Some results on thermalization in 2D CFT and their holographic interpretation

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Based on: GM, R. Sinha, N. Sorokhaibam (1405.6695, 1501.04580), GM, T. Morita (1302.0859), P. Caupta, GM, R. Sinha (1306.4974), ongoing work with S. Paranjape, R. Sinha, N. Sorokhaibam and T. Ugajin

Thermalization/equilibration: search for universality in dynamics

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Single scale model of 2D critical quench: Calabrese-Cardy

Consider 2D dynamics. Assume that

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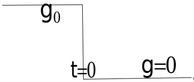
(b) Quench is sudden: $g = g_0 \theta(-t)$



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In this case, the only length scale describing the quench is $\kappa = g_0^{1/(d-\Delta)}$.

At length/distance scales $l, t \gg \kappa$, the initial state is approximated by

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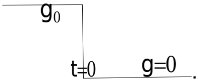

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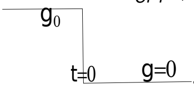

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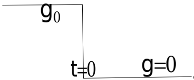

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However, the ratios $\gamma_i/\gamma_j = \Delta_i/\Delta_j$ are universal (determined entirely by the final CFT).

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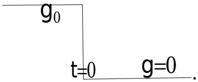
(b) Quench is sudden: $g = g_0 \theta(-t)$ 

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(II) An arbitrary string of local operators, contained within an interval A of size l, also thermalizes (conjecture Cardy 2014)

$$\langle\psi_0|O_i(x_i,t)O_j(x_j,t)...|\psi_0\rangle \xrightarrow{t>t_{eqm}} \mathrm{Tr}\left(O_i(x_i,0)O_j(x_j,0)\rho_\beta\right) + C\, e^{-\gamma_{min}t}$$

The thermal correlator on the right shows usual universality known from critical phenomena. The exponent $\gamma_{min} = 2\pi\Delta_{min}/\beta$, refers to the most relevant operator. (It's assumed that the CFT has a gapped spectrum of the scaling operator).

We will show later how to prove this, using the late time behaviour of the reduced density matrix $\rho_{dyn,A}$ of the interval A.

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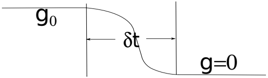

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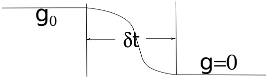

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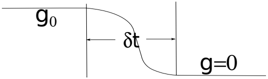

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We propose that in a CFT with additional charges W_n we can solve both issues with the following generalized Calabrese-Cardy (gCC) state

$$|\psi_0\rangle = e^{-\left(\kappa_2 H + \sum_n \kappa_n W_n + \ldots\right)} |Bd\rangle,$$

We will require that W_n are obtained from local currents which are primary or quasiprimary operators.

Multiple cut-off parameters $\leftrightarrow$ multiple scales.

We include integrable conformal theories with ∞ number of conserved charges.

Free scalar quench and the gCC state

- For a scalar field with a time-dependent mass $m^2(t)$ the initial vacuum $|0, in\rangle$ is related to the final vacuum $|0, out\rangle$ by a Bogoliukov transformation

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For a simple quench protocol, e.g. $m^2(t) = m_0^2(1 - \tanh(t/\delta t))/2$, it is easy to explicitly determine $\gamma(k)$ Birrell, Davies 1994, which, expanded in small $|k|/m_0$, $m_0\delta t$, looks like

$$\gamma(k; m_0, \delta t) = -1 + \frac{|k|}{m_0}\left(1 + \frac{\pi^2}{6}(m_0\delta t)^2 + ...\right) - \frac{1}{2}\left(\frac{|k|}{m_0}\right)^2\left(1 + \frac{\pi^2}{3}(m_0\delta t)^2 + ...\right) + ...$$

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By using a variant of the BCH formula, we can write $|0, in\rangle$ in the form:

$$|0, in\rangle = \exp\left[-\frac{1}{m_0}\left(1 + \frac{\pi^2}{6}(m_0\delta t)^2 + ...\right)\sum_k |k| a^\dagger(k) a(k) - \frac{1}{6m_0^3}\left(1 - \frac{\pi^2}{2}(m_0\delta t)^2 + ...\right)\right.$$
$$\left. \times \sum_k |k|^3 a^\dagger(k) a(k) + ...\right] \exp\left[\sum_k a^\dagger_{out}(k) a^\dagger_{out}(-k)]\right| 0, out\rangle,$$

which becomes a generalized CC state, with the boundary state identified as a Dirichlet state and a cut-off for all even W_∞ charges (as expected for a $c = 1$ scalar)

$$\kappa_2 = 1/(2\pi m_0)(1 + \pi^2 (m_0\delta t)^2/6),\ \kappa_3 = 0,\ \kappa_4 = -1/(768\pi m_0^3)(1 - \pi^2(m_0\delta t)^2/2), ...$$

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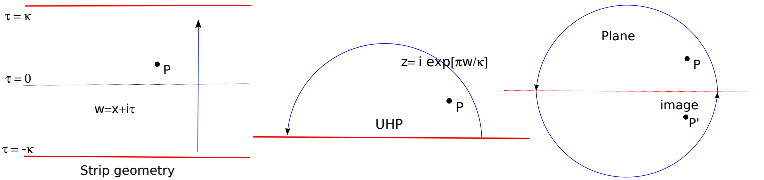

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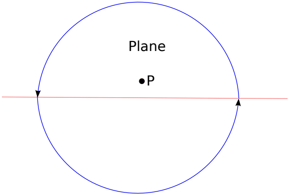

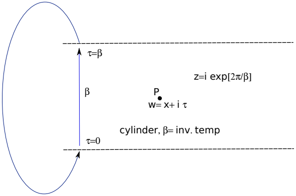

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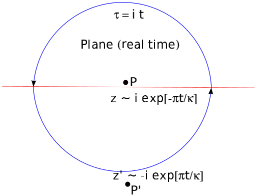

- Thermalization exponent: $\gamma = \frac{\pi}{2\kappa}\Delta = \frac{2\pi}{\beta}\Delta, \Delta = h + \bar{h}$, Equilibrium ensemble ρ_{eqm} given by $\beta = 4\kappa$.

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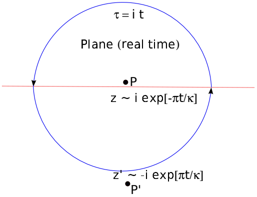

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Same exponent also describes decay of a thermal perturbation.

Review of Calabrese-Cardy

- Calabrese-Cardy state is interpreted in terms of Euclidean τ-evolution Calabrese,Cardy 2005

$$|\psi_0\rangle = e^{-\kappa H}|Bd\rangle$$

Thus, $\langle O(x,\tau)\rangle$ = insertion of O on a strip $\propto \langle O\rangle_{UHP}$ = $\langle O(z)O(z')\rangle_{plane}$

- Disconnected part of two-point function involves $\langle O(z)\rangle_{plane} \propto \langle O(z)\rangle_{cylinder}$, Thermal ensemble, with $\beta \leftrightarrow 2\pi \leftrightarrow 4\kappa$

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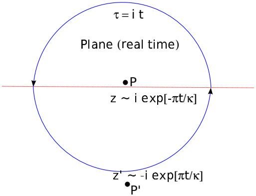

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Generalized CC state

- Do the above results extend to the gCC state

$$|\psi_0\rangle = \exp[-\kappa_2 H - \sum_{n=3}^{\infty} \kappa_n W_n]|Bd\rangle?$$

The W_n's are conserved charges which we assume are obtained from local currents which are primary or quasiprimary operators. We have included the case of integrable CFT's; examples are provided by CFT's carrying a W_∞ algebra.
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$$\langle\psi(t)|O(x)|\psi(t)\rangle \to \mathrm{Tr}(\rho_{eqm} O(x)) + \alpha \exp[-\gamma t]$$

The equilibrium ensemble is related to the cut-off parameters defining the state

$$\rho_{eqm} = \frac{1}{Z}\exp[-\beta H - \sum_n \mu_n W_n] = \rho_{GGE}, \ \kappa = \beta/4, \ \kappa_n = \mu_n/4, n = 3, 4, \ldots,$$

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Thermalization in integrable models

Thermalization of local observables has been discovered over the past eight years in several (massive) integrable models:

Integrable 2D:

Transverse field Ising (Calabrese et al 2005)

$H = -J \sum_{i=1}^{L} [\sigma_i^x \sigma_{i+1}^x + h(t) \sigma_i^z]$

Hard core boson chain (Rigol et al 2007)

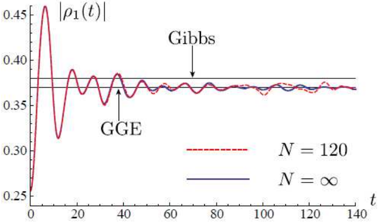
$H = -J \sum_{i=1}^{L(t)} b_i^\dagger b_{i+1} + \mathrm{h.c.}$

Massive Scalar(Sotiriadis, Cardy 2010)

$S = \int d^2x [(\partial\phi)^2 - m^2(t)\phi^2]$

Matrix QM model (Morita, GM 2013)

$S = \int dt [\mathrm{Tr}(U^\dagger \partial_t U + a(t)(U + U^\dagger)]$



(a) $\rho_1(t)$ and GGE vs. Gibbs ensemble

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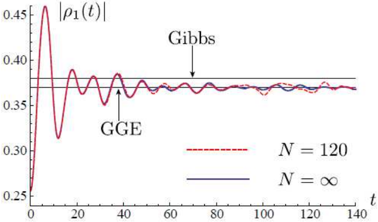

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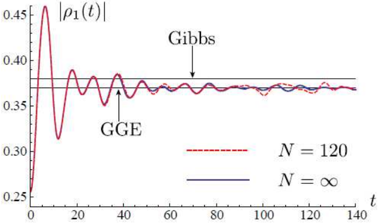

(a) $\rho_1(t)$ and GGE vs. Gibbs ensemble

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For transverse field Ising model, the late time dynamics for $\langle\psi_0(t)|\sigma_i^z\sigma_{i+l}^z|\psi_0(t)$ smoothly interpolates from $t^{-3/2}$ (non-critical quench) to $\exp[-\gamma t]$ (critical quench) GM, Paranjape, Sorokhaibam 2015

Summary of results

- 1. Single operators equilibrate, as follows

$$\langle\psi(t)|O_i(x)|\psi(t)\rangle \to \mathrm{Tr}(\rho_{eqm} O_i(x)) + \alpha_i \;\exp[-\gamma_i t]$$

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$$\gamma_i = \frac{2\pi}{\beta}\left[\Delta_i + \sum_n \tilde{\mu}_n Q_{n,i} + O(\tilde{\mu}^2)\right], \; \tilde{\mu}_n \equiv \frac{\mu_n}{\beta^{n-1}}$$

Here Δ_i and $Q_{n,i}$ are the scaling dimension and W_n-charges carried by O_i.

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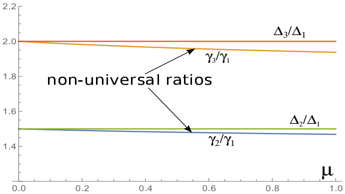
- 4. The above follows from a result about reduced density matrices which we will prove below

$$\rho_{dyn,A}(t) \to \rho_{GGE,A} + \alpha\ e^{-2\gamma_{min} t}$$

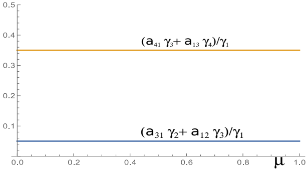
New Universality relations

For the Cardy-Calabrese state, the ratios γ_i/γ_j are universal.
With the generalized CC state, these ratios cease to be universal. For example, in the presence of one extra charge, we have $\gamma_i = \frac{2\pi}{\beta}\left[\Delta_i + \tilde{\mu} Q_i\right]$,

The ratios γ_i/γ_j now depend on μ:



However, it is easy to see that the μ-dependence can be eliminated by considering new ratios such as $(a_{31}\gamma_2 + a_{12}\gamma_3)/\gamma_1$, $(a_{41}\gamma_3 + a_{13}\gamma_4)/\gamma_1$, with $a_{ij} = \Delta_{[i}Q_{j]}$, are independent of μ, and depend only on the spectrum of the final CFT.



Proof of thermalization: one-point function

Recall

$$|\psi_0\rangle = \exp[-\kappa_2 H - \sum_{n=3}^{\infty} \kappa_n W_n]|Bd\rangle ?$$

Consider evaluating

$$\langle\psi_0|O(x)|\psi_0\rangle = \langle Bd| \exp[-\kappa_2 H - \sum_{n=3}^{\infty} \kappa_n W_n] O(x) \exp[-\kappa_2 H - \sum_{n=3}^{\infty} \kappa_n W_n]|Bd\rangle$$

The first term in the exponential represents a Euclidean time evolution; however, there is no such interpretation for the remaining exponentials. Thus, we are forced to expand them and treat them as multiple insertion of charged currents on the strip/UHP.

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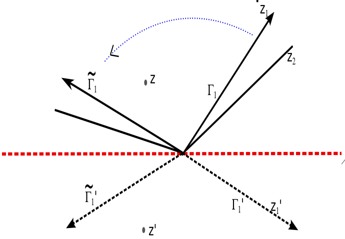

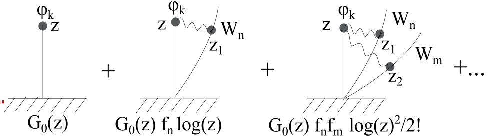

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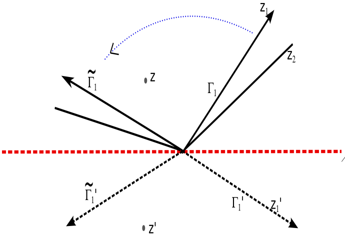

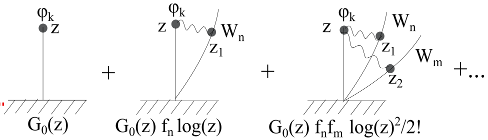

At large times $t \gg \beta$, the Feynman diagrams exponentiate

$$\langle O(t)\rangle_{\kappa_n} = \langle O(t)\rangle_0 \left(1 - f_n\kappa_n t + \frac{(f_n\kappa_n t)^2}{2!} + \ldots\right) = \langle O\rangle_{eqm,\beta,\mu_n} + \alpha \exp[-\gamma_0 t - f_n\mu_n t/4 + O(\mu^2)]$$

General correlators $\langle O_1 O_2 \ldots \rangle$

We will now the extend the above result on thermalization to an arbitrary string of local operators contained in an interval of size l, by following the proposal of Cardy 2014.

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$$
\begin{aligned}
I_A(t) &= \mathrm{Tr}(\hat{\rho}_{A,dynamical}\, \hat{\rho}_{A,eqm}(\beta)) \\
\rho_{A,dynamical}(t) &= \mathrm{Tr}_{\bar{A}}(\rho_{dynamical}(t)),\ \rho_{dynamical} = |\psi(t)\rangle\langle\psi(t)|, \\
\rho_{A,eqm}(\beta) &= \mathrm{Tr}_{\bar{A}}(\rho_{eqm}(\beta)),\ \rho_{eqm} = \frac{1}{Z} e^{-\beta H}
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where $\hat{\rho} = \rho/\sqrt{\mathrm{Tr}\rho^2}$ are square-normalized matrices.

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$$I_A(t) = 1 - \alpha e^{-2\gamma_{min}(t - t_{eqm})} + \text{faster transients}$$

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Implication:

$$\rho_{A,dynamical}(t) \xrightarrow{t > t_{eqm}} \rho_{A,eqm}$$

$$\langle\psi(t)|O_1(x_1)O_2(x_2)....|\psi(t)\rangle \to \mathrm{Tr}(\rho_{eqm,\beta}\, O_1(x_1)O_2(x_2)....) + ...$$

up to terms vanishing as fast as $e^{-\gamma_{min} t}$.

Proof: gluing different geometries— short interval expansion

The thermalization function

$$I_A(t) = \mathrm{Tr}(\hat{\rho}_{A,dynamical}\,\hat{\rho}_{A,eqm}(\beta)) = \frac{\hat{Z}_{sc}}{\sqrt{\hat{Z}_{ss}\hat{Z}_{cc}}}$$

involves gluing a strip and a cylinder along an interval. We compute this by using the short interval expansion (Headrick 2010, Calabrese, Cardy, Tonni 2011) in which each interval is replaced by a direct sum of conformal fields. (GM,Sinha,Sorokhaibam 2015)

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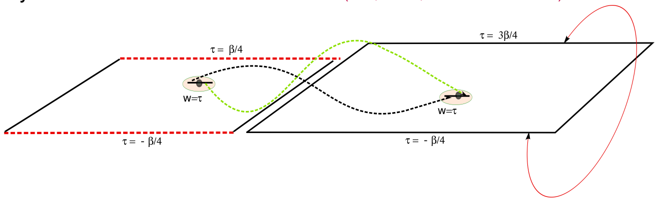

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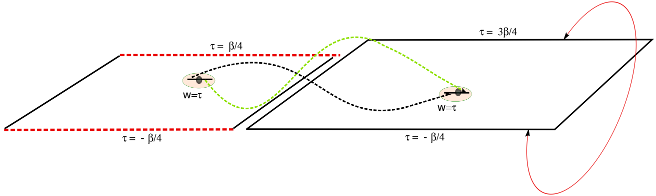

$$\hat{Z}_{sc} = C_{0,0}(1 + S_1^{sc}),\ \ S_1^{sc} = \sum_a \hat{C}_{a,0}(\langle O_a\rangle^{\mu}_{str} + \langle O_a\rangle^{\mu}_{cyl}) + \sum_{ab} \hat{C}_{a,b}\langle O_a\rangle^{\mu}_{str}\langle O_b\rangle^{\mu}_{cyl}$$

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$$\hat{Z}_{cc} = C_{0,0}(1 + S_1^{cc}),\ \ S_1^{cc} = 2\sum_a \hat{C}_{a,0}\langle O_a\rangle^{\mu}_{cyl} + \sum_{ab} \hat{C}_{a,b}\langle O_a\rangle^{\mu}_{cyl}\langle O_b\rangle^{\mu}_{cyl}$$

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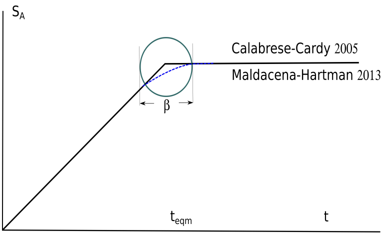
At $t \to \infty$ all one-point functions reduce to thermal one-point function. Thus, $\hat{Z}_{sc} = \hat{Z}_{ss} = \hat{Z}_{ss}$. Hence $I(\infty) = 1$. The slowest transient comes from S_2^{ss} which contains $\langle O_m\rangle^\mu_{str})^2 \sim\sim \exp[-2\gamma_m t]$.

Relation to entanglement entropy



The linear segments of the graph Calabrese-Cardy 2005, Hartman-Maldacena 2013 follow in the factorization limits of a four-twist operator.

Corrections to the above limits involve subleading terms in the twist-field OPE's. In the time interval $|t - t_{eqm}| \lesssim \beta$, using the techniques in the previous slides, we can show that

$$S_{EE}(t) = S_{EE,linear} - Ce^{-2\gamma_{min} t}$$

where γ_{min} refers to the thermalization exponent of the lowest operator appearing in the twist-field OPE.

Thermal correlator

- Consider the following correlator in the GGE

$$G_+(t, l; \beta, \mu) \equiv \frac{1}{Z}\mathrm{Tr}(O(l, t)O(0, 0)e^{-\beta H - \sum_n \mu_n W_n})$$

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By a conformal map, this can be reduced to a correlator on the plane

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- For $\mu = 0$, the above two-point function is given by

$$G_+(t,l;\beta,0) \xrightarrow{t,l \gg \beta} \begin{cases} \text{const } e^{-2\pi t\Delta_k/\beta}, & (t-l) \gg \beta \\ \text{const } e^{-2\pi l\Delta_k/\beta}, & (l-t) \gg \beta \end{cases}$$

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- Consider the following correlator in the GGE

$$G_+(t,l;\beta,\mu) \equiv \frac{1}{Z}\mathrm{Tr}(O(l,t)O(0,0)e^{-\beta H-\sum_n \mu_n W_n})$$

By a conformal map, this can be reduced to a correlator on the plane

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where $b(\mu)$ is time-independent, and is of the form $b(\mu) = 1 + O(\mu)$.

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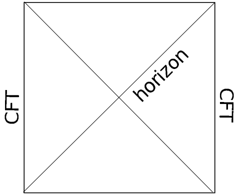
Black hole interpretation of thermal decay

A thermal state in a 2D CFT (admitting a holographic description) is dual to a BTZ black hole. The thermofield double is dual to the eternal black hole. Maldacena 2002.



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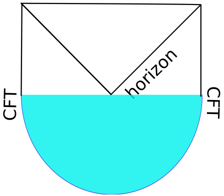

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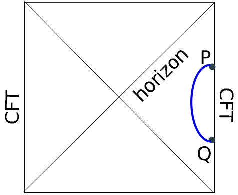
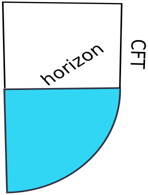


Thermal decay:
(a) Heavy operators: thermal two-point function $\langle O(P)O(Q)\rangle_{CFT,\beta} = e^{-\Delta L(P,Q)} = e^{-\gamma t}$
(b)For light CFT operators, perturbations to the thermal state= perturbation of black hole by a probe field. Hence thermal decay= Quasinormal decay. Explicit calculation of BTZ quasinormal frequency gives Im $(\omega) = \gamma = (2\pi/\beta)\Delta = \gamma_{CFT}$ Sachs 2010

Holographic dual to the CC state

Dynamics: The time-dependent geometry dual to the CC quench state, for $t > 0$, is described by a 'quarter' of the Penrose diagram Maldacena 2002, Takayanagi et al 2010, Aharony et al 2010, Hartman-Maldacena 2013



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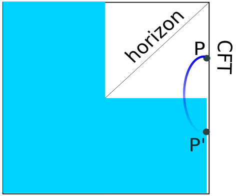

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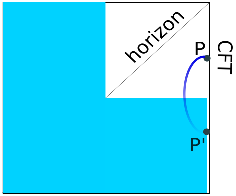

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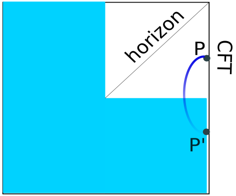

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An alternative holographic picture of a quench is given by a Vaidya spacetime Chesler, Yaffe 2008, Bhattacharya, Minwalla 2009.

The holographic dual to the generalized quench state (gCC)

- We found that the gCC state $\exp[-\kappa_2 H - \sum_n \kappa_n W_n]|Bd\rangle$ thermalizes to an equilibrium ensemble characterized by additional charges/chemical potentials. Such a thermal ensemble should be holographically dual to a black hole with additional charges.

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A simple example of an additional charge is an overall momentum P, for which the correspondence works out. Caputa, GM, R.Sinha 2013

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$$\boxed{\text{thermalization exponent in the generalized quench state = the thermal decay rate in the GGE = (imaginary part of) quasinormal frequency for the corresponding bulk field in a higher-spin black hole}}$$

Cabo-Bizet, Gava, Giraldo-Rivera, Narain 2014

$$\mathrm{Im}\,\omega = \frac{2\pi}{\beta}\left(1 + \lambda + \frac{\tilde{\mu}_3}{3}(1+\lambda)(2+\lambda)\right) = \gamma_{CFT}$$

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We have used $\Delta = 1+\lambda$, and $Q_3 = \frac{1}{3}(1+\lambda)(2+\lambda)$, for the operator dual to the bulk field. Gaberdiel-Gopakumar 2010, Gaberdiel-Hartman 2011, Ammon-Kraus-Gutperle 2011

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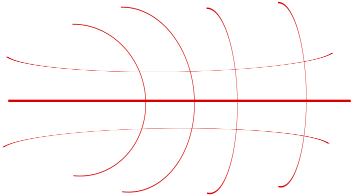

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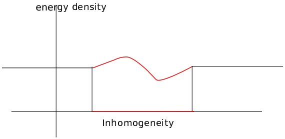

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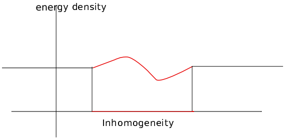
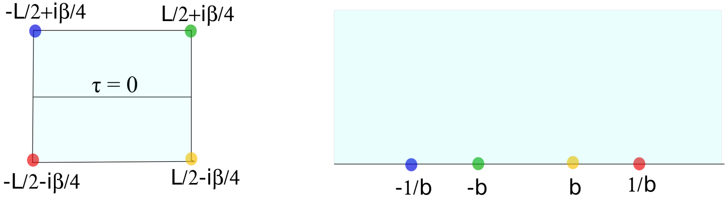


For compact inhomogeneity, limited to a region B, the thermalization results, relevant for a subregion A, hold true after the light crossing time from B to A.

Finite spatial extent— non-thermalization

So far the systems we have considered have infinite spatial extent. With finite boundaries at $x = \pm L/2$, one deals with functional integrals over a rectangular strip. One can apply the technique of conformal maps to reduce the problem to that of an UHP.



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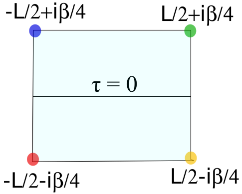
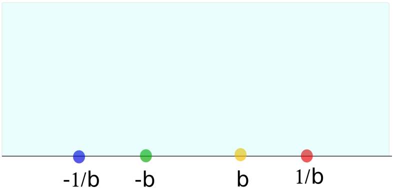
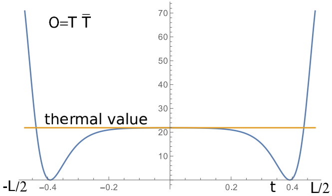




The one-point function of a conformal field typically has a periodic behaviour in time:

GM, Sinha, Ugajin, in Progress



Revival function

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However, CFT's with holographic duals may not have this property generally, in which case, one will have thermalization even in compact space. This will be dual to black hole formation.

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- Thermalization works for an arbitrary string of operators, as long as they are contained in a finite interval.

Conclusions— contd.

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- The holographic dual to the generalized CC state appears to be a higher spin black hole. The CFT thermalization exponent exactly agrees with the quasinormal frequency of the black hole.

Open questions

- Are details of the quench protocol remembered even in the absence of additional charges? (“no hair”)
- Generalization to Vaidya spacetimes.
- Generalization to higher dimensions.
- Connection to short time dynamics Das, Gallante, Myers 2014-15
- Can one prove similarly general statements about late time dynamics of massive theories?