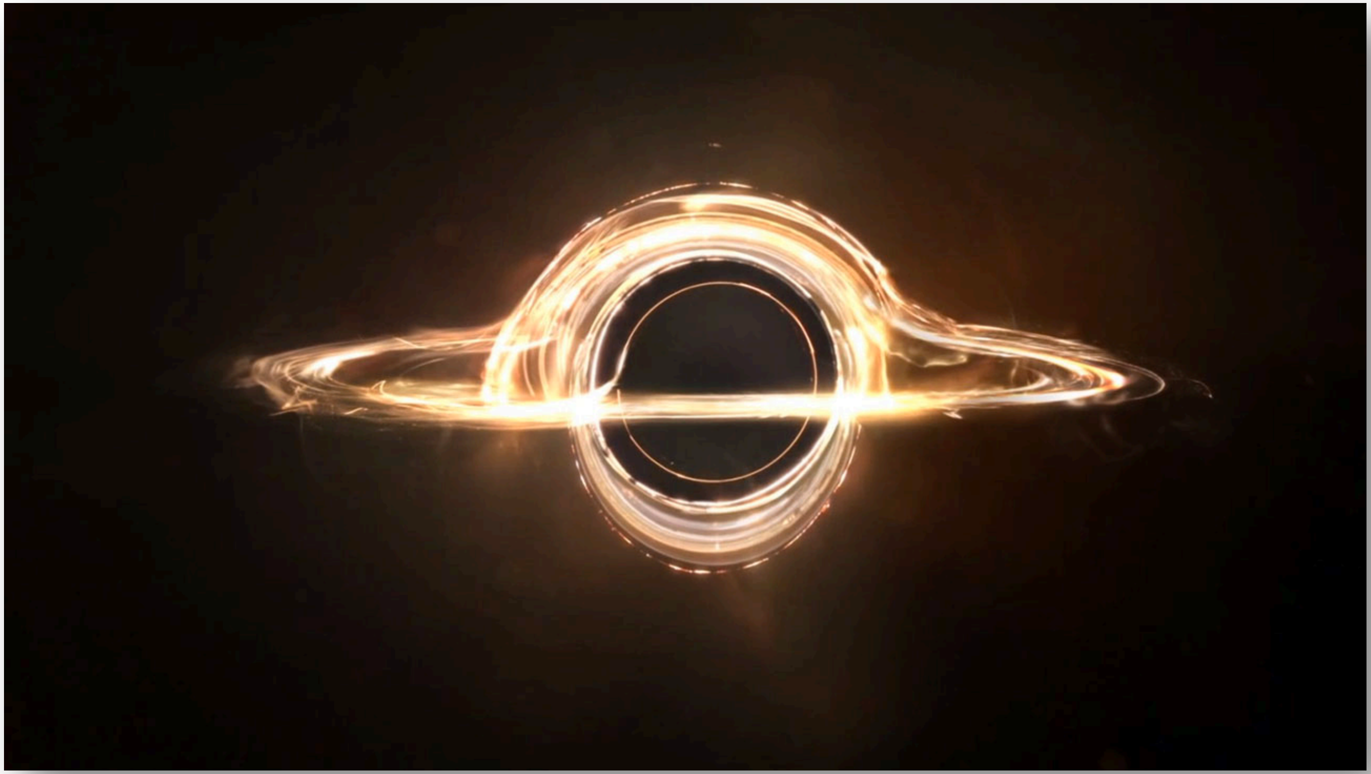
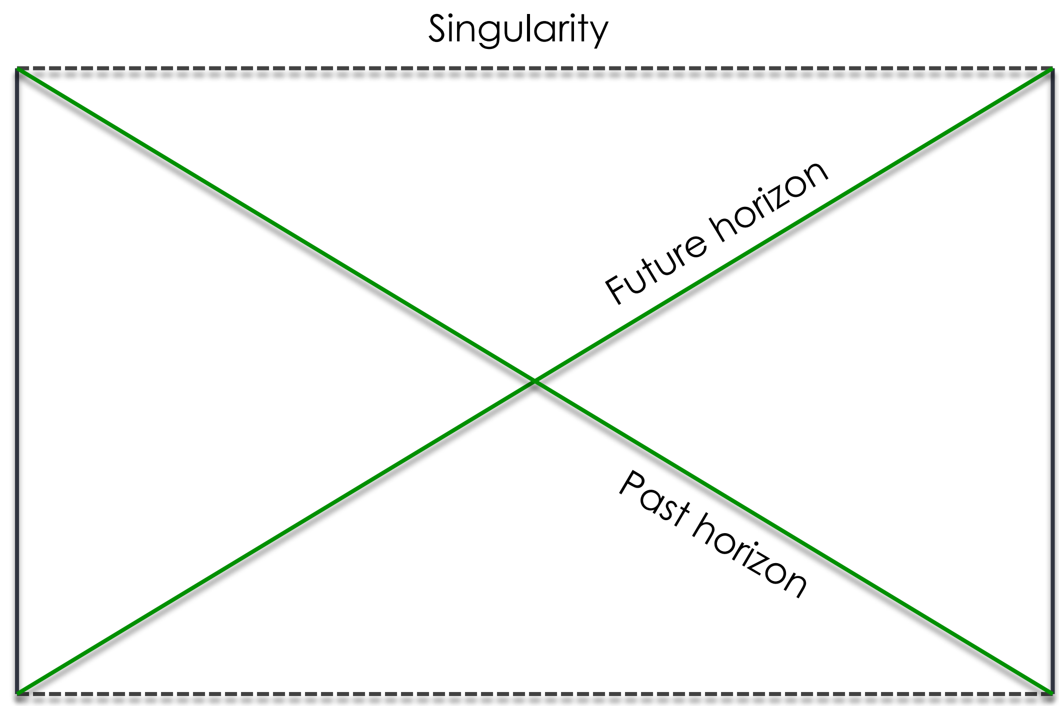
WILSON LINES IN ADS$_3$/CFT$_2$

Strings 2015

Alejandra Castro, University of Amsterdam

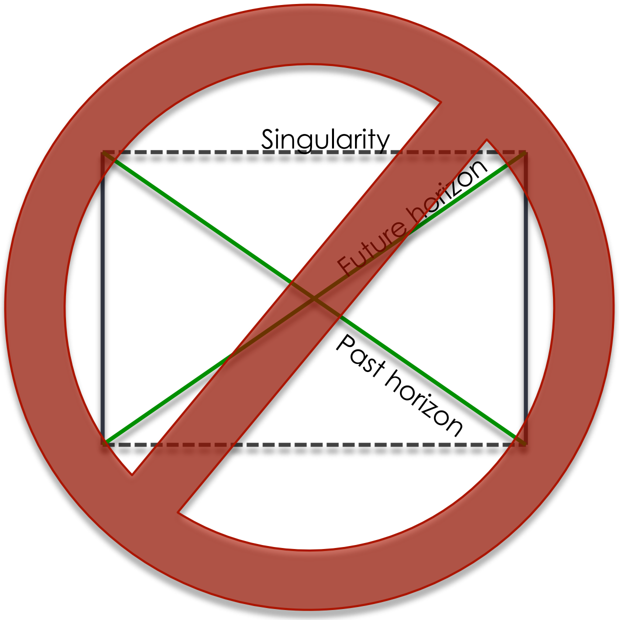


Hollywood’s conception of black holes



Relativist's conception of a black hole

Not every gravitational theory admits a geometrical description (or at least not an obvious one).



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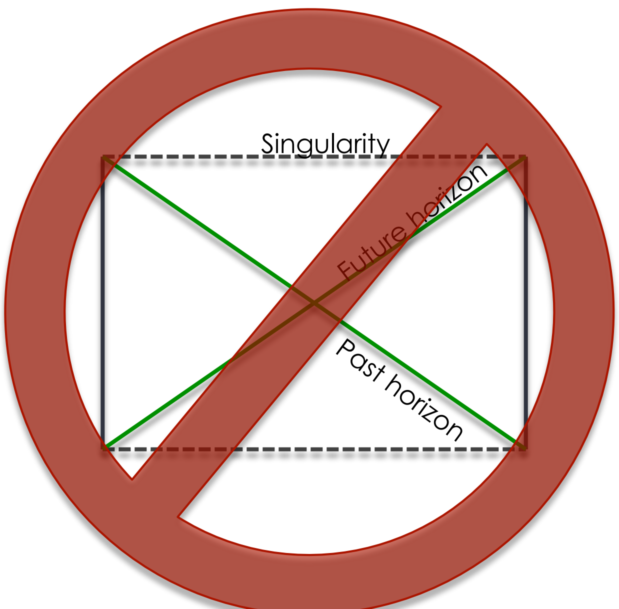
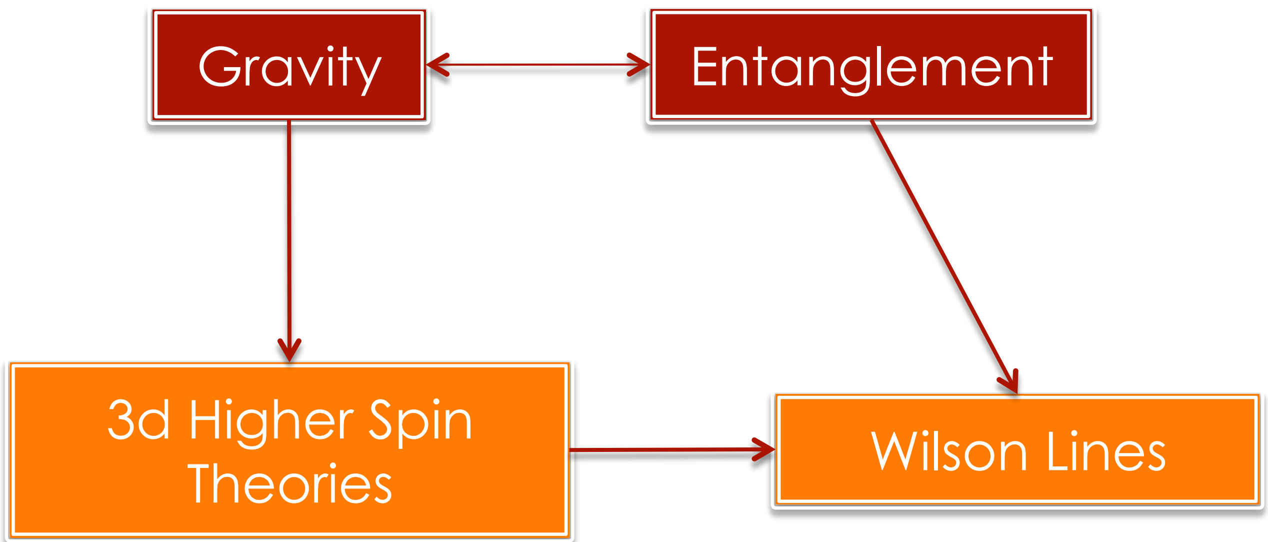


If geometry is lacking,
you might question the relation among:

Gravity ⟷ Thermodynamics

Gravity ⟷ Entanglement

This talk will report on recent progress related to …



Collaborators

Based on work with:

- Martin Ammon & Nabil Iqbal 1306.4338 [hep-th]
- Eva Llabres 1410.2870 [hep-th]
- Jan de Boer, Eliot Hijano, Juan Jottar & Per Kraus 1412.7520 [hep-th]

In progress with:

- Fotios Dimitrakopoulos, Eva Llabres & Nabil Iqbal
- Max Banados, Alberto Faraggi & Juan Jottar

Outline

1. Gravity & Chern-Simons Theory

2. Wilson Lines

3. Black Holes Revisited

4. Outlook

1. Gravity & Chern-Simons

An old story revived

3d Gravity

In 2+1 dimensions, we have the **luxury** of casting general relativity in terms of:

[Acucharro & Townsend; Witten]

Einstein-Hilbert: Metric, curvature

OR

Chern-Simons: Flat connections

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↓

Inclusion of massless higher spin fields is straightforward!

Higher Spin Theories

How to interpret Chern-Simons theory as a theory of gravity?

$$S_{CS}[\mathcal{A}] = \frac{k}{4\pi}\int_{\mathcal{M}} \mathrm{Tr}\left(\mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right)$$

It is not just a matter of actions and equations of motion.
Other important **INPUTS** are:

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Other important **INPUTS** are:

1. **Gauge Group:**
Organization of the massless modes

$$\mathcal{A} \in G = \underbrace{G_L \times G_R}_{SL(2,\mathbb{R}) \times SL(2,\mathbb{R})}$$

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2. Boundary Conditions:
Setup the AdS/CFT dictionary

$$\mathcal{A} - \mathcal{A}_{\mathrm{AdS}} = O(1)$$

Observables

How do we quantify and classify solutions?

Observables

1. Asymptotic Symmetry Group and Ward Identities

$$SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \longrightarrow \mathrm{Vir} \times \mathrm{Vir}$$

[Brown & Henneaux]

$$SL(N,\mathbb{R}) \times SL(N,\mathbb{R}) \longrightarrow \mathcal{W}_N \times \mathcal{W}_N$$

[Campoleoni et al]

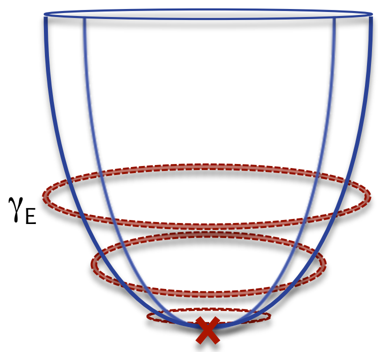
$$\mathrm{hs}[\lambda] \times \mathrm{hs}[\lambda] \longrightarrow \mathcal{W}_\infty[\lambda] \times \mathcal{W}_\infty[\lambda]$$

[Henneaux & Rey; Gaberdiel & Hartman]

With central charge: $\boxed{c = \frac{3\ell}{2G_3} = 6k}$

Observables

2. Non-perturbative Properties



$$\mathrm{Hol}_{\gamma_E}(\mathcal{A}) = \mathcal{P}\exp\left(\oint_{\gamma_E} \mathcal{A}\right)$$

a. Trivial holonomy along thermal cycle: **Euclidean Black Hole**

b. Trivial holonomy along spatial cycle: **Conical Defect**

[Kraus & Gutperle, AC et al, Compere et al, Bunster et al, …]

3. Partition Functions, 1-loop Determinants, … (ask me later)

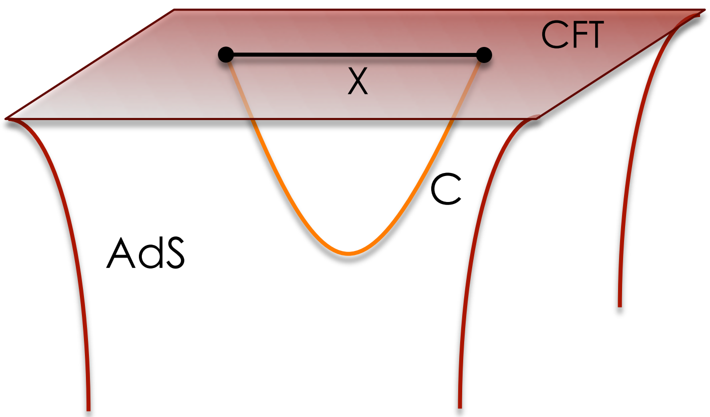
Quick summary

- We are working with a Chern-Simons formulation of higher spin theories.
- It is clear how to identify & organize finite charge configurations (ASG).
- It is clear how to construct Euclidean black holes and quantify its thermodynamics properties.
- None of the above required, nor involved, a metric.

2. Wilson Lines

Building more observables

Entanglement Entropy



[Ryu & Takayanagi]

Anti-de Sitter Gravity

Minimal areas
(geodesic in 3d)

$\longleftrightarrow$

Conformal Field theory

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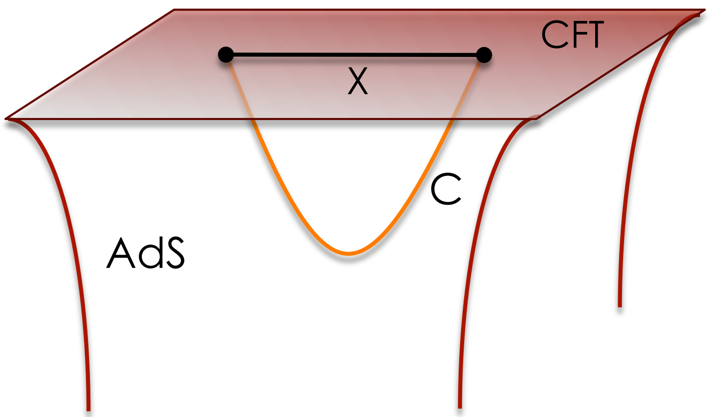

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Anti-de Sitter Gravity
Minimal areas
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$\longleftrightarrow$

Conformal Field theory
Entanglement Entropy

This prescription clearly fails in higher spin gravity.
What is a geodesic in a theory without a metric?

Wilson line & massive probes

Think in terms of the connection. Think massive.

[Ammon, AC & Iqbal; de Boer & Jottar]

$$W_{\mathcal{R}}(C) = \mathrm{tr}_{\mathcal{R}}\left(\mathcal{P}\exp\oint_C \mathcal{A}\right) = \int \mathcal{D}U \exp[-S(U;\mathcal{A})_C]$$

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Encodes quantum numbers of the probe. Here it is an infinite dimensional representation of G.

Auxiliary field.
Captures dynamics of the probe.

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Wilson loop encodes the dynamics of a massive point particle.
Natural replacement of geodesic equation.

Features

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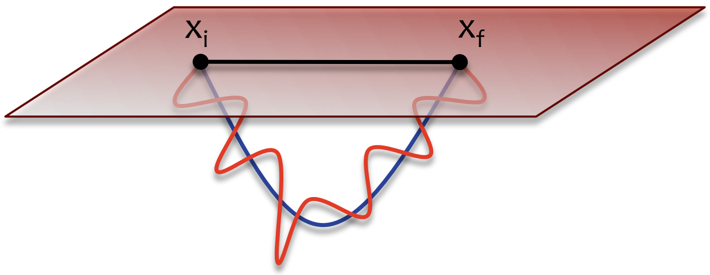
1. For SL(2) connections, the Wilson line reproduces the geodesic length.

$$W_{\mathcal{R}}(x_i, x_f) \sim \exp\left(-\underbrace{\sqrt{2c_2}}_{\text{Casimir of the representation (i.e. mass of the particle)}} L(x_i, x_f)\right)$$



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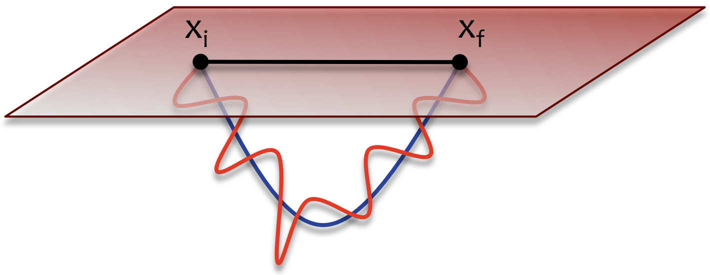
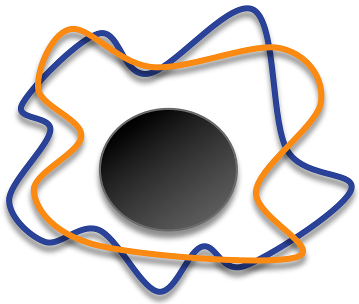


2. For "black hole" connections, the Wilson loop gives thermal entropy.

$$S_{\text{thermal}} = -\log\left(W_{\mathcal{R}}(\underbrace{C}_{\substack{\text{Closed spatial cycle} \\ \text{(non-trivial)}}})\right)$$



The proposal

$$W_{\mathcal{R}}(C_{ij}) = \langle i|\mathcal{P}\exp\int_{C_{ij}}\mathcal{A}|j\rangle$$

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a. Mass, no rotation, no spin charge

$$L_0|U\rangle = \bar{L}_0|U\rangle = h|U\rangle$$

$$2c_2 = 4h(h-1)$$

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b. Mass, no rotation, spin-3 charge

$$W_0|U\rangle = \bar{W}_0|U\rangle = w|U\rangle$$

$$c_3 = \frac{3}{8}w\left(h^2 - \frac{1}{4}w^2\right)$$

$$\boxed{S_3 = -\log\left(W_{\mathcal{R}_{2,3}}(C_{ij})\right)}$$

[Hijano & Kraus]

Developments

Can we prove our proposal?

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Excellent agreement!

Developments

Large central charge limit, we have the correspondence

$$\underbrace{\langle \mathcal{O}(\infty)\sigma(x)\tilde{\sigma}(1)\mathcal{O}(0)\rangle}_{\text{Approximated by vacuum black}} = \underbrace{e^{-S_{\mathrm{CS,bulk}}}}_{\text{Approximated by classical action}}$$

Developments

Large central charge limit, we have the correspondence

$$\underbrace{\langle \mathcal{O}(\infty)\sigma(x)\tilde{\sigma}(1)\mathcal{O}(0)\rangle}_{\substack{\text{Approximated by} \\ \text{the vacuum block}}} = \underbrace{e^{-S_{\mathrm{CS},\mathbf{bulk}}}}_{\substack{\text{Approximated by} \\ \text{bulk classical action}}}$$

Reason: both governed by same monodromy problem (ODE)

Equivalent statements hold in 3d Gravity; the metric formulation just requires a bit more work.

[Hartman; Faulkner]

Developments

Still large central charge limit, scale dimension of operators:

$$\frac{\Delta_{\mathcal{O}}}{c} = O(1)$$ **Heavy (e.g. black hole)**

$$\langle \mathcal{O}(\infty)\sigma(x)\tilde{\sigma}(1)\mathcal{O}(0)\rangle = e^{-W_{\mathcal{R}}(C_{1,x})}$$

$$\frac{\Delta_{\sigma}}{c} = O(n-1)$$ **Light (e.g. twist fields)**

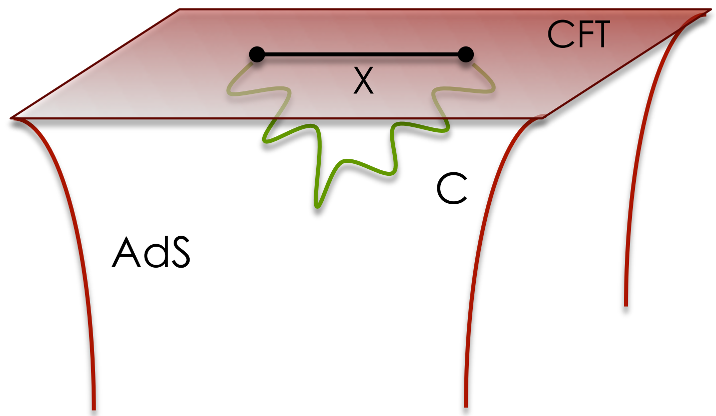
Vacuum conformal block is computed by a **bulk Wilson line** probing an asymptotically AdS_3 background with higher spin fields excited.

3. Black Holes Revisited

Penrose diagrams in higher spin theory

Until now we have only discussed physics at one boundary.

And we have spectacular agreement with the CFT.



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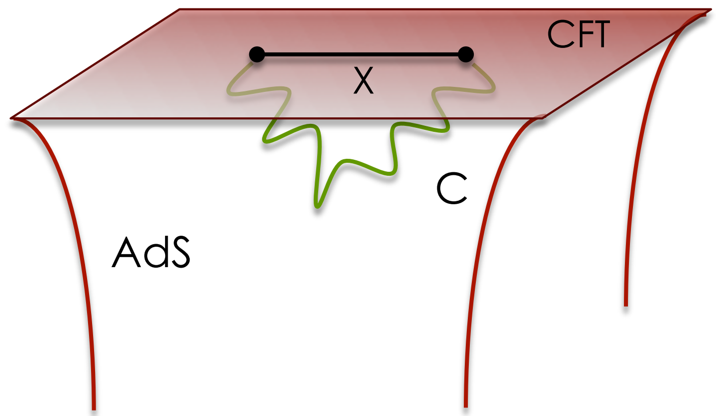
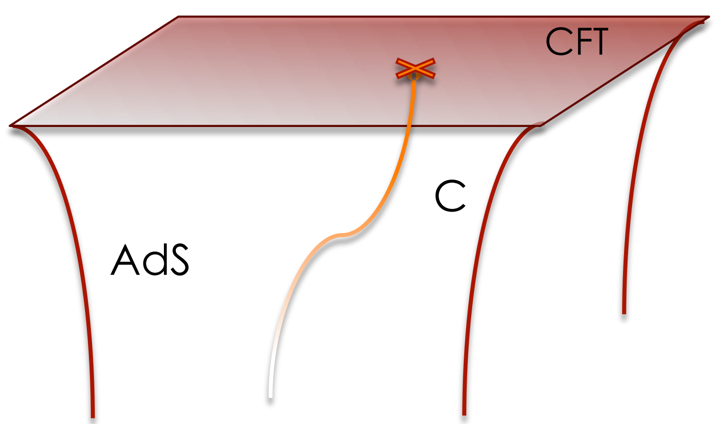
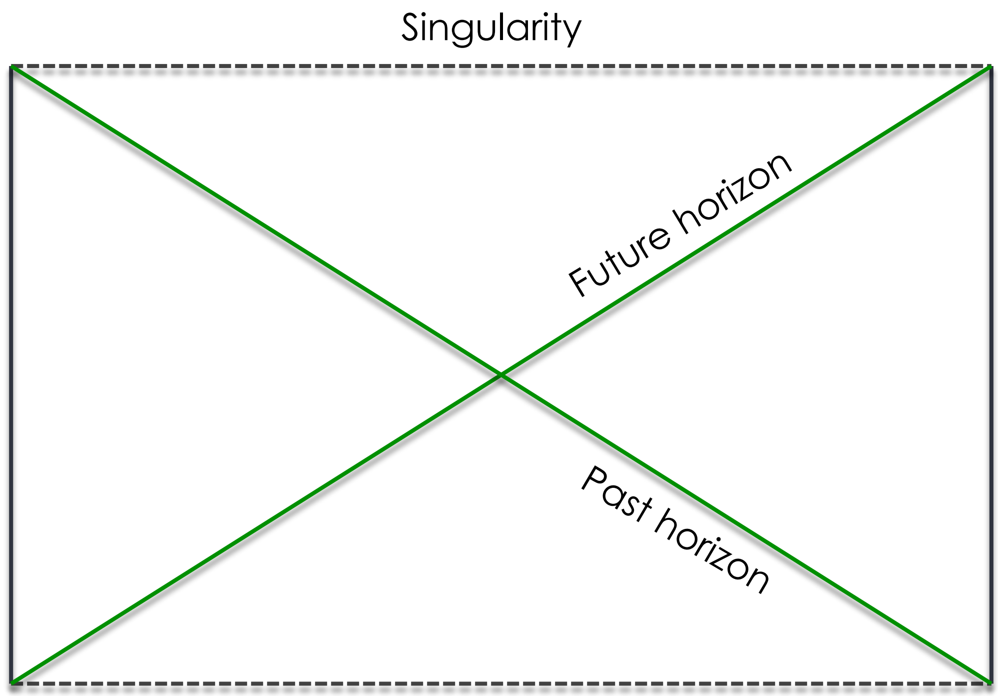


What if one end goes loose?



Does Chern-Simons theory know that an eternal black hole has two boundaries?



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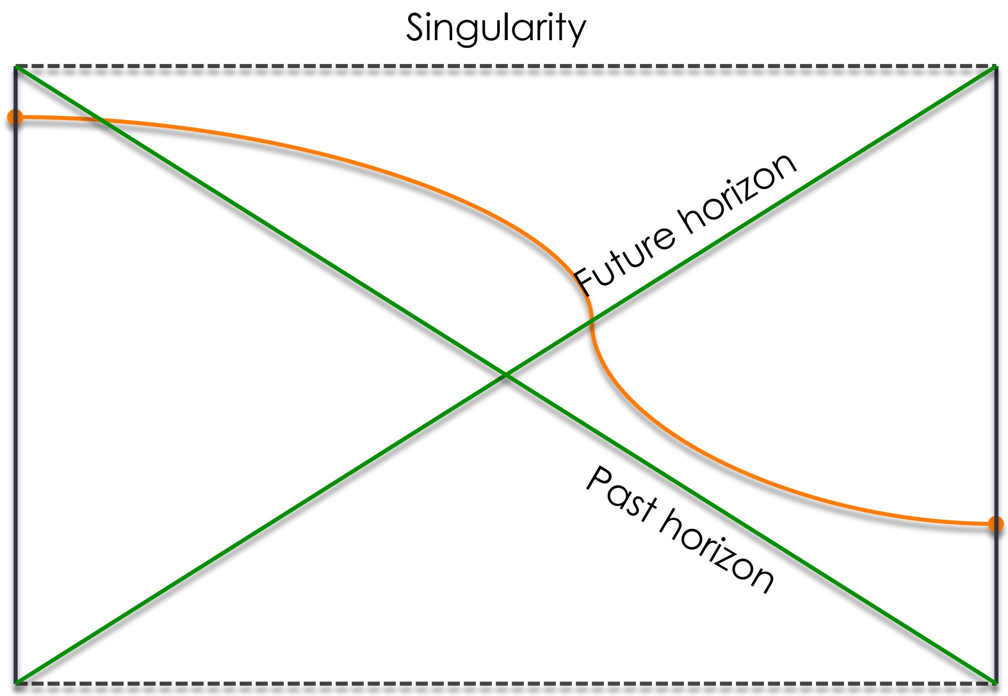


If yes, how does Chern-Simons know?

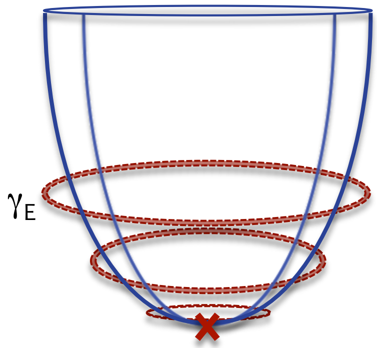
Using a Wilson Line, we should be able to reconstruct and define a notion of Penrose diagram.

Lorentzian HS Black Holes

A possible definition of Lorentzian black hole:

An eternal black is a thermo-field double state in the CFT.

Whereas an Euclidean black holes satisfies:

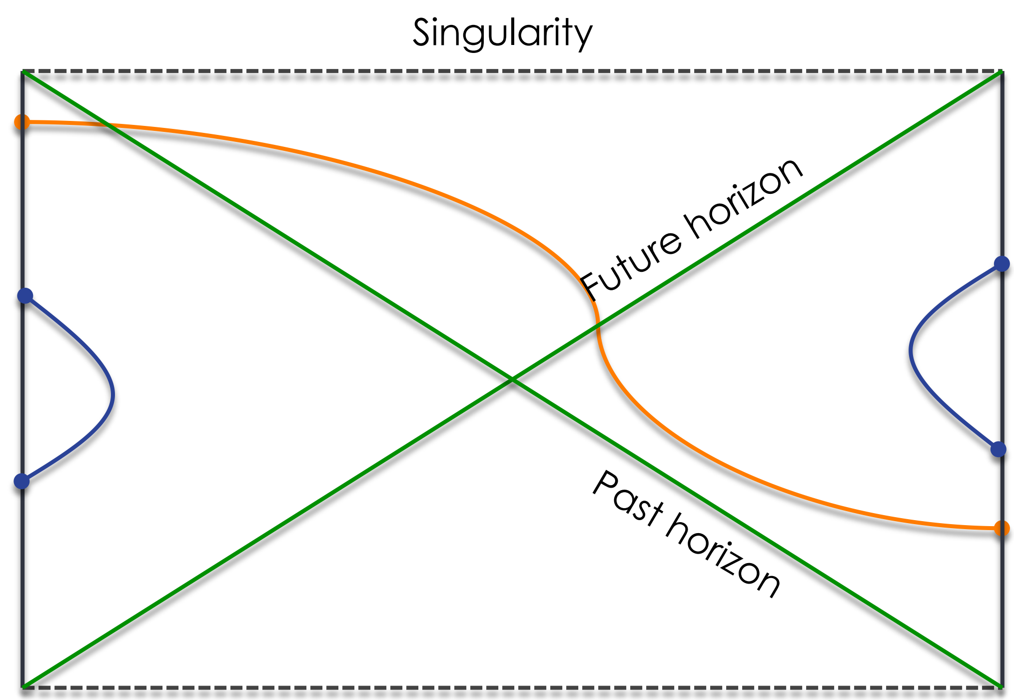


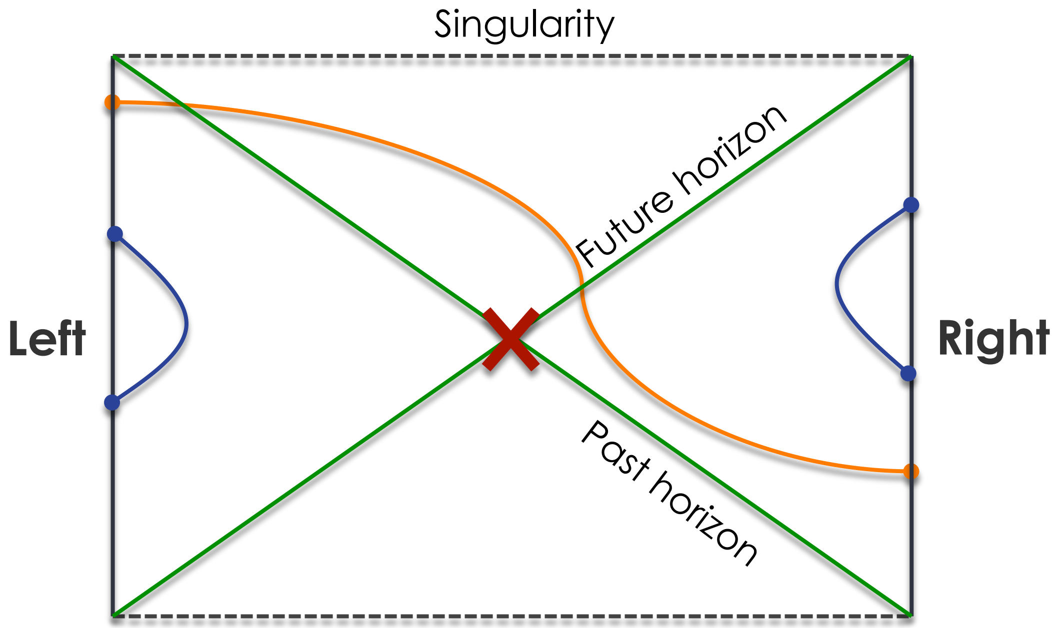
$$\mathrm{Hol}_{\gamma_E}(\mathcal{A}) = \mathcal{P}\exp\left(\oint_{\gamma_E}\mathcal{A}\right)$$

An eternal black is a thermo-field double state in the CFT.

$$|\psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2}(E_n+\mu Q_n)} |Un\rangle_L |n\rangle_R$$

This imposes more restrictions than Euclidean regularity conditions





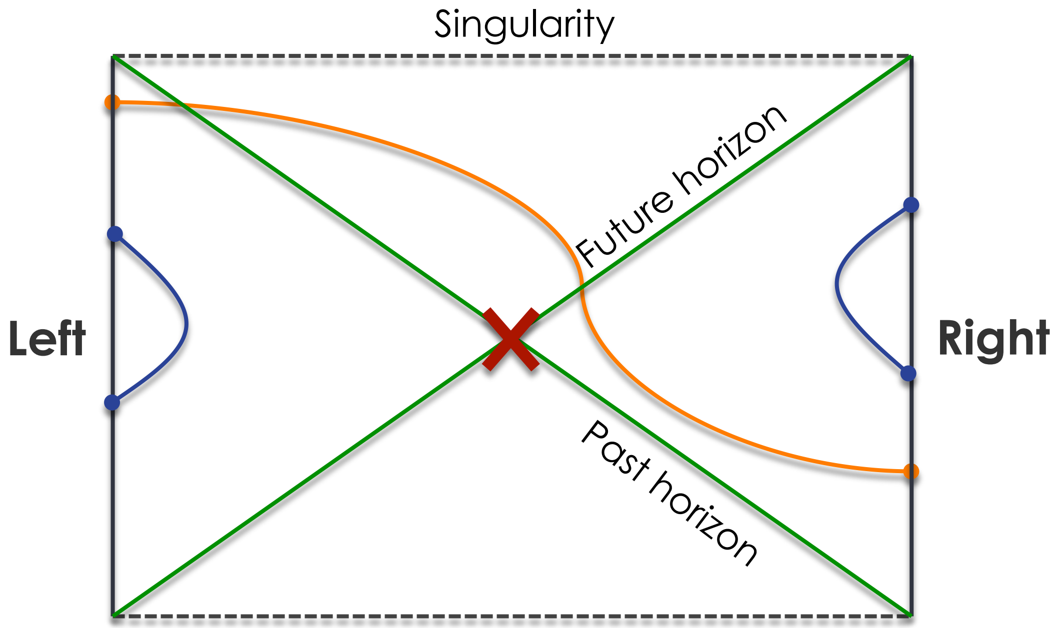
Testing this proposal is difficult: non-trivial to have test particle. Important prior work made use of Vasiliev scalar field.

[Kraus & Perlmutter]

Problem as we pose it: given

$$\mathcal{A} = b^{-1}(\rho)\Big(a(x,t) + d\Big)b(\rho)$$

what is the appropriate **causal diagram**?



The Wilson line should behave according to our definition.

1. Signal of a bifurcation point.
2. Left-Left sided correlator should equal Right-Right sided.
3. Left-Right correlator should obey appropriate KMS conditions.

$$\mathcal{A} = b^{-1}(\rho)\Big(a(x,t) + d\Big)b(\rho)$$

4. Outlook

Conclusions

1. Higher spin theories unveil new features and new challenges in quantum gravity.

2. We replaced and generalized the notion of distances.

Wilson Lines & Entanglement

3. Plethora of quantitative checks within AdS/CFT.

Chern-Simons versus Toda Theory

In Progress

1. Global interpretation of Lorentzian solutions.
The Wilson line is allowing us to decode the bulk.

2. Extremal Black Holes: appropriate definition, characteristic and BPS features.

THANK YOU!