Black holes in the 1/D expansion

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Strings 2015
ICTS-TIFR, Bengaluru
June 22 - June 26, 2015

Black holes are very important objects in **GR**,

but they do not appear in the fundamental formulation of the theory

They're non-linear, extended field configurations with complicated dynamics

Strings are very important objects in **YM** theories,

but they do not appear in the fundamental formulation of the theory

They're non-linear, extended field configurations with complicated dynamics

Strings *become the basic* objects in the **large N** limit of SU(**N**) YM

In this limit, YM can be reformulated using worldsheet variables

Strings are still extended objects, but their dynamics simplifies drastically

Is there a limit of GR in which Black Hole dynamics simplifies a lot?

Yes, the limit of **large D**

any other parameter?

Is there a limit in which GR can be formulated with black holes as the basic (extended) objects?

Maybe, the limit of large D

BH in *D* dimensions

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

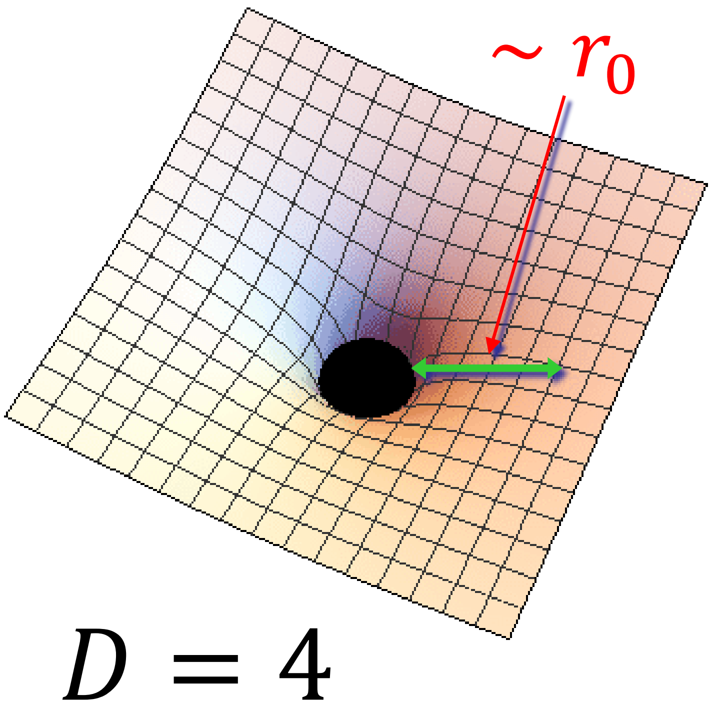
r_0 is the only scale

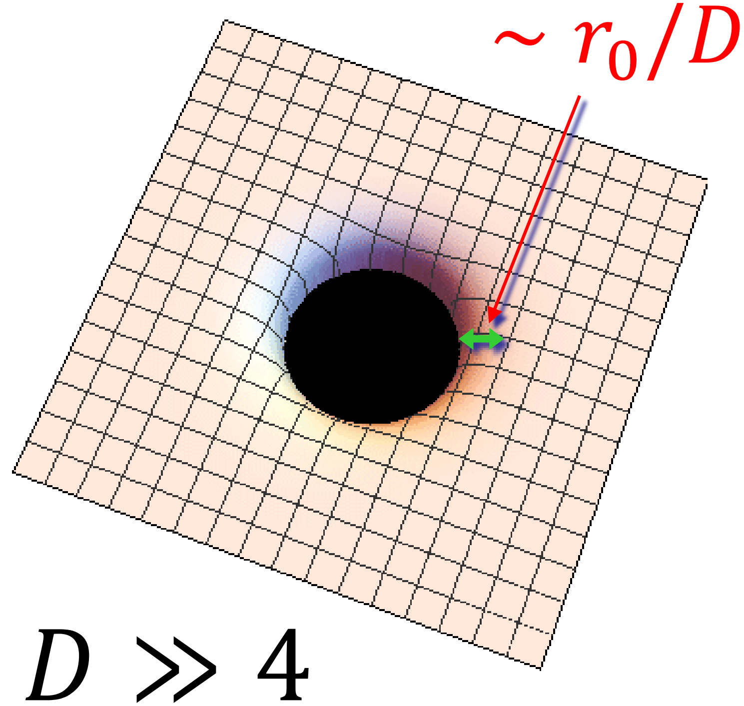
BH in D dimensions

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

r_0 is the only scale
when D is fixed

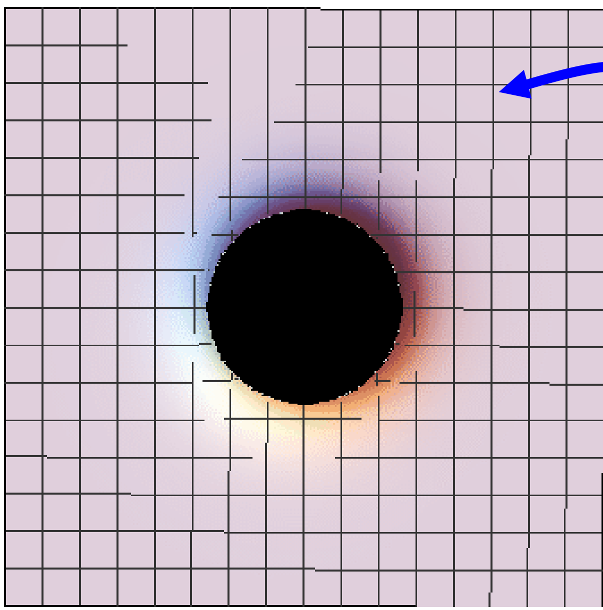
Large-D ⇒ localization of gravitational field





Hierarchy of scales: $\frac{r_0}{D} \ll r_0$

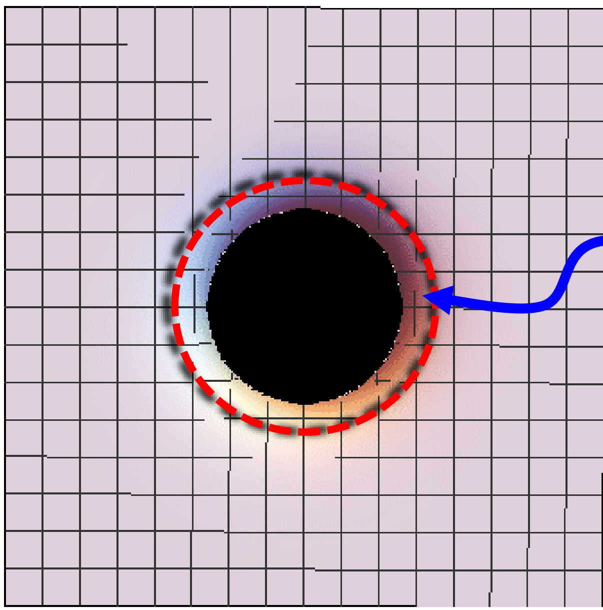
$$r \gg \frac{r_0}{D} \Rightarrow \left(\frac{r_0}{r}\right)^{D-3} \to 0$$



Flat space

"**Far**" region

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \iff r - r_0 \lesssim \frac{r_0}{D}$$



$r - r_0 \sim \frac{r_0}{D}$

$r \ll r_0$: "**Near-horizon**" region

Near-horizon geometry

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\left.\begin{aligned} \left(\frac{r}{r_0}\right)^{D-3} &= \cosh^2\rho \\ t_{near} &= \frac{D}{2r_0} t \end{aligned}\right\} \begin{array}{l}\text{finite} \\ \text{as } D \to \infty\end{array}$$

Near-horizon geometry

2d string bh

$$ds^2_{nh} \to \frac{4r_0^2}{D^2}\left(-\tanh^2\rho \;\, dt^2_{near} + d\rho^2\right)$$

$$+\, r_0^2\, (\cosh\rho)^{4/D}\, d\Omega^2_{D-2}$$

2d dilaton

'string length' $\ell_s \sim \frac{r_0}{D}$

Near-horizon universality

2d string bh = near-horizon geometry
of **all neutral non-extremal bhs**

RE+Grumiller+Tanabe

rotation = local boost

(along horizon)

cosmo const = 2d bh mass-shift

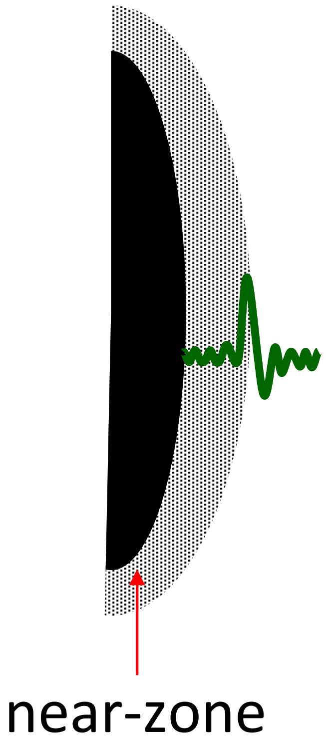
Does this help understand/solve bh dynamics?

Black hole perturbations

Large-D limit splits quasinormal modes into two sets

Dias+Hartnett+Santos

RE+Suzuki+Tanabe



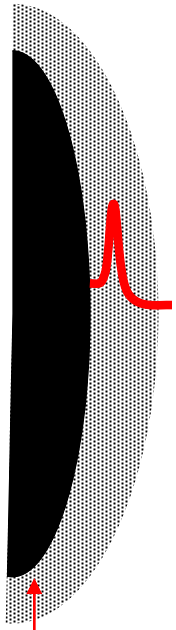
‘Boring’ modes

High frequency: $\omega \sim D/r_0$

Non-normalizable in near-zone

Almost **featureless oscillations of a hole in flat space**

Stable



‘Interesting’ modes

Low frequency: $\omega \sim D^0/r_0$

Normalizable in near-zone:

decoupled from far-zone

Capture ***instabilities*** and ***hydro***

Black hole perturbations

Quasinormal modes of Schw-(A)dS bhs

Gregory-Laflamme instability of black branes

Ultraspinning instability of rotating bhs

All solved analytically

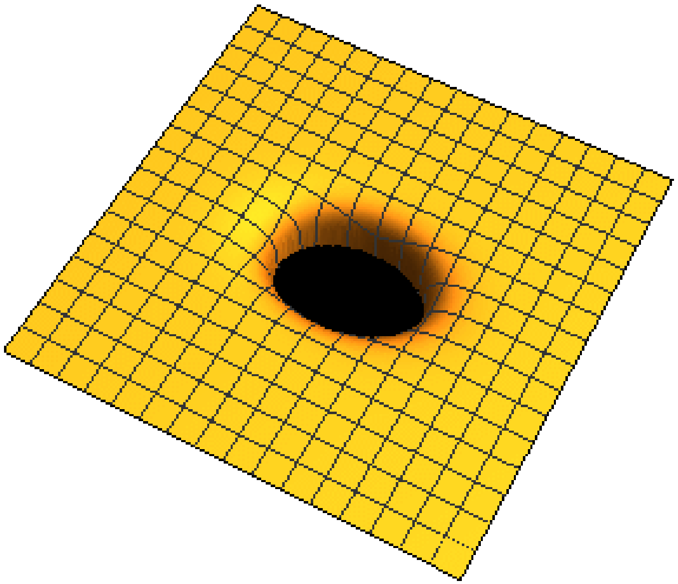
to several orders in 1/D

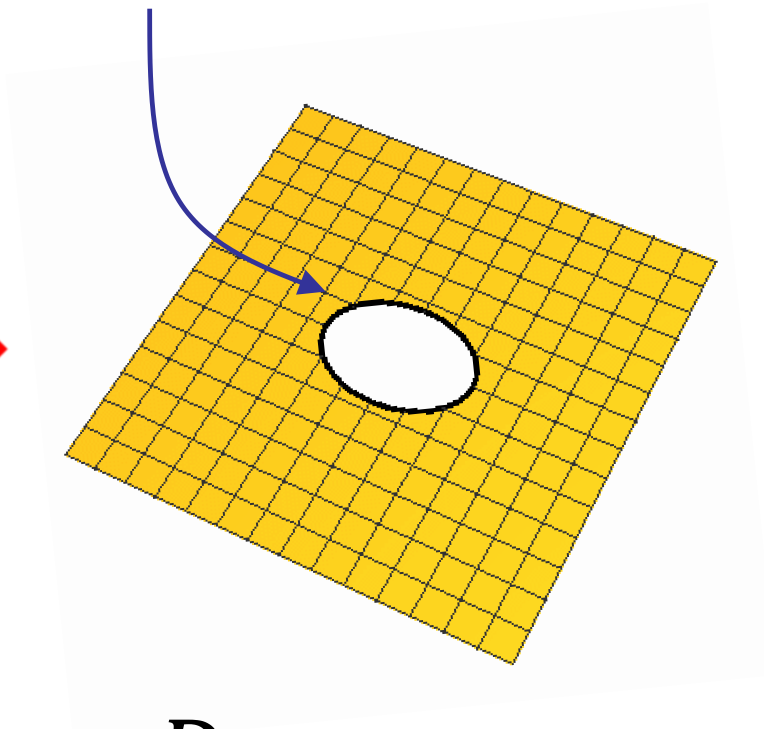
Fully non-linear GR

@ large D

Replace bh $\longrightarrow$ Surface in empty background

What's the dynamics of this surface?





$D \gg 4$

$D \to \infty$

Large D Effective Theory

Solve Einstein equations in near-horizon

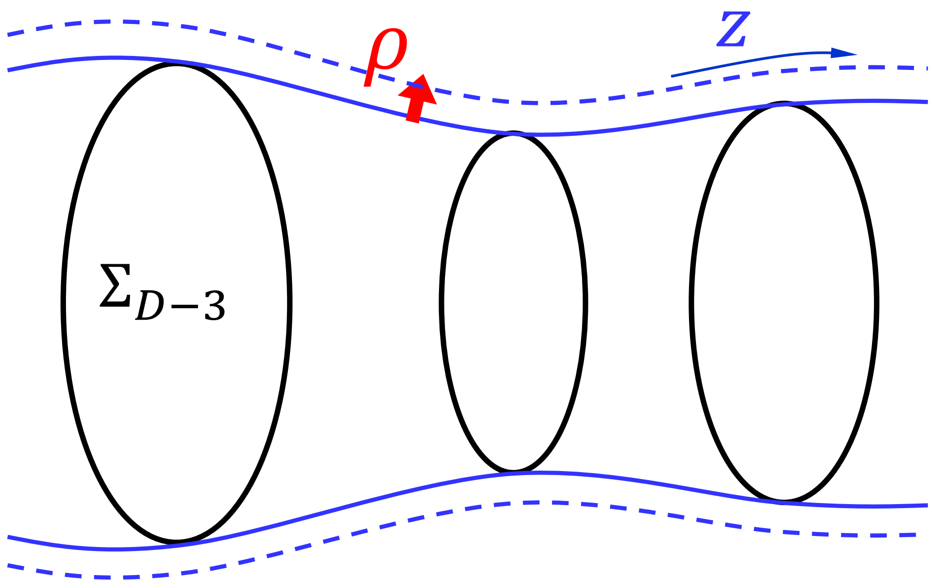
→ *Effective theory*

for the 'slow' decoupling modes

Gradient hierarchy

$\perp$ Horizon: $\partial_\rho \sim D$

$\parallel$ Horizon: $\partial_z \sim 1$



$$ds^2 = \frac{N^2(\rho,x)}{D^2} d\rho^2 + g_{ab}(\rho,x)dx^a dx^b$$

Einstein equations (w/ Λ)

$$K^a_b = \frac{D}{N} g^{ac} \partial_\rho g_{cb}$$ ➡ def: extrinsic curvature

$$\frac{D}{N}\partial_\rho K^a_b + K\,K^a_b = R^a_b + \frac{\delta^a_b}{L^2} - \frac{\nabla^a \nabla_b N}{N}$$ ➡ radial evolution eqn

$$R - K^2 + K^\mu_\nu K^\nu_\mu + \frac{(D-1)(D-2)}{L^2} = 0$$

$$\nabla_\nu K^\nu_\mu - \nabla_\mu K = 0$$

➡ ρ-independent constraints

Solved for stationary configurations

and for some time-dependent systems

RE+Shiromizu+Suzuki+Tanabe+Tanaka

Suzuki+Tanabe

A different (and time-dependent) formulation by

Bhattacharyya + De + Minwalla + Mohan + Saha

cf. Sayantani Bhattacharyya's talk

Stationary solution

Soap-bubble equation (redshifted)

$$\boxed{K = 2\gamma\kappa}$$

$K =$ **extrinsic curvature** trace

$\gamma =$ **redshift** factor on ‘membrane’

Lorentz boost from rotation

\+ gravitational redshift from background

$\kappa =$ **surface gravity**

Static soap bubble in Minkowski (AdS) =

Schwarzschild (AdS) BH

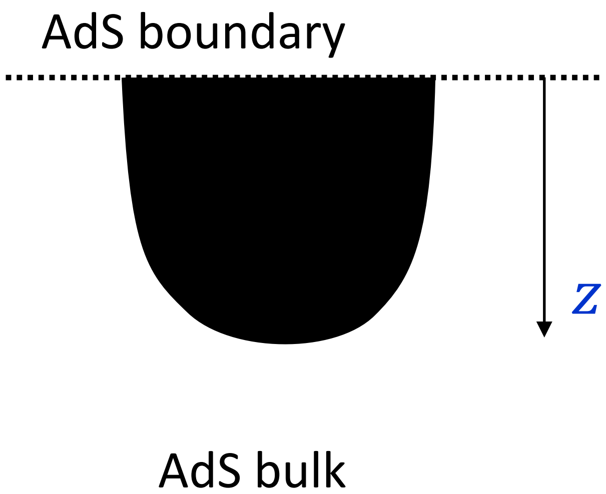
Rotating soap bubble =

Myers-Perry rotating BH

Black droplets

Black hole at boundary of AdS

Hubeny+Marolf+Rangamani



dual to CFT in BH background

Numerical solution:

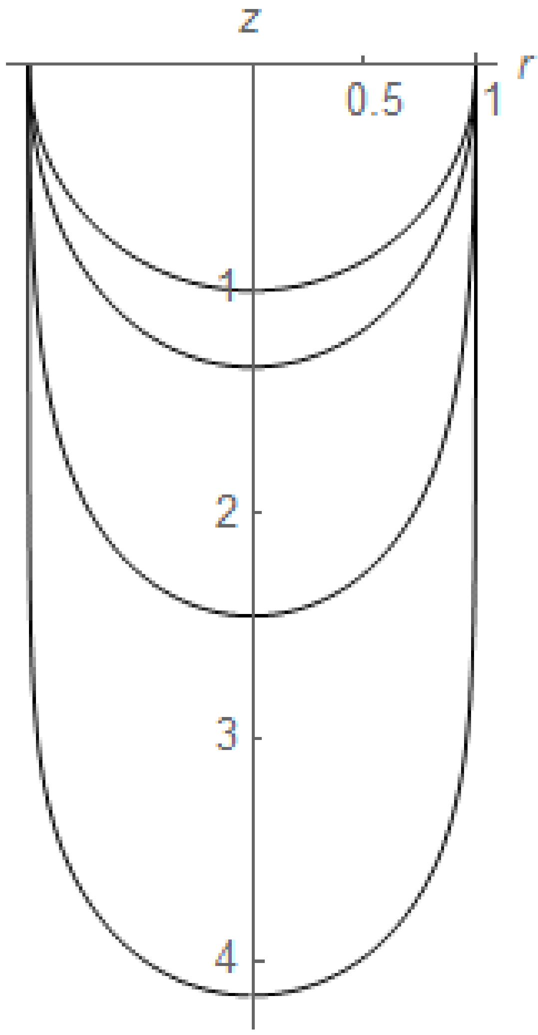
Figueras+Lucietti+Wiseman

Numerical code @ large D

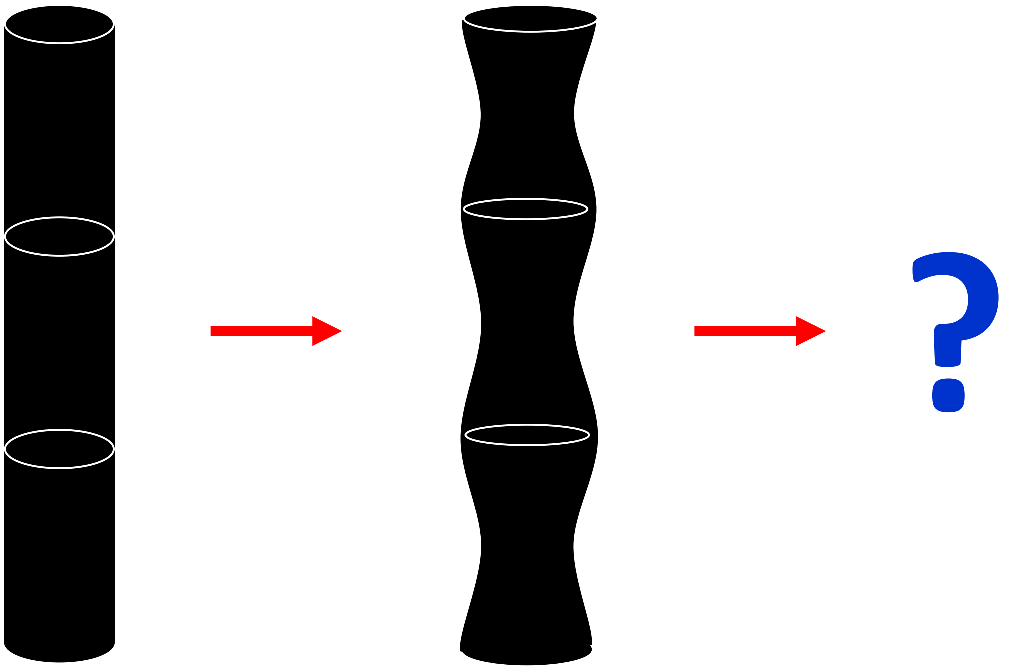
```
zmin = 0.0001;
zmax = 0.67;
r0 = 0.5;
```

$$\text{NDSolve}\left[\left\{r'[z] == -\frac{z}{r[z]}\,\frac{1+\sqrt{r[z]^2+z^2\left(1-r[z]^2\right)}}{1-z^2},\ r[\text{zmin}] == \text{r0}\right\},\ r,\ \{z,\ \text{zmin},\ \text{zmax}\}\right]$$

Black droplets

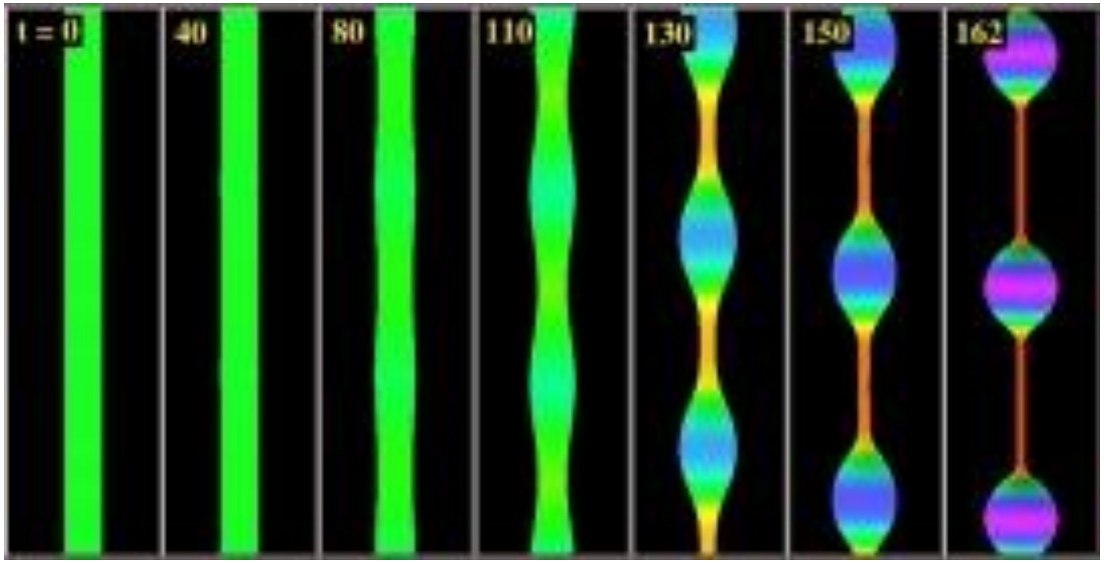


Black String Instability: Evolution and Endpoint



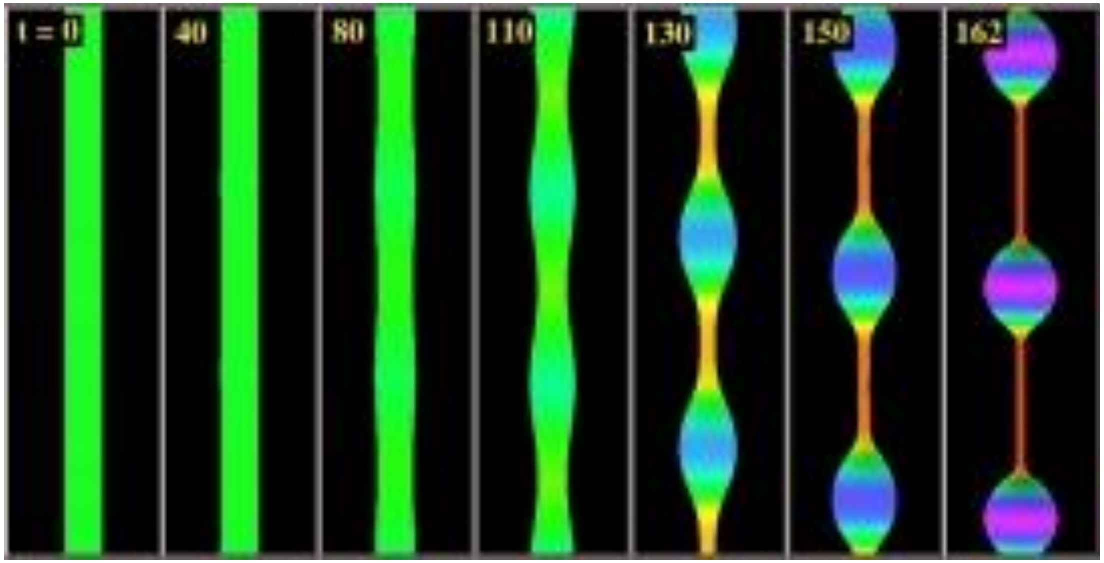
Gregory+Laflamme

5-dimensional black string



Lehner and Pretorius

5-dimensional black string



Lehner and Pretorius

100 000 CPU hours

2 months on 100 processors

Several open issues, *eg*

Can we understand (eg, analytically) the late-time asymptotic configuration?

Is this the generic endpoint of black brane instabilities?

Several open issues, *eg*

Can we understand (eg, analytically) the late-time asymptotic configuration?

NO

Is this the generic endpoint of black brane instabilities?

Critical dimension for non-uniform black strings

$$D = D_* \simeq 13.5$$

E Sorkin

$D < D_*$: weak non-uniformity decreases area

$\Rightarrow$ ~~non-unif~~ bs endpoint

$D > D_*$: weak non-uniformity increases area

$\Rightarrow$ possible endpoint at non-unif bs

Non-linear dynamics of black strings at large D

Collective variables:

$m(t,z)$ = mass density of black string (local area)

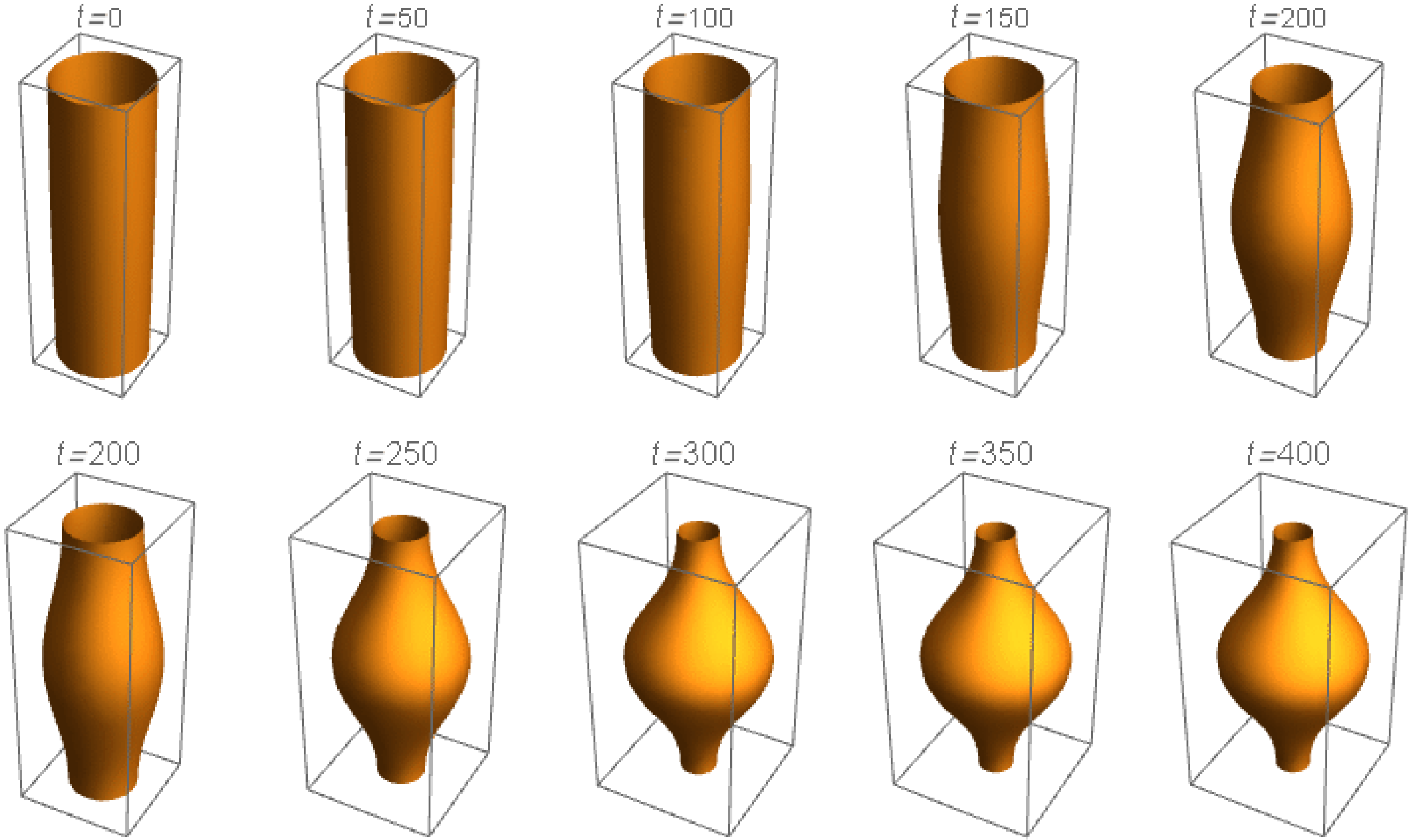
$p(t,z)$ = momentum density (local boost)

$$\boxed{\begin{aligned} \partial_t m(t,z) - \partial_z^2 m(t,z) &= -\partial_z p(t,z) \\ \partial_t p(t,z) - \partial_z^2 p(t,z) &= \partial_z m(t,z) - \partial_z\left(\frac{p^2}{m}\right) \end{aligned}}$$

Code for black string evolution

```
eq1 = ∂_t m[t, z] - ∂_{z,z} m[t, z] + ∂_z p[t, z];
eq2 = ∂_t p[t, z] - ∂_{z,z} p[t, z] - ∂_z m[t, z] + ∂_z (p[t, z]^2 / m[t, z]);
pde = {eq1 == 0, eq2 == 0};
tmax = 400; k = .98; Mstep = .1;
L = 2 π / k;
δm = 0.01 Exp[-4 z^2];
δp = 0;
icbc = {m[0, z] == 1 + δm, p[0, z] == δp, m[t, -L/2] == m[t, L/2], p[t, -L/2] == p[t, L/2]};
NDSolve[{pde, icbc}, {m, p}, {t, 0, tmax}, {z, -L/2, L/2}, MaxStepSize → Mstep]
```

Takes $O(1)$ seconds to produce a solution



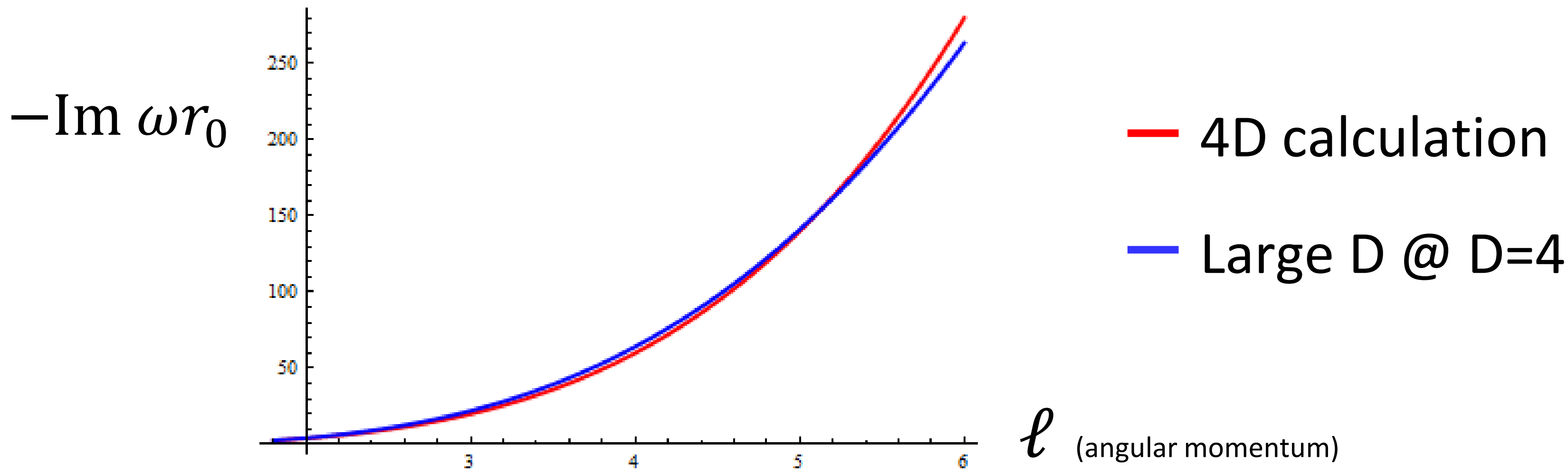
Endpoint: **stable non-uniform** black string

Horowitz+Maeda

Conclusion

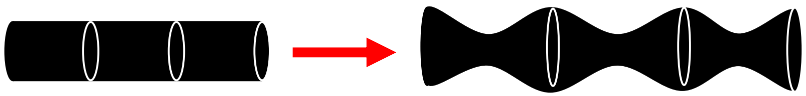
The 1/D expansion is *the* approach for efficiently solving the dynamics of black hole horizons

Quasinormal frequency in $D = 4$ (vector-type)



Calculation up to $\frac{1}{D^3}$: 6% accuracy in $D = 4$

Threshold mode of black string in $D = n + 4$



$$k_{GL} = \sqrt{n}\left(1 - \frac{1}{2n} + \frac{7}{8n^2} + \left(2\zeta(3) - \frac{25}{16}\right)\frac{1}{n^3} + \left(\frac{363}{128} - 5\zeta(3)\right)\frac{1}{n^4} + \mathcal{O}(n^{-5})\right)$$

$$k_{GL}|_{n=2} = 1.238$$

1.269 (numerical)

2.4% accuracy

Critical dimension for black string instability
(1^{st} to 2^{nd} order transition)

Large D:

Suzuki+Tanabe

Leading order: $D = 13$

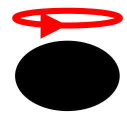
Next-to-leading-order: $D \simeq 13.5$

Numerical: $D \simeq 13.5$

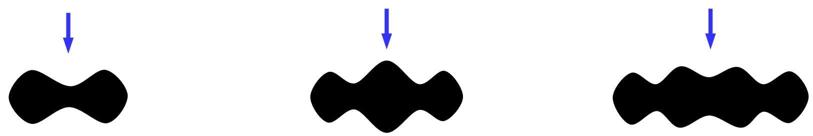
E Sorkin

Ultraspinning bifurcations of Myers-Perry black holes

Suzuki+Tanabe



$$\frac{a}{r_+} = \sqrt{3}, \qquad \sqrt{5}, \qquad \sqrt{7}, \ldots$$



Numerical: $\frac{a}{r_+} = 1.77, \qquad 2.27, \qquad 2.72 \ldots$ (D=8)

Dias et al

Error $\lesssim$ 2.7%