A topological gauge theory for the entropy current

R. Loganayagam

School of Natural Sciences,
Institute for Advanced Study, Princeton.
Work done with Mukund Rangamani, Felix Haehl
[1412.1090], [1502.00636] & [work in progress].

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Motivation

- For me, this story begins a few years ago, when we were trying to understand how parity odd part of Cardy formula generalises to higher dimensions.

  Kristan Jensen, R. L., Amos Yarom. arXiv:1311.2935 [hep-th]

- We noticed that parity odd Cardy formula can be stated as an anomaly of a new abelian (say $U(1)_T$) gauge field coupled to the entropy current with temperature as its chemical potential.
- It was a curious (and a convenient) fact which I thought had no broader consequences.

Motivation II

- But, already there were evidences pointing to a broader structure.
- In his paper on the famous black hole entropy formula, Wald had argued long ago that BH entropy is the $U(1)_T$ charge on the horizon.
- In a paper which came immediately after ours, Sayantani constructed an entropy current for time-independent fluid configurations (hydrostatics) which can immediately be interpreted as the $U(1)_T$ current.

Sayantani Bhattacharyya. arXiv:1312.0220 [hep-th]

Motivation III

- Recently, we gave a complete classification of fluid equations admitting an entropy current [cf. Mukund's talk]

  Felix M. Haehl, R. L., Mukund Rangamani.arXiv:1412.1090 (summary)

  Felix M. Haehl, R. L., Mukund Rangamani. arXiv:1502.00636 (detailed)

- The entropy current in our classification occurs naturally as the $U(1)_T$ current.
- Further $U(1)_T$ seemed to implement in fluid equations, Kubo-Martin-Schwinger (KMS) like conditions (analytic continuation of Euclidean periodicity to real time).

The $U(1)_T$ conjecture

We will take these hints seriously and propose the following :

A fluid (or its dual black hole) is characterised by the emergence of a new abelian $U(1)_T$ gauge symmetry.

Its microscopic origins lie in KMS conditions.

Its macroscopic manifestation is entropy and its current is the entropy current.

Fluids do not have any propagating $U(1)_T$ photon. So we add

Its dynamics is of a topological BF type gauge theory coupled to an appropriate sigma model of fluid modes .

This talk is about to how this would work and what its consequences would be for both fluid dynamics and BH dynamics

Schwinger Keldysh field theory

- Let us start our story with a field theory whose fluid (or dual BHs) we want to describe.
- Interested in mixed state evolution (in canonical ensemble).
- Correct formalism for this is Schwinger-Keldysh field theories (also in-in formalism, doubled path integral, path integral for density matrix).

Fundamental features of SK theories

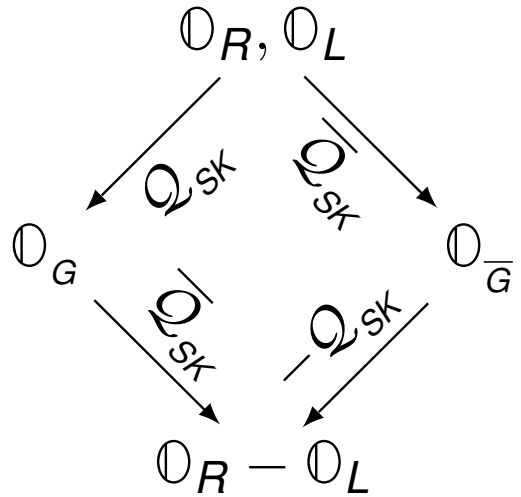
1. **Doubling** : All fields and symmetries occur in pairs (called left and right fields/symmetries)
2. **Topological limit** : Often stated as ordering prescription on the correlators. A more useful version is

   Correlators of difference operators are topological.

   SK version of Veltman's 'largest time equation'. **Encode unitarity/causality.**
3. **KMS conditions** : for thermal/near-thermal systems. *Non-local* conditions which guarantee correct analytic continuation to euclidean periodic correlators. **Encode the right entanglement.**

Topological limit via Quadrupling

- Easiest way to realise the second feature is in fact to **quadruple** by adding ghost pairs to SK pairs .
- Grassmann odd, nilpotent, mutually anti-commuting $\{\mathcal{Q}_{SK}, \overline{\mathcal{Q}}_{SK}\}$ such that



- All known SK effective theories use this quadrupling to achieve topological limit.

Long distance limit

- We want to figure out what is the long distance description of this SK theory.
- Standard belief is that this gives fluid dynamics - but how this happens is not understood.
- We will propose the following happens :
  1. **Doubled symmetries** are spontaneously broken to their diagonal. *Fluid dynamics is the sigma model of resultant goldstone modes (coresponding to broken difference symmetries).*
  2. There is a **topological limit** of this sigma model which is inherited from microscopic theory.
  3. **KMS conditions** become local at long distances and can be recast as an emergent $U(1)_T$ gauge invariance. Entropy current is the corresponding gauge current.

Long distance limit II

- Our goal : A $U(1)_T$ gauged sigma model (with an appropriate target space) which is a deformation of a $U(1)_T$ gauged *topological* sigma model with $\{\mathcal{Q}_{SK}, \overline{\mathcal{Q}}_{SK}\}$.
- Next step : does this give fluid dynamics ? Does $U(1)_T$ current agree with entropy current ?
- **Answer** : A qualified **Yes** !
- The sigma model itself is straightforward to write down and we have done various checks which work out very well.
- A detailed classification of allowed terms and their exact mapping to fluid transport would convert this to an unqualified Yes - work in progress.

The $U(1)_T$ gauge dynamics

- Will briefly describe some features of our sigma model in the topological limit.
- Presence of $\{\mathcal{Q}_{SK}, \overline{\mathcal{Q}}_{SK}\}$ means this is a *balanced* TQFT

R. Dijkgraaf, G. Moore. arXiv:9608169 [hep-th]

- The paradigmatic gauge theory which is also a balanced TQFT is the Vafa-Witten twist of $\mathcal{N} = 4_{4d}$ SYM which localises on to Anti-Self-Dual instantons with $F^{+}_{ab} = 0$.

Cumrun Vafa, Edward Witten. arXiv:9408074 [hep-th]

- We require localisation to hydrostatics with $(F_{ab})_{U(1)_T} = 0$. Given the amount of topological invariance required, one basically gets a *BF* type theory which is Vafa-Witten $\mathcal{N} = 4_{4d}$ without self-duality condition.

The $U(1)_T$ gauge dynamics II

Let us pause a bit to see the remarkable statement we have argued ourselves into from very reasonable assumptions

**Within every relativistic fluid,
entropy dynamics is associated with
a topological gauge theory which is
a twisted cousin of $\mathcal{N} = 4_{4d}$ SYM .**

- This obviously calls for further exploration and checks.
- One crucial check is whether it reproduces the correct anomaly induced transport and parity odd Cardy formula.
- In the remaining time, let me turn to speculations on the gravitational side and the implications to black hole physics.

Gravitational story

- Let us try now to implement the three features of Schwinger-Keldysh to BH description

  Doubling, Topological limit & **KMS conditions.**

- The first two conditions imply that we should begin with a doubled topological string theory which localises on to Hartle-Hawking vacuum and then deform it by sources.
- This is the search for a Ooguri-Strominger-Vafa (OSV) type statement for large AdS BHs dual to CFT fluids.
- The doubled topological string theory of OSV should in general replaced by some kind of a 'balanced topological string theory'

Gravitational story II

- We should now implement KMS conditions. Let us do this by taking seriously the idea that $U(1)_T$ emerges in IR.
- This can be implemented in string theory by putting an IR brane carrying this $U(1)_T$ gauge theory.
- The statement that the $U(1)_T$ gauge theory looks very similar to $\mathcal{N} = 4_{4d}$ SYM suggests that this $U(1)_T$ brane is an appropriate topological version of D3 brane.
- Thus we have arrived at another interesting statement

Large AdS_5 BHs are described by a deformation of a balanced topological *IIB* theory along with a 3-Brane carrying '$U(1)_T$ open strings'. It is dual to $\mathcal{N} = 4_{4d}$ SYM fluid dynamical sigma model .

Gravitational story III

- This is essentially 'membrane paradigm' with the $U(1)_T$ open strings realizing the 'strings on the horizon' idea of Susskind and others.
- In the fluid limit, this might make precise the membrane paradigm ideas by

  Dominik Nickel, Dam T. Son. arXiv:1009.3094
- It would obviously be enormously interesting to make these ideas precise.
- A good check would be to derive Wald formula by localisation or more ambitiously, a coherent description of the BH interior.

Gravitational story IV

- Let me end with a fun application which illustrates how various BH ideas fit nicely into this story.
- Since entropy is the $U(1)_T$ charge, the objects carrying entropy are naturally endpoints of '$U(1)_T$ open string' .
- Comparing this against $ER = EPR$ idea of Maldacena-Susskind, we identify '$U(1)_T$ open string' with their 'quantum wormhole'.
- A large number of $U(1)_T$ open strings stretched between two locations should then be dual to a wormhole via usual re-interpretation of open string diagrams as closed string diagrams.

Conclusions

- We took seriously Schwinger-Keldysh, its topological limit and an emergent gauge theory for the entropy current.
- And this leads to many tantalising statements and a calculable formalism for fluids (and presumably BHs).
- This is in fact a broad programme, **the fluid manifesto** if you wish, to understand fluid effective theories and the effective string theories for BHs.
- While we work out the computational details, we intend to put out a short paper with an outline of these ideas soon.
- We hope that a detailed examination of these ideas will lead us to answers to more mystifying questions like small BHs and dS gravity.

Thank you !