Constraints on Inflationary Correlators From Conformal Invariance

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Based on:

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2) A. Ghosh, N. Kundu, S. Raju and SPT, JHEP, 1407, 2014, 011.

3) N. Kundu, A. Shukla and SPT, JHEP 1504, 2015, 061.

4) N. Kundu, A. Shukla and SPT, In Prep.

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Outline

- Introduction
- The general approach (with motivation from AdS/CFT)
- Specific Correlators
- General Ward Identities
- Conclusions

Introduction

- Inflation is an attractive idea.
- It explains the approximate homogeneity and isotropy of the universe.
- And also gives rise to small perturbations, which can explain the fluctuations seen in the CMB and provide the seed for structure formation.

Introduction

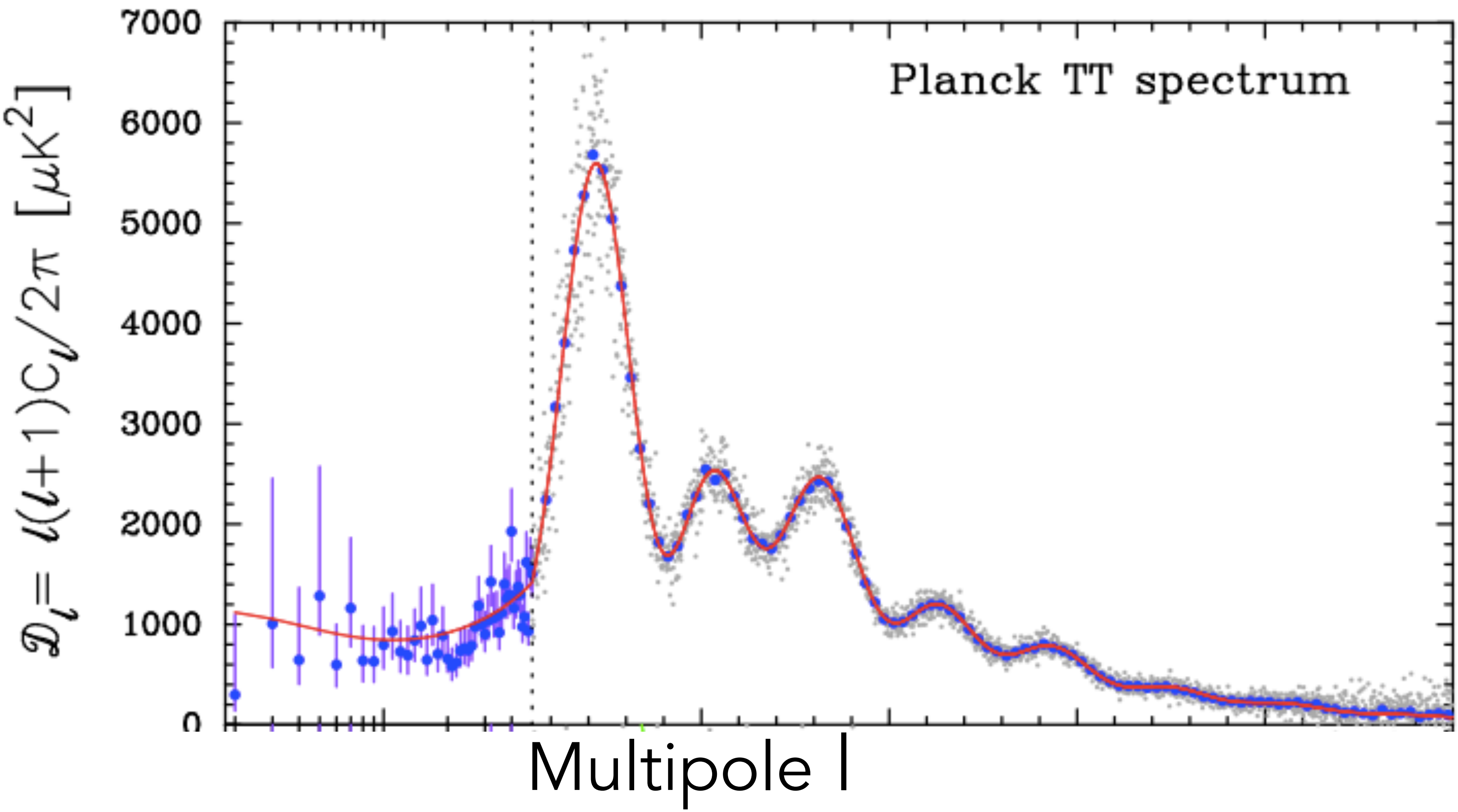
- These perturbations are generated due to the rapidly evolving universe.
- They are quantum effects involving gravity.
- Thus clearly of interest in any study of quantum gravity.

Introduction

- The two point correlator (for scalar perturbations) has been observed.

- Subsequent progress will hopefully lead to an observation of non-Gaussianity (three point function etc).

Planck TT spectrum (2013)



Introduction

During inflation universe was approximately deSitter space.

The symmetry group of 4-dim. deSitter space is SO(1,4)

$$ds^2 = -dt^2 + e^{2Ht}(dx^2 + dy^2 + dz^2)$$

H: Hubble parameter

Introduction

The question we will ask :

What constraints do the symmetries impose on correlation functions of the quantum perturbations produced inflation?

Our analysis will include the small breaking of the symmetries

Introduction

Such a symmetry based analysis has the advantage of being robust and model independent.

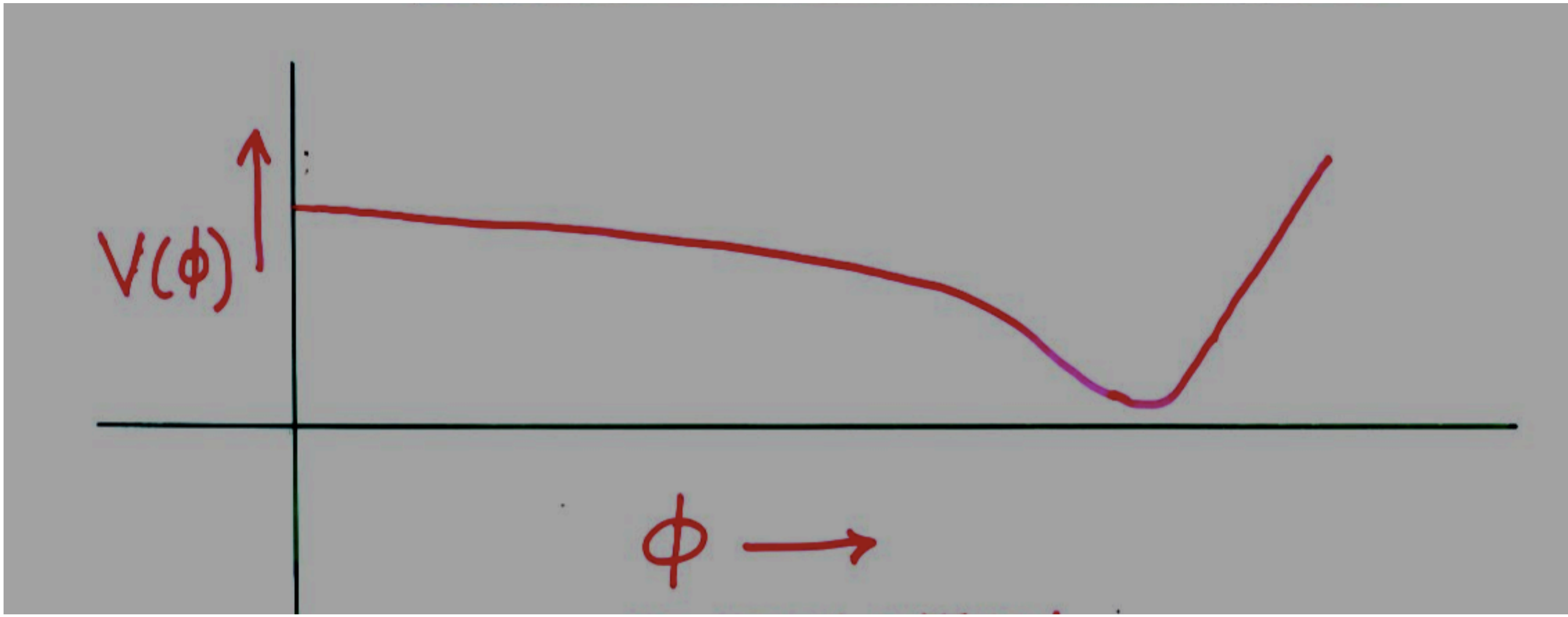
Introduction

The basic idea in a large class of inflation models :

- An additional scalar field, the inflaton.

- Slowly varying potential.

(we will only consider models of single field inflation)



Scalar Field: Φ Inflaton

Canonical Slow Roll Model:

$$S = \int d^4x \sqrt{-g} \frac{1}{16\pi G} [R - \frac{1}{2} (\nabla \phi)^2 - V(\phi)]$$

Two derivative terms

However, higher derivative corrections may be important.

$$S = \int d^4x \sqrt{-g} \frac{1}{16\pi G} \left[R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) + \frac{R^2}{\Lambda^2} + \frac{R^3}{\Lambda^4} + \frac{(\partial^2\phi)^2}{\Lambda^2} + \cdot\right.$$

If $\quad H \sim \Lambda$

This could happen e.g., in weakly String Theory

$$\Lambda \sim M_{st}$$

$$H \sim M_{st} \ll M_{Pl}$$

$$g_s \ll 1$$

Introduction

- Given our poor understanding of string theory in time dependent situations we cannot directly calculate the resulting quantum perturbations in such a situation today.

- However symmetry considerations should still hold and conclusions obtained from them should apply here too.

Introduction

In fact we will find that symmetry based considerations can be quite powerful.

Symmetries

The symmetries of deSitter space:

$$ds^2 = -dt^2 + e^{2Ht}(dx^i)^2$$

SO(1,4): $\frac{5.4}{2} = 10$ generators

Translations: 3 $x^i \to x^i + c^i$

Rotations: 3 $x^i \to R^i_j x^j$

Scaling: 1 $t \to t + \frac{1}{H}\ln(\lambda), x^i \to \frac{x^i}{\lambda}$

Symmetries

Special Conformal Transformations : 3

$$x^i \to x^i - 2(b_j x^j)x^i + b^i\left(\sum_j (x^j)^2 - \frac{e^{-2Ht}}{H^2}\right),$$

$$t \to t + 2\frac{b_j x^j}{H},$$

Symmetries

The SO(1,4) symmetries of dS_4 are the same as those of a three dimensional Euclidean CFT.

We will call this symmetry group the conformal group.

Symmetries

- However, we will not assume any dS-CFT correspondence.
- Rather the observations relating symmetries of dS and CFT's will only serve to organise our discussion of symmetries in the inflationary context.

Symmetries

During Inflation the conformal symmetries are not exact.

But breaking is small.

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1 \qquad \delta = -\frac{H\ddot{H}}{\dot{H}^2} \ll 1$$

$$\epsilon_1 = \frac{\dot{\phi}^2}{H^2} \ll 1$$

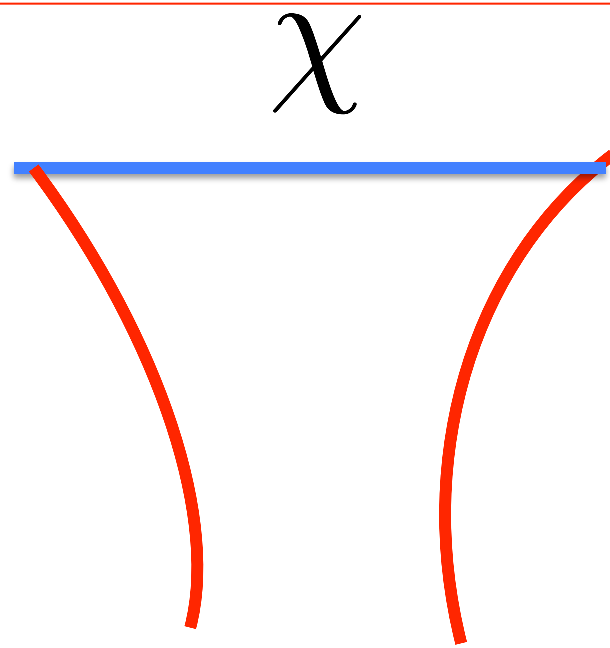
Wave Function

The wave function is a useful way to organise the discussion of symmetries.

Once the constraints imposed by symmetries on the wave function are understood, all constraints on correlators follow.

Wave Function

We will be interested in the wave function at late times: $\Psi[\chi]$



$\Psi[\chi]$

Functional of perturbations

Wave Function

Late time: when modes of interest have exited the horizon and stopped evolving.

$$\chi = \chi(t) e^{i\vec{k}\cdot\vec{x}}$$

$$\frac{k}{a} \ll H$$

$$a = e^{Ht}$$

Massless fields then become time independent due to Hubble friction.

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + \frac{k^2}{a^2}\delta\phi = 0$$

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} \simeq 0$$

$$\delta\phi = const$$

Symmetry Considerations

For a system which is close to Gaussian we can expand the wave function as follows:

$$\Psi[\chi] = exp(-\frac{1}{2}\int d^3xd^3y\chi(x)\chi(y) < O(x)O(y) >$$
$$+\frac{1}{6}\int d^3xd^3yd^3z\chi(x)\chi(y)\chi(z) < O(x)O(y)O(z) > + \cdots)$$

$< (O(x)O(y) >, < O(x)O(y)O(z) >$ for now are just coefficients which determine the wave function.

Symmetry Considerations

- Symmetry considerations lead to conditions on the coefficient functions.

- These will turn out to be identical to the Ward identities of conformal invariance satisfied by the corresponding correlator in a CFT.

Symmetry Considerations

- In this way the study of constraints imposed by the conformal symmetries on the wave function will be mapped to a study of constraints on correlations in a CFT.

- This is the central idea behind the analysis.

Symmetry Considerations

- The breaking of conformal invariance will give rise to corrections to these Ward identities.

- The resulting relations will be the analogue of the Callan Symanzik equations.

The Wave Function

The wave function on a spatial slice can be obtained by carrying out a path integral

$$\Psi[\chi] = \int_{initial}^{\chi} D\chi \;\; e^{iS}$$

The Wave Function

We will choose Bunch Davies initial conditions. These preserve conformal invariance.

By appropriate rescaling the action can be written as:

$$S = (\frac{M_{Pl}^2}{H^2})\tilde{S}$$

Since gravity waves produced during inflation have not been observed

$$\frac{M_{Pl}^2}{H^2} \geq\sim 10^{12} \gg 1$$

Thus the path integral

$$\Psi[\chi] = \int^{\chi} D[\chi] e^{i\left[\left(\frac{M_{Pl}^2}{H^2}\right)\tilde{S}\right]}$$

Can be carried out in the semi classical approximation.

By solving the equations of motion subject to the boundary conditions.

The Wave Function

Loop effects are unimportant but α' effects may be significant.

Analogy with AdS/CFT

The wave function is analogous to the partition function in AdS/CFT.

$$\Psi[\chi] \leftrightarrow Z[\chi]$$

Late time value of perturbation

Source in boundary field theory

Analogy with AdS/CFT

In fact, situation is analogous to the large N limit in AdS/CFT

$$\frac{M_{Pl}^2}{H^2} \leftrightarrow N$$

Analogy with AdS/CFT

One important difference.

Observables are expectation values to be calculated from the wave function:

$$< \chi\chi >= \int [D\chi]\ \chi\ \chi |\Psi|^2$$

Derivation of the Ward Identities

The Perturbations

The SO(3) rotational symmetry can be used to classify perturbations:

1) Spin 2: tensor perturbations

2) Spin 0 : scalar perturbations (arises from a mixing of the inflaton perturbation and metric)

Derivation of Ward Identities

Metric in ADM form:

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

Set N=1; $N^i = 0$

(synchronous gauge)

Derivation Of Ward Identities

EOM for N, N^i must still be imposed.

These lead to conditions ensuring that the wave function is invariant under residual gauge transformations which preserve synchronous gauge.

Derivation of Ward Identities

N^i EOM: spatial reparametrisations

$$x^i \to x^i + \epsilon^i(\vec{x})$$

N EOM: time reparametrisations

$$t \to t + \epsilon(\mathbf{x})$$

(accompanying spatial reparametrisation vanishes at late times)

Derivation Of Ward Identities

- These conditions then lead to the ward identities of conformal invariance.
- More generally the identities including the breaking of conformal invariance.

N EOM: time reparametrisations

$$t \to t + \epsilon(\mathbf{x})$$

Accompanying spatial transformation vanishes at late time

$$x^i \to x^i + \partial_i \epsilon \int dt \frac{1}{a^2(t)}$$

Ward Identities

For example in dS space:

$$ds^2 = -dt^2 + e^{2Ht}[\delta_{ij} + \gamma_{ij}]dx^i dx^j$$

$$ds^2 = -dt^2 + e^{2Ht}[\delta_{ij} + 2\zeta\delta_{ij} + \hat{\gamma}_{ij}]dx^i dx^j$$

Wave Function:

$$\Psi[\gamma_{ij}] = e^{-\left[\int d^3x_1 d^3x_2 \gamma_{ij}(x_1)\gamma_{kl}(x_2) < T^{ij}(x_1)T^{kl}(x_2) > + \cdots\right]}$$

Ward Identities

Invariance of Ψ under time reparametrisation $\zeta \to \zeta + H\epsilon(\mathbf{x})$ leads to condition

$$T_i^i = 0$$

More correctly, coefficient functions containing T_i^i vanish.

$$< T_i^i \; T_{ij} \; T_{kl} \ldots >= 0$$

Ward Identities

In inflationary background

$$\phi = \bar{\phi}(t) + \delta\phi$$

$$\Psi[\gamma_{ij}, \delta\phi] =$$

$$exp\Big[- \int d^3x_1, d^3x_2 \gamma_{ij}(x_1)\gamma_{kl}(x_2) < T^{ij} T^{kl} >$$

$$- \int d^3x_1, d^3x_2 \delta\phi(x_1)\delta\phi(x_2) < O(x_1)O(x_2) > + \cdots \Big]$$

In inflationary background

$$\phi = \bar{\phi}(t) + \delta\phi$$

Now under $t \to t + \epsilon(\mathbf{x})$

$$\zeta \to \zeta + H\epsilon(\mathbf{x})$$

$$\delta\phi \to \dot{\bar{\phi}}\epsilon + \delta\phi$$

Inflationary Background

Invariance of wave function leads to

$$T_i^i + \frac{\dot{\bar{\phi}}}{H} O = 0$$

This is analogous to a CFT perturbed by an operator which breaks conformal invariance.

$$\beta = -\frac{\dot{\bar{\phi}}}{H}$$

Specific Correlators

This relation between T^i_i and O provides a convenient way to compute coefficient functions of T^i_i to leading order in slow roll approximation

$$< T^i_i \; T^i_i >= (\frac{\dot{\phi}}{H})^2 \underbrace{< OO >}_{\text{can be calculated in dS space}}$$

Specific Correlators

Mata, Raju, Trivedi, 1211.5482

The coefficient function $< OOT_{ij} >$
Is completely fixed by conformal invariance.

O Corresponds to an operator of dimension 3.
T_{ij} to the stress tensor.

Specific Correlator

Result:

$$< \zeta(\vec{k_1})\zeta(\vec{k_2})\gamma_s(\vec{k_3}) >= (2\pi)^2\delta(\sum_{\vec{k}_i} \frac{8}{\Pi_i(2k_i^3)} P_s P_T \epsilon^{sij} k_{1i} k_{2j} S$$

$$S = (k_1 + k_2 + k_3) - \frac{\sum_{i>j} k_i k_j}{(k_1 + k_2 + k_3)} - \frac{k_1 k_2 k_3}{(k_1 + k_2 + k_3)^2}$$

$$\gamma_s = \gamma_{ij} e^{sij}$$

Specific Correlators

Result:

$$< \zeta(k_1)\zeta(k_2) >= (2\pi)^3\delta^3(\sum(\vec{k}_1 + \vec{k}_2)P_s\frac{1}{k_1^3}$$

$$< \gamma_s(\vec{k}_1)\gamma_{s'}(\vec{k}_2) >= (2\pi)^3\delta(\vec{k}_1 + \vec{k}_2)P_T\frac{\delta_{s,s'}}{k_1^3}$$

Specific Correlators

Result has a detailed functional form.

If observed would be a direct check of a central assumption of inflation namely approximate conformal invariance.

(Includes contact terms which can be important).

Specific Correlators

Caveat: ζ actually refers to the Sassaki -Mukhanov variable.

Conserved outside the horizon.

In the gauge $\delta\phi = 0$ it is ζ

Specific Correlators

Observationally the most significant non-gaussianity is from the scalar 3-pt function.

In this case the breaking of conformal invariance cannot be neglected, even for leading order answer.

Kundu, Shukla, Trivedi, arXiv:1410.2606

Ward identities:

$$\left(\sum_{a=1}^{3}\boldsymbol{k}_a\cdot\frac{\partial}{\partial\boldsymbol{k}_a}\right)\langle O(\boldsymbol{k}_1)O(\boldsymbol{k}_2)O(\boldsymbol{k}_3)\rangle = \frac{\dot{\bar{\phi}}}{H}\,\langle O(\boldsymbol{k}_1)O(\boldsymbol{k}_2)O(\boldsymbol{k}_3)O(\boldsymbol{k}_4)\rangle\Big|_{\boldsymbol{k}_4\to 0}.$$

$$\begin{aligned}\mathcal{L}^{b}_{\boldsymbol{k}_1}\langle O(\boldsymbol{k}_1)O(\boldsymbol{k}_2)O(\boldsymbol{k}_3)\rangle' + \mathcal{L}^{b}_{\boldsymbol{k}_2}\langle O(\boldsymbol{k}_1)O(\boldsymbol{k}_2)O(\boldsymbol{k}_3)\rangle' + \mathcal{L}^{b}_{\boldsymbol{k}_3}\langle O(\boldsymbol{k}_1)O(\boldsymbol{k}_2)O(\boldsymbol{k}_3)\rangle' \\ = 2\,\frac{\dot{\bar{\phi}}}{H}\left[\boldsymbol{b}\cdot\frac{\partial}{\partial\boldsymbol{k}_4}\right]\left\{\langle O(\boldsymbol{k}_1)O(\boldsymbol{k}_2)O(\boldsymbol{k}_3)O(\boldsymbol{k}_4)\rangle'\Big|_{\boldsymbol{k}_4\to 0}\right\},\end{aligned}$$

Specific Correlators

Linear equations for <OOO> with source term determined by <OOOO> in suitable limit.

General solution :

$$< OOO >= S_H + S_I$$

homogenous

inhomogeneous

Three Point Scalar Correlator

S_I Uniquely determined by <OOOO>

To leading order <OOOO> can be calculated in conformal limit.

S_H completely fixed from conformal invariance. Corresponds to the 3 pt function of a marginal scalar.

Three Point Scalar Correlator

In this way 3 –pt function is fixed by data obtained in the conformally invariant limit.

In slow roll approx. generically S_I will dominate over S_H

Resulting answer not unique, because 4 pt function <OOOO> is not unique.

Three Point Scalar Correlator

Generically <OOOO> will not vanish in conformally invariant limit.

Thus, $$S_I \sim O(\frac{\dot{\phi}}{H})$$

And $$< OOO > \sim O(\frac{\dot{\phi}}{H})$$

This will be the extent of the suppression for the 3 pt. function.

Scalar Three Point Function

Roughly, although functional form is different, generically we learn from the ward identities that

$$f_{NL} \sim \epsilon$$

Thus the Non-Gaussianity will be suppressed.

Hard to observe in CMB.

Scalar Three Point Function

But might be observed through galaxy surveys.

Or 21 cm physics.

$$f_{NL} \sim \epsilon \sim 10^{-2}$$

A firm prediction just from symmetries.

Scalar Three Point Function

Arkani-Hamed, Maldacena: arXiv: 1503.08043

Could be bigger.

If there are particles with $M \sim H$
With enhanced couplings to inflaton
(compared to graviton).

The four point function <OOOO> will then be bigger. And so will the 3 pt function.

Scalar Three Point Function

Arkani-Hamed, Maldacena: arXiv: 1503.08043

The four point function <OOOO> can be enhanced if there are extra particles with

$$M \sim H$$

Which have couplings to the inflaton that are enhanced (compared to those of the graviton).

Then the 3 pt function will also be enhanced.

Scalar Three Point Function

Or if the scalar sector in inflation breaks conformal invariance in a significant way, e.g. DBI inflation etc.

General Ward Identities

Puzzle:

4 point scalar correlator in dS space does not seem to satisfy the ward identities of conformal transformations.

Serry, Sloth, Vernizzi, JCAP 0701 (2007) 0903 (2009) 018.

Issue :

Ghosh, Kundu, Raju, SPT, arXiv: 1401.1406

We are computing local correlators in a theory of quantum gravity.

$$< \zeta(x^1)\zeta(x^2)\cdots\zeta(x^n) >$$

These are well defined only after gauge fixing.

A general conformal transformation will lead to a change of gauge

Resolution :

The conformal transformation must be accompanied by a compensating coordinate transformation that restores the gauge.

The resulting ward identities are then satisfied.

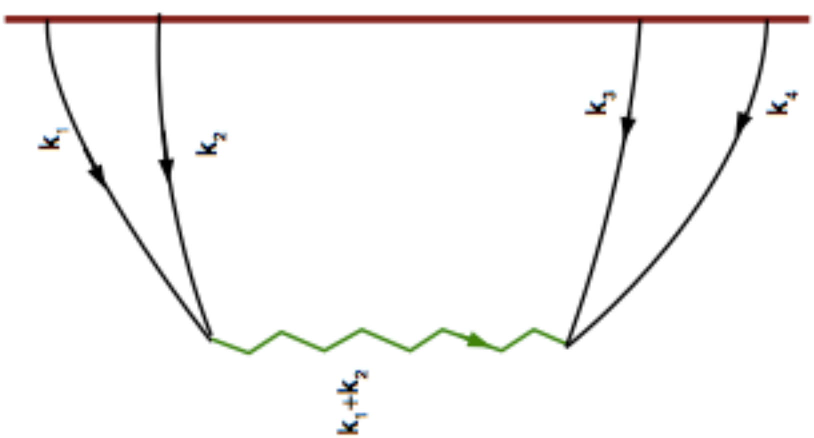
In Practical Terms:

Additional gauge fixing is needed in going from the wave function to expectation values.

$$< \zeta(x^1)\zeta(x^2)\cdots\zeta(x^n) >= \int D[\gamma_{ij}]|\Psi|^2\zeta(x^1)\zeta(x^2)\cdots\zeta(x^n)$$

Gauge fixing needed to make sum over metrics well defined.

Can be important even for calculating scalar correlators.



General Ward Identities

General Ward Identities valid to all orders in the slow roll expansion can now be written down.

Nilay Kundu, Ashish Shukla, SPT, In Prep.

Scale Invariance

$$\left(3n + \sum_{a=1}^{n} k_a \frac{\partial}{\partial k_a}\right) \langle \zeta(\boldsymbol{k}_1) \cdots \zeta(\boldsymbol{k}_n) \rangle = - \frac{1}{\langle \zeta(\boldsymbol{k}_{n+1}) \zeta(-\boldsymbol{k}_{n+1}) \rangle'} \langle \zeta(\boldsymbol{k}_1) \cdots \zeta(\boldsymbol{k}_{n+1}) \rangle \Big|_{\boldsymbol{k}_{n+1} \to 0},$$

Maldacena Consistency Conditions

Special Conformal Transformations

$$\langle \delta(\zeta(\boldsymbol{k}_1)) \cdots \zeta(\boldsymbol{k}_n) \rangle + \cdots + \langle \zeta(\boldsymbol{k}_1) \cdots \delta(\zeta(\boldsymbol{k}_n)) \rangle = \\ -2\left(\boldsymbol{b} \cdot \frac{\partial}{\partial \boldsymbol{k}_{n+1}}\right) \frac{\langle \zeta(\boldsymbol{k}_1) \cdots \zeta(\boldsymbol{k}_{n+1}) \rangle}{\langle \zeta(\boldsymbol{k}_{n+1}) \zeta(-\boldsymbol{k}_{n+1}) \rangle'} \bigg|_{\boldsymbol{k}_{n+1} \to 0},$$

$$\delta(\zeta(\boldsymbol{k})) = \widehat{\mathcal{L}}_{\boldsymbol{k}}^{\boldsymbol{b}} \, \zeta(\boldsymbol{k}) + 6\, b^m k^i \int \frac{d^3\tilde{\boldsymbol{k}}}{(2\pi)^3} \frac{1}{\tilde{k}^2} \zeta(\boldsymbol{k} - \tilde{\boldsymbol{k}}) \, \widehat{\gamma}_{im}(\tilde{\boldsymbol{k}}) \\ + 2\, b^m k^i \int \frac{d^3\tilde{\boldsymbol{k}}}{(2\pi)^3} \frac{1}{\tilde{k}^2} \widehat{\gamma}_{ij}(\boldsymbol{k} - \tilde{\boldsymbol{k}}) \, \widehat{\gamma}_{jm}(\tilde{\boldsymbol{k}}),$$

Summary

- Approximate Conformal Invariance is a powerful constraint.

- Hopefully Non-Gaussianity will be observed and we will be able to test whether the early universe was approximately conformally invariant.

Summary

- Methods can be extended to DBI inflation etc.

- Perhaps these considerations can help also in formulating a more precise dS/CFT correspondence.

GMRT TIFR

World's Largest Telescope At Meter Wavelengths