On Compactifications of 6d (1,0) Theories

Cumrun Vafa

Strings 2015
ICTS, Bangalore, India

June 23, 2015

Based on:

arXiv:1504.08348

Geometric Engineering, Mirror Symmetry and 6d (1,0) -> 4d, N=2

Michele Del Zotto, Cumrun Vafa, Dan Xie

Related Work:

arXiv:1501.01031

6d N=(1,0) theories on T^2 and class S theories: part I

Kantaro Ohmori, Hiroyuki Shimizu, Yuji Tachikawa, Kazuya Yonekura

Recent Developments have led (potentially) to a full classification of SCFT's in 6 dimensions.

See Jonathan Heckman's talk

These include the (2,0) ADE theories as well as a vast number of (1,0) theories.

It is natural to ask what compactifications of these theories lead to in lower dimensions with different amounts of supersymmetries. In this talk I will focus on toroidal compactificaitons to 4d.

Famous Example: Toroidal compactification of 6d (2,0) SCFT

6d ADE (2,0) theories → 4d ADE N=4 SYM

This geometrizes the SL(2,Z) symmetry of 4d N=4

[Witten]

How about (1,0)?

If we compactify (1,0) theories on T^2,

we get 4d, theories with N=2 and so we can ask:

Are there 4d N=2 theories that also enjoy SL(2,Z) duality symmetry that come from such theories?

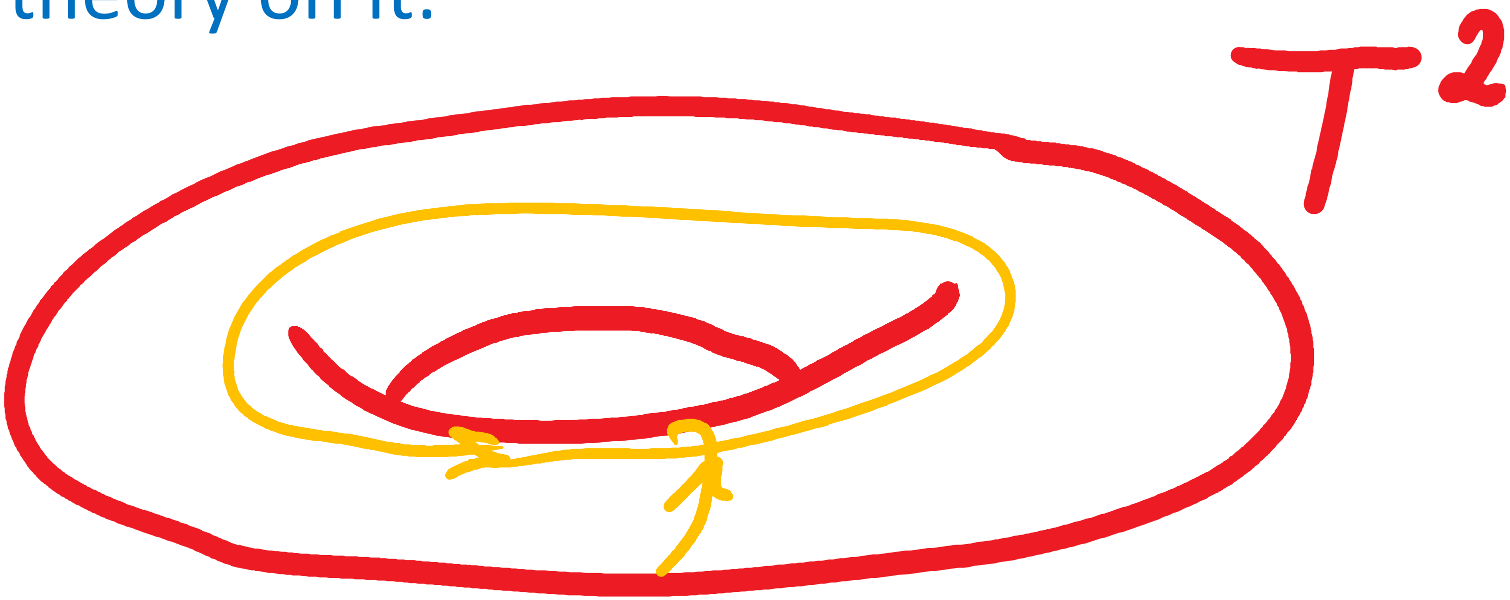
In fact we should first ask:

Are there 4d N=2 theories with SL(2,Z) symmetry?

The answer is yes! The first known example is in the original work of [Seiberg-Witten]: SU(2) gauge theory with 4 fundamental matter. There are other theories discovered as well:

SU quiver theories for affine A $\rightarrow$
Moduli of points on T^2 [Witten]
SU quiver theories for affine D,E $\rightarrow$
Moduli of flat D,E-connections on T^2 [Katz,Mayr,V]
Do these have a 6d explanation, like the more familiar N=4 case?

Let us consider a toroidal geometry and compactify 6d (1,0) theory on it:



We can in addition consider turning on Wilson lines corresponding to gauge or global symmetries.
The resulting 4d theory will depend on this data.

Question: What are interesting 4d IR fixed points?

There are many possible 4d theories we may end up with. We could aim to keep a maximal amount of structure of the 6d theory intact.
There are two broad classes of theories we end up with:

1) Preserve the global symmetries of the 6d theory.

2) Complex structure of the torus is an exactly marginal deformation in the 4d theory with manifest SL(2,Z) duality.

It turns out it is not possible to do both!

Basic Strategy:

1-Geometric engineering as F-theory on CY 3-folds X.

2-Compactification on a circle: M-thoery on X.

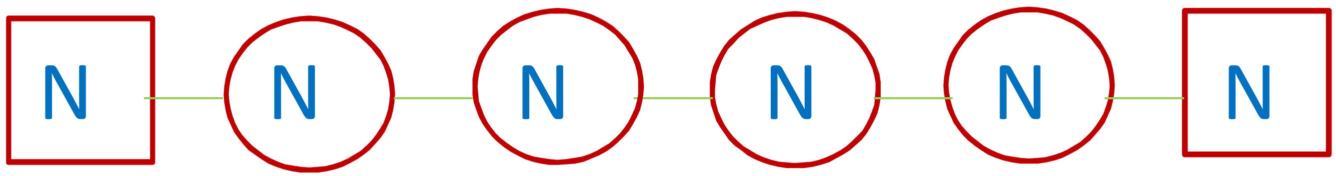
3-Compactification on another circle: IIA on X.

4-Mirror symmetry leads to IIB on mirror geometry Y.

5-Vacuum geometry of the N=2 theory emerges from mirror geometry Y.

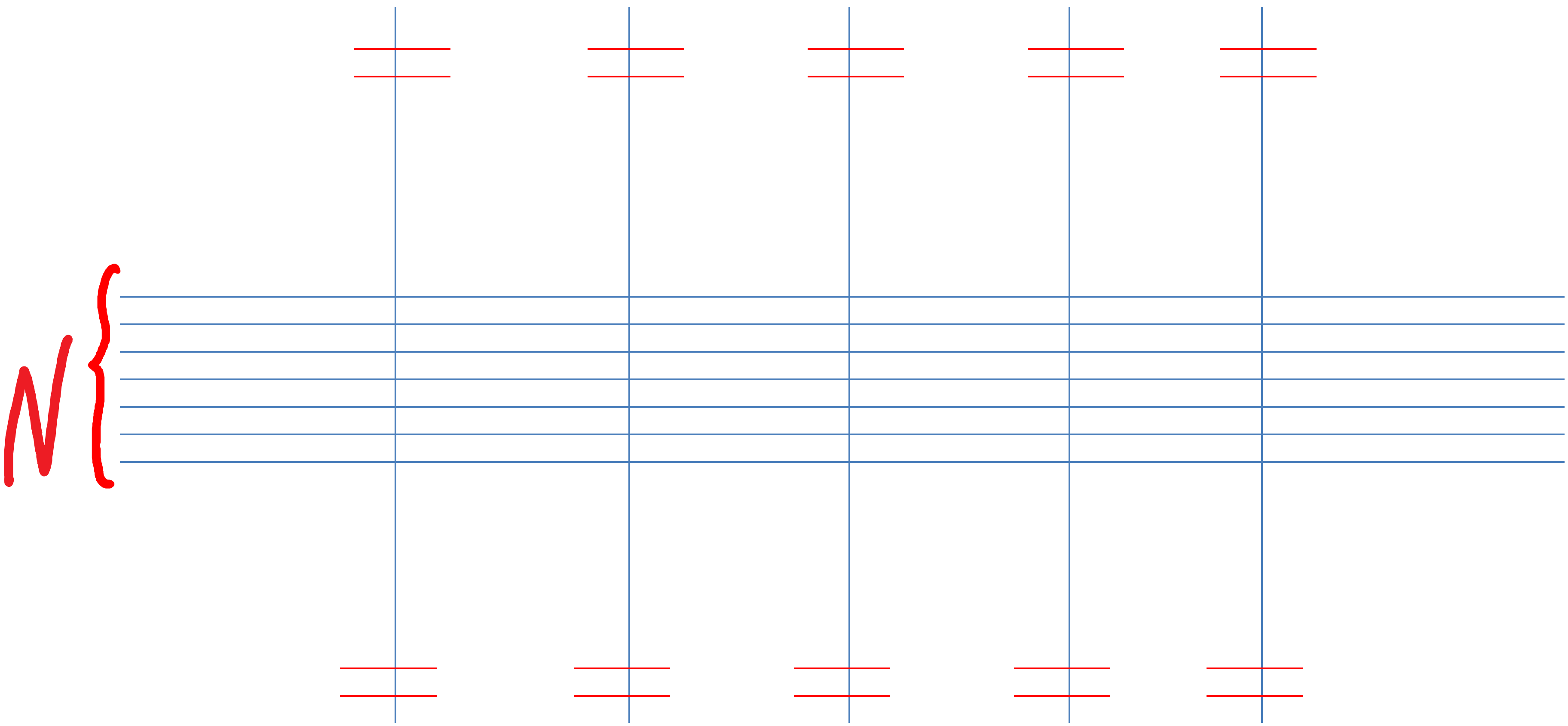
We start with the simplest class of examples (A-type) where we can also use brane techniques and then proceed to more complex examples where mirror symmetry techniques lead to the emergent N=2 vacuum geometry.

A simple class of 6d (1,0) theories:
(self-intersection of the spheres = -2)



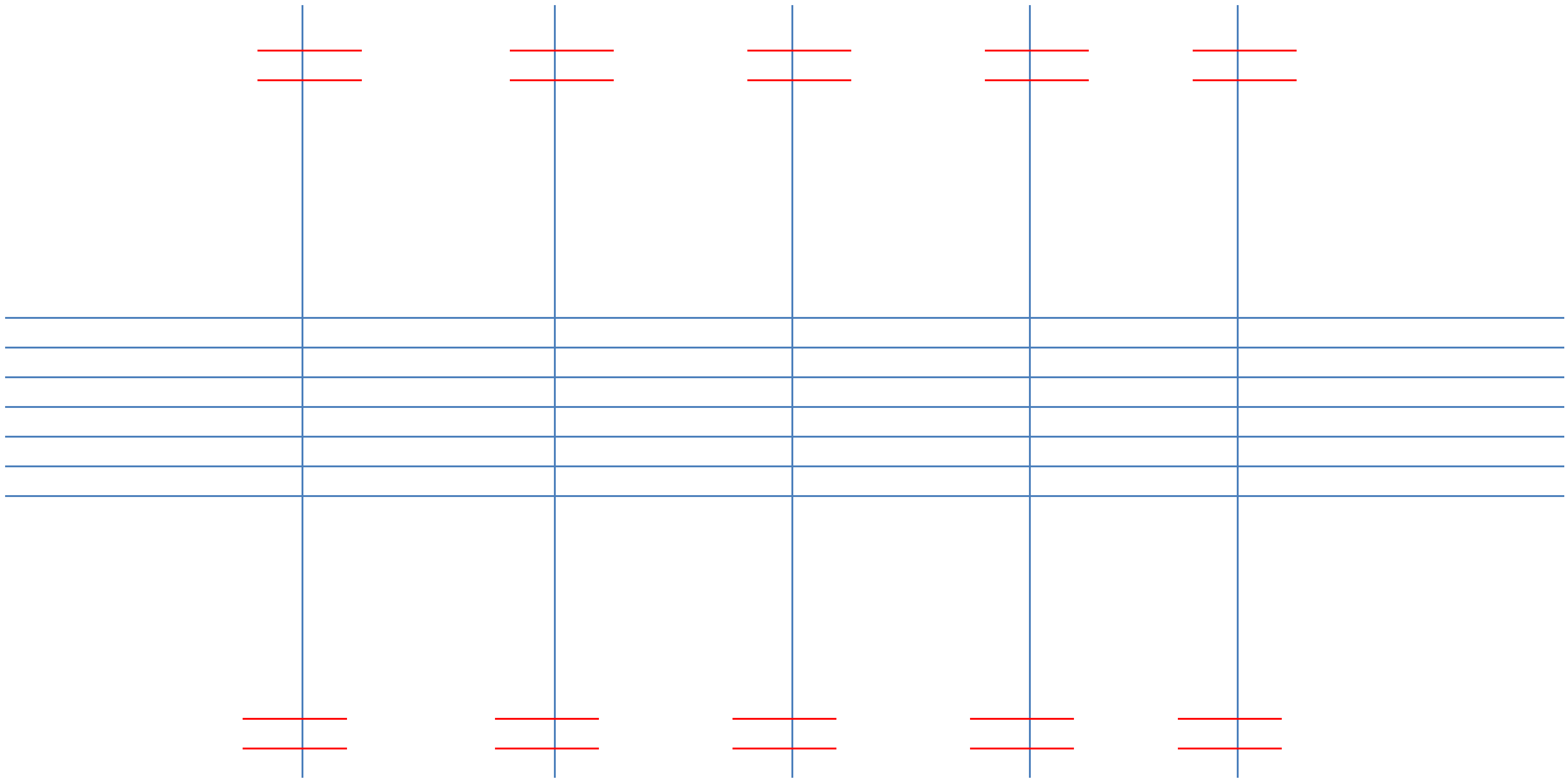
This theory has SU(N) x SU(N) global symmetry.

F-theory on an elliptic 3-fold engineers this theory. Compactification on a circle leads to M-theory on the same elliptic 3-fold where the elliptic fiber has size 1/R [Haghighat et. al.]:

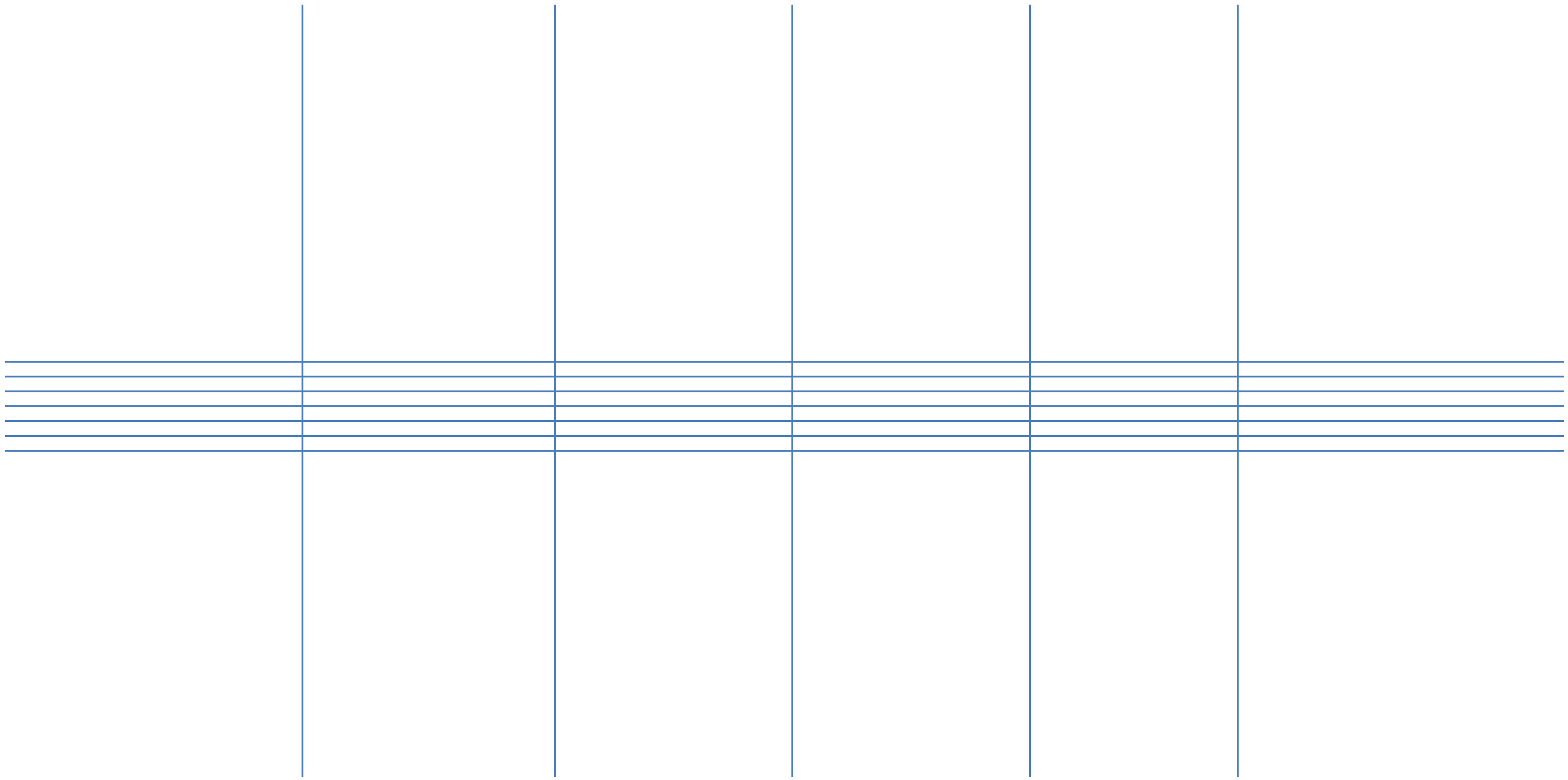


This is the toric skeleton of the 3-fold which is also dual to a web of (p,q) 5-brane in 5d. The size of elliptic fiber is 1/R.

The most obvious way of going down one more dimension leads to



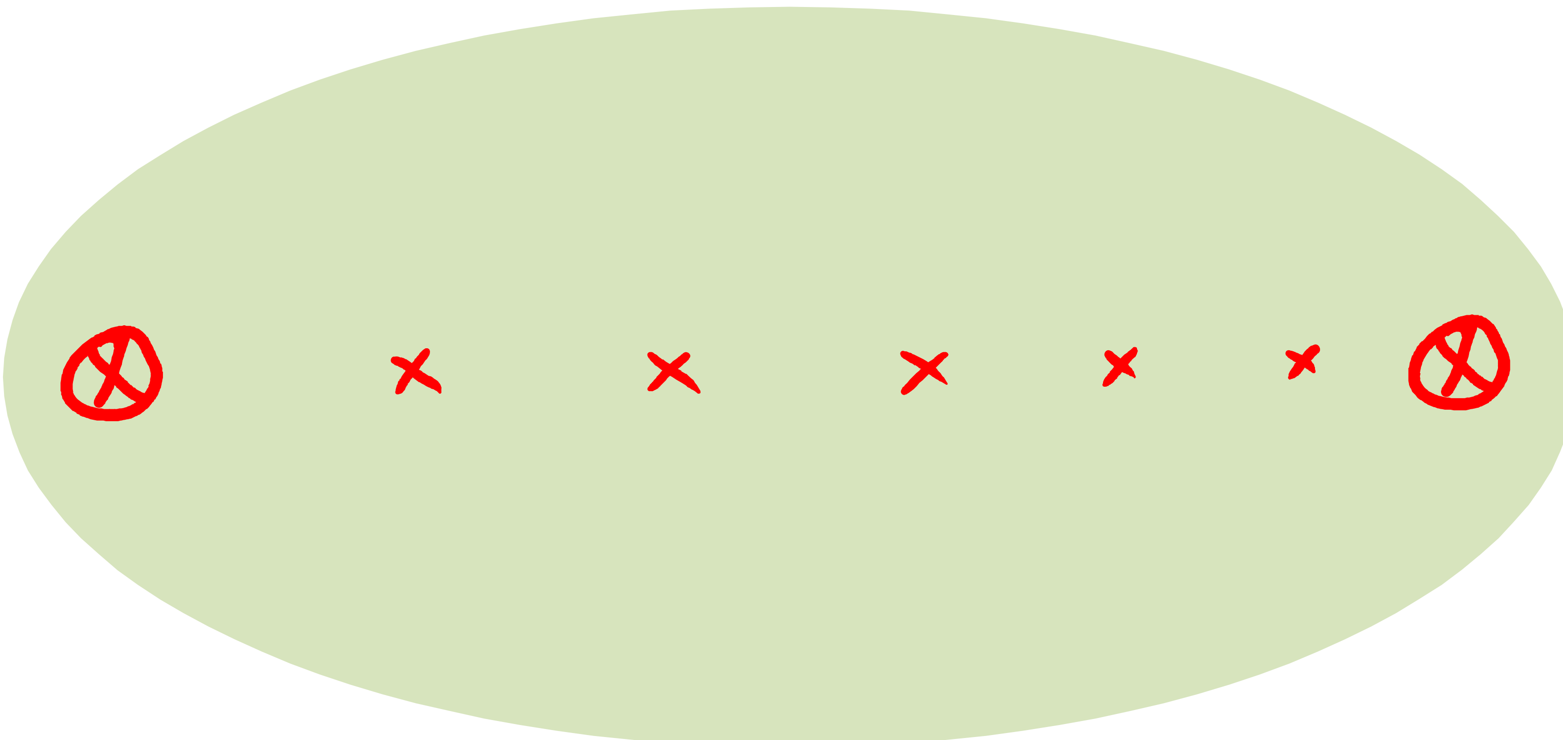
The most obvious way of going down one more dimension leads to



D4(horizontal)-NS5 (vertical) system

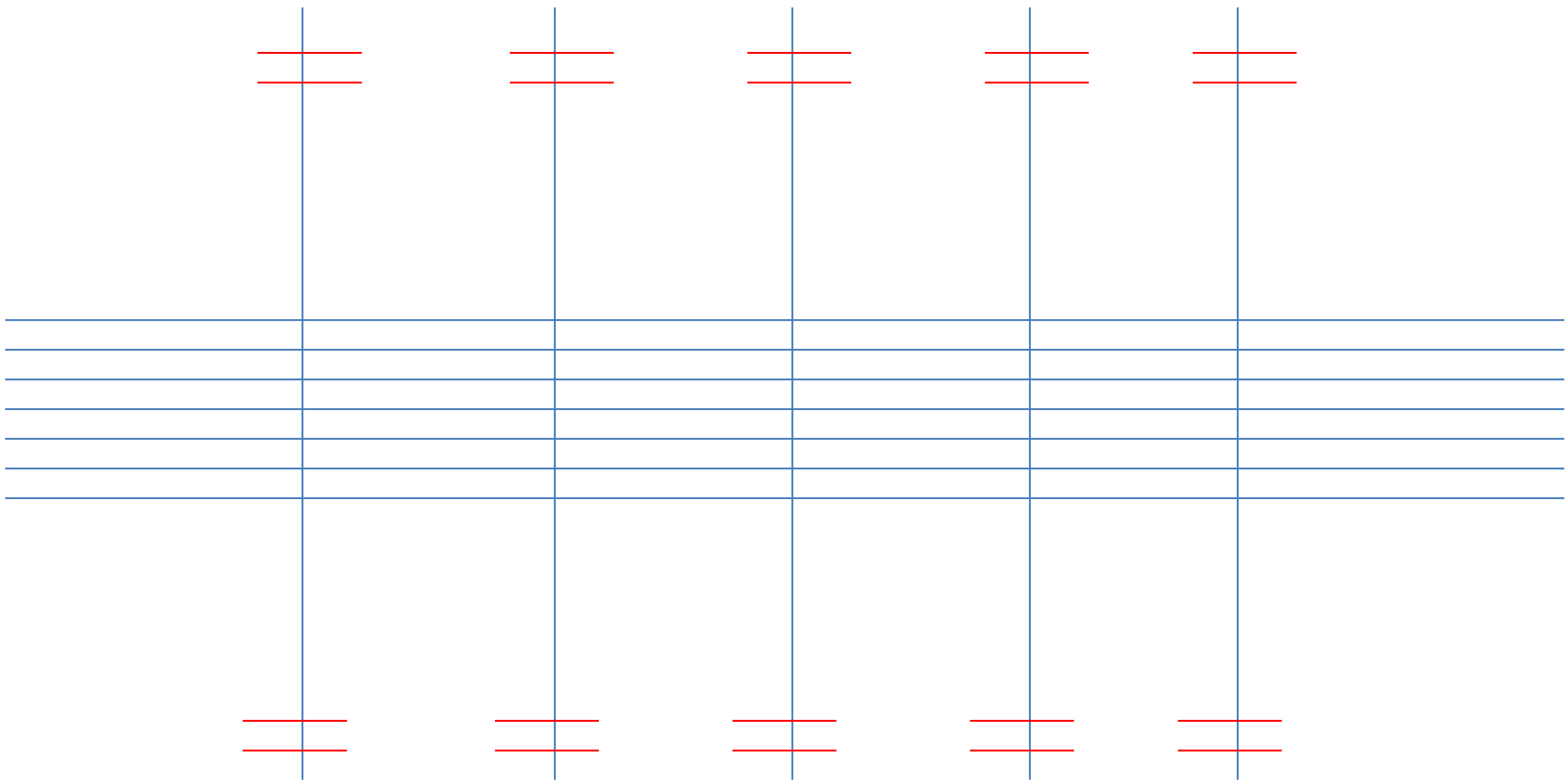
Same quiver as in 6d with SU(N) X SU(N) symmetry

Modular transformation of torus does not act in IR

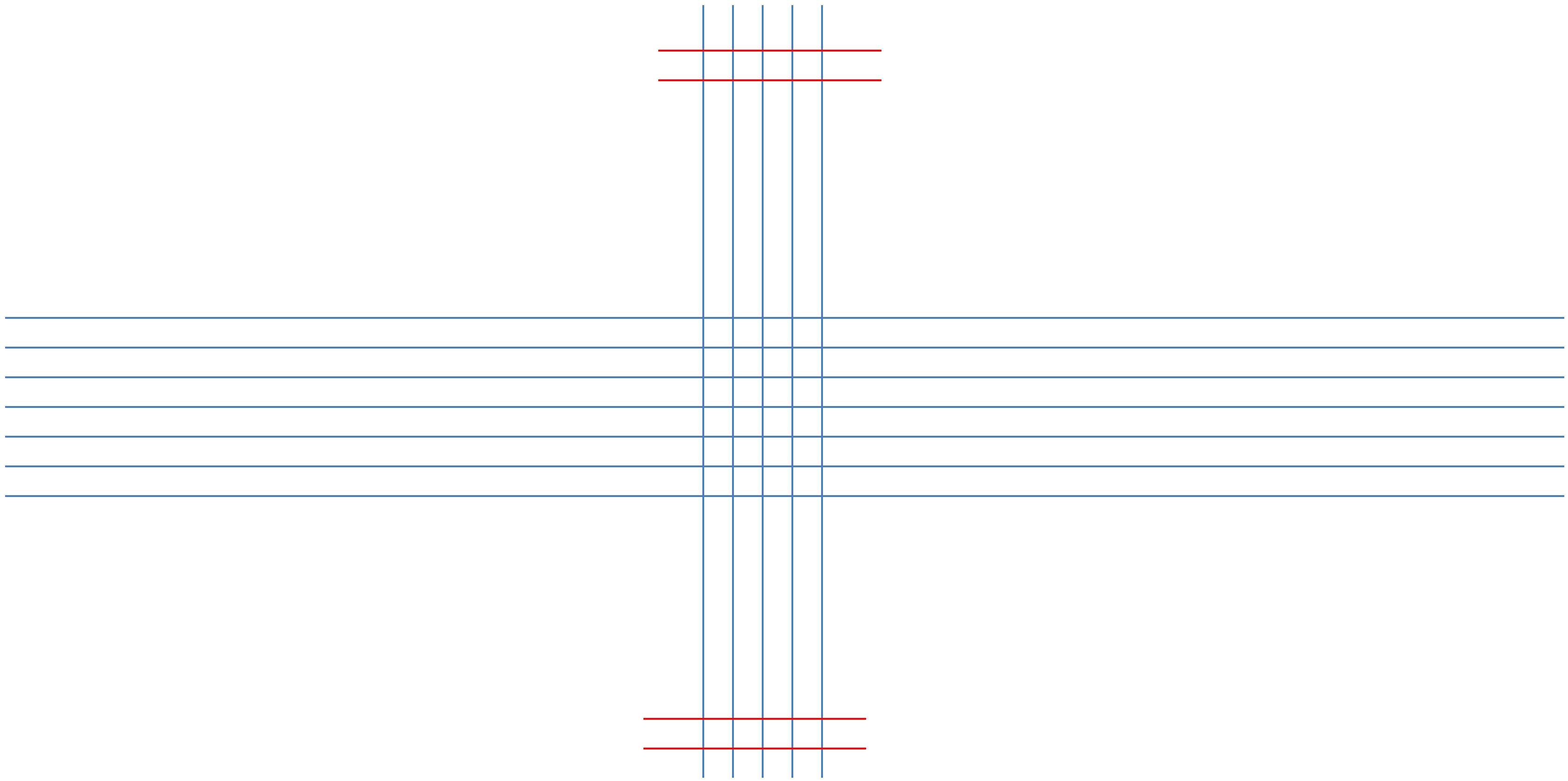


A genus 0 Class S theory of A-type with 2 full punctures +simple punctures

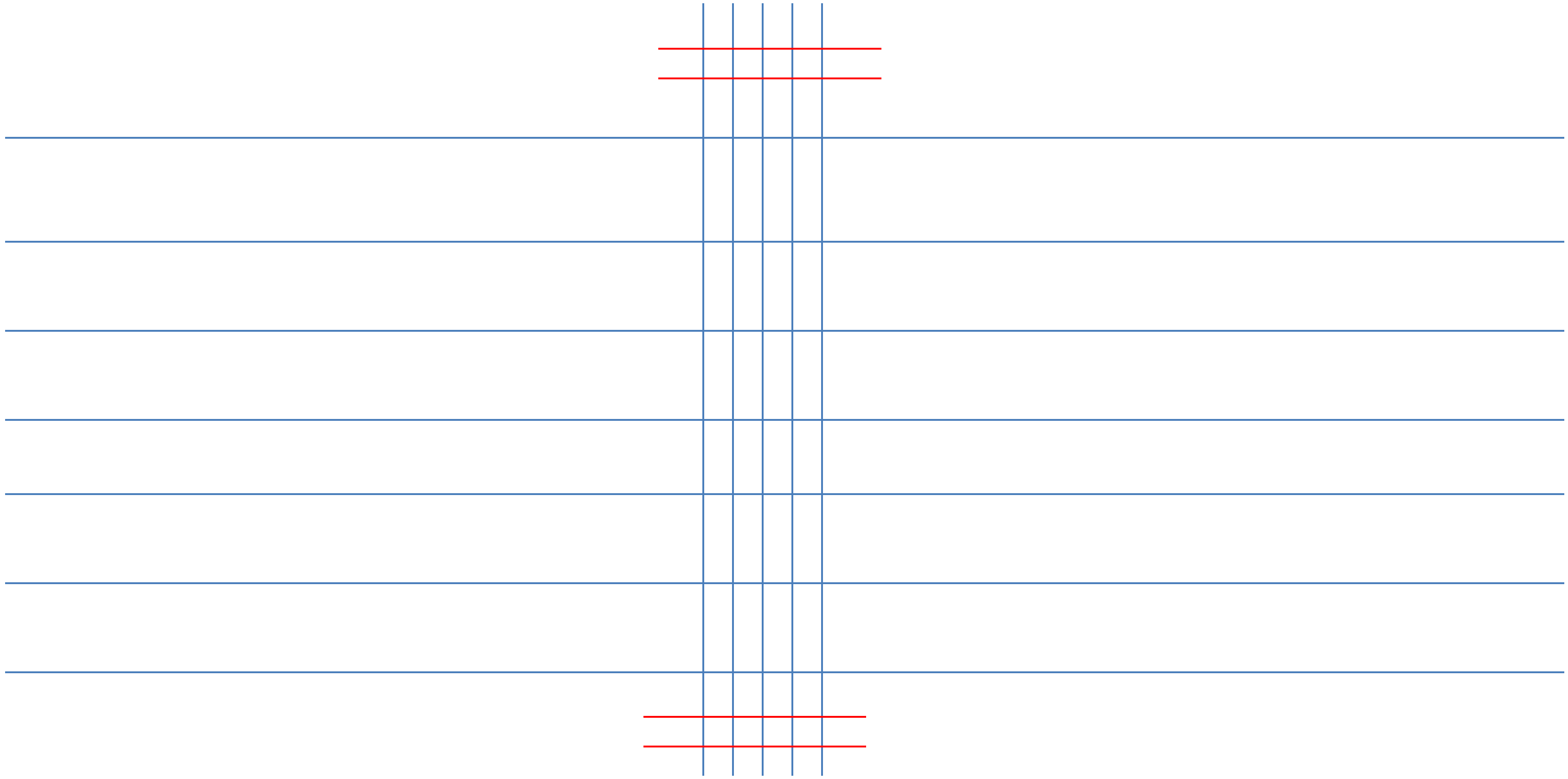
But there are other limits:



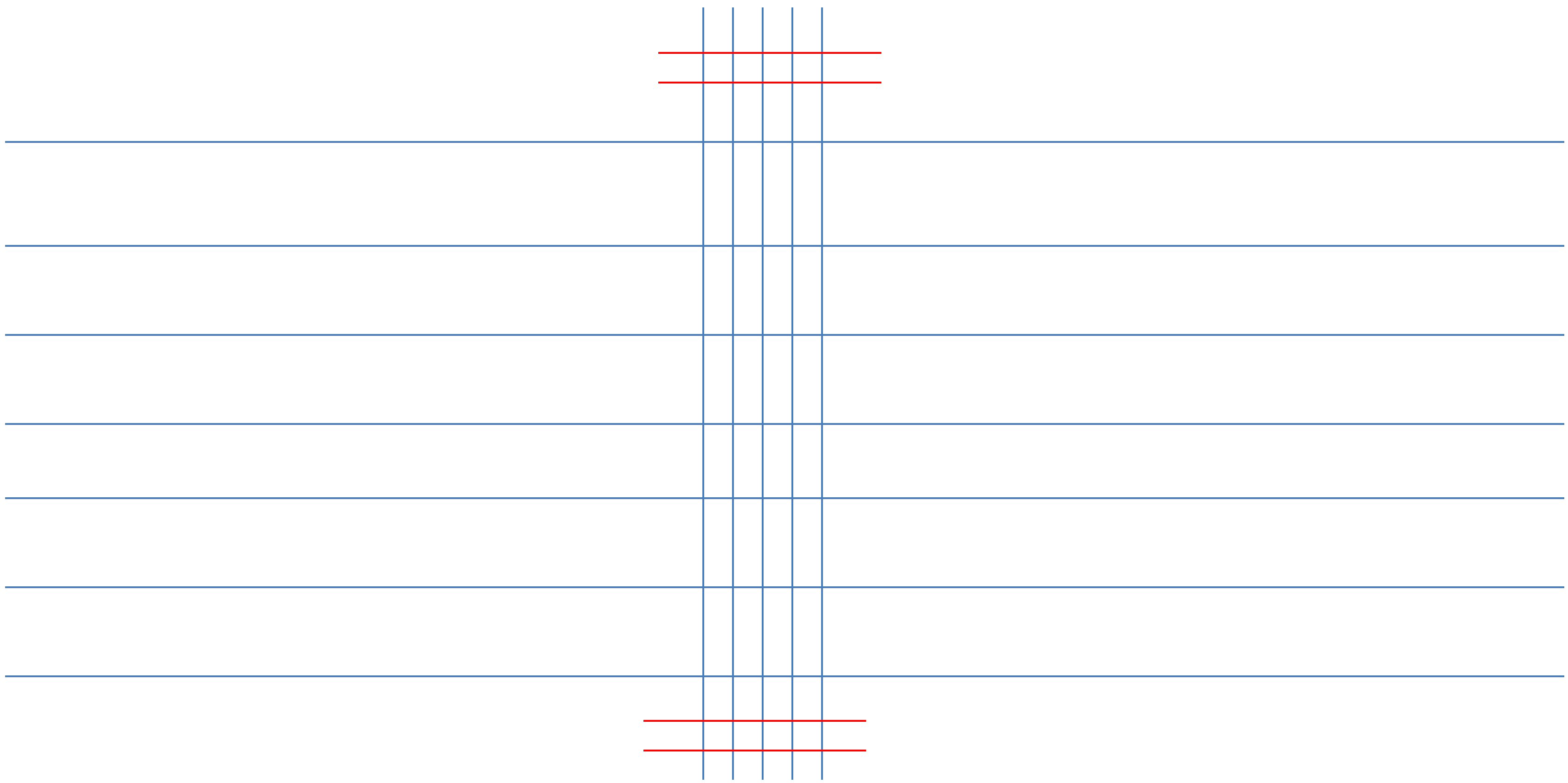
But there are other limits:



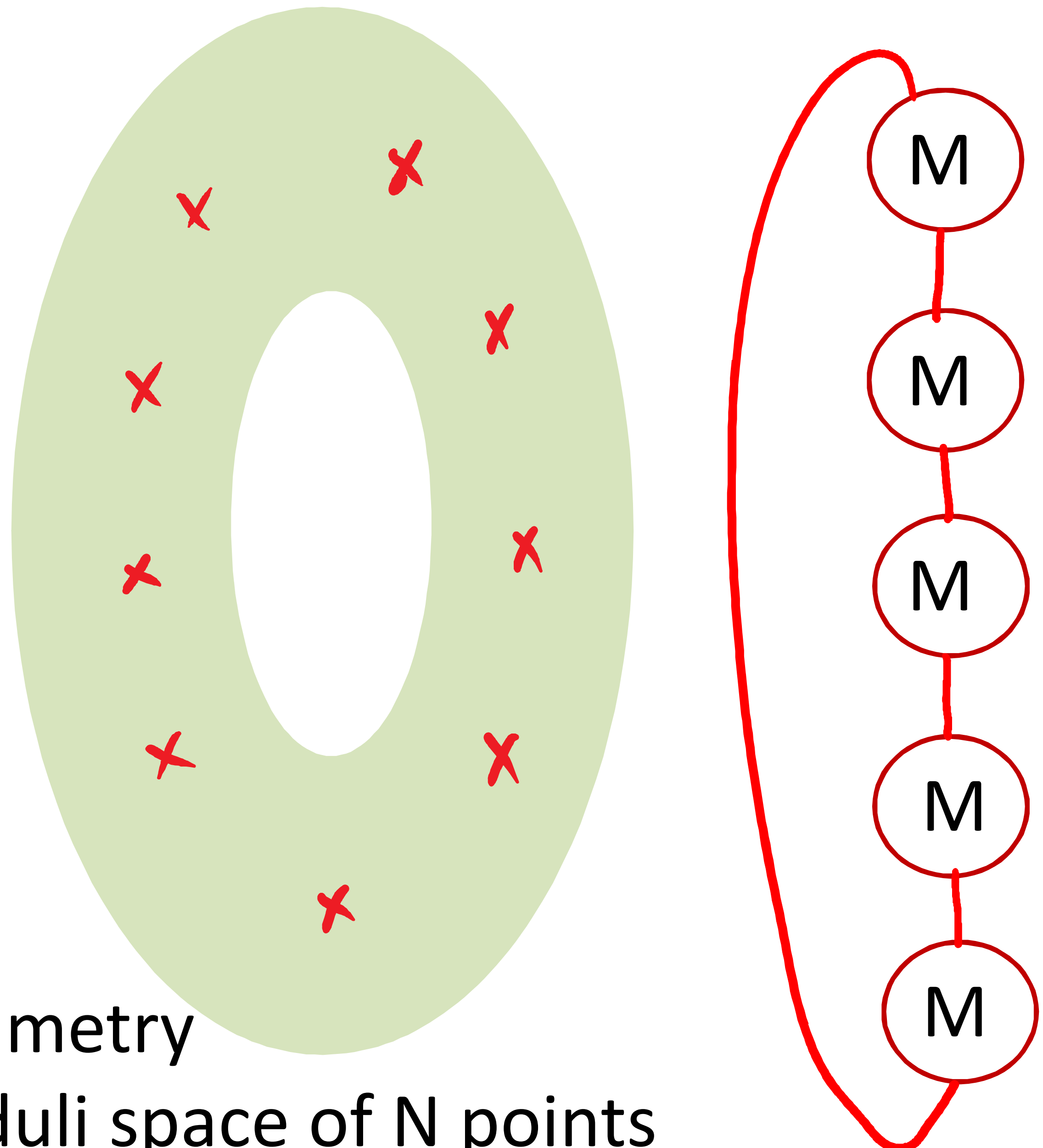
But there are other limits:



But there are other limits:



And now we can view vertical lines as D5 branes and horizontal lines as NS5-brane. Then in 4d:

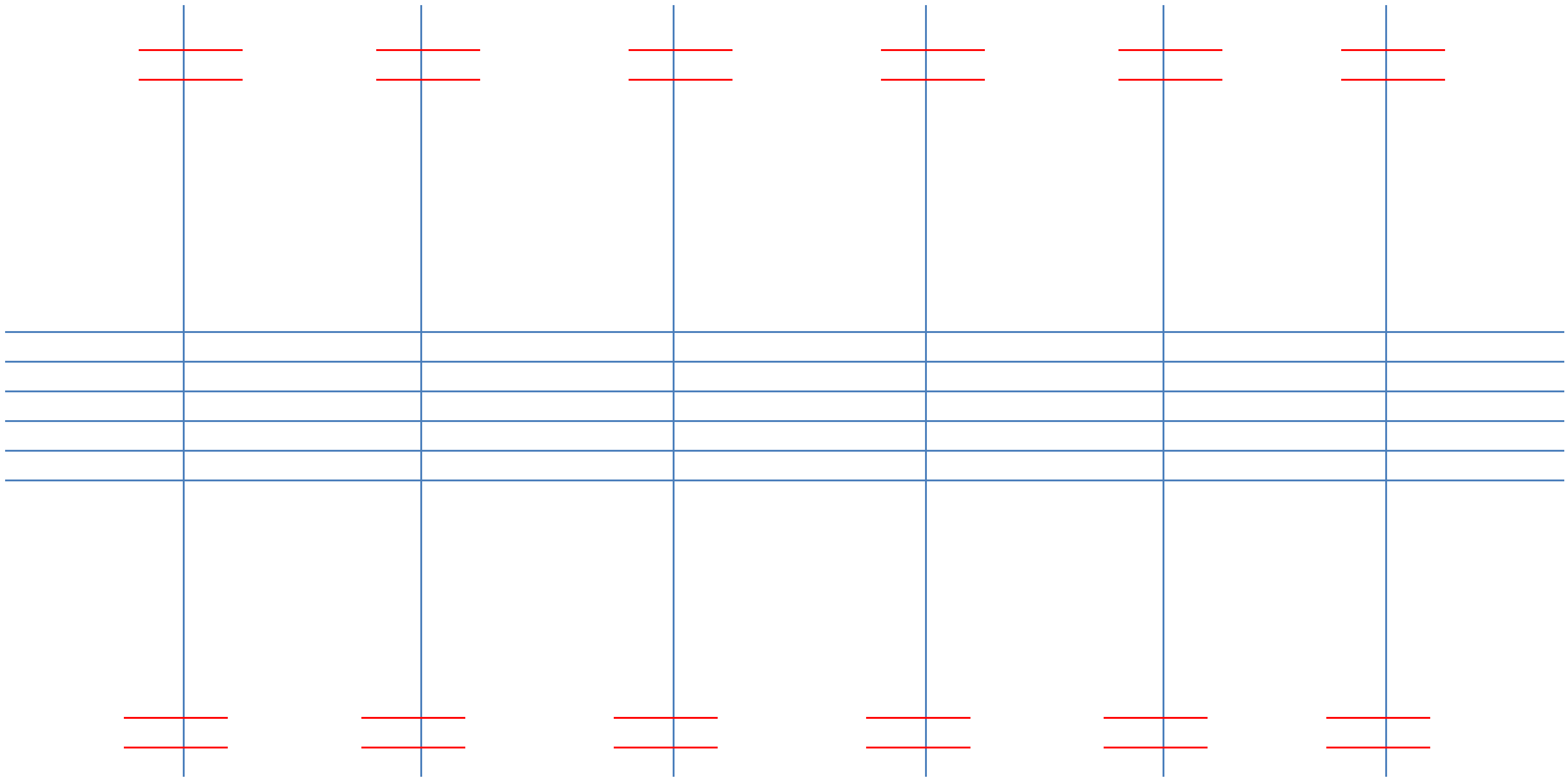


Now SL(2,Z) symmetry
is manifest: Moduli space of N points
on T^2 = Moduli space of flat SU(N) on T^2

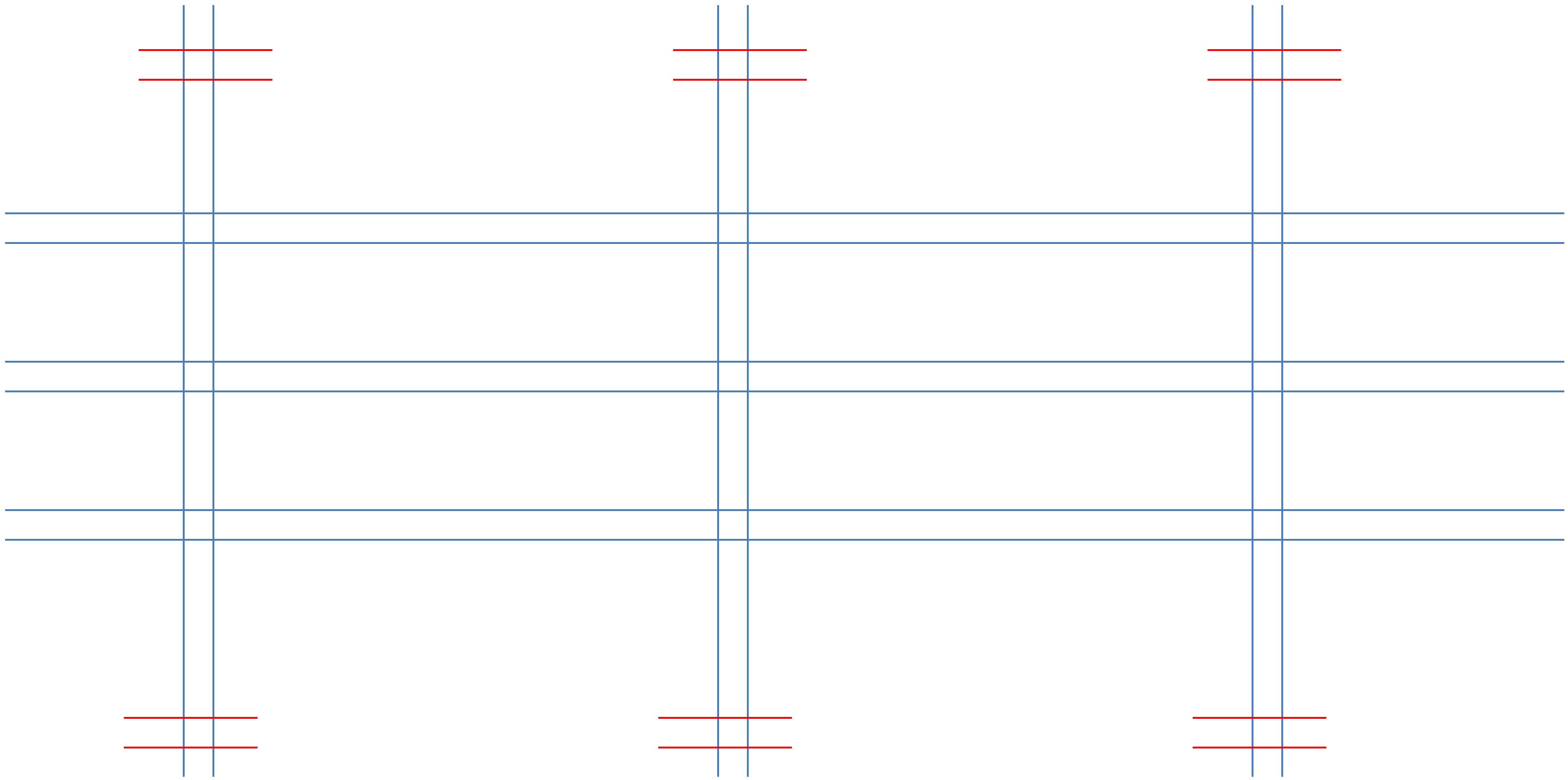
We can interpret this result as follows:
The global symmetry SU(N) x SU(N)
has been first broken to the diagonal SU(N) and secondly, by turning on flat SU(N) Wilson lines to U(1)'s.

SL(2,Z) symmetry has become manifest at the expense of sacrificing the 6d global symmetries which are now reflected in the moduli space of the 4d theory.

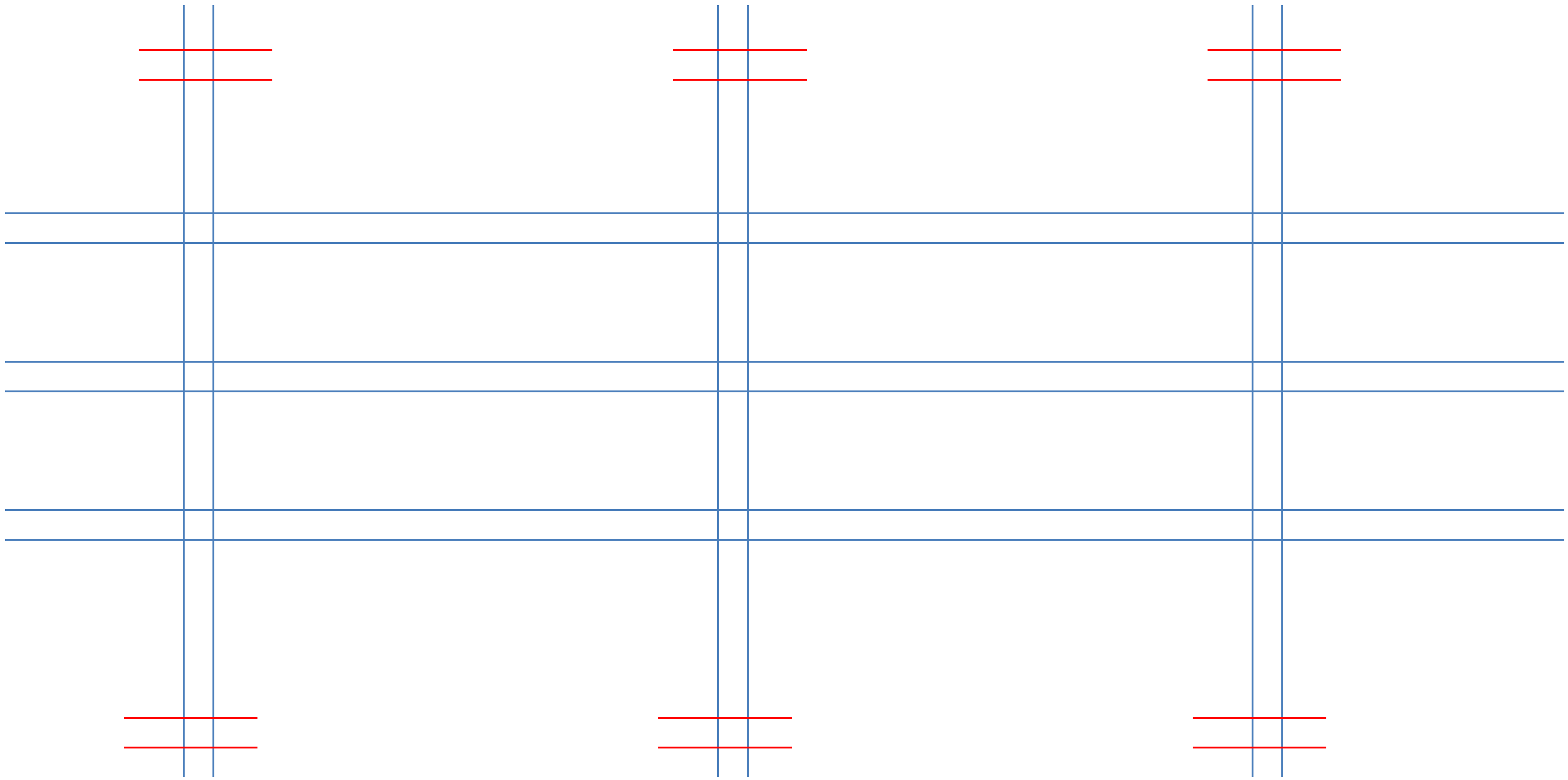
However there are other 4d limits:



However there are other 4d limits:

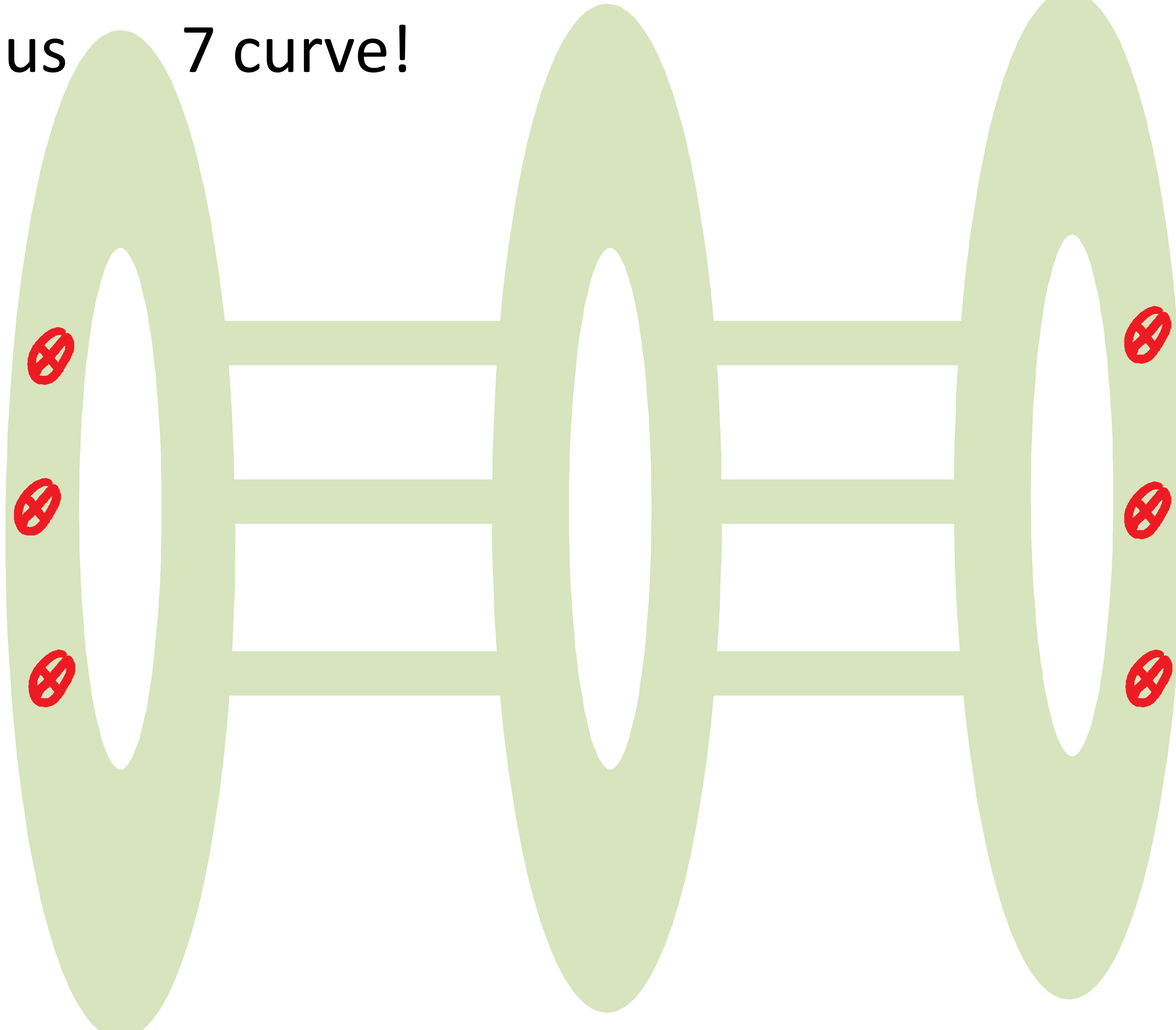


However there are other 4d limits:

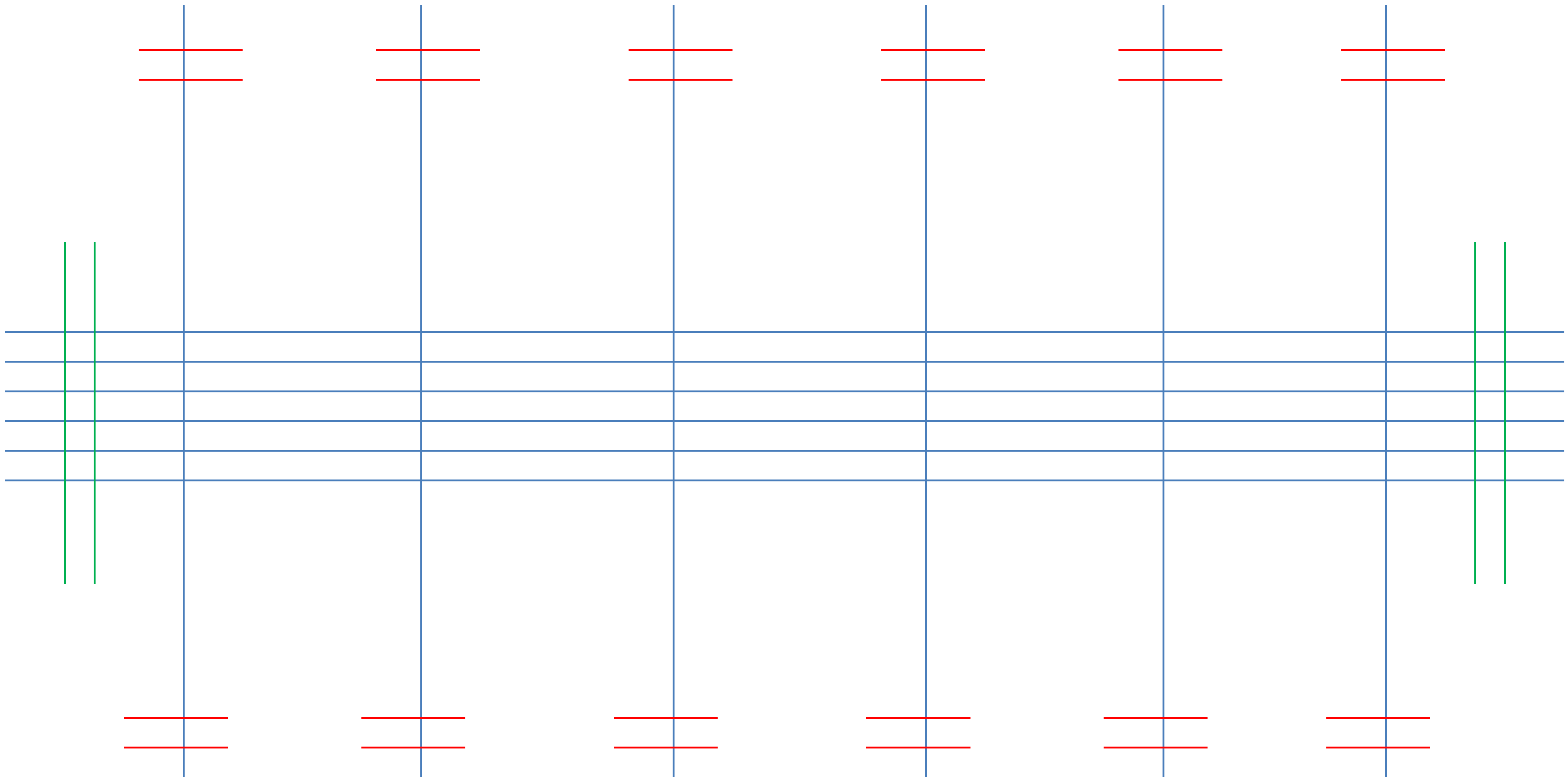


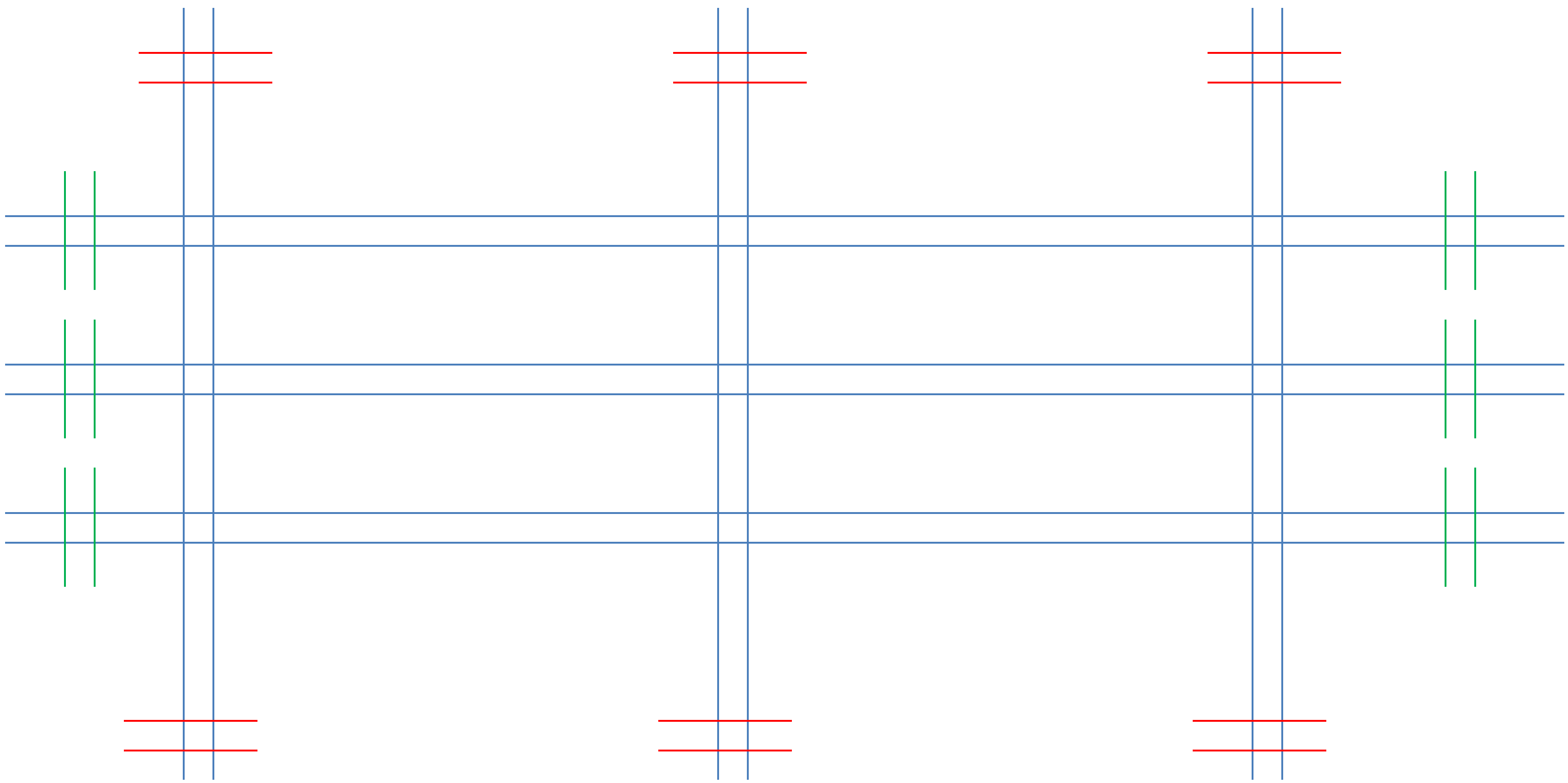
We see the outlines of a Riemann surface!

Class S-theory of type A_1 with 6 full punctures on a genus 7 curve!

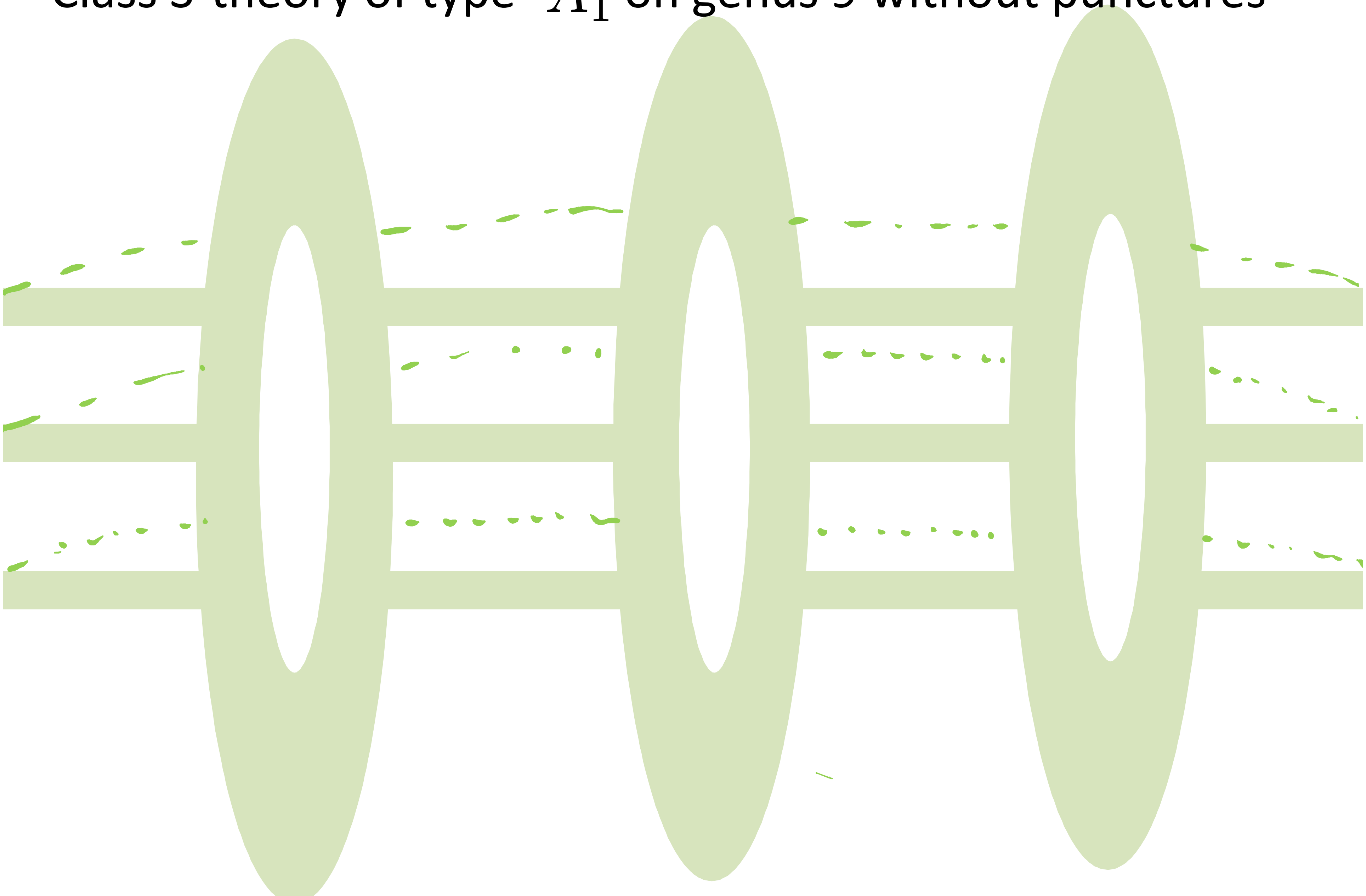


Little string version:





Class S-theory of type A_1 on genus 9 without punctures



Note that we can get arbitrary genus class S theory from compactification of 6d theories, but we land on special moduli of the 4d theory.

This leads to a purely
‘field theoretic notion of swampland':

Suppose we start from a higher dimensional QFT and wish to obtain a particular lower dimensional QFT, allowed by supersymmetry and other symmetry considerations. Which ones can we actually obtain? If so do we land on arbitrary moduli for such theories or a subspace?

We now turn to a large set of examples:

Orbifolds in type IIB:

$$\mathbf{C}^2/¡$$

When

$$¡ \subset SU(2)$$

is the (2,0) theory. But even if ¡ is not in SU(2) we can preserve SUSY in the F-theory setup, with non-trivial elliptic fibration. In fact all F-theory models are of this type. A simple subset of these:

F-theory Orbifolds

We consider F-theory orbifolds of the type:

$$\frac{T^2 \times \mathbf{C}^2}{¡}$$

Where the action of ¡ on T^2 can only involve

$$\mathbf{Z}_2, \mathbf{Z}_3, \mathbf{Z}_4, \mathbf{Z}_6$$

due to the possible symmetries of T^2 .

Example 1:

$$¡ \;= \langle Z_k, Z_{Nk} \rangle \qquad k = 2, 3, 4, 6$$

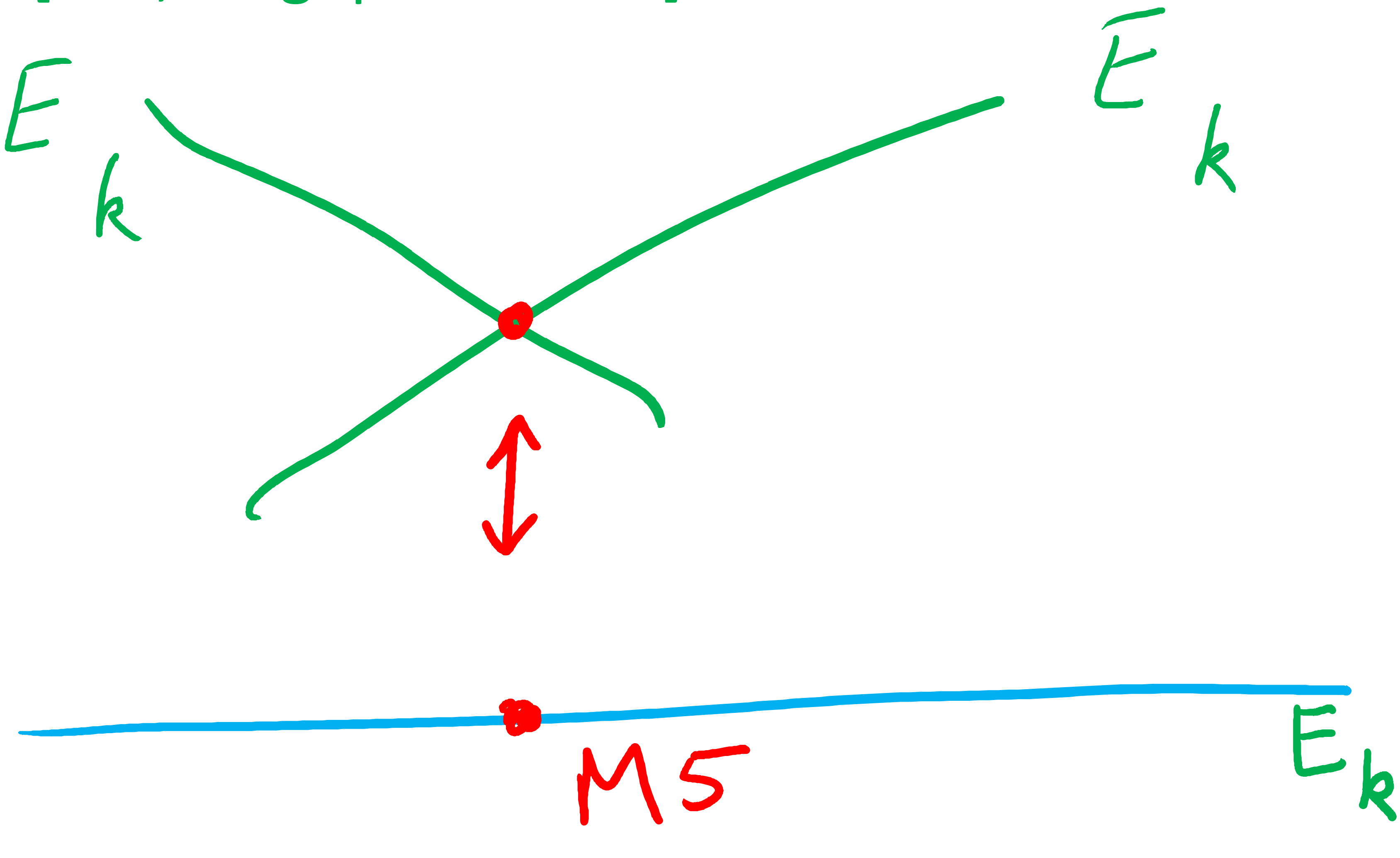
$$Z_k = \langle (a; 1, a^{-1}) \rangle \qquad Z_{Nk} = \langle (1; b, b^{-1}) \rangle$$

It turns out that this is equivalent in the M-theory setup, to N (M5 branes) probing D4,E6,E7,E8 singularities (for k=2,3,4,6 respectively).

To see this, consider the case N=1. This corresponds to having two Z_k singularities in the base intersect at a point. On the other hand for k=2,3,4,6 these give 7-branes of type D4,E6,E7,E8:

For k=2,3,4,6 these give rise to 7-branes of type D4,E6,E7,E8

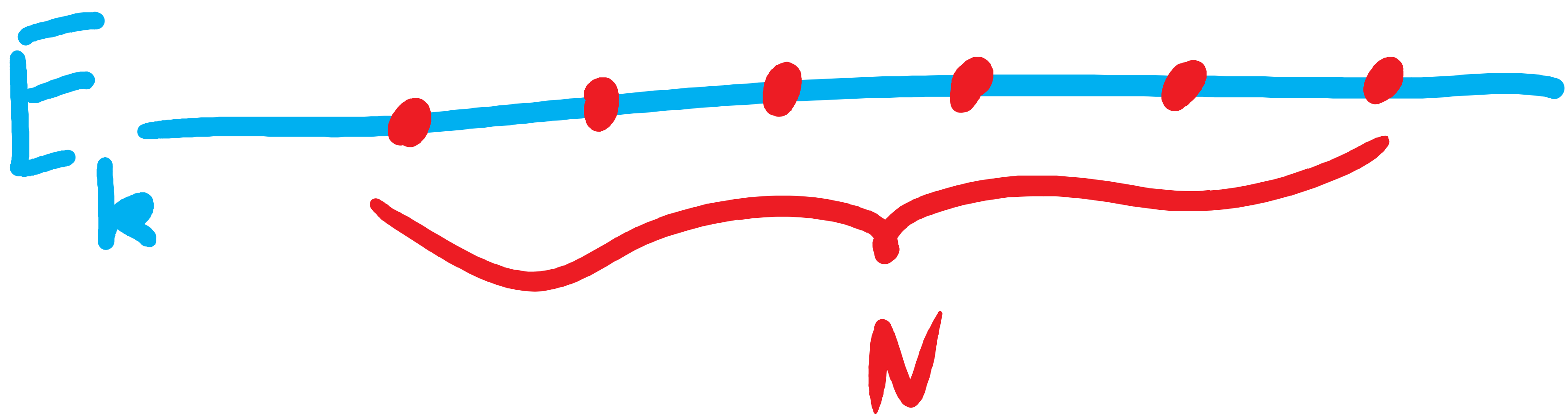
[Sen,Dasgupta+Mukhi]



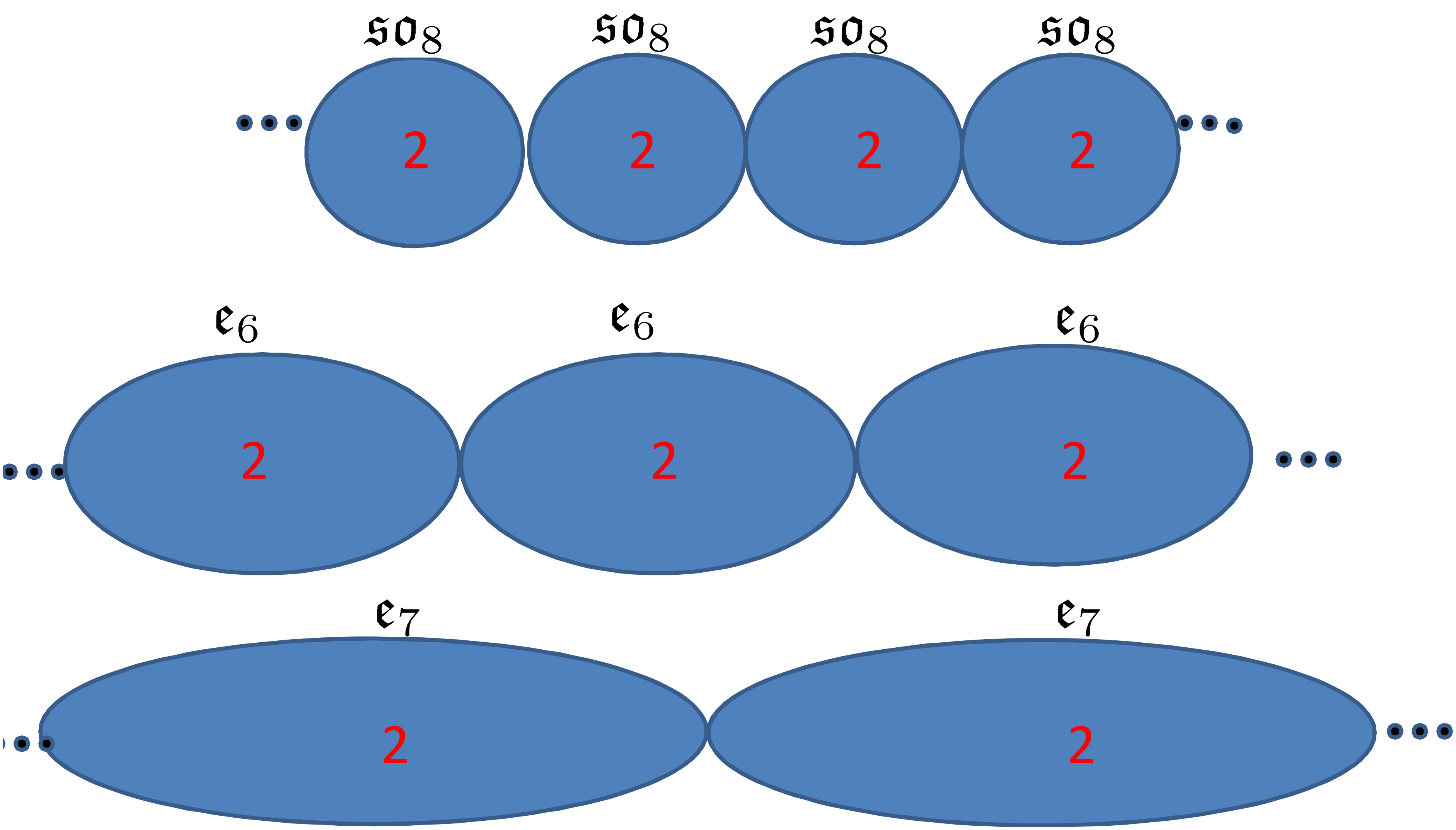
Going back to the more general case

$$Z_k = \langle (a; 1, a^{-1}) \rangle \qquad Z_{Nk} = \langle (1; b, b^{-1}) \rangle$$

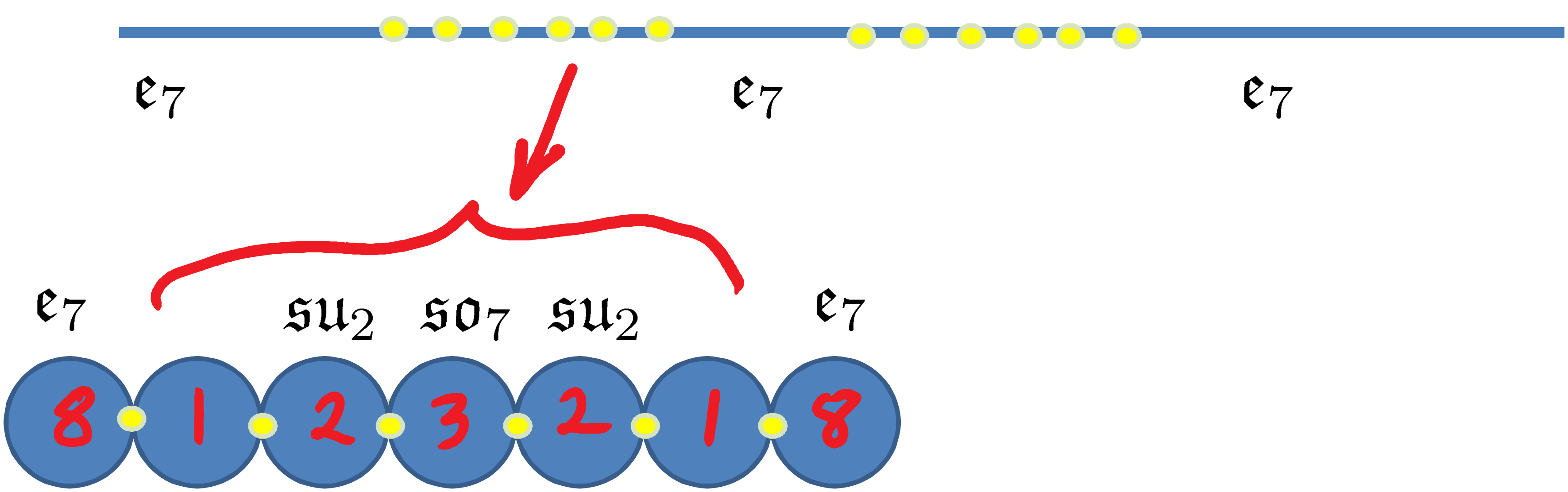
introduces additional A_{N-1} singularity which in M-theory setup corresponds to adding N M5 branes probing D4,E6,E7,E8 singularities leading to N conformal matters of the corresponding type:



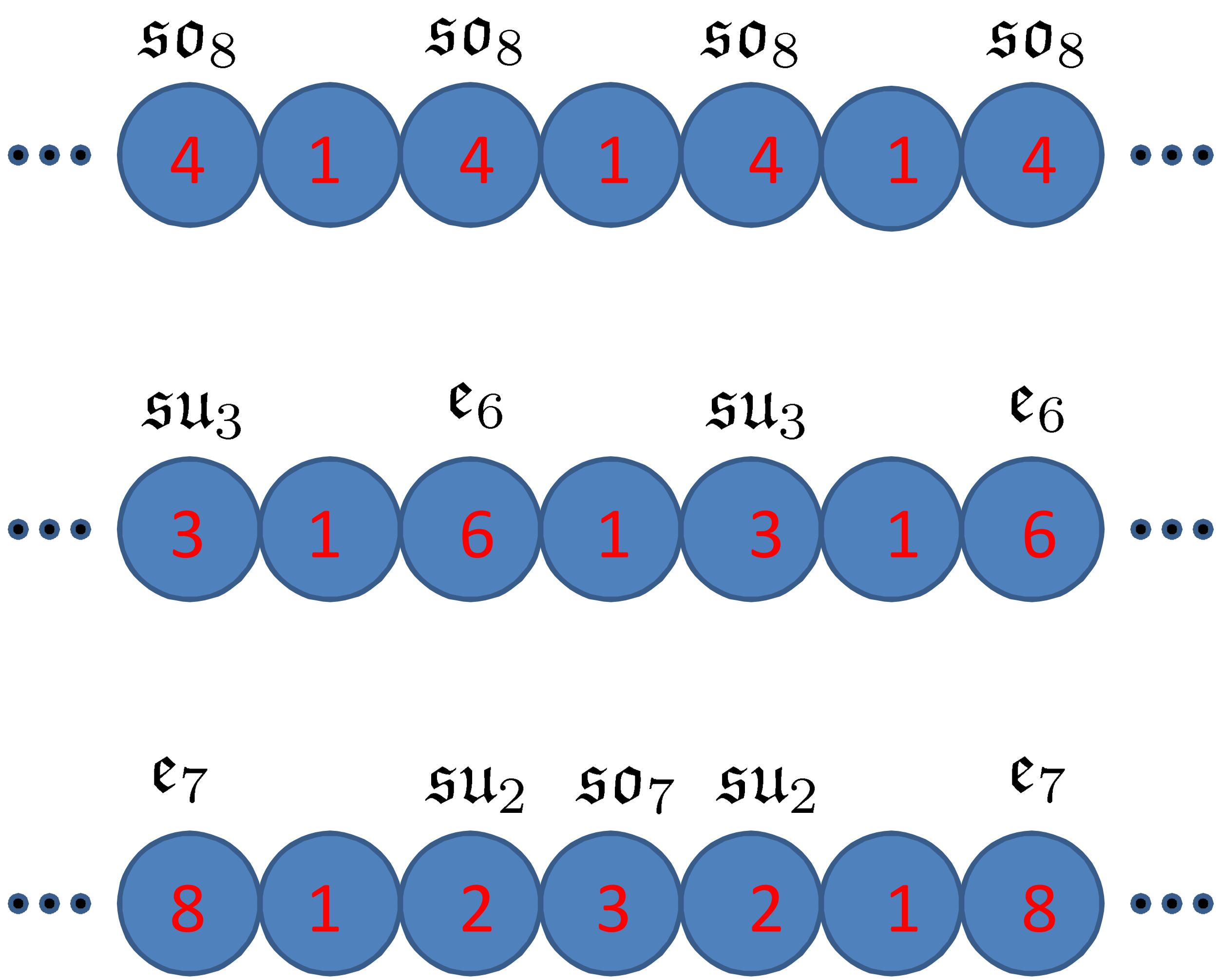
The Base Geometry



Complicated tensor branch: Fractionation of M5 brane. For example:



bifundamental 'E7 matter' =SCFT


$\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$
4 1 4 1 4 1 4
$\mathfrak{su}_3$ $\mathfrak{e}_6$ $\mathfrak{su}_3$ $\mathfrak{e}_6$
3 1 6 1 3 1 6
$\mathfrak{e}_7$ $\mathfrak{su}_2$ $\mathfrak{so}_7$ $\mathfrak{su}_2$ $\mathfrak{e}_7$
8 1 2 3 2 1 8

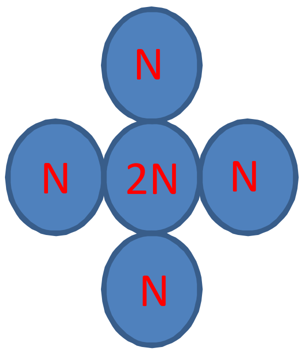
Compactifying on two circles, down to 4d,
we can use mirror symmetry to find the effective theory:

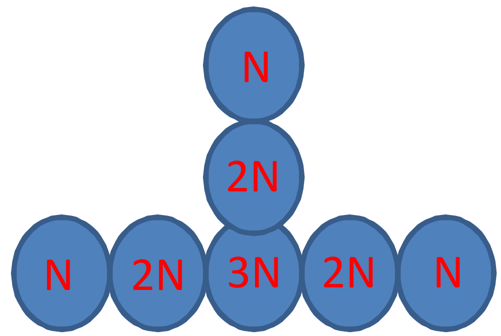
For example for E6 case we get the CY 3-fold mirror geometry

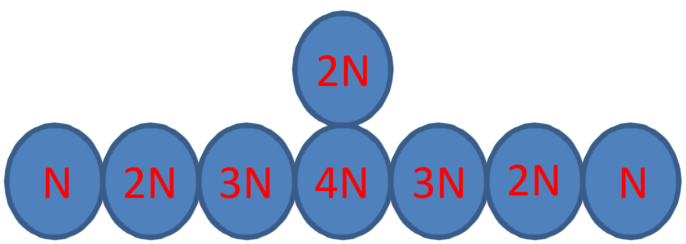
$$x_1^3 + x_2^3 + x_3^3 + y_1^{3N} + y_2^{3N} + deformations = 0$$

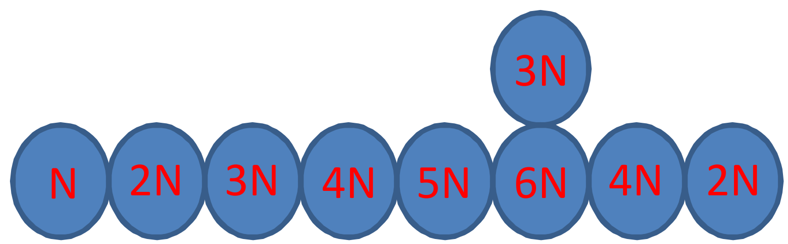
And depending on the choice of deformations we can obtain different IR fixed points. There are two distinguished ones again: One preserving SL(2,Z) structure and the other preserving G x G global symmetry:

1) N M5->D4->D3 branes probing singuarlities lead to quiver theories where SL(2,Z) structure is preserved and the moduli space is the same as flat connections on torus.



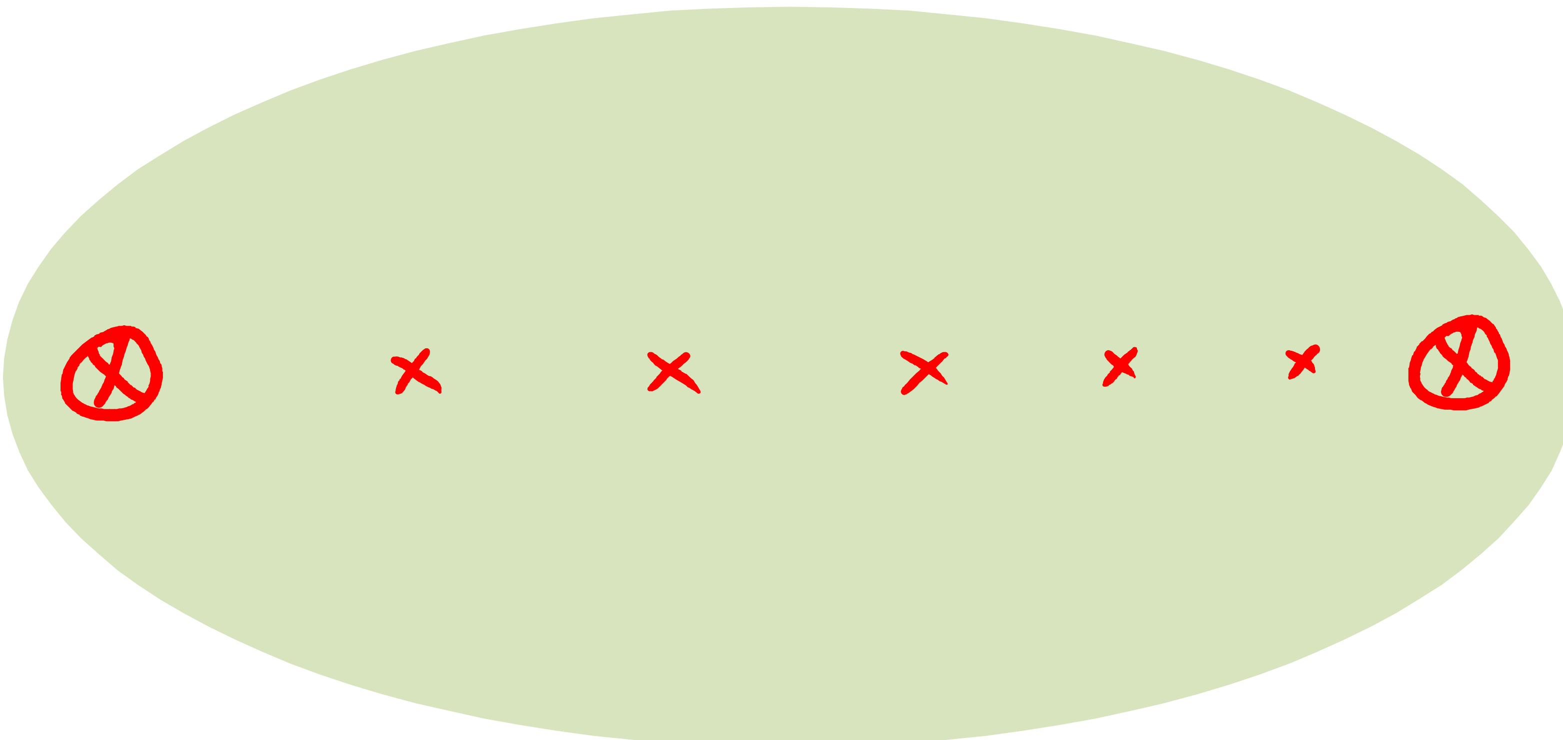






The original example of SU(2) with 4 doublets which enjoys SL(2,Z) symmetry is the special case of D4 affine quiver and we see here that it arises from toroidal compactification of a 6d (1,0) theory: In M-theory setup it arises from the toroidal compactification of a single M5 brane probing D4 singularity. Thus we see that its SL(2,Z) symmetry can be explained geometrically.

The second possibility, where G x G is preserved



A genus 0 Class S theory of E-type with 2 full punctures +simple punctures

For E6 case this emerges from mirror geometry and deformations leading to

$$x_1^3 + x_2^2 g + x_3 g^2 = 0 \qquad g(x_3, y_1, y_2) = x_3 + y_1^N + y_2^N + defs.$$

which has a curve of E6 singularity located at

$$x_1 = x_2 = g = 0$$

and the full punctures are at $y_1 = 0, y_2 = 0$
and the minimal punctures at the N points where

$$x_3 = 0$$

Just as in the A-case, by tuning parameters differently we can end up with different class S theories. For example, for N conformal matter of the E8 by tuning parameters we get the mirror CY geometry given by

$$x_1^3 + x_2^3 + (x_3^2 + f_{2N}(y_1, y_2))^3 = 0$$

which corresponds to a Class S-theory of SO(8) type on the hyperelliptic curve

$$x_3^2 + f_{2N}(y_1, 1) = 0$$

of genus g=N-1 with 4+2N punctures.

There are other N=2 supersymmetric models known in 4d which are expected to enjoy SL(2,Z) duality based on their BPS structure.
These include unconventional N=2 matter systems Dp(G), which are indexed by an integer p and an ADE group G and are generalization of Argyres-Douglas D-type theories enjoying G as a global symmetry [Cecotti+DelZotto, Xie].

$$G - -D_2(G) \oplus D_2(G) \oplus D_2(G) \oplus D_2(G)$$

$$G - -D_3(G) \oplus D_3(G) \oplus D_3(G)$$

$$G - -D_2(G) \oplus D_4(G) \oplus D_4(G)$$

$$G - -D_2(G) \oplus D_3(G) \oplus D_6(G)$$

It is natural to ask if these theories also have an exact SL(2,Z) acting on their moduli space and whether they come from toroidal compactification of some 6d SCFT's.

This is indeed the case.

$$¡ = \langle Z_k, ¡_G \rangle \qquad k = 2, 3, 4, 6$$

$$Z_k : \langle (a^2; a^{-1}, a^{-1}) \rangle \qquad a^{2k} = 1$$

$$¡_G : \langle 1; ¡_{ADE} \rangle$$

This class generalizes the D4,E6,E7,E8 affine quiver theories.

We can use this and mirror symmetry to solve for the vacuum geometry of these N=2 theories in 4 dimensions. Mirror geometry leads to a Landau-Ginzburg model with superpotential:

$$W_{LG} = W_{E_k}(x_1, x_2, x_3) + W_{ADE}(z_1, z_2, z_3)$$

$$W_{E_6} = x_1^3 + x_2^3 + x_3^3 + a x_1 x_2 x_3$$

$$W_{E_7} = x_1^2 + x_2^4 + x_3^4 + a x_1 x_2 x_3$$

$$W_{E_8} = x_1^2 + x_2^3 + x_3^6 + a x_1 x_2 x_3$$

In general this is NOT a CY 3-fold (6 variables!) , but one can use the periods of the LG to compute the exact vacuum geometry of these 4d, N=2 theories.

A special case of these theories, with G=SU(2):

$$SU(2)--D_2(SU(2))\oplus D_2(SU(2))\oplus D_2(SU(2))\oplus D_2(SU(2))$$
$$SU(2)--D_3(SU(2))\oplus D_3(SU(2))\oplus D_3(SU(2))$$
$$SU(2)--D_2(SU(2))\oplus D_4(SU(2))\oplus D_4(SU(2))$$
$$SU(2)--D_2(SU(2))\oplus D_3(SU(2))\oplus D_6(SU(2))$$

These are among the 'complete' N=2 theories [Cecotti+V].
Correspond to compactification of 6d theories with a 1-dimensional tensor branch on spheres:
O(-4),O(-6),O(-8),O(-12)
with 6d gauge groups,
SO(8), E6, E7, E8

Conclusion

Toroidal compactification of 6d (1,0) superconformal theories lead to insights about 4d N=2 theories:

1-Distinct 4d N=2 theories can arise from a single 6d theory.

2-SL(2,Z) symmetry of the 4d, N=2 theories can be explained geometrically, just as is the case for 4d, N=4 theories.

It would be very interesting to study more examples and also classify the rich class these theories are expected to lead to when reducing SUSY to N=1 in 4d.