A State-Dependent Construction of the Black Hole Interior

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References

Based on work with Kyriakos Papadodimas (CERN & Groningen)

1. "Comments on the Necessity and Implications of State-Dependence in the Black Hole Interior", arXiv: 1503.08825.
2. "Local Operators in the Eternal Black Hole", arXiv:1502.06692.
3. "State-Dependent Bulk-Boundary Maps and Black Hole Complementarity", arXiv: 1310.6335.
4. "The Black Hole Interior in AdS/CFT and the Information Paradox", arXiv:1310.6335.
5. "The unreasonable effectiveness of exponentially suppressed corrections in preserving information"
6. "An Infalling Observer in AdS/CFT",arXiv:1211.6767.

and work in progress with Souvik Banerjee (Groningen), Prashant Samantray (IIT-Indore), Sudip Ghosh (ICTS-TIFR).

Context

- The context for this talk is the Information Paradox. In its modern avatar, this turns into the question:

  "Can AdS/CFT describe the BH Interior?"

  [Mathur, Almheiri, Marolf, Polchinski, Sully, Stanford, 2009-2015]

- This version is not restricted to evaporating BHs.

- Paradox extends to the Eternal Black Hole.

  [Kyriakos Papadodimas, S.R., 2015]

Overview

- Resolution: Paradox can be complete resolved using a state-dependent map between interior bulk observables and boundary observables.

  [K.P., S.R, 2013–15]

- New Consequences: This construction of the interior leads to a precise version of ER=EPR conjecture of Maldacena and Susskind.

Outline

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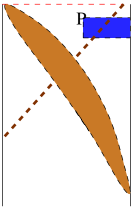
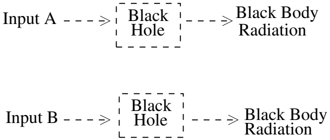
The Old Information Paradox



- In the shaded patch, physics is independent of details of collapse.

$$\langle a^{\dagger}_{\omega} a_{\omega'} \rangle = \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}} \delta(\omega - \omega')$$

- Suggests that for different inputs, we get the same output.



Resolution to the Old Information Paradox

- Very small corrections of the order of e^{-S} can restore unitarity. [Maldacena, 2001]

- Pure density matrix in a very large system can mimic a thermal density matrix to extreme accuracy

$$\mathrm{Tr}\left(\rho_{\text{pure}} A_\alpha\right) = \frac{1}{\mathcal{Z}} \mathrm{Tr}\left(e^{-\beta H} A_\alpha\right) + \mathrm{O}\left(e^{-\frac{S}{2}}\right),$$

for a large class of observables A_α.

- Another way to state this is

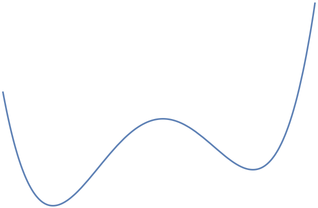
$$\rho_{\text{pure}} = \frac{1}{\mathcal{Z}} e^{-\beta H} + e^{-S} \rho_{\text{corr}}; \quad \rho_{\text{pure}}^2 = \rho_{\text{pure}}$$

Information can be encoded in tiny correlations between the Hawking quanta.

Path Integral Perspective

- Effective field theory insufficient to control such corrections.
- A semi-classical spacetime is a saddle point of the QG path-integral.

$$\mathcal{Z} = \int e^{-S} \mathcal{D} g_{\mu\nu}$$



- Perturbative effective field theory (used to derive the Hawking answer) is an asymptotic series expansion of this path-integral.
- Non-perturbatively, the notion of local spacetime breaks down.

Outline

Revisiting the Information Paradox

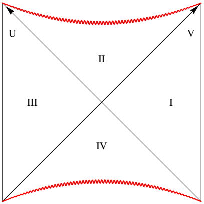
- Recent developments have challenged the notion that small corrections can resolve the information paradox.

  [Mathur, Almheiri, Marolf, Polchinski, Sully, Stanford, 2009–13]

- We will review an extension of these arguments to the eternal black hole.

- Resolution of the paradox in the eternal black hole provides strong evidence that a similar resolution applies to the single-sided case.

Thermofield Doubled State



- Eternal black hole is dual to an entangled state of two CFTs

$$|\Psi_{\text{tfd}}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_E e^{-\frac{\beta E}{2}} |E, E\rangle$$

[Maldacena, 2001]

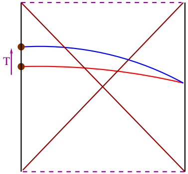
- On the left boundary, we identify

$$t_{\text{sch}} = -t_{CFT,L}$$

Time Shifted States

- We now consider time-shifted states
$$|\Psi_{\rm T}\rangle = e^{i H_L T} |\Psi_{\rm tfd}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum e^{i \phi_E - \frac{\beta E}{2}} |E, E\rangle$$

- Geometrically, this corresponds to a large diffeomorphism that dies off at the right-boundary but not left-boundary.



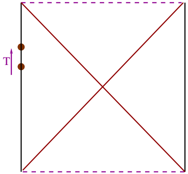
For example

$$\begin{aligned} U &\to U\left[\gamma \hat{\theta}(-X) + \hat{\theta}(X)\right]; \quad X = V - U \\ V &\to \frac{V}{\gamma}\left[\hat{\theta}(-X) + \gamma \hat{\theta}(X)\right]; \qquad \gamma = e^{\frac{2\pi T}{\beta}}. \end{aligned}$$

with $\hat{\theta}$ a smooth version of the theta function.

Smoothness of Time Shifted States

- Large diffs that differ by trivial diffs are equivalent. Use this to undo the diff everywhere, except infinitesimally close to the boundary.



- Makes it clear that the large diffs leaves intrinsic properties of the geometry invariant, but just slides the boundary.

Therefore, the states $|\Psi_{\text{T}}\rangle = e^{iH_L T}|\Psi_{\text{tfd}}\rangle$ are also smooth but glued differently to the boundary

$$t_{\text{sch}} = -t_{CFT,L} + T.$$

Long Time Shifts

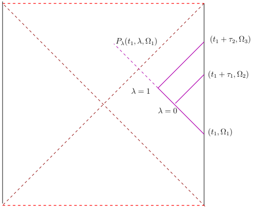
The states $|\Psi_{\rm T}\rangle = e^{iH_L T}|\Psi_{\rm tfd}\rangle$ are smooth, even for $T \sim e^{N^2}$

- Strong conclusion relies only on equivalence of Hamiltonian evolution and diffeomorphisms; not on eom.
- Equivalent to statement that an infalling observer from the right observes the same geometry at arbitrarily late time.
- Equivalent to statement that there is no natural common origin of time on the two sides.

Relational Observables

- AdS boundary allows us to define quasi-local diffeomorphism invariant observables.

- Consider the following process: jump from the boundary, wait for a certain proper time, measure the field.



- Behind the horizon, we write

$$\phi(t, \Omega, \lambda) = \sum_{\omega, m} \mathcal{O}_{\omega,m} e^{-i\omega t} g^{(1)}_{\omega,m}(\Omega, \lambda) + \boxed{\widetilde{\mathcal{O}}_{\omega,m}} e^{i\omega t} g^{(2)}_{\omega,m}(\Omega, \lambda) + \text{h.c},$$

Central Question

- Can we find a CFT operator $\phi(x)$, so that for generic T

$$\boxed{\langle \Psi_{\rm T} | \phi(x_1) \ldots \phi(x_n) | \Psi_{\rm T} \rangle = G(x_1 \ldots x_n)}$$

  with $|\Psi_{\rm T}\rangle = e^{i H_L T} |\Psi_{\rm tfd}\rangle$.

- On the RHS, G is the Green function as computed by semi-classical effective field theory in the eternal black hole background.

- Well-posed question about existence of operators in the CFT.

Modern Information Paradox

- By extending work of AMPSS, can show that $\nexists \phi(x)$ s.t. $\forall T$

$$\langle \Psi_{\rm T} | \phi(x_1) \ldots \phi(x_n) | \Psi_{\rm T} \rangle = G(x_1 \ldots x_n).$$

[K.P,S.R., 2015]

- Reason is, roughly, that states $|\Psi_{\rm T}\rangle$ are overcomplete.

- Seems different from the information paradox, but principal paradox is still

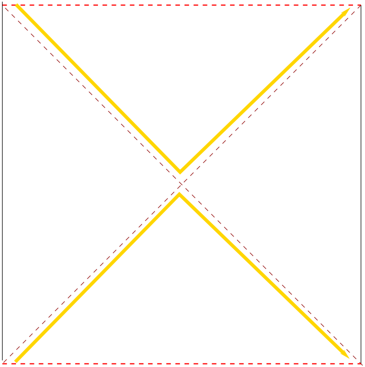
Unitarity vs Effective Field Theory

Some Possibilities

- AdS $\neq$ CFT (a.k.a. superselection sectors)?

  [Marolf, Wall, 2012]

- Eternal Black Hole also has firewalls?

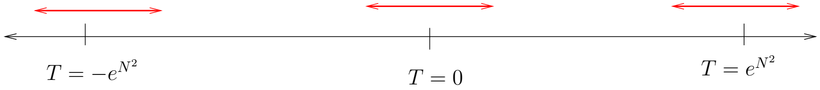


But both possibilities above contradict explicit computations that look inside the horizon of the eternal BH.

[Hartman, Maldacena, Kraus, Ooguri, Shenker]

Proposed Resolution: State Dependence

- No need to use the "same operator" for all T: state-dependence.
- We use one operator $\phi^{\{0\}}$ in a range of states about $T = 0$.
- After exponentially long T, we switch to another operator $\phi^{\{T\}}$



Outline

The Little Hilbert Space

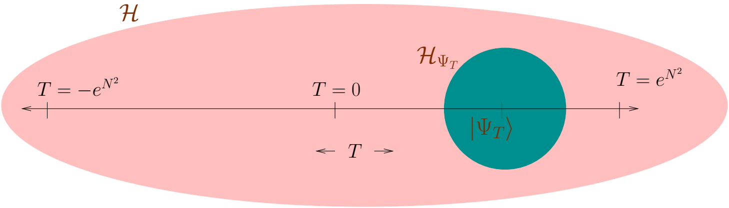
- $|\Psi_T\rangle \equiv$ Black Hole Microstate
- Little Hilbert Space: all possible effective field theory excitations of $|\Psi_T\rangle$

$$\mathcal{H}_{\Psi_T} = \mathcal{A}|\Psi_T\rangle,$$

$$\mathcal{A} = \text{span}\{\mathcal{O}_{\omega_1},\ \mathcal{O}_{\omega_1}\mathcal{O}_{\omega_2}, \ldots, \mathcal{O}_{\omega_1}\mathcal{O}_{\omega_2}\ldots\mathcal{O}_{\omega_K}\}.$$

with

$$\omega_m \ll \mathcal{N}, \quad K \ll \mathcal{N}$$



Definition of $\widetilde{\mathcal{O}}_\omega$

- Define $\widetilde{\mathcal{O}}_\omega$ precisely within H_Ψ

$$SA_\alpha|\Psi\rangle = A_\alpha^\dagger|\Psi\rangle$$

and

$$\boxed{\widetilde{\mathcal{O}}_\omega = S\Delta^{\frac{-1}{2}}\mathcal{O}_\omega\Delta^{\frac{1}{2}}S}$$

[KP, SR, 2013]

- This is closely related to the isomorphism used in Tomita-Takesaki theory.

- $\phi(t, \Omega, \lambda)$ constructed using this $\widetilde{\mathcal{O}}_\omega$ is a linear operator on H_Ψ and has the correct effective field theory correlators

Obtaining a Smooth Horizon

- In eternal BH, can carry out this program explicitly

$$\widetilde{\mathcal{O}}_\omega = \sqrt{\frac{C}{\pi\beta^2}} \int_{-T_{\text{cut}}}^{T_{\text{cut}}} \mathcal{O}_{L\omega}(T_i) P_{\mathcal{H}_{\Psi_{T_i}}} dT_i,$$

- Smear with appropriate mode functions

$$\phi(x) = \sum_\omega \left[\mathcal{O}_\omega\, g_\omega^{(1)}(x) + \widetilde{\mathcal{O}}_\omega\, g_\omega^{(2)}(x) + \text{h.c.}\right]$$

Obtaining a Smooth Horizon

- We can probe the interior with correlators of these operators.

$$\langle \Psi_{\mathrm{T}} | \phi(x_1) \ldots \phi(x_n) | \Psi_{\mathrm{T}} \rangle = G(x_1, \ldots x_n)$$

- Explicitly consistent with perturbative fields propagating on a weakly curved spacetime near the horizon.

This construction explicitly resolves the information paradox for the eternal black hole. Problem of overcompleteness is resolved by state-dependence.

Outline

New Applications

- Apart from resolving the information paradox, we can apply this construction of the interior to other entangled systems.
- Leads to a natural formulation of "ER=EPR". (Originally proposed as relation between entanglement and wormholes by Maldacena and Susskind.)

Simple Operators and the Little Hilbert Space for Entangled Systems

- Consider an entangled state

$$|\Psi_{\text{en}}\rangle = \sum \alpha_i |\widetilde{\Psi}_i\rangle \otimes |\Psi_i\rangle$$

- Expand the set of simple operators to include both left and right operators.

$$\mathcal{A}, \quad \mathcal{A}_L, \quad \mathcal{A}_{\text{prod}} = \mathcal{A}_L \otimes \mathcal{A}$$

- Therefore the little Hilbert space is

$$\mathcal{H}_{\Psi_{\text{en}}} = \mathcal{A}_{\text{prod}} |\Psi_{\text{en}}\rangle$$

Mirrors for Entangled States

- $$\mathcal{H}_{\Psi_{\text{en}}} = \mathcal{A}_{\text{prod}} |\Psi^j_{\text{en}}\rangle = \bigoplus_j \mathcal{A} |\Psi^j_{\text{en}}\rangle$$

- We may have null relations due to entanglement. For example, in the thermofield $\mathcal{O}_{L,\omega}|\Psi_{\text{tfd}}\rangle = e^{\frac{-\beta\omega}{2}} \mathcal{O}^{\dagger}_{\omega}|\Psi_{\text{tfd}}\rangle$.

$$\mathcal{H}_{\Psi_{\text{en}}} \not\cong \mathcal{A}_{\text{prod}}$$

- Number of terms in the sum depends on number of null relations involving both left and right operators acting on the state.

- Define

$$S = \sum_j S_j$$

ER=EPR from Mirror Operators

Are correlators of right-relational observables affected by simple unitaries on the left?

- With $U_L = e^{iA_L}$, compare

$$\langle \Psi_{\rm en} | U_L^\dagger \phi(x_1)\phi(x_2)\ldots\phi(x_n) U_L | \Psi_{\rm en}\rangle$$

and

$$\langle \Psi_{\rm en} | \phi(x_1)\phi(x_2)\ldots\phi(x_n) | \Psi_{\rm en}\rangle$$

- Equivalent diagnostic is

Commutator	$[\widetilde{\mathcal{O}}_\omega, \mathcal{O}^\dagger_{L,\omega}] \vert \Psi_{\rm en}\rangle$
Two-Pt Function	$\langle \Psi_{\rm en} \vert \widetilde{\mathcal{O}}_\omega \mathcal{O}^\dagger_{L,\omega} \vert \Psi_{\rm en}\rangle$

- We can apply this diagnostic to several examples.

Standard Wormhole for the Thermofield

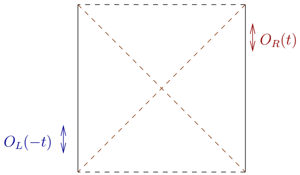
- Consider the thermofield double state

$$|\Psi_{\text{tfd}}\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\frac{\beta E}{2}} |E, E\rangle$$

- We find the commutator and two point function

$$[\mathcal{O}^{\dagger}_{L\omega}, \widetilde{\mathcal{O}}_{\omega}] \doteq C_{\omega}, \quad \langle \Psi_{\text{tfd}} | \widetilde{\mathcal{O}}_{\omega} \mathcal{O}^{\dagger}_{L\omega} | \Psi_{\text{tfd}} \rangle = G_{\beta,\omega}.$$

- By computing these and other correlators we can obtain a picture of the dual geometry.



Generic Entanglement: "Long Wormhole"

- Consider a more "generic" entangled state

$$|\Psi_{\text{gen}}\rangle = U_L^{\text{arb}} |\Psi_{\text{tfd}}\rangle$$

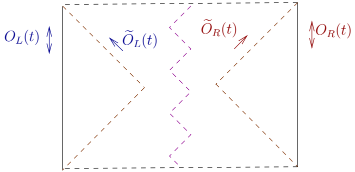
- Now we find that

$$[\widetilde{\mathcal{O}}_\omega, \mathcal{O}^\dagger_{L\omega}] \doteq \text{O}\left(e^{-\frac{S}{2}}\right), \quad \langle \Psi_{\text{gen}} | \widetilde{\mathcal{O}}_\omega \mathcal{O}^\dagger_{L\omega} | \Psi_{\text{gen}} \rangle = \text{O}\left(e^{-\frac{S}{2}}\right).$$

- But for some generic CFT operator on the left

$$\langle \Psi_{\text{gen}} | [Y_L, \widetilde{\mathcal{O}}_\omega] |^2 | \Psi_{\text{gen}} \rangle = \text{O}(1).$$

- These correlators suggest a dual geometric picture.



Microcanonical Double: Low Band-Pass Wormhole

- Consider the microcanonical doubled state.

$$|\Psi_{\text{md}}\rangle = \frac{1}{\sqrt{\mathcal{D}}} \sum_{E_i = E - \Delta}^{E_i = E + \Delta} |E_i, E_i\rangle.$$

  Take $\beta \Delta \ll 1$. This leads to a low pass wormhole!

- For low frequencies, $\omega \ll \Delta$,

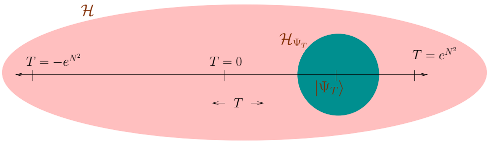
$$[\widetilde{\mathcal{O}}_\omega, \mathcal{O}^\dagger_{L\omega}] = C_\beta(\omega), \quad \langle \Psi_{\text{md}} | \widetilde{\mathcal{O}}_\omega \mathcal{O}^\dagger_{L\omega} | \Psi_{\text{md}} \rangle = G_\beta(\omega).$$

- But for high frequencies, $\omega \gg \Delta$,

$$[\widetilde{\mathcal{O}}_\omega, \mathcal{O}^\dagger_{L\omega}] = 0 + \mathrm{O}\left(\frac{\omega}{\Delta}\right), \quad \langle \Psi_{\text{md}} | \widetilde{\mathcal{O}}_\omega \mathcal{O}^\dagger_{L\omega} | \Psi_{\text{md}} \rangle = 0 + \mathrm{O}\left(\frac{\omega}{\Delta}\right).$$

Outline

Why is State-Dependence Surprising?

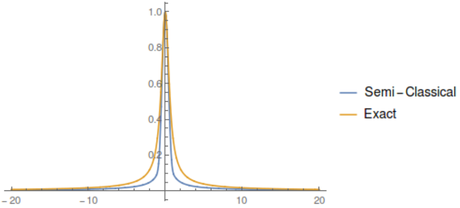


- Canonical gravity suggests that relationally defined observables are linear operators on the H-space.
- Canonical gravity intuition valid for the little Hilbert space $\Rightarrow$ no infalling observer can detect state-dependence within EFT.
- Violations of Born Rule $\leftrightarrow$ conceptually defined local bulk observables correspond to non-linear CFT operators when considered on the full H-space

Failure of Canonical Intuition in TFD

- CFT inner-product has a fat tail invisible to canonical gravity.

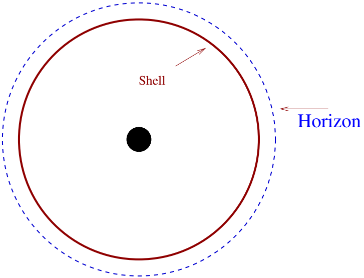
$$|\langle\Psi_{\rm tfd}|\Psi_{\rm T}\rangle| = e^{\frac{-CT^2}{2\beta^2}}, \quad T \ll 1$$
$$|\langle\Psi_{\rm tfd}|\Psi_{\rm T}\rangle| = \mathrm{O}\left(e^{\frac{-S}{2}}\right), \quad T \gg 1$$



$$\widetilde{\mathcal{O}}_{\omega} = \sqrt{\frac{C}{\pi\beta^2}} \int_{-T_{\rm cut}}^{T_{\rm cut}} \mathcal{O}_{L\omega}(T_i) P_{\mathcal{H}_{\Psi_{T_i}}} dT_i,$$

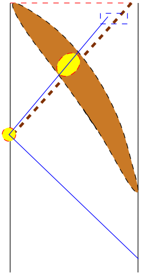
- State-independence $\Leftrightarrow$ $T_{\rm cut} \to \infty$; Prevented by fat-tail.

Marolf-Polchinski Shells



- For every black hole with empty interior at late times, consider configuration with a ultra-relativistic shell just inside the horizon.
- Binding energy with BH cancels rest+kinetic energy of shell $\Rightarrow$ increase in AdM energy is small.
- So, Marolf-Polchinski claim that as many states with firewalls as states with smooth interiors $\Rightarrow$ mirror operators must be singular.

Back-Reaction



- However, the M-P states have singularities in the past.
- So, gravitational back-reaction important in out-of-time order correlator

$$\langle \Psi | U_{\rm MP}^{\dagger} \phi(x_{\rm out}) \phi(x_{\rm in}) U_{\rm MP} | \Psi \rangle = \langle \Psi | \phi(x_{\rm out}) \phi(x_{\rm in}) | \Psi \rangle ?$$

Other Examples of State-Dependence

- The Ryu-Takayanagi formula, and other attempts to relate geometry to entanglement are state-dependent.

$$\frac{1}{4G_N}\langle A(R)\rangle = S_{\text{ent}}(R)$$

- On the LHS, $A(R)$ is an operator in canonical gravity. But, easy to show that

$$\nexists X, \text{ s.t.} \langle X\rangle = S_{\text{ent}}(R)$$

- Similar comments hold for ER=EPR, and other relations involving complexity and geometry etc.

- This does not prove that geometric quantities are state dependent, but is suggestive.

Summary

- Modern Information Paradox can be rephrased as a question about the existence of CFT operators dual to bulk fields.
- Paradox extends to the eternal black hole.
- Can be resolved using a state-dependent map between boundary and bulk fields.
- This map can be written down precisely.
- Leads naturally to ER=EPR.

Appendix

More on back-reaction

- Naive MP prediction is that

$$\langle \Psi | e^{-iN_\omega \theta} \widetilde{a}_{\omega'} a_\omega e^{iN_\omega \theta} | \Psi \rangle = \frac{e^{\frac{-\beta\omega}{2}} e^{i\theta\omega}}{1 - e^{-\beta\omega}} \delta(\omega - \omega')$$

- Naively, one may also have thought that

$$\langle \Psi | e^{-iH_L T} \widetilde{a}_{\omega'} a_\omega e^{iH_L T} | \Psi \rangle = \boxed{e^{i\omega T}} \frac{e^{\frac{-\beta\omega}{2}}}{1 - e^{-\beta\omega}} \delta(\omega - \omega')$$

But this is incorrect.

- Phase operator is more non-trivial, but note that the naive prediction is clearly suspect in the presence of back-reaction.

Properties of Mirror Modes

- From analysis of large diffeomorphisms, we find

$$\langle \Psi_{\rm T} | A_R [H, \widetilde{\mathcal{O}}_{\omega}] | \Psi_{\rm T} \rangle = \omega \langle \Psi_{\rm T} | A_R \omega \widetilde{\mathcal{O}}_{\omega} | \Psi_{\rm T} \rangle.$$

- Demanding that two-pt function of ϕ agrees with effective field theory

$$\langle \Psi_{\rm T} | \widetilde{\mathcal{O}}_{\omega} \widetilde{\mathcal{O}}_{\omega}^{\dagger} | \Psi_{\rm T} \rangle = e^{\beta\omega} \langle \Psi_{\rm T} | \widetilde{\mathcal{O}}_{\omega}^{\dagger} \widetilde{\mathcal{O}}_{\omega} | \Psi_{\rm T} \rangle$$

Long Time Average

- Long Time Average $\Rightarrow$ cross-terms drop out.

$$\frac{1}{T_{\rm av}}\int_{-T_{\rm av}}^{T_{\rm av}} \langle \Psi_{\rm T}|X|\Psi_{\rm T}\rangle dT = \frac{1}{Z} e^{-\beta E}\langle E, E|X|E, E\rangle$$
$$+\frac{e^{-\frac{\beta(E+F)}{2}}}{Z T_{\rm av}} \sin\left[(E-F)T_{\rm av}\right]\sum_{E,F}\langle E, E|X|F, F\rangle$$

- From a Laplace Transform

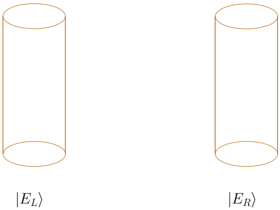
$$\langle E, E|\widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger|E, E\rangle = e^{-\beta\omega}\langle E, E|\widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega|E, E\rangle$$
$$\langle E, E|A_R[H, \widetilde{\mathcal{O}}_\omega]|E, E\rangle = \omega\langle E, E|A_R\widetilde{\mathcal{O}}_\omega|E, E\rangle.$$

No Wormhole in Eigenstate Pairs

- QUESTION: How can state-independent operators describe smooth effective field theory in eigenstate pairs — which have no entanglement — if they cannot do so in single eigenstates.

- To sharpen this: assume that there is no "wormhole" in eigenstate pairs.

$$\langle E, E| U_L^{\dagger} \phi(x_1) \ldots \phi(x_n) U_L |E, E\rangle = \langle E, E| \phi(x_1) \ldots \phi(x_n) |E, E\rangle, \forall U_L$$



The "Occupancy" Paradox for the Eternal Black Hole

- Now, we can set up a version of the AMPSS paradox.

$$\langle \Omega, E | \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger | \Omega, E \rangle \approx \frac{1}{Z(\beta)} \mathrm{Tr}_R (e^{-\beta H_R} \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger)$$
$$= \frac{1}{Z(\beta)} e^{-\beta\omega} \mathrm{Tr}(e^{-\beta H} \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega) \approx e^{-\beta\omega} \langle \Omega, E | \widetilde{\mathcal{O}}_\omega^\dagger \widetilde{\mathcal{O}}_\omega | \Omega, E \rangle ?$$

- Here, we use equivalence of microcanonical and canonical ensembles, cyclicity of trace and commutator with Hamiltonian.

- If we also use

$$\langle \Omega, E | [\widetilde{\mathcal{O}}_\omega, \widetilde{\mathcal{O}}_\omega^\dagger] | \Omega, E \rangle > 0$$

  then this suggests

$$\langle \Omega, E | \widetilde{\mathcal{O}}_\omega \widetilde{\mathcal{O}}_\omega^\dagger | \Omega, E \rangle < 0 ?$$