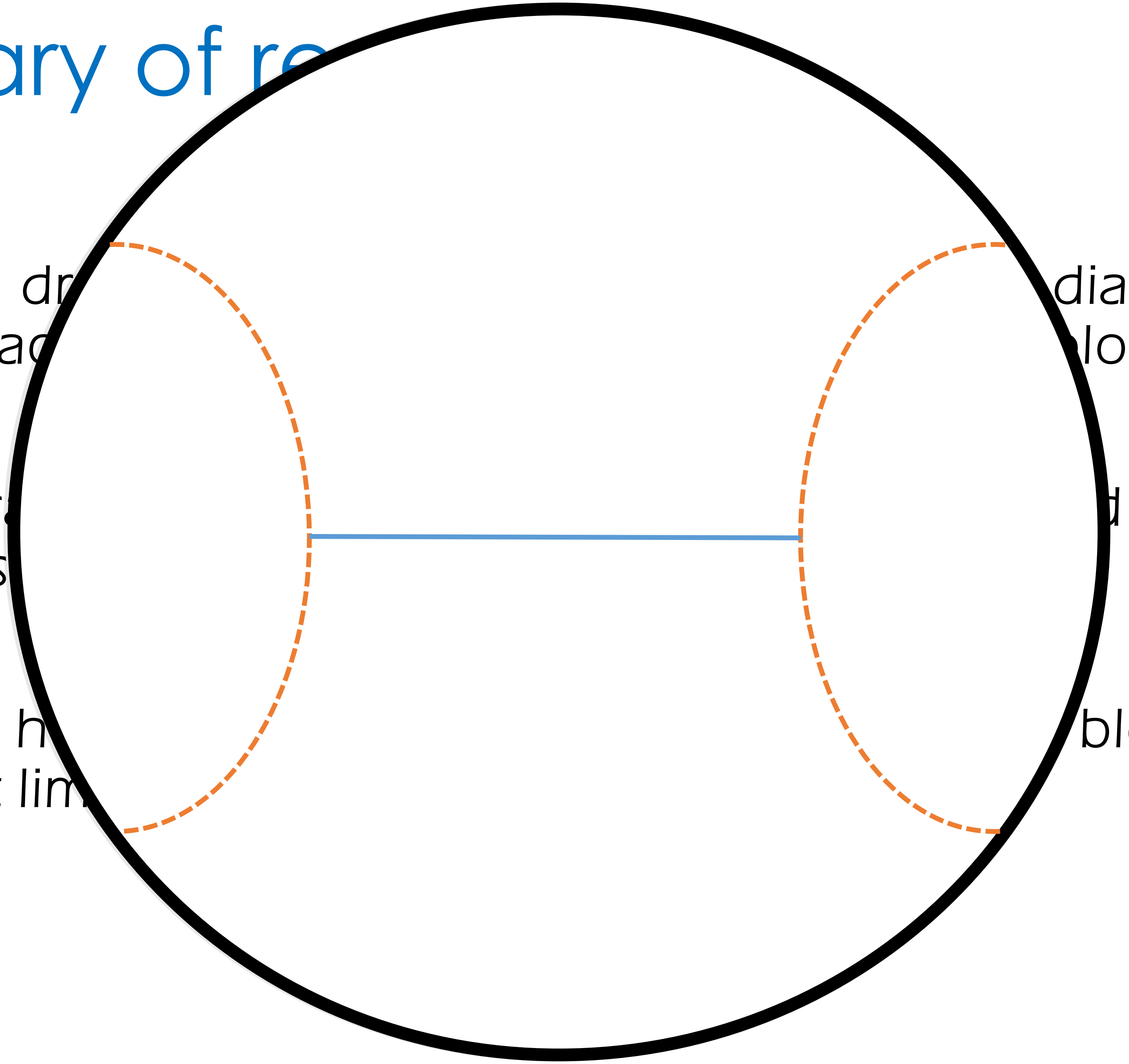
- *Should* a d-dimensional block have a gravity dual? On the one hand, it is not a semiclassical object. On the other, it is determined by conformal symmetry.

Summary of results

- A new and dramatically simpler treatment of Witten diagrams in position space, that exhibits the dual CFT conformal block expansion.
- The holographic dual of a generic conformal block in d spacetime dimensions.
- In d=2, the holographic dual of a Virasoro conformal block in a heavy-light limit of large central charge.

Summary of re

- A new and dr diagrams in position spac block expansion.
- The hologr d spacetime dimensions
- In d=2, the h block in a heavy-light lim



Outline

Witten diagrams and the geometry of conformal blocks

Holographic duals of large c Virasoro blocks

Based on work with E. Hijano, P. Kraus, R. Snively (1507.xxxxx)

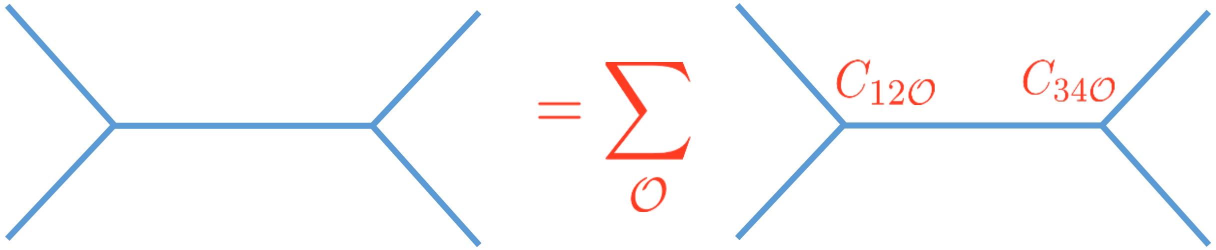
The many faces of a conformal block

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = (\text{Power law}) \times \sum_{\mathcal{O}} C_{12}{}^{\mathcal{O}} C_{34\mathcal{O}} G_{\Delta,s}(u,v)$$

Cross-ratios:

$$\left(u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} \right)$$

- Blocks with external scalars:
  - Have series and integral representations
  - Are hypergeometric in even d
  - Obey an SO(d,2) Casimir equation
  - Can be well-approximated by a sum over poles



- Spinning blocks:
  - Are only partly known
  - Also admit recursion relations

[Ferrara, Gatto, Grillo; BPZ; Zamolodchikov; Dolan, Osborn; Kos, Poland, Simmons-Duffin; Costa, Penedones, Poland, Rychkov; ...]

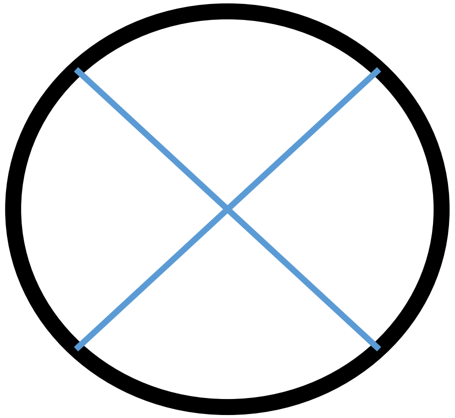
CFT spectrum at large N

- CFTs with Einstein-like gravity duals have the following light spectrum:
  - A finite density of light, low-spin single-trace operators.
  - Their multi-trace composites

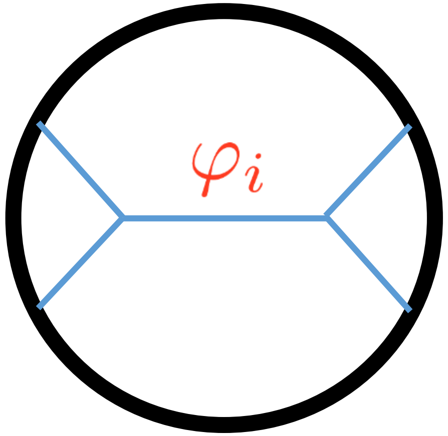
$$\mathcal{O}_i$$
$$[\mathcal{O}_i\mathcal{O}_j]_{n,\ell} \equiv \mathcal{O}_i \Box^n \partial^{\mu_1} \cdots \partial^{\mu_\ell} \mathcal{O}_j$$
$$\cdots$$

- Order-by-order in 1/N, these fields furnish crossing-symmetric correlators.

$$\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4 \rangle\big|_{1/N^2} =$$



$$+\sum_i$$



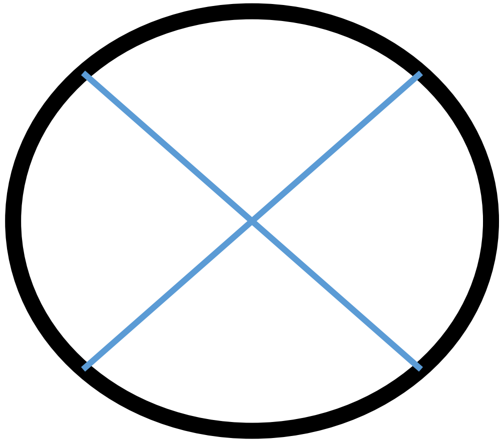
$$+\ (\text{crossed})$$

- Witten diagrams = 1/N expansion

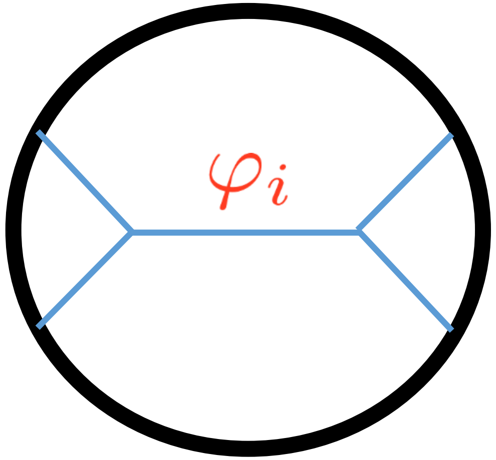
[GKPW; Heemskerk, Penedones, Polchinski, Sully; El-Showk, Papadodimas]

Witten diagrams primer

$$\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle\big|_{1/N^2} =$$



$$+\sum_i$$



$$+ \text{(crossed)}$$

- "D-functions" in x-space
- Polynomial M(s,t)
- Contains double-trace blocks

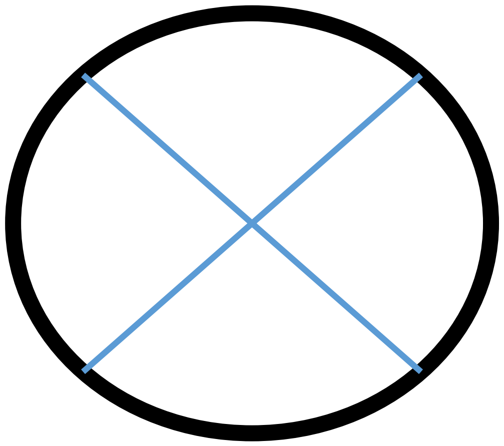
- Sum of D-functions in x-space
- Meromorphic M(s,t)
- Contains single-, double-trace blocks

[GKPW; D'Hoker, Freedman, Mathur, Matusis, Rastelli; Liu, Tseytlin; Arutyunov, Frolov, Petkou; Dolan, Osborn; ...]

[Penedones; El-Showk, Papadodimas; Fitzpatrick, Kaplan, Penedones, Raju, van Rees; Paulos; Fitzpatrick, Kaplan; Raju; Costa, Goncalves, Penedones; Bekaert, Erdmenger, Ponomarev, Sleight]

Witten diagrams primer

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \big|_{1/N^2} =$$



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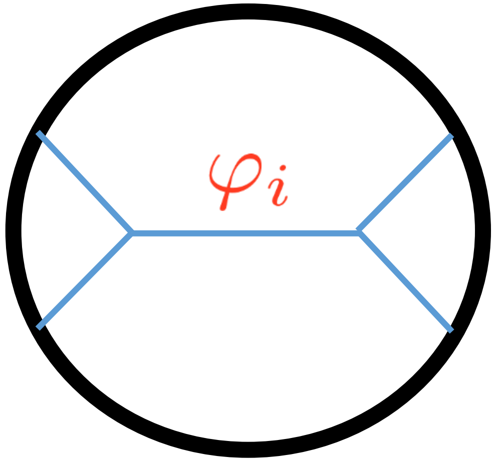


$$+ \text{(crossed)}$$

<u>What has been computed?</u>

- 1998-2002: Scalar contact; s=0,1,2 exchange between external scalars. Computed in double-OPE expansion…
  - Where are the blocks?

- Recent: split representation of bulk-to-bulk propagators; Mellin amplitudes…
  - Technically involved, not in position space, or both.

Geodesics to the rescue

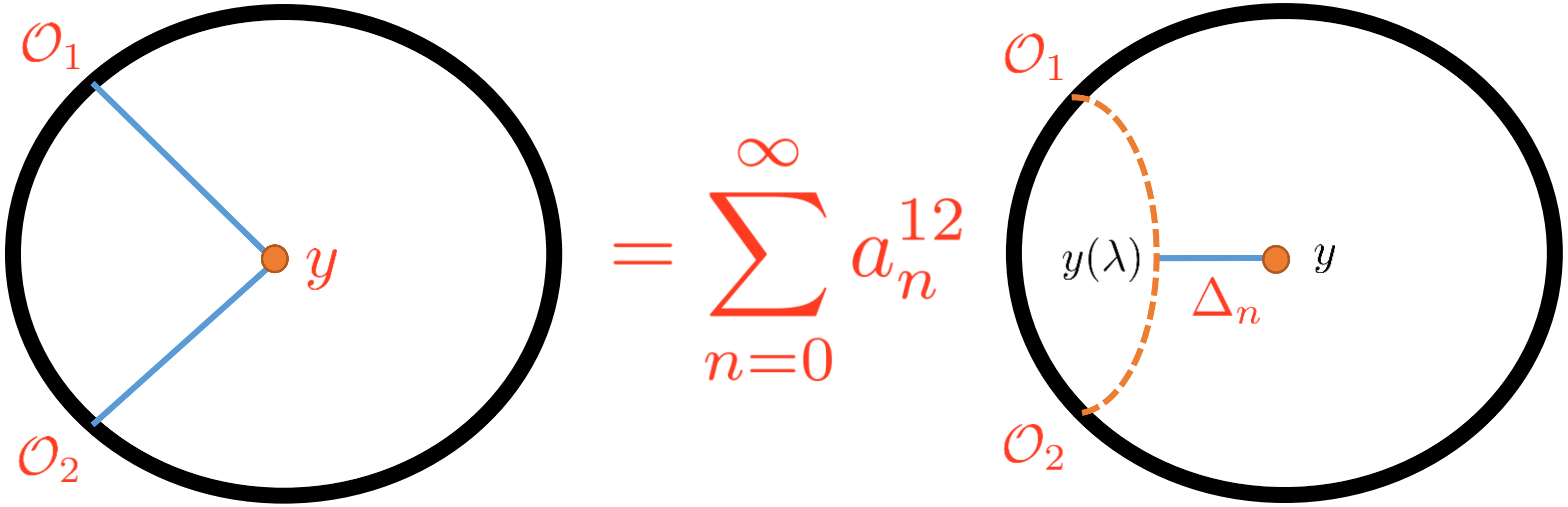
- To proceed, we introduce the following identity obeyed by AdS scalar propagators:

$$G_{b\partial}(x_1,y)G_{b\partial}(x_2,y) = \sum_{n=0}^{\infty} a_n^{12}\phi_{12;\Delta_n}(y)$$

where

$$\phi_{12;\Delta_n}(y) = \int_\lambda G_{b\partial}(x_1,y(\lambda))G_{b\partial}(x_2,y(\lambda))G_{bb}(y(\lambda),y;\Delta_n)$$

$$\Delta_n = \Delta_1 + \Delta_2 + 2n$$



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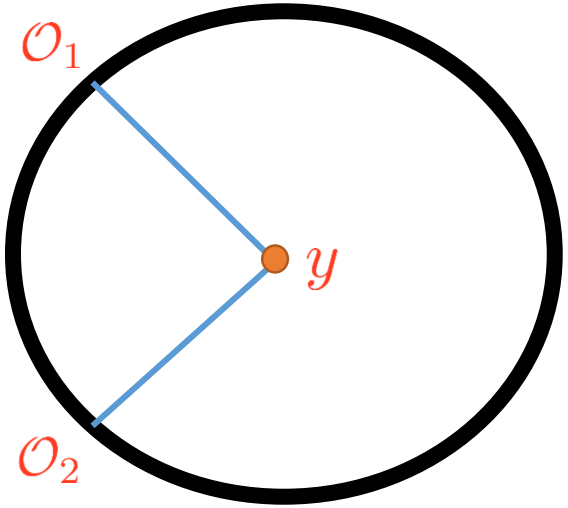
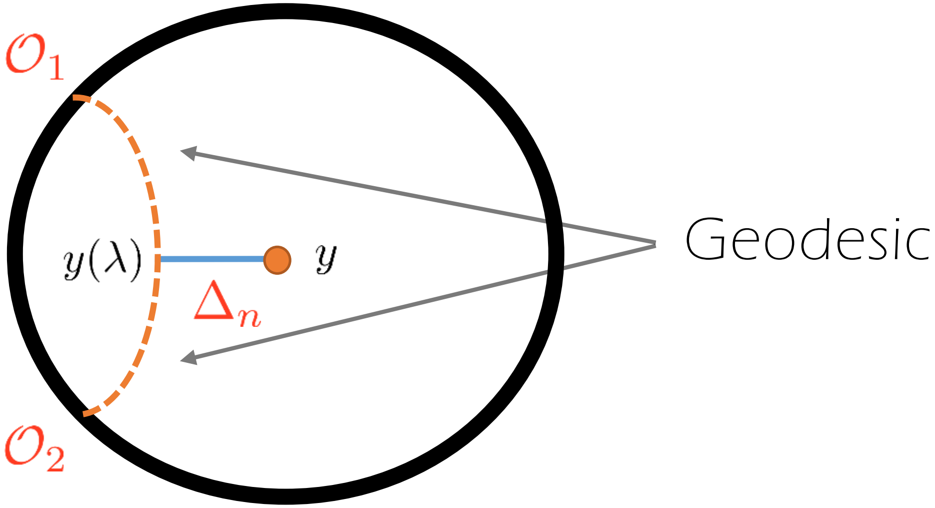


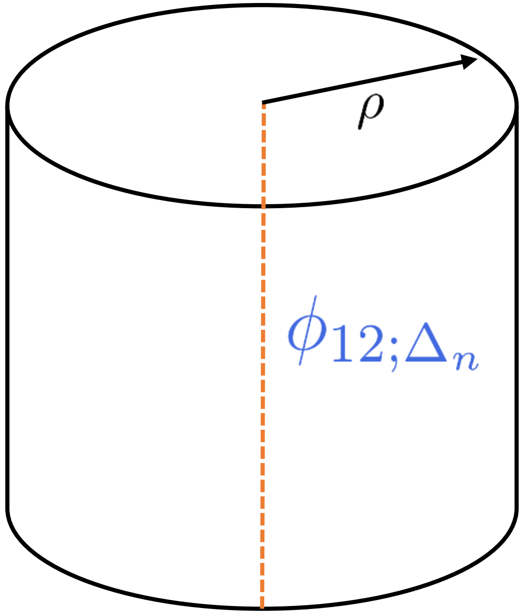
$$= \sum_{n=0}^{\infty} a_n^{12}$$



Geodesics to the rescue

$$G_{b\partial}(x_1, y) G_{b\partial}(x_2, y) = \sum_{n=0}^{\infty} a_n^{12} \phi_{12;\Delta_n}(y)$$

- We can think of phi as normalizable solution of the scalar wave equation with a geodesic source.
  - In global AdS:



$$\Delta_n = \Delta_1 + \Delta_2 + 2n$$

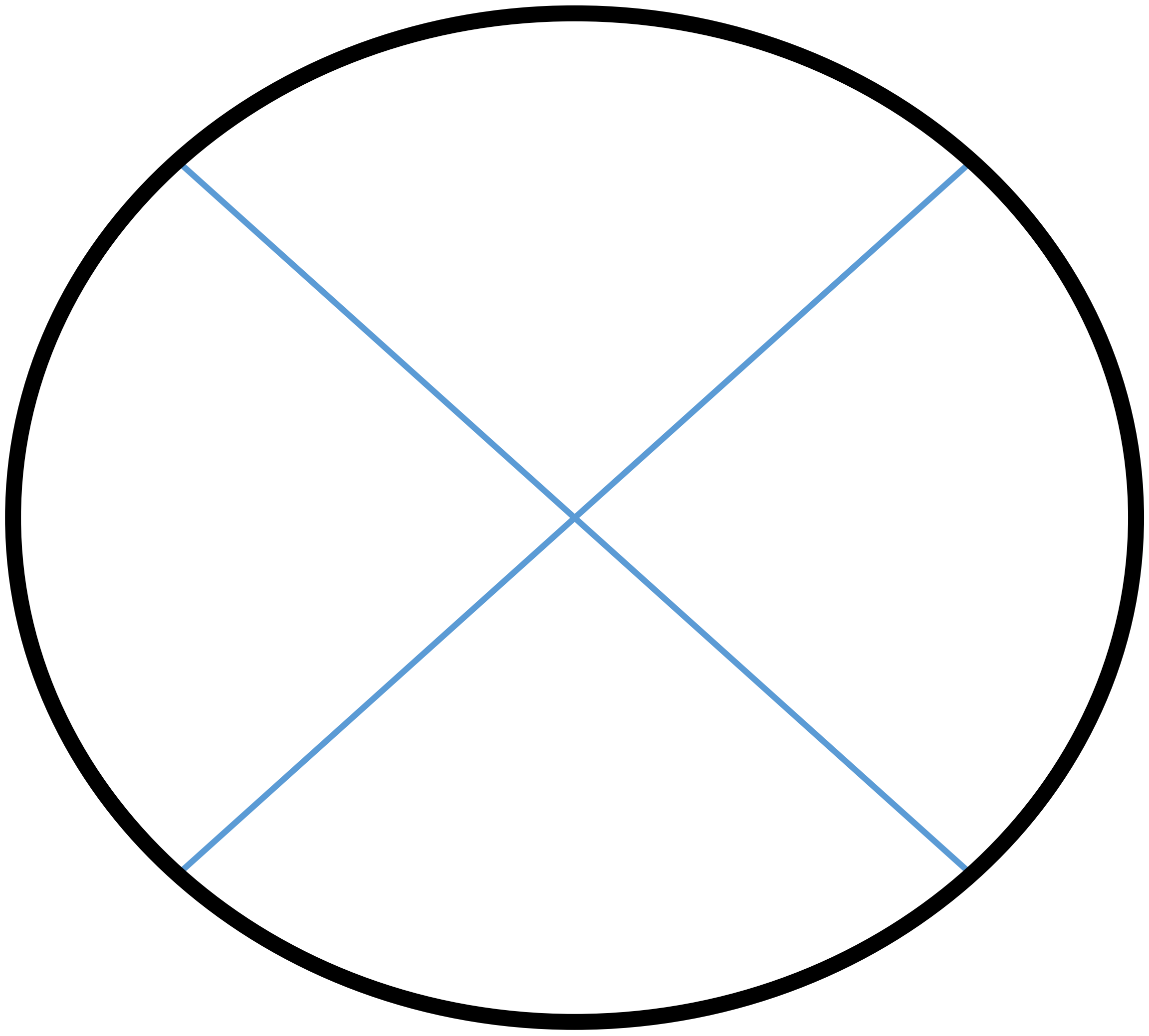
$$(\cos\rho)^{\Delta_1+\Delta_2} = \sum_{n=0}^{\infty} a_n^{12} (\cos\rho)^{\Delta_n} {}_2F_1\left(\frac{\Delta_n + \Delta_{12}}{2}, \frac{\Delta_n - \Delta_{12}}{2}; \Delta_n - \frac{d-2}{2}; \cos^2\rho\right)$$

We now apply this identity to Witten diagrams.

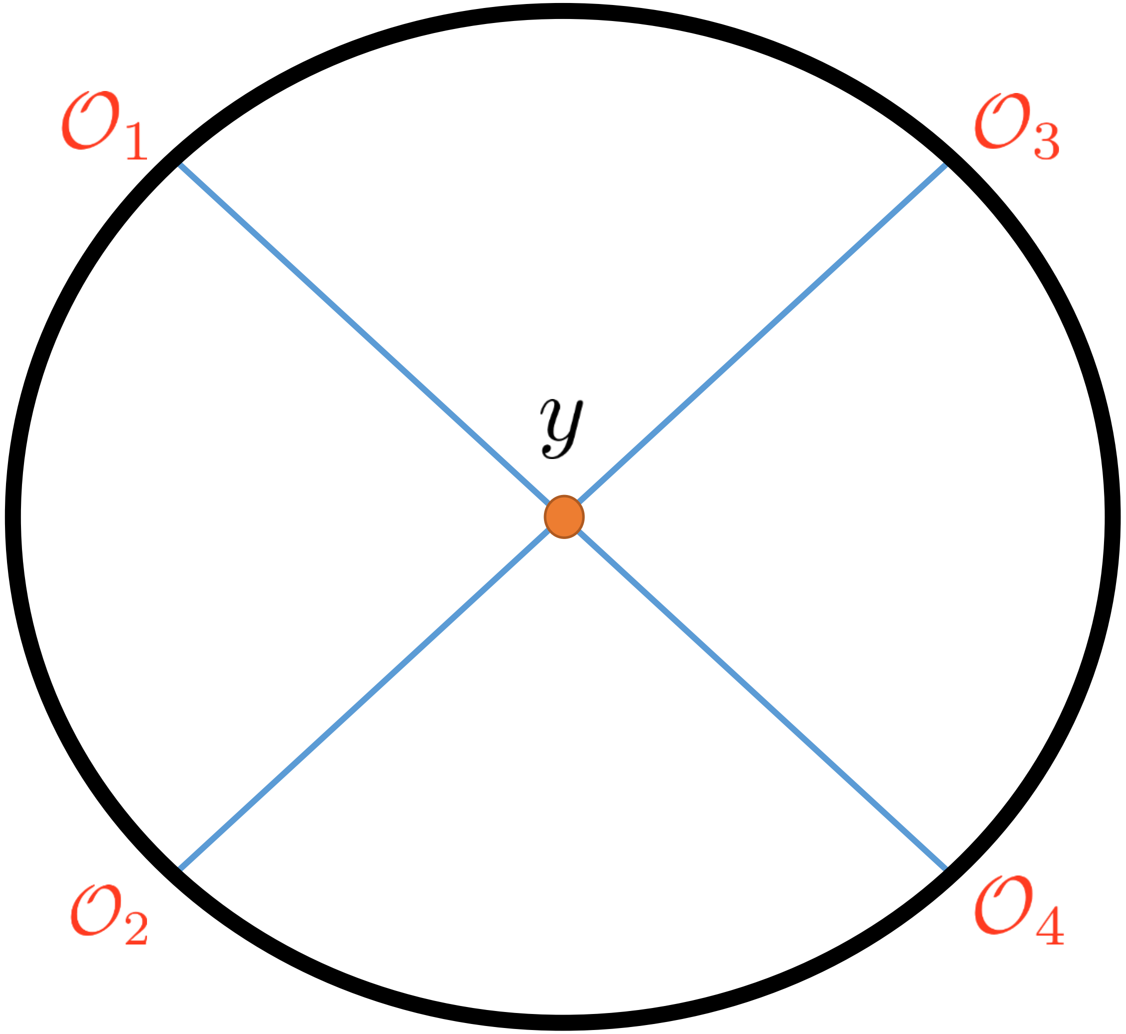
As we will see, the geometric representation of a conformal block will naturally emerge.

I. Scalar contact diagram
II. Scalar exchange diagram
III. Legs
IV. Loops
V. Spin

I. Scalar contact diagram



I. Scalar contact diagram



$$= \int_y G_{b\partial}(x_1, y) G_{b\partial}(x_2, y) G_{b\partial}(x_3, y) G_{b\partial}(x_4, y)$$

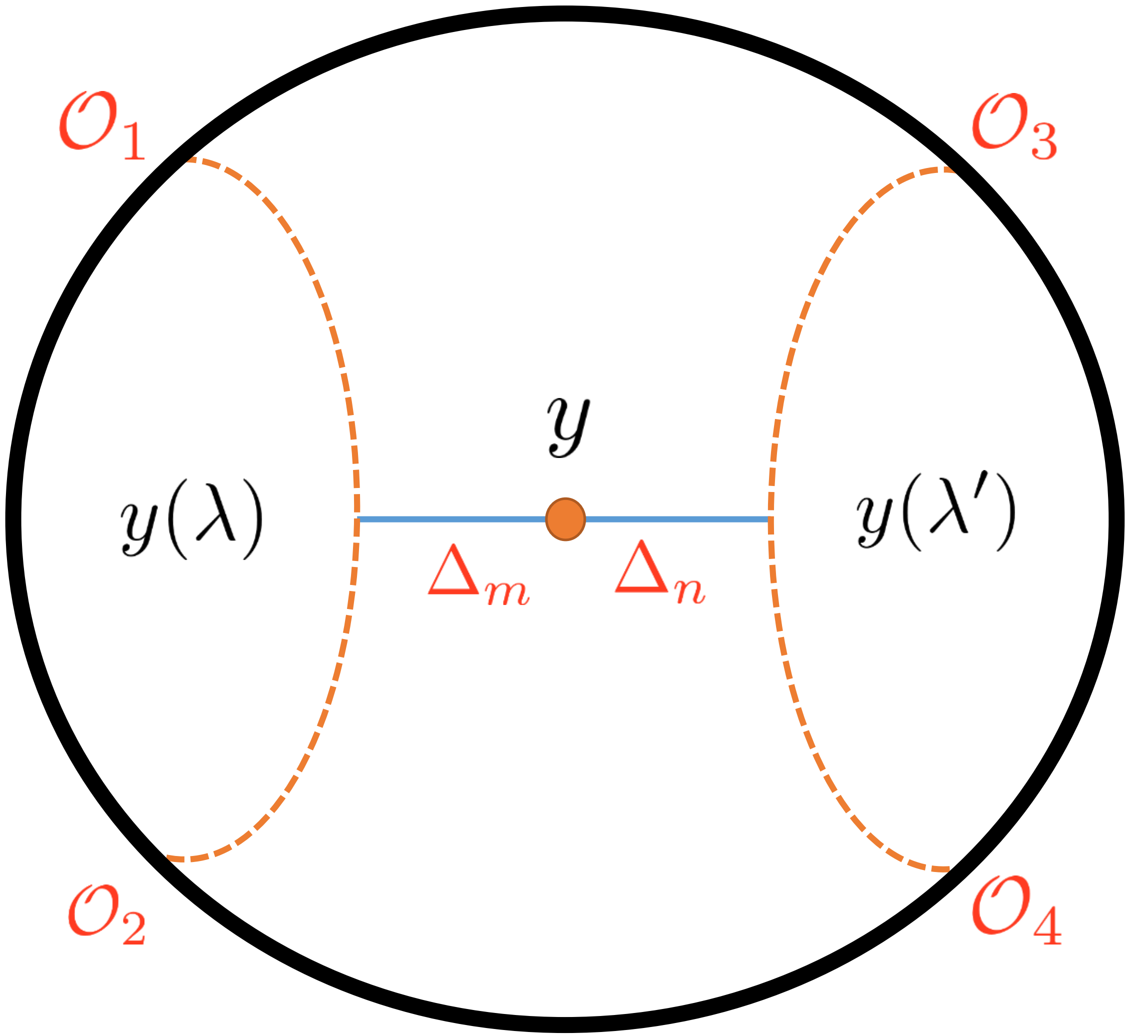
$$= D_{\Delta_1 \Delta_2 \Delta_3 \Delta_4}(x_i)$$

(Dimensions and positions suppressed: $D\mathcal{O}_i(x_i) = \Delta_i \mathcal{O}(x_i)$)

$$m^2 R_{\text{AdS}}^2 = \Delta(\Delta - d)$$

I. Scalar contact diagram

$$\sum_{m,n}$$



1. Use geodesic identity twice:

$$\Delta_m = \Delta_1 + \Delta_2 + 2m, \;\; m = 0, 1, 2, \ldots$$

$$\Delta_n = \Delta_3 + \Delta_4 + 2n, \;\; n = 0, 1, 2, \ldots$$

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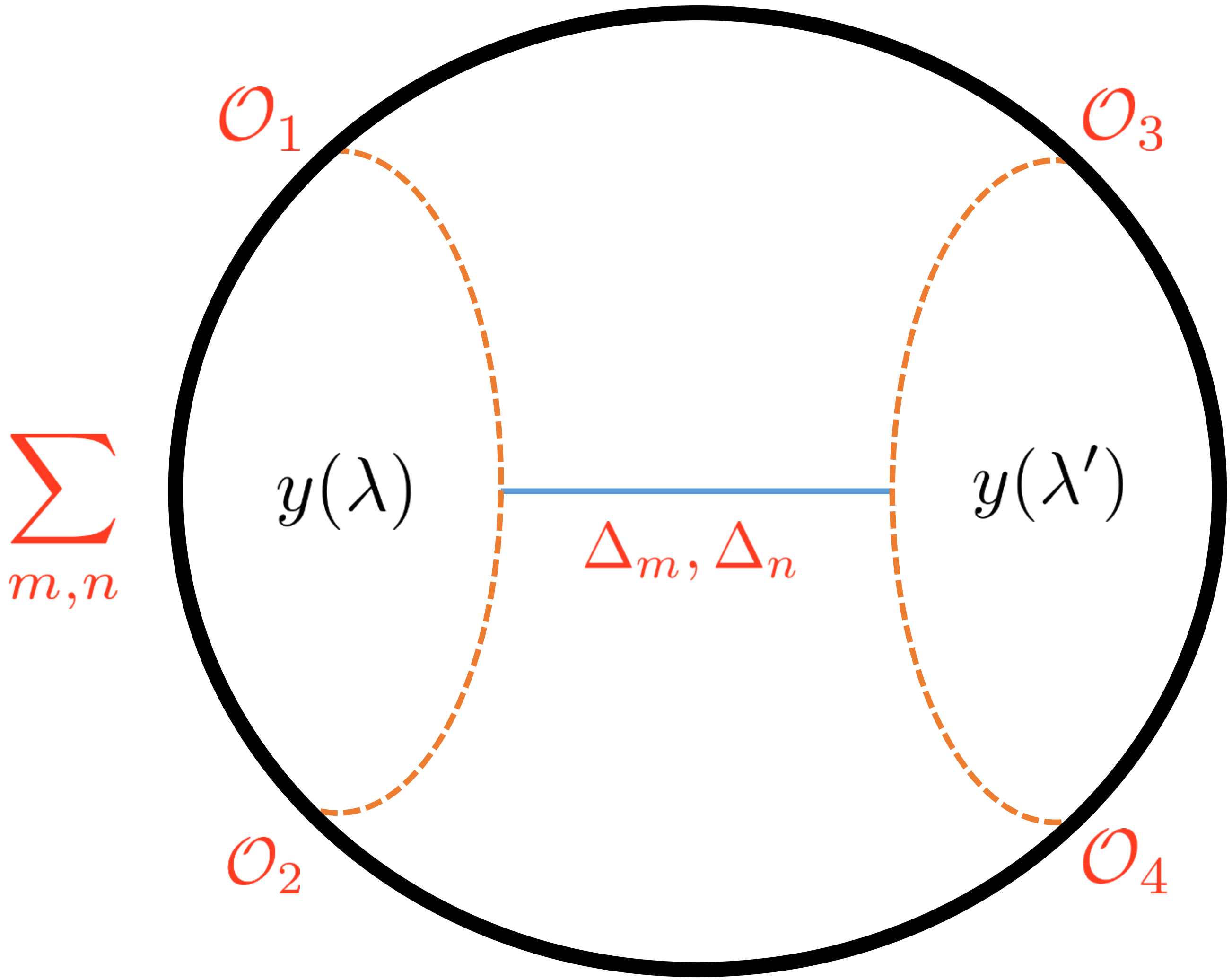

1. Use geodesic identity twice:

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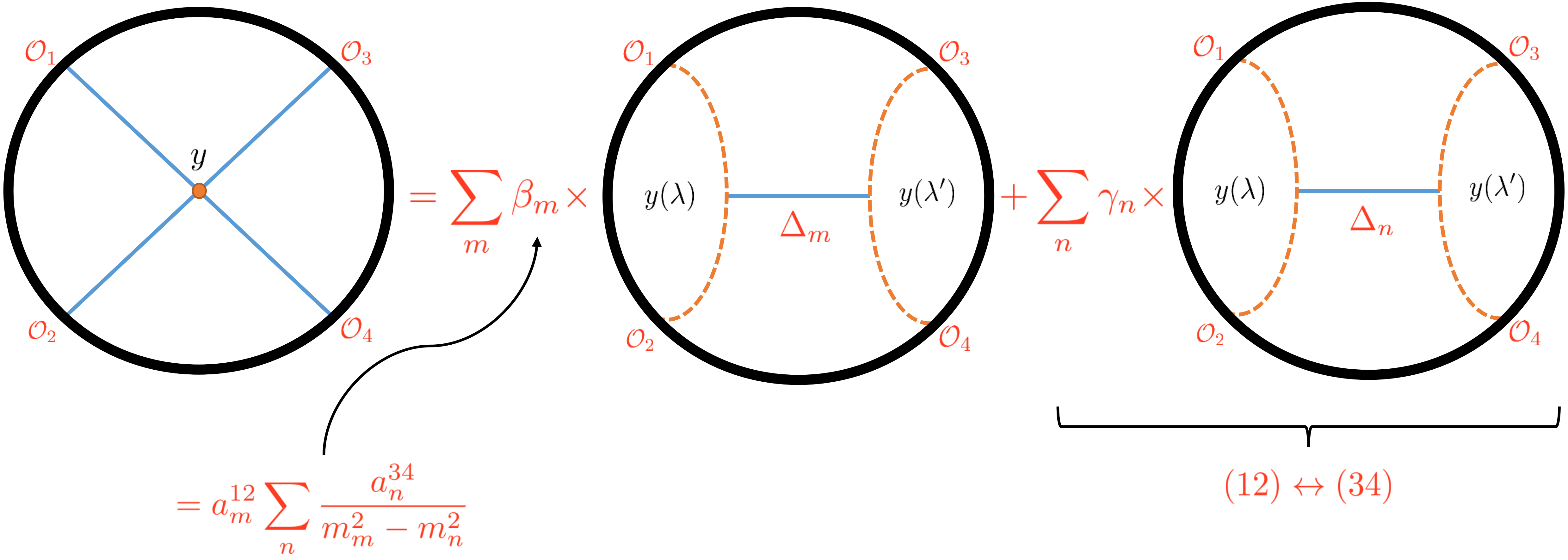
$$\Delta_n = \Delta_3 + \Delta_4 + 2n, \ \ n = 0, 1, 2, \ldots$$

2. Use algebra:

$$\int_y \langle y(\lambda)| \frac{1}{\nabla^2 - m_m^2} |y\rangle\langle y| \frac{1}{\nabla^2 - m_n^2} |y(\lambda')\rangle$$
$$= \langle y(\lambda)| \left(\frac{1}{\nabla^2 - m_m^2} - \frac{1}{\nabla^2 - m_n^2} \right) |y(\lambda')\rangle$$
$$\times \frac{1}{m_m^2 - m_n^2}$$

I. Scalar contact diagram



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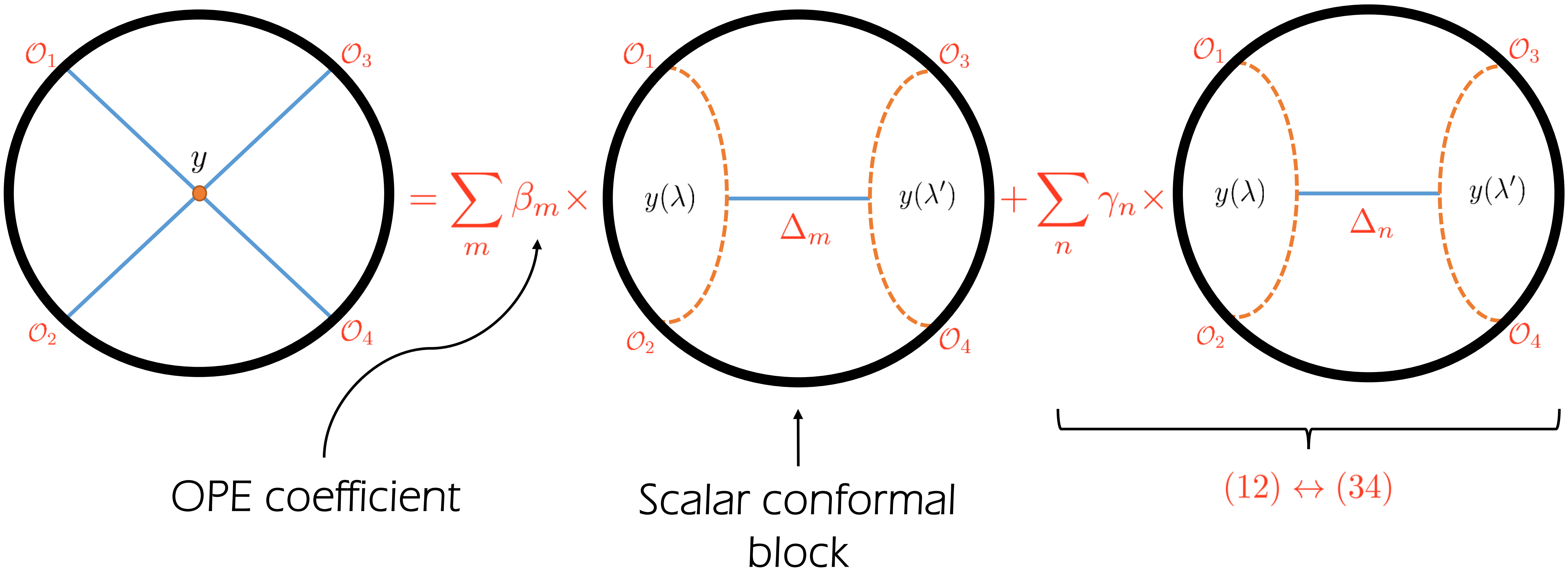
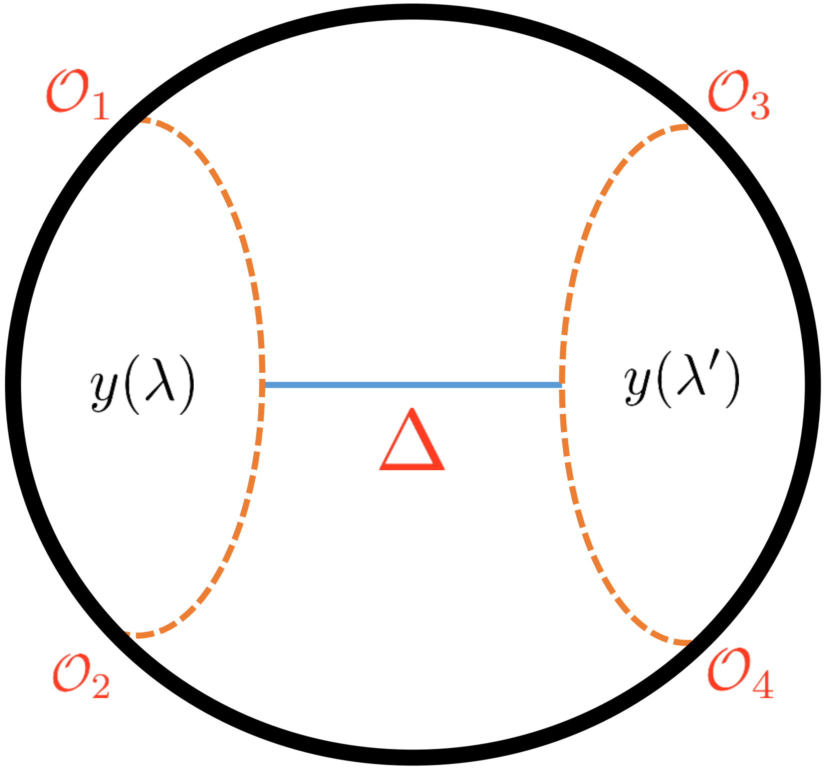


The holographic dual of a conformal block

$$G_{\Delta,0}(x_i) =$$



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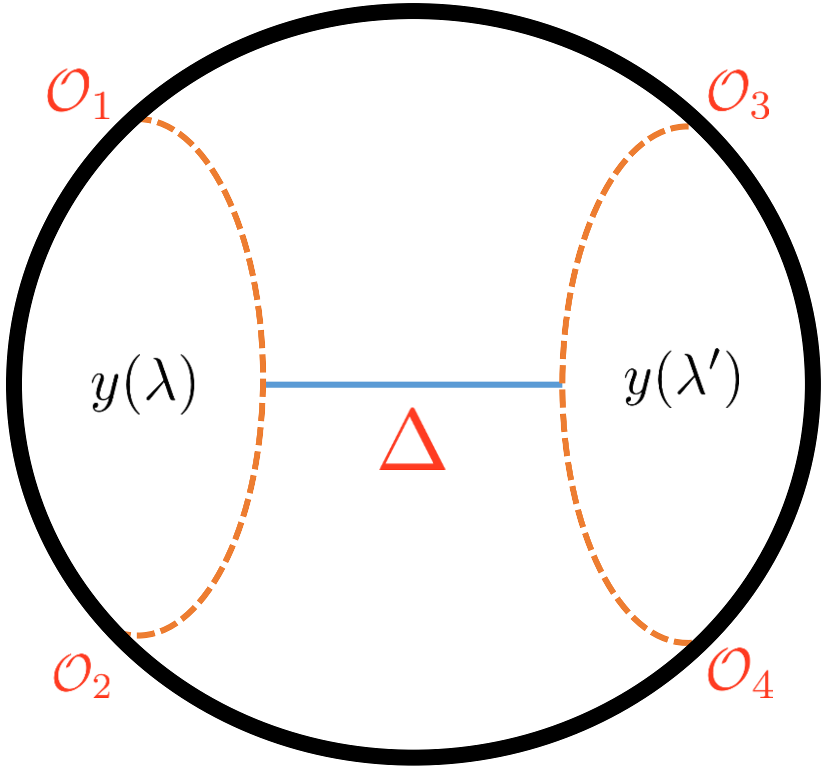
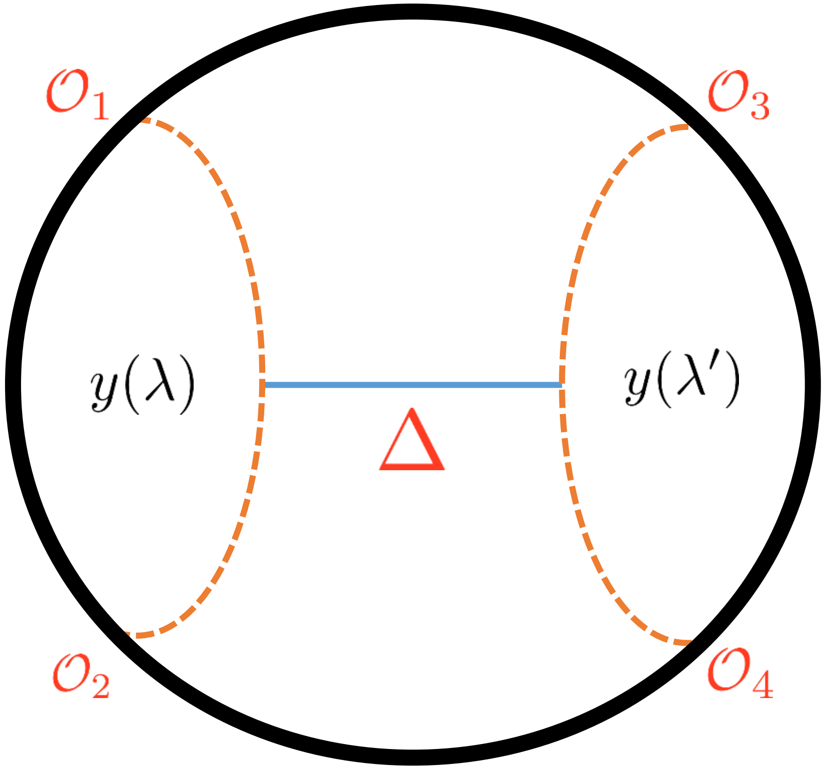


This geometrizes the original integral representation of the block:

$$G_{\Delta,0}(x_i) = \int_\lambda \int_{\lambda'} G_{b\partial}(y(\lambda), x_1) G_{b\partial}(y(\lambda), x_2) \times G_{bb}(y(\lambda), y(\lambda'); \Delta) \times G_{b\partial}(y(\lambda'), x_3) G_{b\partial}(y(\lambda'), x_4)$$

The holographic dual of a conformal block

$$G_{\Delta,0}(x_i) =$$



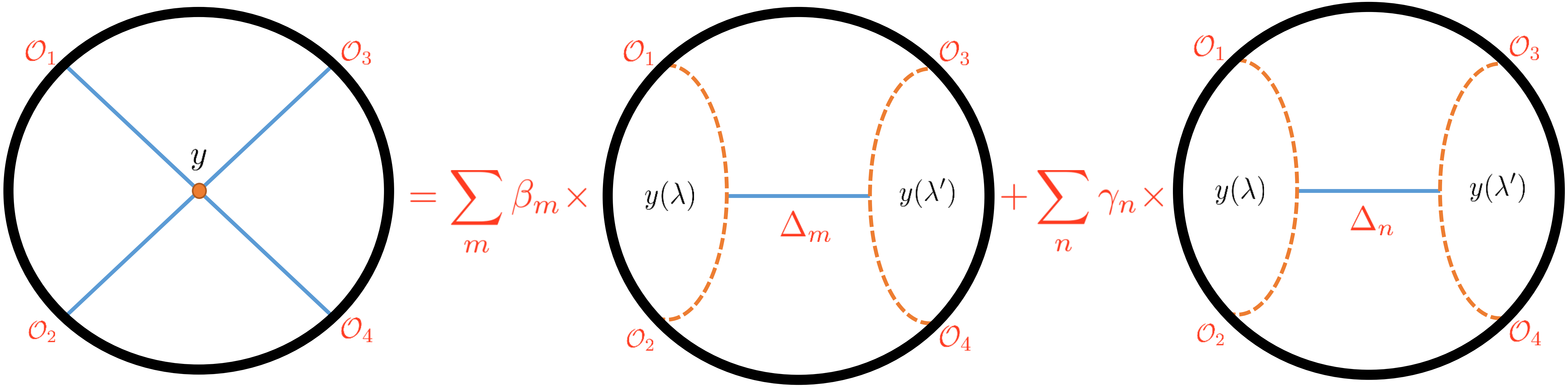
This geometrizes the original integral representation of the block:

$$G_{\Delta,0}(u,v) = u^{\Delta-\Delta_3-\Delta_4} \int_0^1 d\sigma \sigma^{\frac{\Delta-\Delta_{34}-2}{2}} (1-\sigma)^{\frac{\Delta+\Delta_{34}-2}{2}} (v+\sigma(1-v))^{\frac{-\Delta+\Delta_{12}}{2}}$$

$$\times \, {}_2F_1\left(\frac{\Delta+\Delta_{12}}{2}, \frac{\Delta-\Delta_{12}}{2}; \Delta - \frac{d-2}{2}; \frac{u\sigma(1-\sigma)}{v+\sigma(1-v)}\right)$$

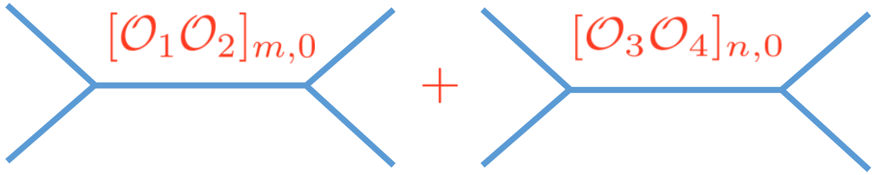
[Ferrara, Gatto, Grillo, Parisi '72]

I. Scalar contact diagram



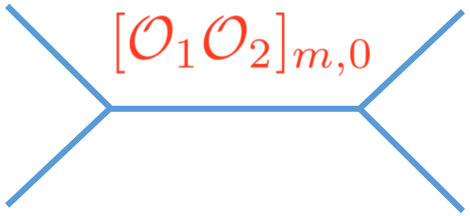
<u>Final result:</u>

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(x_i) = \sum_m \underbrace{\beta_m G_{\Delta_m,0}(x_i) + \sum_n}_{\text{Double-trace exchanges:}} \gamma_n G_{\Delta_n,0}(x_i)$$

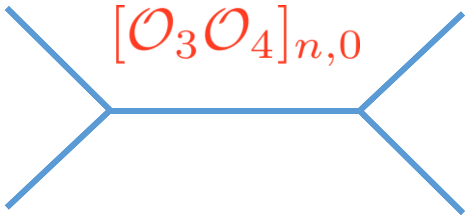


I. Scalar contact diagram: comments

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(x_i) = \sum_m$$



$$+ \sum_n$$



- No *integration* needed!

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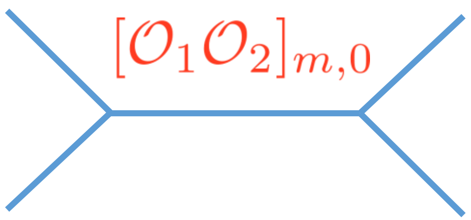

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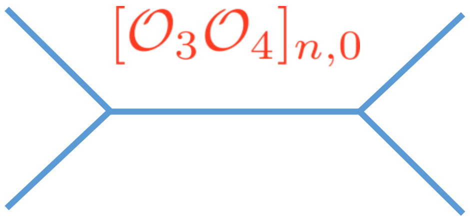

- No integration needed!
- A geometric decomposition of any D-function into scalar blocks.
  - This is useful beyond holography:

$$\langle \mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\rangle \approx (\text{free}) + \lambda \overline{D}_{1111}(x_i) + O(\lambda^2)$$

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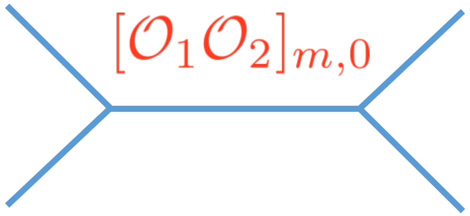

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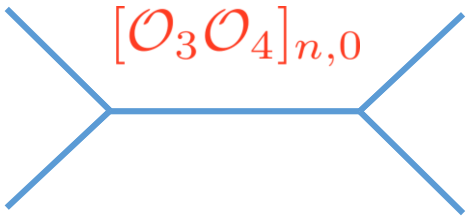

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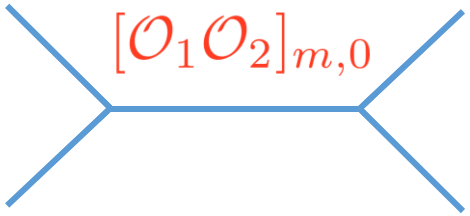
- For non-generic operator dimensions, appearance of logs is immediate:

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(x_i) = \sum_n \underbrace{\left(2a_n^{12}\sum_{m\neq n}\frac{a_m^{34}}{m_n^2 - m_m^2}\right)}_{P_1(n)} G_{\Delta_n,0}(x_i) + \underbrace{\left(\frac{a_n^{12}a_n^{34}}{\partial_n m_n^2}\right)}_{P_0(n)\gamma_1(n)} \partial_n G_{\Delta_n,0}(x_i)$$

$u^n \log u(1+\ldots)$

I. Scalar contact diagram: comments

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(x_i) = \sum_m$$



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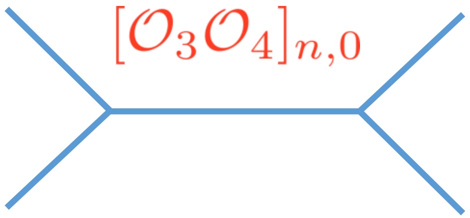

- No integration needed!
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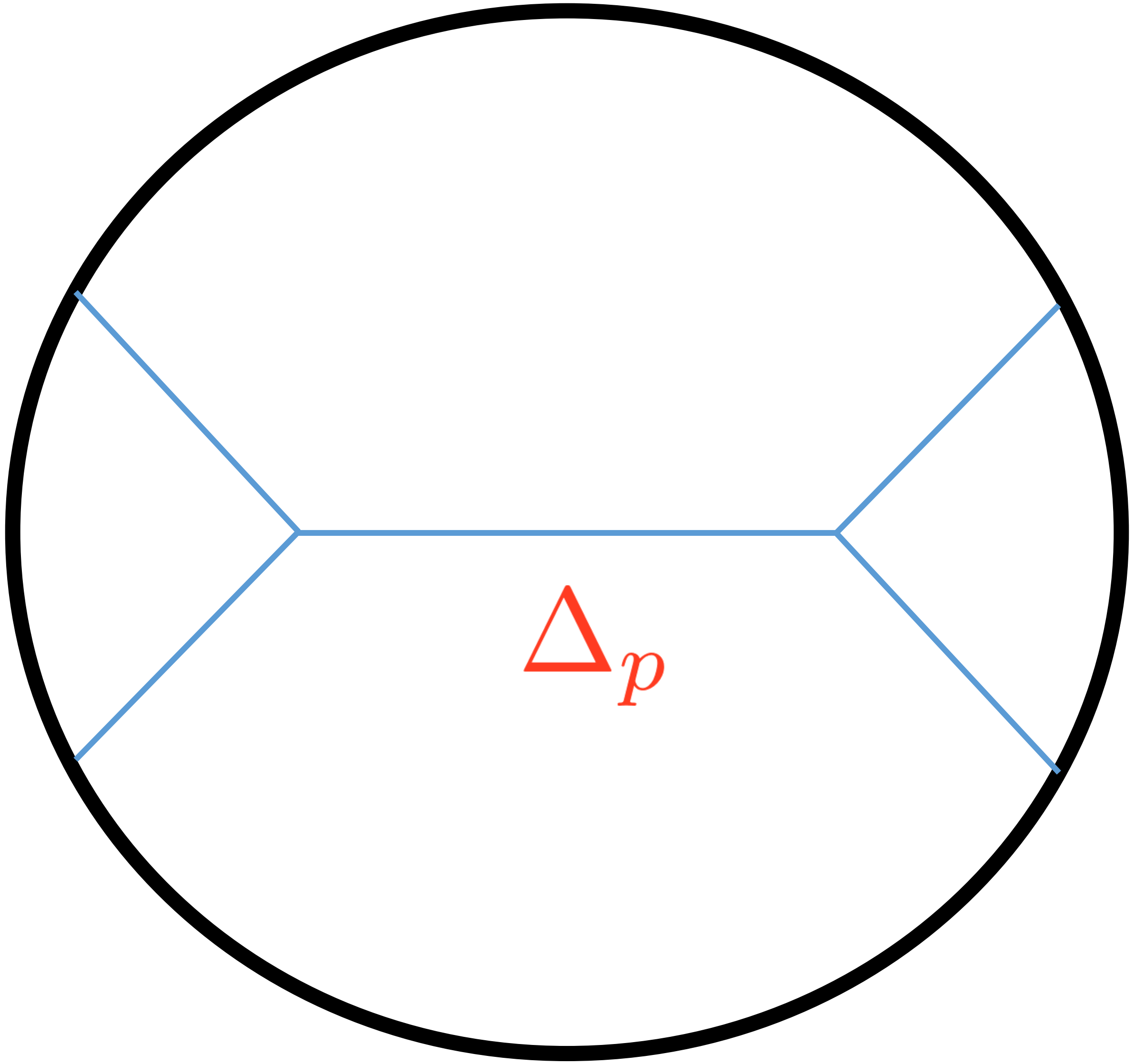
$$\langle \mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\rangle \approx (\text{free}) + \lambda \overline{D}_{1111}(x_i) + O(\lambda^2)$$

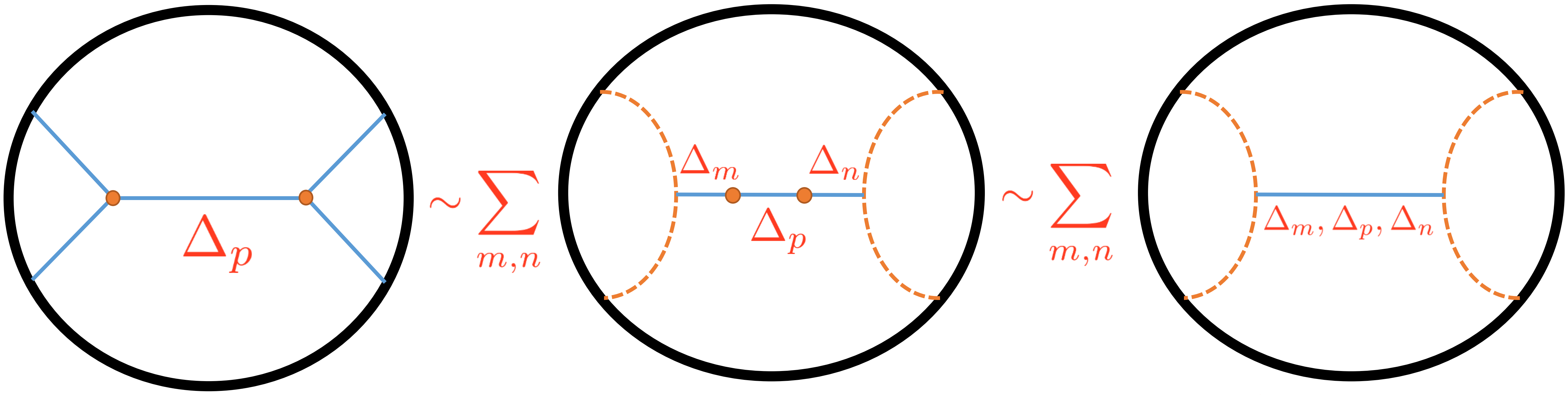
- Two Mellin comments:

1. OPE coeffs, anomalous dimensions have also been derived via Mellin space.
2. Despite its exponential Mellin representation, a conformal block does have a geometric interpretation in AdS.

II. Scalar exchange diagram



II. Scalar exchange diagram



$$=$$

$$\underbrace{\alpha_p G_{\Delta_p,0}(x_i)}_{\text{Single-trace}} + \sum_m \underbrace{\beta_m G_{\Delta_m,0}(x_i) + \sum_n \gamma_n G_{\Delta_n,0}(x_i)}_{\text{Double-trace}}$$

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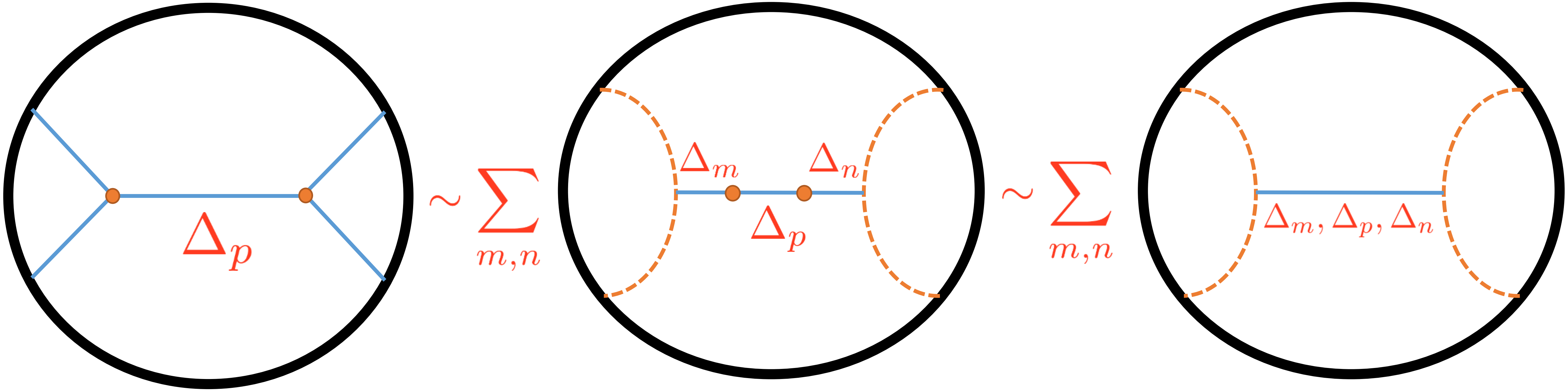
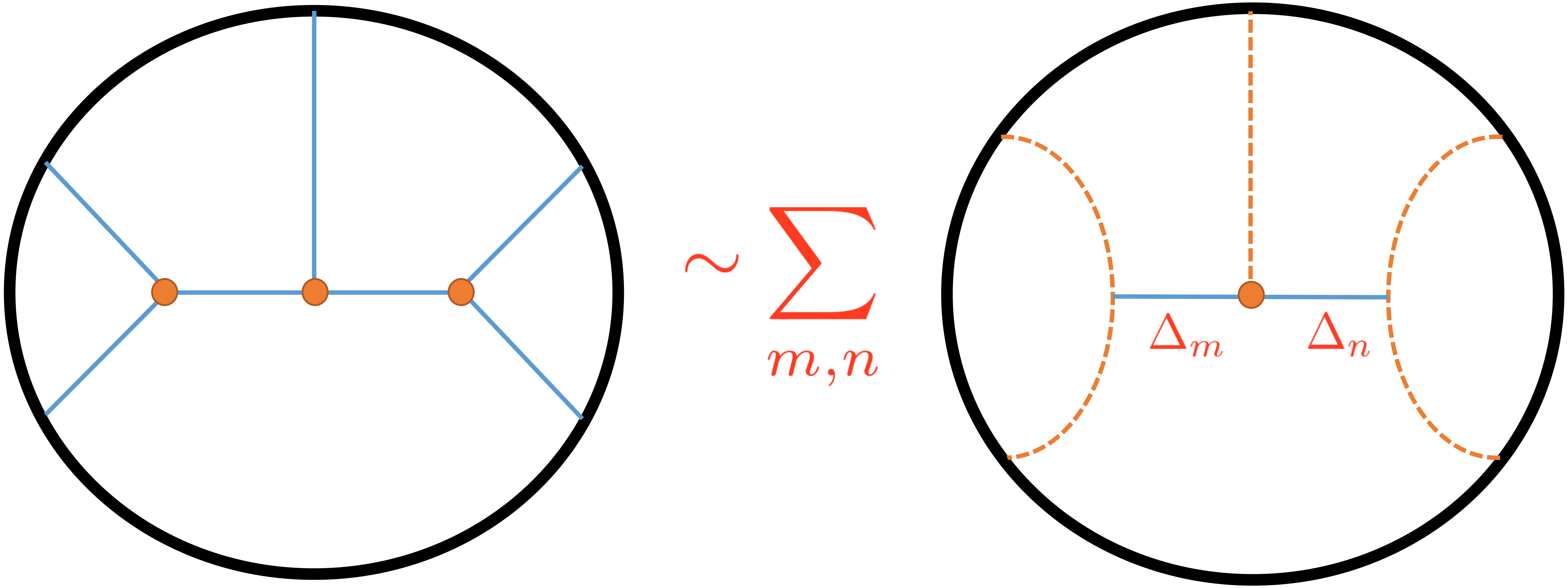


$$\alpha_p G_{\Delta_p,0}(x_i) + \sum_m \beta_m G_{\Delta_m,0}(x_i) + \sum_n \gamma_n G_{\Delta_n,0}(x_i)$$

We derive a simple relation between exchange and contact OPE coefficients: $\beta_m^{\text{Exchange}} = \beta_m^{\text{Contact}} \times \dfrac{1}{m_m^2 - m_p^2}$

III. Legs

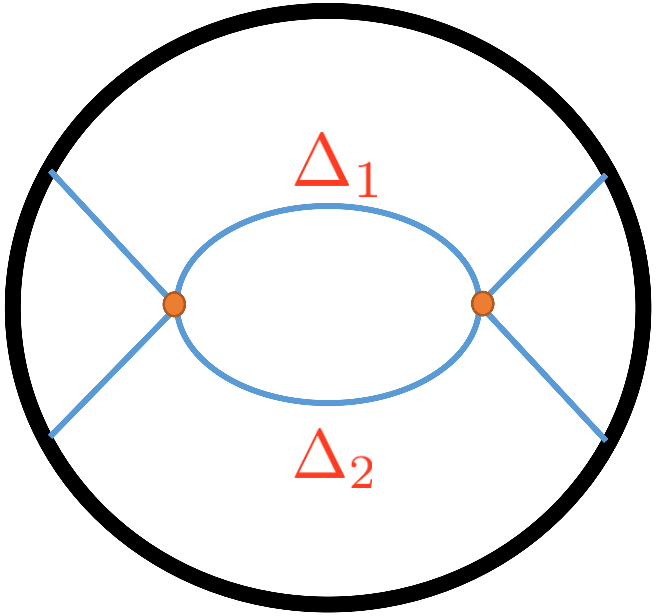
- These techniques apply to any number of external legs.



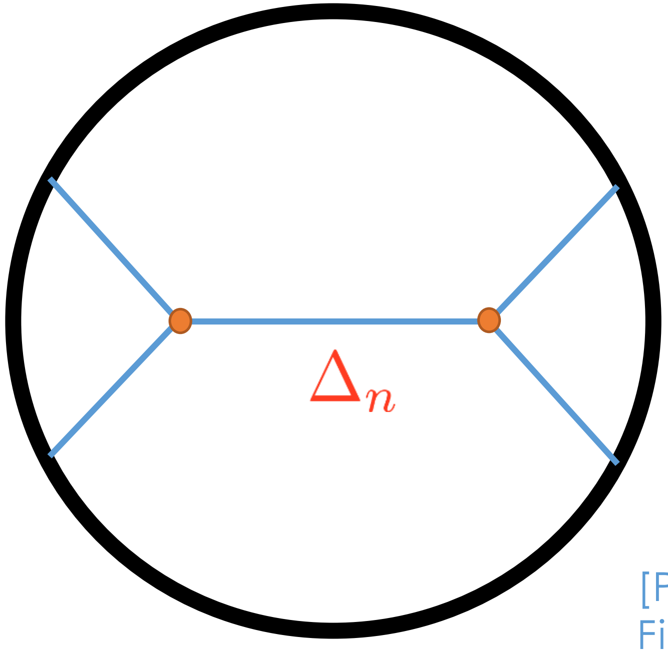
Holographic dual of a
5-point block

IV. Loops

- Certain loop graphs can be written as infinite sums over tree-level graphs, and hence can be easily decomposed:



$$= \sum_{n=0}^{\infty} f_{\Delta_1 \Delta_2}(n) \times$$



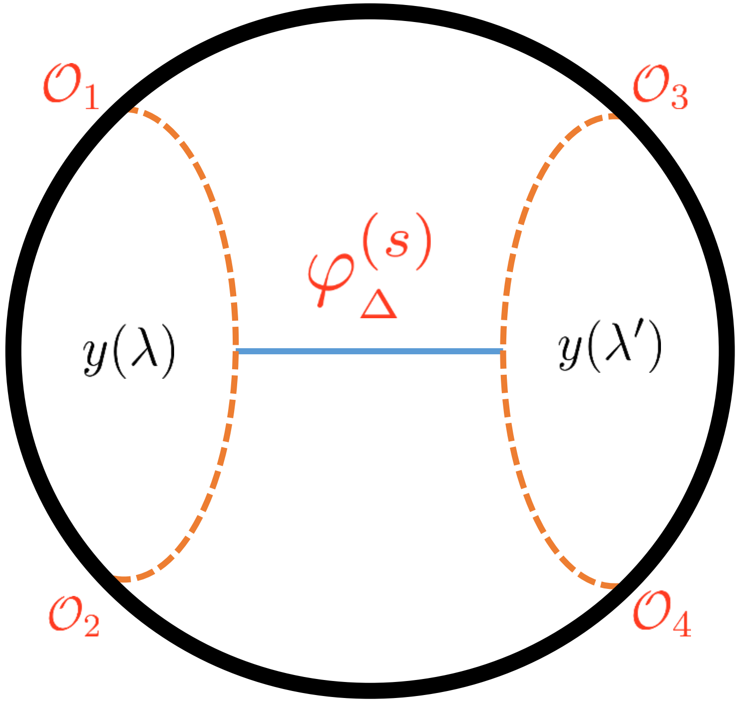
[Penedones; Fitzpatrick, Kaplan]

- General loop diagrams will contain interesting structures.

V. Spin

- The holographic dual of a conformal block for symmetric tensor exchange is:

$$G_{\Delta,s}(x_i) =$$



- That is, pull the spin-s propagator back to the geodesics. Schematically,

$$G_{\Delta,s}(x_i) = \int_\lambda \int_{\lambda'} G_{b\partial}(y(\lambda), x_1) G_{b\partial}(y(\lambda), x_2) \times G_{bb}^{(s)}(y(\lambda), y(\lambda'); \Delta) \times G_{b\partial}(y(\lambda'), x_3) G_{b\partial}(y(\lambda'), x_4)$$

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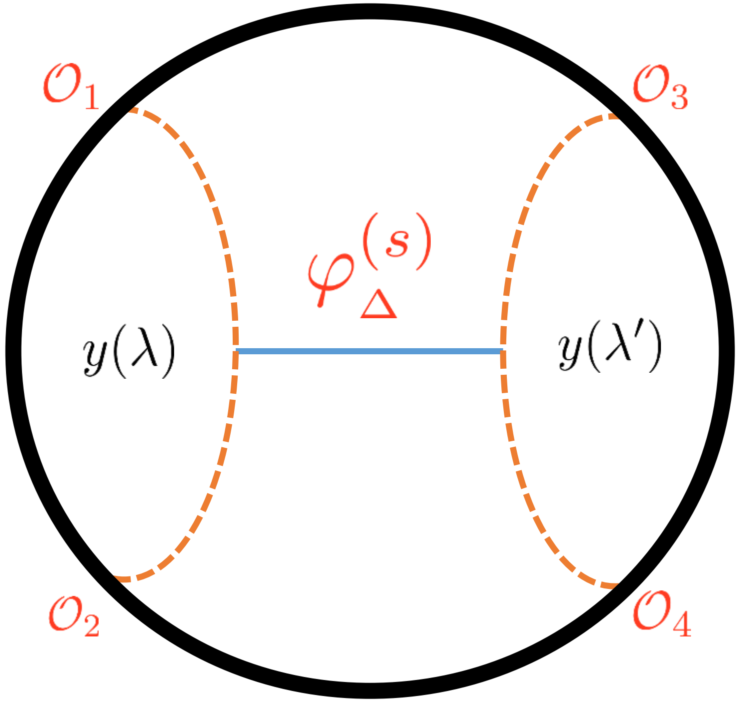

✓

Solves conformal
Casimir equation

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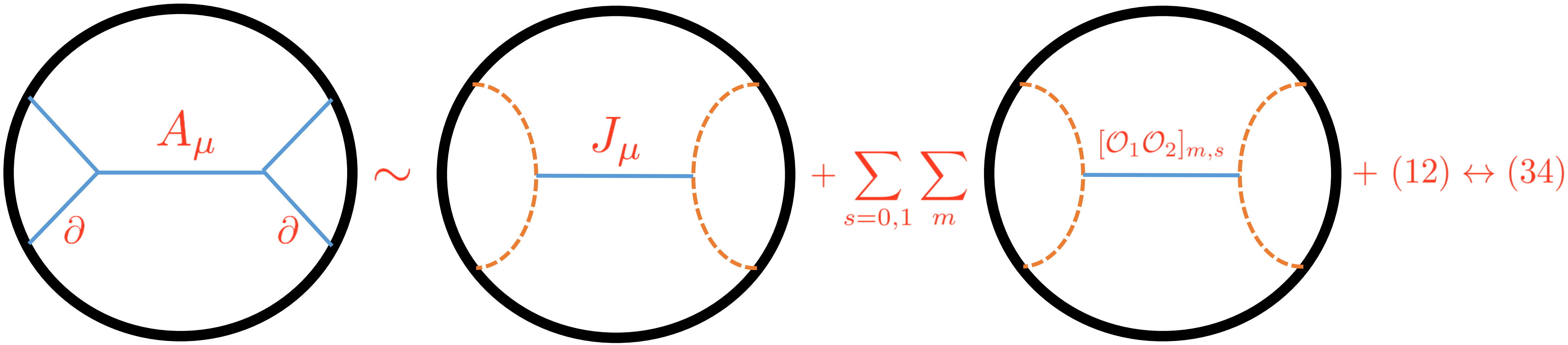
V. Spin

- The spin exchange block appears in massive STT spin-s exchange, or scalar contact diagrams with derivative vertices.



- Decomposition of all contact diagrams in scalar theory is in progress.

- Graviton exchange is straightforward!

Outline

Witten diagrams and the geometry of conformal blocks

Holographic duals of large c Virasoro blocks

Virasoro conformal blocks

$$SO(2,2) \simeq SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \subset \text{Vir} \times \text{Vir}$$

- Virasoro blocks contain contributions of full Virasoro conformal families:

$$\langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\mathcal{O}_3(z_3,\bar{z}_3)\mathcal{O}_4(z_4,\bar{z}_4)\rangle = (\text{Power law}) \times \sum_p C_{12p}{C^p}_{34}|\underline{\mathcal{F}(c,h_i,h_p,z)}|^2$$

- The blocks are known in a series expansion in small z [Alba, Fateev, Litvinov, Tarnopolsky]
- Better yet, the global decomposition of the blocks is also known, by solving Zamolodchikov's recursion relation:

$$\mathcal{F}(c,h_i,h_p,z) \propto \sum_{O_q \in M(c,h_p)} C_{12q}{C^q}_{34}\, z^{h_q}\, {}_2F_1(h_q+h_{12},h_q+h_{34};2h_q;z)$$

[EP]

Quasiprimaries:
O_p, $:TO_p:$, …

Known functions
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Virasoro conformal blocks @ large c

h_i, h_p fixed

$\frac{h_1}{c}, \frac{h_2}{c} \sim 1$
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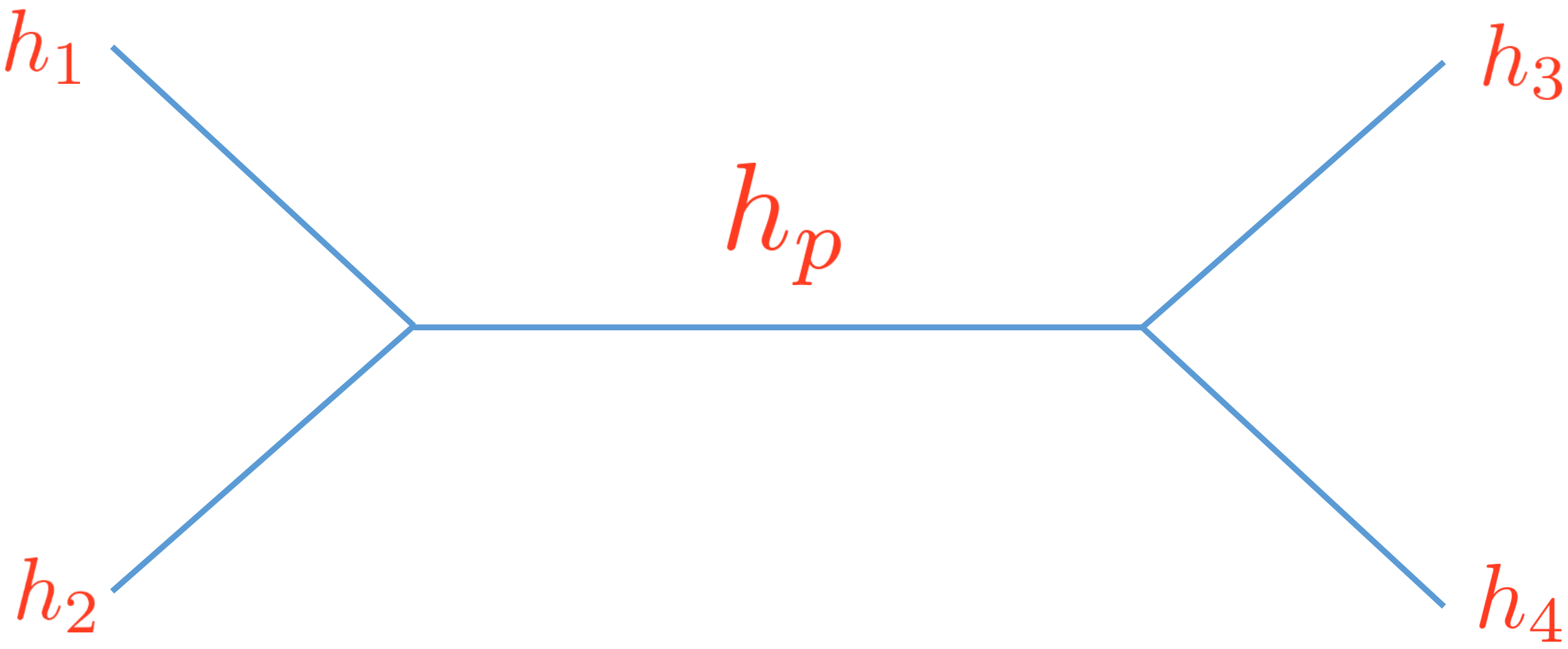

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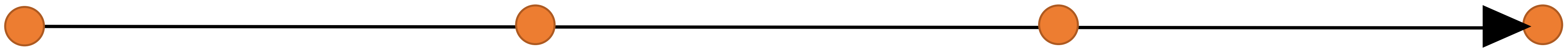

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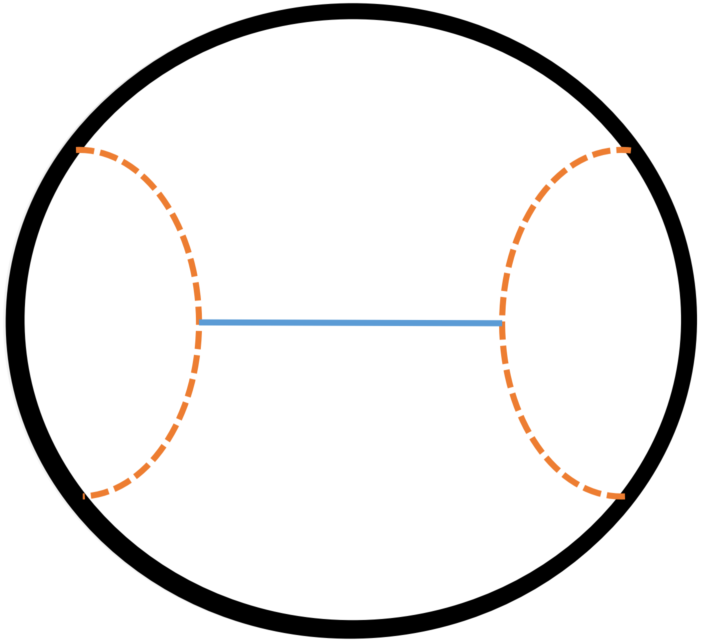
Global block:

$$\mathcal{F} \approx z^{h_p}\,{}_2F_1(h_p + h_{12}, h_p + h_{34}; 2h_p; z)$$



[Zamolodchikov]

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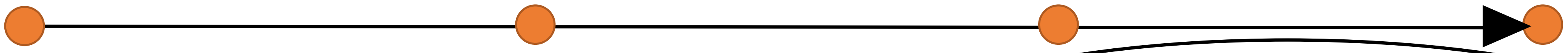

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f fixed by monodromy problem, soluble only perturbatively.

[Zamolodchikov]

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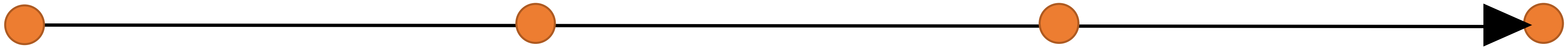

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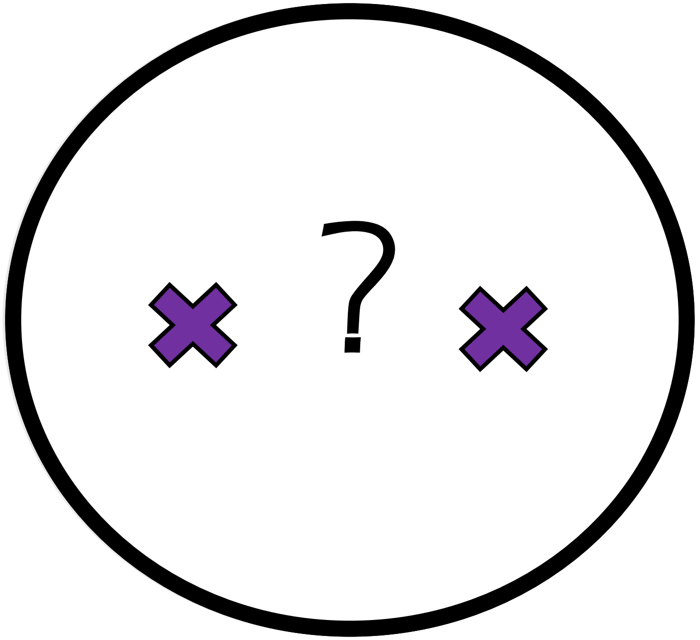

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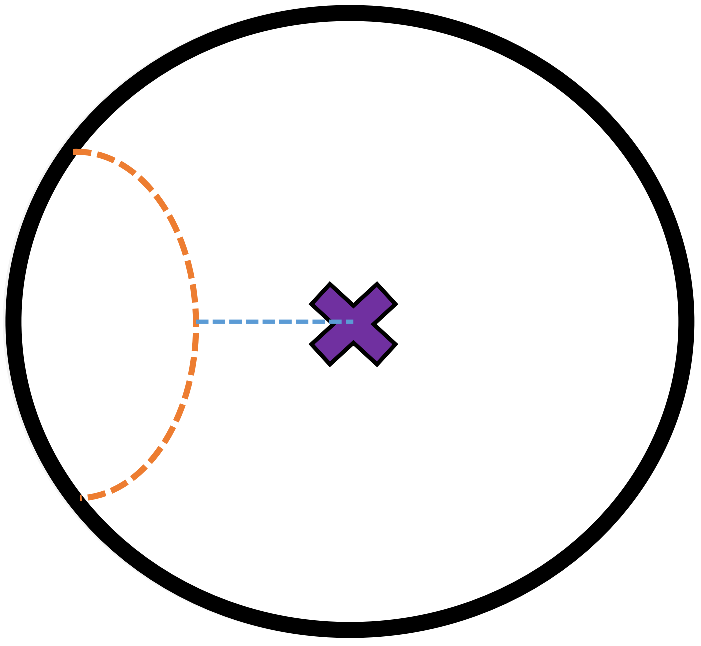

[Fitzpatrick, Kaplan, Walters; Hijano, Kraus, Snively; Alkalaev, Belavin]

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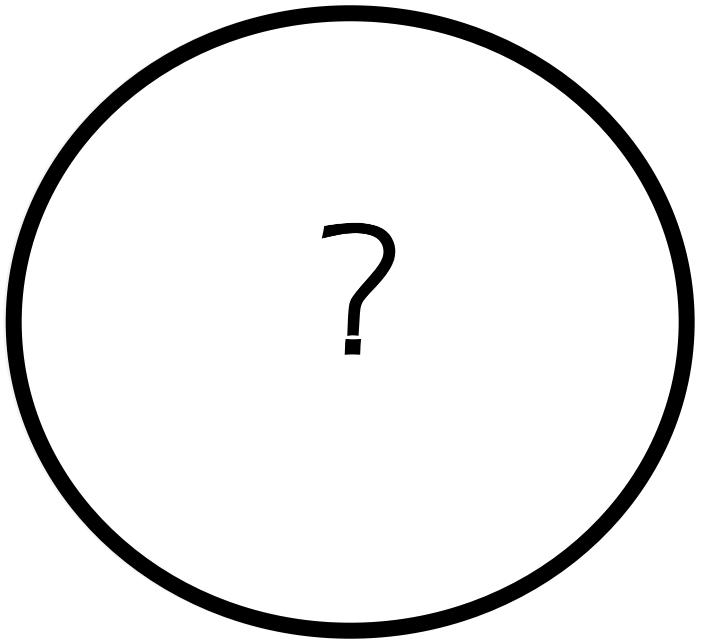
<u>Heavy-light limit:</u>

$\mathcal{F} =$ Global block in new background set up by heavy operators $\longleftrightarrow$



[Fitzpatrick, Kaplan, Walters]

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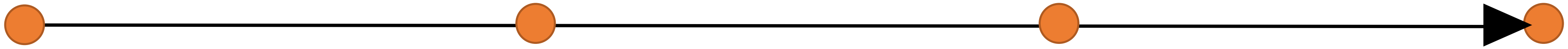

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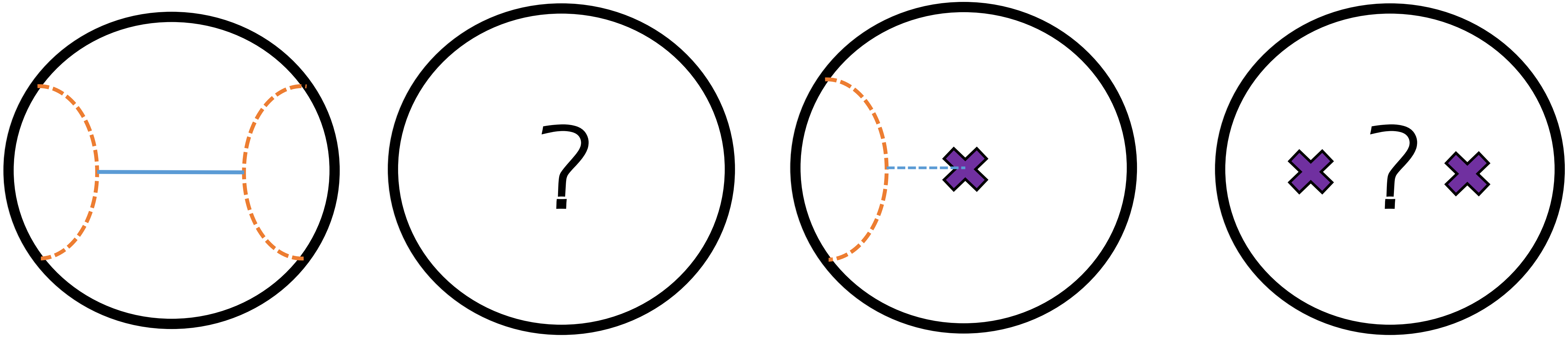
Holographic duals of large c Virasoro blocks:



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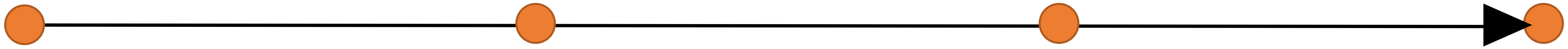

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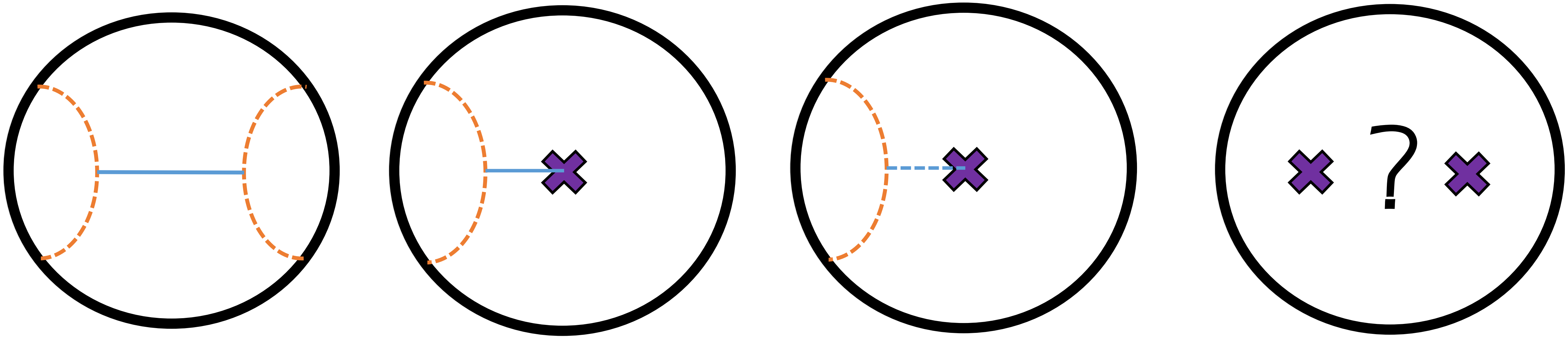

Future directions

- Further reformulation of AdS perturbation theory:
  - Loops
  - External spin
  - Graviton 4-point function: what are the holographic tensor structures?
- Geometric derivations of new results for conformal blocks:
  - Mixed symmetry exchange
  - Corrections to large spin, fixed twist

- What is the holographic dual of a generic Virasoro block?
- Are there analogs in non-AdS backgrounds, say, dS?
- What object in quantum gravity decomposes into conformal blocks?