

# Memories, Asymptotic Symmetries and Soft Theorems

Strings2015

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Bangalore

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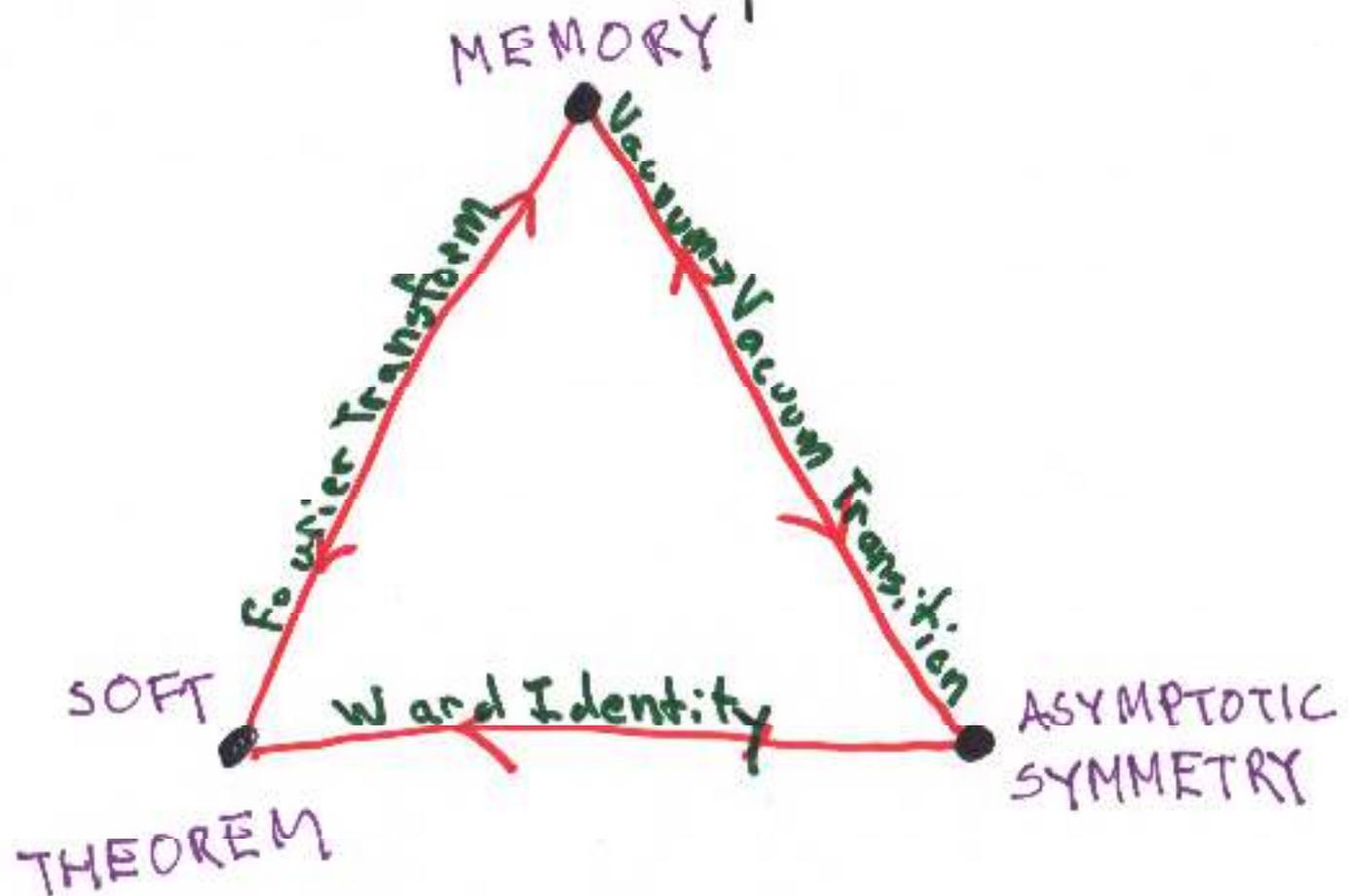
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And contributions from:

Casali, Skinner, Lipstein, Mason, Chen, Huang, Wen, Avery, Schwab, Bianchi, Guerrieri, Campiglia, Du, Luo, Zlotnikov, Adamo, Volovich, He, Yaun, Di Vecchia, Marotta, Mojaza, Mohd, Vasquez-Mozo, Bern, Vera, Broedel, Rosso, Plefka, Liu, Lambert, Fu, Wang, Kalousios, Rojas, White, Davies, Nohles, De Leeuw, Rosso, Davies, Barnich, Banks, Hyun, Park, Yi, .....

This talk considers a **triangle** of exact equivalence relations between three classes of physical phenomena, each studied for for over half a century:



Many copies of the **triangle** echo throughout physics:

- a) QED, Yang-Mills, gravity...
- b) leading, subleading, subsubleading...
- c) classical, quantum
- d) Minkowski,  $d=2,3,4,5,\dots$ , deSitter,...
- e)  $N=0$  susy,  $N=1$  susy, ...
- ⋮

Each case is different and interesting. The symmetries are  $\infty$ -dimensional, and imply an  $\infty$  of new, physically observable, conserved charges. In most cases, only one corner of the **triangle** was known prior to a year ago.

# MOTIVATIONS

0. Explore/understand rich structure of the deep infrared.
1. Organize soft sectors of QED/QCD, improve jet/collider computations?
2. Minkowski space holography?
3. Explain "miraculous" structures encountered in amplitudes program?
4. Universality/inevitability of string theory?

The **triangle** has consequences for the black hole information puzzle. At the end we will explain that, in contrast to popular belief, physical black holes carry an infinite amount of measurable hair.

## SPECIFIC EXAMPLE OF THIS TALK

Many copies of the **triangle** echo throughout physics:

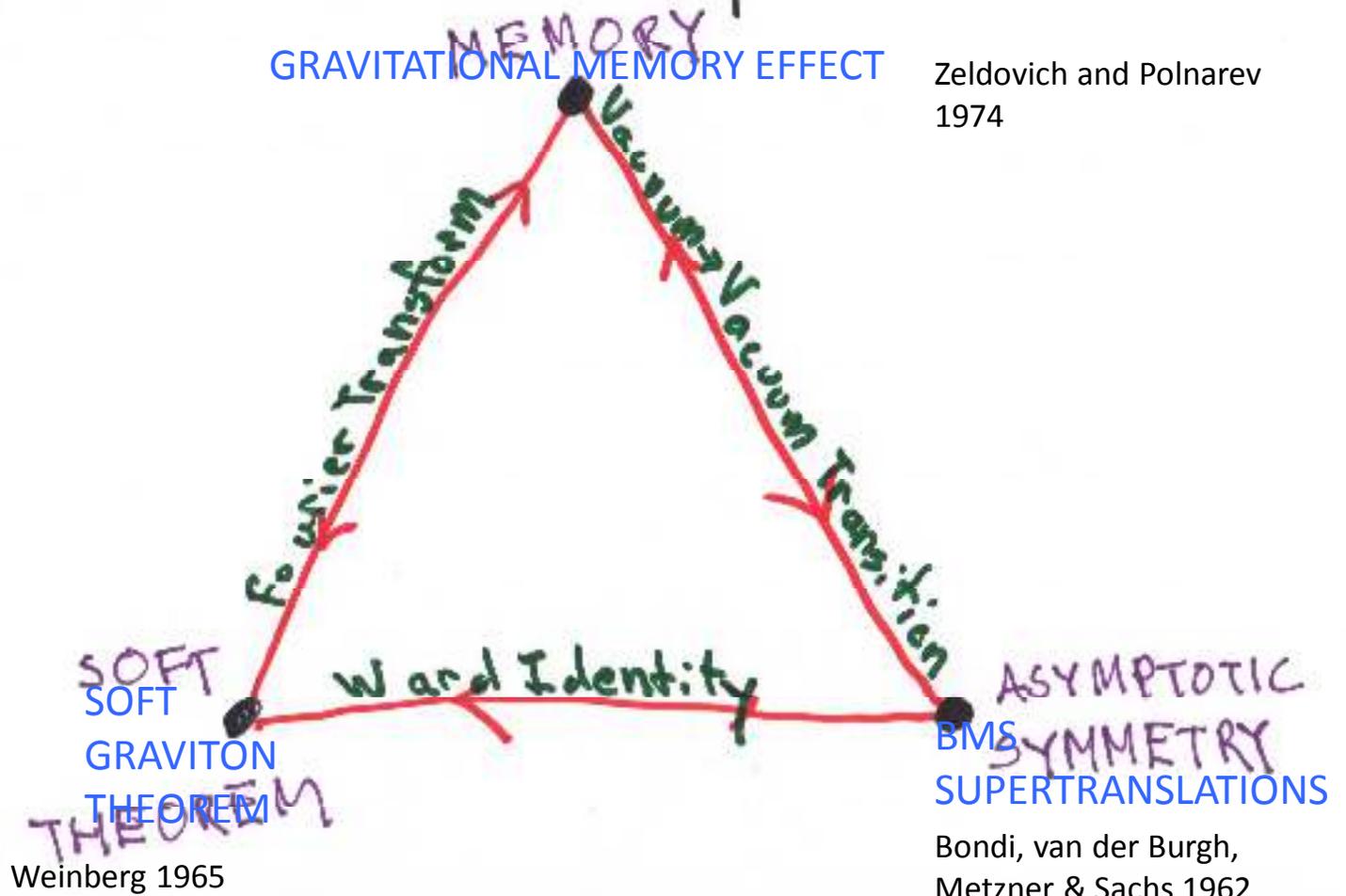
- a) QED, Yang-Mills, [redacted] ...
- b) [redacted] subleading, subsubleading ...
- c) [redacted]
- d) [redacted]  $d=2,3,5,\dots$ , deSitter, ...
- e) [redacted]  $N=1$  Susy, ...

THIS IS ODDLY THE BEST UNDERSTOOD OF ALL CASES!

Each case is different and interesting. The symmetries are  $\infty$ -dimensional, and imply an  $\infty$  of new, physically observable, conserved charges. In most cases, only one corner of the **triangle** was known prior to a year ago.

SPECIFIC EXAMPLE

This talk considers a **triangle** of exact equivalence relations between three classes of physical phenomena, each studied for for over half a century:





# Soft Graviton Theorem

Weinberg  
1965

$$\begin{aligned} & \left( \sum_{k=1}^n \frac{\epsilon_{\mu\nu} p_k^\mu p_k^\nu}{q \cdot p_k} \right) \\ & + \mathcal{O}(q^0) \end{aligned}$$

Exact & universal formula with simple diagrammatic derivation.



# Gravitational Memory

Zeldovich & Polnarev '74  
Braginsky & Thorne '87  
Christodoulou '91  
retarded

AF Minkowski metric:

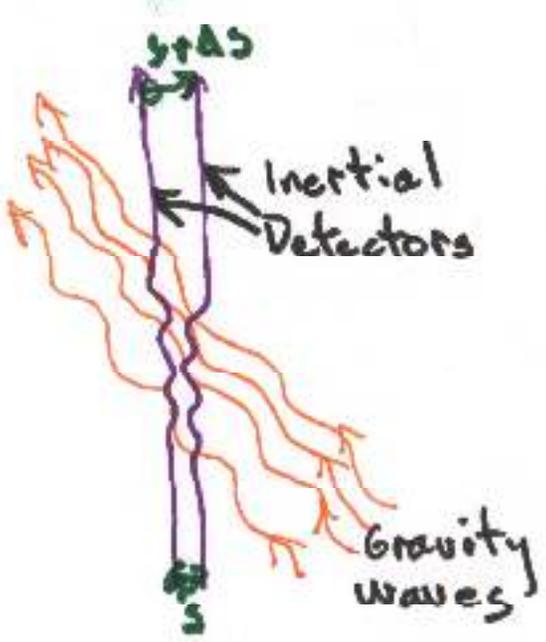
$$ds^2 = -du^2 - 2du dr + r^2 \gamma_{z\bar{z}} dz d\bar{z} + r C_{zz} dz^2 + r C_{\bar{z}\bar{z}} d\bar{z}^2 + \dots$$

Flat iff  $C_{zz} = -2\partial_z^2 C(z, \bar{z})$  gravity waves

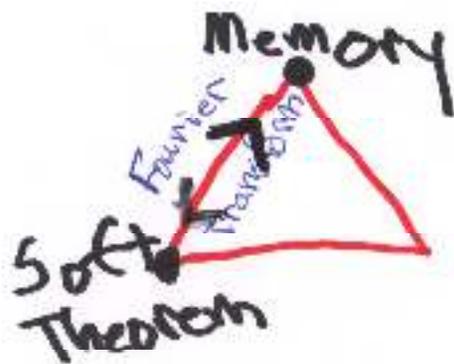
Integrated geodesic deviation equation:

$$\Delta s^{\bar{z}} = \frac{1}{2r} \gamma^{z\bar{z}} \Delta C_{zz} s^z$$

final - initial = change in  $C_{zz}$



DC effect. Quite possibly measured in coming decades. Pulsar timing array?



"Infrared Photons & Gravitons"  
S. Weinberg, 1965

"Gravitational Wave-Burst  
with Memory and  
Experimental Prospects"  
V. Braginsky and K.S. Thorne 1987

The dominance of the  $1/(p \cdot q)$  pole in (2.5) implies that the effect of attaching one soft-graviton line to an arbitrary diagram is to supply a factor equal to the sum of (2.5) over all external lines in the diagram.

**Softweird**

$$(2\pi G)^{10} \sum_i \kappa_i p_i^\mu p_i^\nu / [p_i \cdot q - i\epsilon_i] \quad (2.7)$$

permanent change in the gravitational-wave field (the burst's memory)  $\delta h_{ij}^{TT}$  is equal to the transverse, traceless (TT) part<sup>10</sup> of the time-independent, Coulomb-type,  $1/r$  field of the final system minus that of the initial system. If  $P^A$  is the 4-momentum of mass  $A$  of the system and  $P_i^A$  is a spatial component of that 4-momentum in the rest frame of the distant observer, and if  $\mathbf{k}$  is the past-directed null 4-vector from observer to source, then  $\delta h_{ij}^{TT}$  has the following form:

$$\delta h_{ij}^{TT} = \delta \left( \sum_A \frac{\kappa P_i^A P_j^A}{k \cdot P^A} \right)^{TT} \quad (1)$$

Here we use units with  $G = c = 1$ . In the observer's local Car-



Changing notations, using  $\theta(u) = \int \frac{d\omega}{\omega} \frac{e^{i\omega u}}{2\pi i}$  and elementary particles  $\rightarrow$  black holes, stars!

Memory effect provides physical method to measure soft gravitons.



# Asymptotic Supertranslation

## Symmetry

Bondi, van der Burgh  
Metzner  
Sachs 1962

AS 2013  
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Angle dependent <sup>super</sup> translations

$$u \rightarrow u + f(z, \bar{z}) + \dots$$

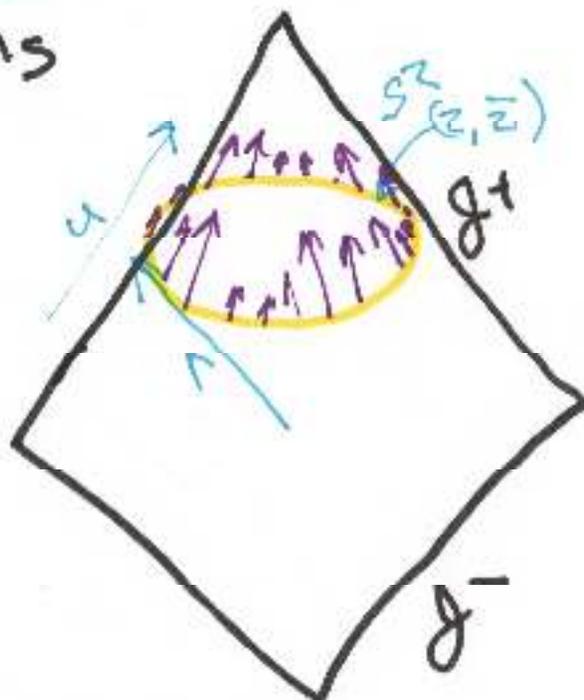
$$z \rightarrow z + \frac{1}{r} \gamma^2 \bar{z} \partial_{\bar{z}} f(z, \bar{z}) + \dots$$

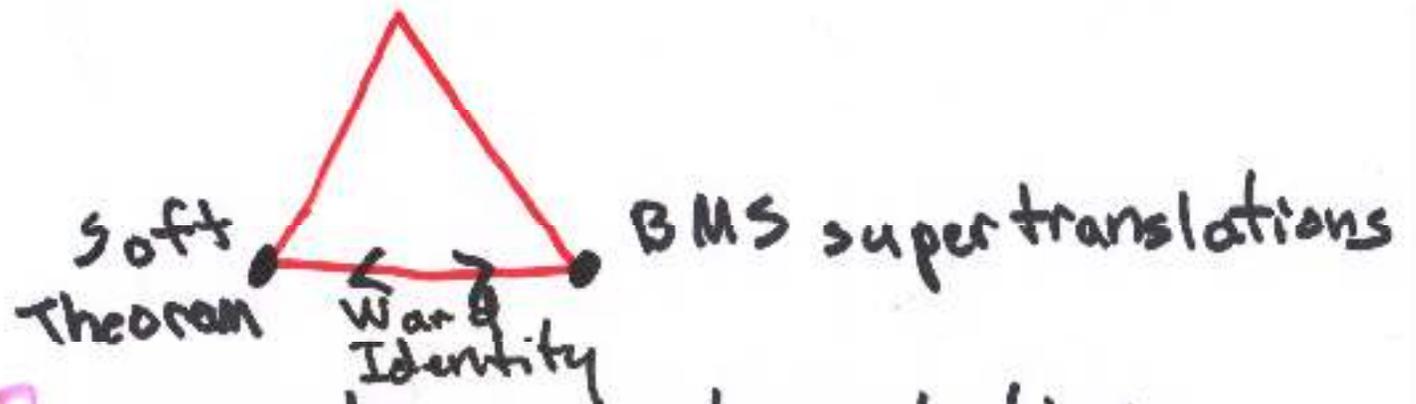
act **nontrivially** on  
the physical phase  
space, while preserving  
AF form of metric.

$$C_{zz} \rightarrow C_{zz} + 2D_z^2 f$$

is the goldstone boson.

**Flat metrics not all equivalent. Energy = 0  
but angular momentum  $\neq 0$ .**





Let  $B$  generate supertranslations.  
Then

$$B\mathcal{L} - \mathcal{L}B = 0$$

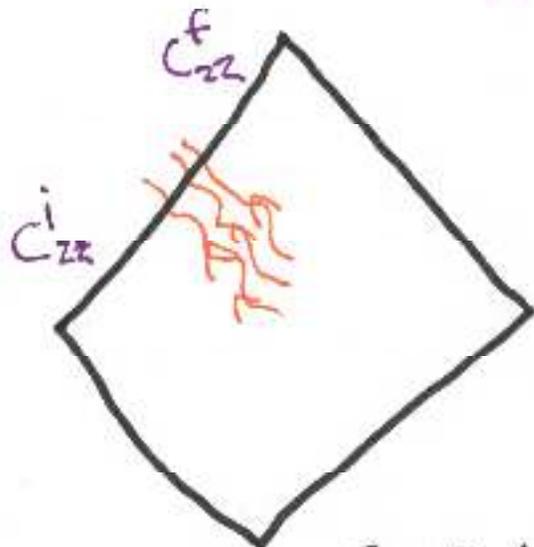
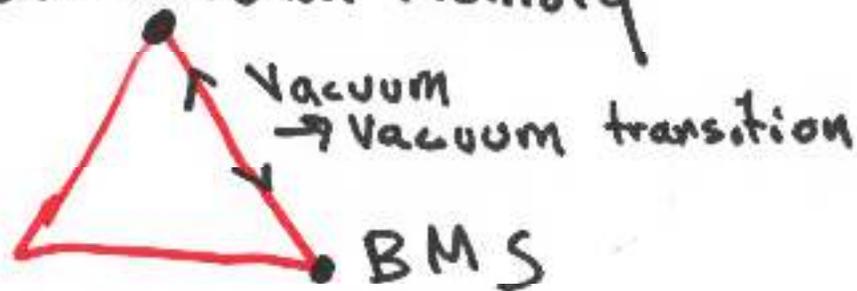
moreover

$$\langle \text{out} | B\Delta - \Delta B | \text{in} \rangle$$

is precisely Weinberg's soft graviton theorem.  $\Delta$  of associated conserved quantities is total energy at each incoming angle.

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# Gravitational Memory



The passage of energy induces a ~~to~~ BMS vacuum transition

with

$$C_{zz}^f - C_{zz}^i = -2D_z^2 f$$

$$f(t, \bar{z}) \sim \int d^2w \ln|z-w|^2 \int du T_{uu}(w)$$

Radiation pulse = domain wall separating BMS equivalent vacuum. Initial & final detector positions differ by BMS diffeo



This is gravitational memory!

1411.5745  
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## Implications for the information puzzle

- (i) Black holes carry  $\infty$  BMS hair.
- (ii) Hawking radiation is constrained by an  $\infty$  of conservation laws.
- (iii) The zero-energy vacuum is polydegenerate and can store  $n$  information.



## CONCLUSION

The infrared structure of gauge and gravitational theories is richer than previously realized. We have much to understand.