Self-dual strings in 6d and instanton counting for 5d SYM

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[1] Postech

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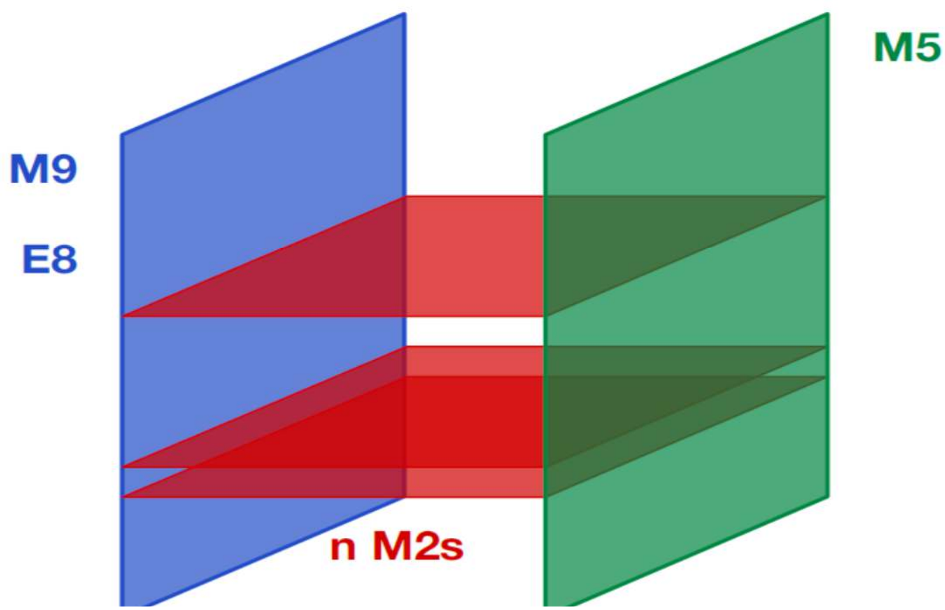
- Based on two papers
- General instanton counting and 5d SCFT
  by Chiung Hwang, Joonho Kim, Seok Kim and JP
- Elliptic genus of E-strings
  by Joonho Kim, Seok Kim, Kimyeong Lee, JP, Cumrun Vafa
- Exploring aspects of 5d, 6d SCFTs using 1d, 2d field theory

Outline

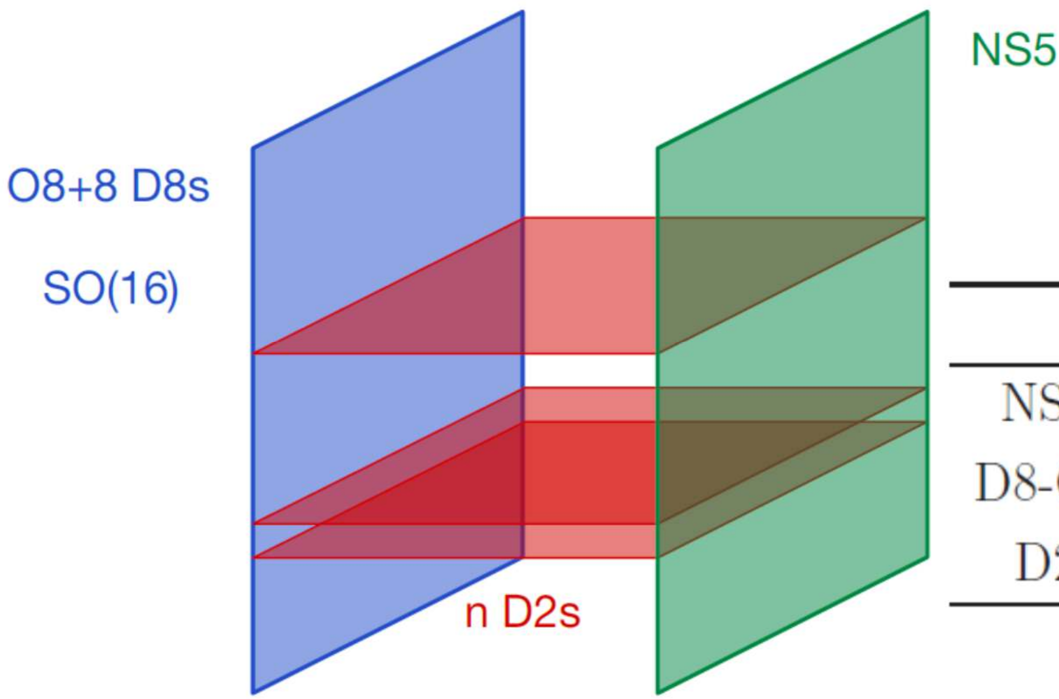
- Self-dual strings in 6d SCFT
- Motivation for the instanton counting for 5d SYM
- Details of (0,4) ADHM Quantum Mechanics
- Conclusions

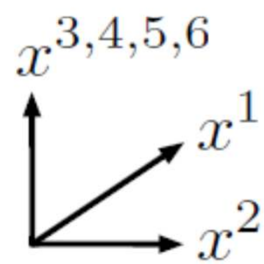
Self-dual string

- Many interesting (0,1) SCFTs arise in the M theory or F-theory setting (cf. Vafa, Heckman's talk)
- Focus on (0,1) E_8 theory
- Coincidence limit of M5 and M9 produces the tensionless self-dual string
- which is an interacting 2D SCFT
- How can we describe such self-dual string theories?



- Use the related gauge theory configuration
- and take $g_{2YM} \to \infty$ limit to obtain the 2d SCFT
- which corresponds to the decompactifying limit in Type IIA setup





	0	1	2	3	4	5	6	7	8	9
NS5	•	•		•	•	•	•			
D8-O8	•	•		•	•	•	•	•	•	•
D2	•	•	•							

- The resulting 2D theory has (0,4) world-sheet SUSY
- The symmetry is
  $SO(4)_{3456} \times SO(3)_{789} \sim SU(2)_L \times SU(2)_R \times SU(2)_I$
  doublet indices $\alpha, \beta \quad \dot{\alpha}, \dot{\beta} \quad A, B$

- matter content

$$\begin{array}{rcl} \text{vector} & : & O(n) \text{ antisymmetric} \quad (A_\mu,\ \lambda_+^{\dot{\alpha}A}) \\ \text{hyper} & : & O(n) \text{ symmetric} \quad (\varphi_{\alpha\dot{\beta}},\ \lambda_-^{\alpha A}) \\ \text{Fermi} & : & O(n) \times SO(16) \text{ bifundamental} \quad \Psi_l \end{array}$$

- interactions determined by (0, 4) SUSY

- We work out the elliptic genus as an evidence for the proposal
  (For the (0,2) 2D gauge theory, the elliptic genus was worked out by Gadde, Gukov and Benini, Eager, Hori, Tachikawa)
- With a choice of (0,2) worldsheet SUSY, the elliptic genus is given by

$$Z_n(q, \epsilon_{1,2}, m_l) = \mathrm{Tr}_{\mathrm{RR}}\left[(-1)^F q^{H_L} \bar{q}^{H_R} e^{2\pi i \epsilon_1 (J_1 + J_I)} e^{2\pi i \epsilon_2 (J_2 + J_I)} \prod_{l=1}^{8} e^{2\pi i m_l F_l}\right]$$

- J_1, J_2 Cartans of $SO(4)_{3456}$, J_I Cartans of $SU(2)_{789}$, F_l are the Cartans of $SO(16)$
- elliptic genus gives the BPS spectrum of 6d theory and all-genus topological string amplitudes on related CY_3 (F-theory, M-theory setup)
- important check: E_8 symmetry in IR

single string

- $O(1) \sim Z_2$ and we have four discrete holonomies on T^2

$$Z_1 = \sum_{i=1}^{4} \frac{Z_{1(i)}}{2} = -\frac{\Theta(q, m_l)}{\eta^6 \theta_1(\epsilon_1)\theta_1(\epsilon_2)}$$

- where $\Theta(q, m_l)$ is the E_8 theta function
- the sum over discrete holonomies is the same as that of R, NS sectors with GSO projections to obtain $E_8(\times E_8)$ heterotic string out of the free fermion formalism

- We call it E-string

Higher E-string

- Two E-string: $O(2)$ gauge theory has 7 holonomy sectors

$$Z_2(\tau, \epsilon_{1,2}, m_l) = \frac{1}{2} Z_{2(0)} + \frac{1}{4} \sum_{a=1}^{6} Z_{2(a)}$$

$$Z_{2(0)} = \frac{1}{2\eta^{12}\theta_1(\epsilon_1)\theta_1(\epsilon_2)} \sum_{i=1}^{4} \left[\frac{\prod_{l=1}^{8} \theta_i(m_l \pm \frac{\epsilon_1}{2})}{\theta_1(2\epsilon_1)\theta_1(\epsilon_2 - \epsilon_1)} + \frac{\prod_{l=1}^{8} \theta_i(m_l \pm \frac{\epsilon_2}{2})}{\theta_1(2\epsilon_2)\theta_1(\epsilon_1 - \epsilon_2)} \right]$$

$$Z_{2(1)} = \frac{\theta_2(0)\theta_2(2\epsilon_+) \prod_{l=1}^{8} \theta_1(m_l)\theta_2(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_2(\epsilon_1)\theta_2(\epsilon_2)} \quad , \quad Z_{2(2)} = \frac{\theta_2(0)\theta_2(2\epsilon_+) \prod_{l=1}^{8} \theta_3(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_2(\epsilon_1)\theta_2(\epsilon_2)}$$

$$Z_{2(3)} = \frac{\theta_4(0)\theta_4(2\epsilon_+) \prod_{l=1}^{8} \theta_1(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_4(\epsilon_1)\theta_4(\epsilon_2)} \quad , \quad Z_{2(4)} = \frac{\theta_4(0)\theta_4(2\epsilon_+) \prod_{l=1}^{8} \theta_2(m_l)\theta_3(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_4(\epsilon_1)\theta_4(\epsilon_2)}$$

$$Z_{2(5)} = \frac{\theta_3(0)\theta_3(2\epsilon_+) \prod_{l=1}^{8} \theta_1(m_l)\theta_3(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_3(\epsilon_1)\theta_3(\epsilon_2)} \quad , \quad Z_{2(6)} = \frac{\theta_3(0)\theta_3(2\epsilon_+) \prod_{l=1}^{8} \theta_2(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_3(\epsilon_1)\theta_3(\epsilon_2)}$$

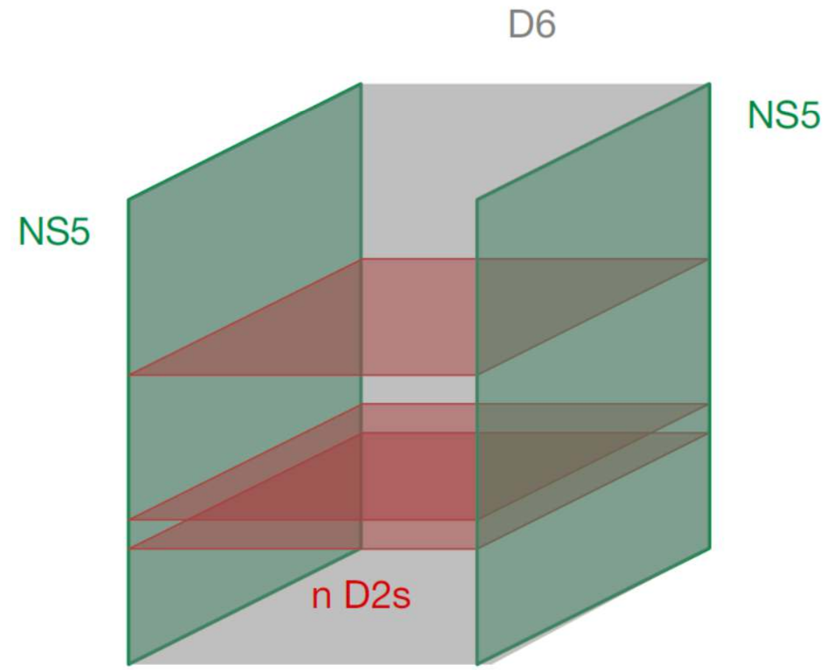
- Coincides with the previous result of Haghighat, Lockhart, Vafa obtained using the E_8 symmetry with low genus expansion, where E_8 symmetry is manifest

$$
\begin{aligned}
Z_2 \;=\; & \frac{1}{576\eta^{12}\theta_1(\epsilon_1)\theta_1(\epsilon_2)\theta_1(\epsilon_2-\epsilon_1)\theta_1(2\epsilon_1)}\Big[4A_1^2(\phi_{0,1}(\epsilon_1)^2-E_4\theta_{-2,1}(\epsilon_1)^2) \\
& +3A_2(E_4^2\phi_{-2,1}(\epsilon_1)^2-E_6\phi_{-2,1}(\epsilon_1)\phi_{0,1}(\epsilon_1))+5B_2(E_6\phi_{-2,1}(\epsilon_1)^2-E_4\phi_{-2,1}(\epsilon_1)\phi_{0,1}(\epsilon_1))\Big] \\
& +(\epsilon_1\leftrightarrow\epsilon_2)
\end{aligned}
$$

- Higher string results coincide with the known results of topological string amplitudes

More self-dual strings

- M strings: M2 branes between two M5 branes worked out by Haghighat, Kozcaz, Lockhart, Vafa before E string
- Type IIA description



- 2D (0,4) theory with U(n) vector multiplet, one adjoint hyper, one fundamental hyper, one fundamental Fermi

More self-dual strings

- F theory on ellptic CY_3 with the base $O(-n) \to P^1$
- Self-dual string is D3 wrapping on a shrinking 2-cycle
- $n = 1$ E-string $n = 2$ M-string (Witten 1996)
- $n = 4$ admits perturbative IIB orientifold description (Bershadsky, Vafa 1997)
  on C^2/Z_2 with $\Omega\Pi$ where $\Pi : z_1 \to z_1 \; z_2 \to -z_2$
- The resulting (0,4) 2D theory is $Sp(k)$ gauge theory with $Sp(k)$ vector multiplets, $Sp(k) \times SO(8+2p)$ bifundamental hyper, $Sp(k) \times Sp(p)$ bifundamental Fermi
  (Haghighat, Klemm, Lockhart, Vafa after E-string)

Motivation for Instanton Counting for 5d SYM

- Some of SCFTs in 5d, 6d can be obtained by considering the strong coupling limit of SYM in 5d
- One can obtain the partition function or the index of SCFTs in 5d, 6d
- from the localization of 5d SYM on S^5, $S^4 \times S^1$
- though 5d SYM is not renormalizable

Motivation continued

- All of these are reduced to the evaluation of instanton partition of Nekrasov on $S^1 \times R^4$ with various groups and with various matters
- e.g. It's known that N=1 5d $Sp(1)$ with $N_f = 5, 6, 7, 8$, one antisymmetric hyper
  exhibit E_6, E_7, E_8 global symmetry
- $Sp(1)$ with $N_f = 5, 6, 7, 8$ nonrenormalizable, no connections to 4d theory, how to define the instanton calculus?
- Related fact: 5d $U(1) N = 1^*$ Nekrasov partition function necessarily involves small instanton singualrities
- Crucial to obtain the index of 1 M5-brane (0,2) theory

Motivation continued

- For the problem of the ambiguities in UV completion, we use the string theory as a guide
- Viewed 5d SYM as D4 systen, we consider D0-D4 SUSY Quantum mechanics as a proposal to compute the Nekrasov partition function (modulo removing string DOF irrelevant to field theory)

Message

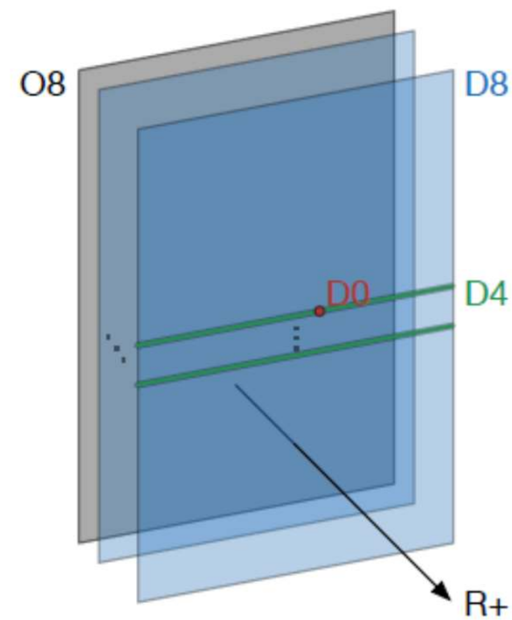
- We solve the previously unsolved technical problem
- evaluation of the Nekrasov partition function is reduced to the evaluation of the residue
- Contour for this evaluation for general cases not known
- 5d N=1 vector multiplets worked out by Nekrasov
- 5d N=1 hypermultiplet of arbitrary representation ?
- We give the systematic derivation of the contour using ADHM QM
- related to evaluation of 2d elliptic genus using Jeffrey-Kirwan residue via dimensional reduction (Benini, Eager, Hori, Tachikawa I, II) + some additional subtleties
- Thereby establishing the basic tools to explore various 5d, 6d SCFTs
- Related work: Hori, Kim, Yi and Cordova, Shao

ADHM Quantum Mechanics

- Interested in 5d N=1 gauge theories (8 supercharges) and their instantons
- topological $U(1)_T$ charge
  $k = \int TrF \wedge F \in Z$
- These are particles in 5d
- Instantons preserve 1/2 of SUSY $\bar{Q}^A_{\dot{\alpha}}$ (0,4) SUSY
- could be bound states of elementary partcles and instantonic particles
  $M = \frac{k}{g^2_{YM}} + Trv\Pi$
- v scalar vev of Coulomb branch, Π electric charge

ADHM Quantum Mechanics as D0-D4 systems

- (0, 4) SUSY Quantum Mechanics obtained from (0,4) 2D SUSY
- Multiplet structure follows from 2D
- D4 Gauge group G= U(N), Sp(N), SO(N)
- D0 Gauge group $\hat{G} = U(k), O(k), Sp(k)$, k being instanton number
- D0-D4 bifundamental hypers of $G \times \hat{G}$
- D0-D0 adj, symmetric, antisymmetric hypers $\hat{G} = U(k), O(k), Sp(k)$ position moduli of instantons
- D0-D8 : Fermi multiplets



Witten Index of ADHM Quantum Mechanics

- Compute the index $\leftarrow$ (0,2) elliptic genus of 2D
- $Z_{inst} = \sum Z_k q^k$

$$Z^k_{\mathrm{QM}}(\epsilon_1, \epsilon_2, \alpha_i, z) = \mathrm{Tr}\left[(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{-\epsilon_1(J_1+J_R)} e^{-\epsilon_2(J_2+J_R)} e^{-\alpha_i \Pi_i} e^{-z\cdot F}\right]$$

- J_1, J_2 Cartans of SO(4) J_R Cartan of $SU(2)_I$
  α_i chemical potential of electric charge Π_i, F flavor symmety

Witten Index of ADHM Quantum Mechanics continued

- zero mode integral A_t holonomy and scalar ϕ in the vector multiplet
- $\phi^I = \varphi^I + iA^I$ give rise to cylinder geometry $I = 1 \ldots rankG$

$$Z = \frac{1}{|W|}\oint e^{\kappa \mathrm{tr}(\phi)} Z_{\text{1-loop}} = \frac{1}{|W|}\oint e^{\kappa \mathrm{tr}(\phi)} Z_V \prod_{\Phi} Z_\Phi \prod_{\Psi} Z_\Psi$$

- (0,2) vector $\quad Z_V = \prod_{\alpha \in \mathrm{root}} 2\sinh\frac{\alpha(\phi)}{2} \prod_{I=1}^{r} \frac{d\phi_I}{2\pi i}$
- (0,2) Fermi $\quad Z_\Psi = \prod_{\rho \in R_\Psi} 2\sinh\left(\frac{\rho(\phi) + J\epsilon_+ + Fz}{2}\right)$
- (0,2) chiral $\quad Z_\Phi = \prod_{\rho \in R_\Phi} \frac{1}{2\sinh\left(\frac{\rho(\phi)+J\epsilon_+ + F\cdot z}{2}\right)}$
- (0,4) hyper= 2 (0,2) chiral

- Dangerous regions
  a.two noncompact regions of cylinder
  b. poles from chiral multiplets $\rho(\phi_*) + J\epsilon_+ + Fz = 0$
- poles from chiral multiplets; regularized by D terms and work out the pole regions carefully

Multi-dimensional residues

- Given by Jeffrey-Kirwan residues
- Choose $\eta = -\zeta$ (FI) if needed

$$\text{JK-Res}(\mathbf{Q}_*, \eta) \frac{d\phi_1 \wedge \cdots \wedge d\phi_r}{Q_{j_1}(\phi) \cdots Q_{j_r}(\phi)} = \begin{cases} |\det(Q_{j_1}, \cdots, Q_{j_r})|^{-1} & \text{if } \eta \in \text{Cone}(Q_{j_1}, \cdots, Q_{j_r}) \\ 0 & \text{otherwise} \end{cases}$$

$$Z = \frac{1}{|W|} \sum_{\phi_*} \text{JK-Res}(\mathbf{Q}(\phi_*), \eta)\ Z_{\text{1-loop}}(\phi, \epsilon_+, z)$$

Heuristic Derivation of Contour

- $exp(Q_i\phi) = exp(-J_i\epsilon_+)$
  For vectors $J_i > 0$ residue is inside the unit circle for positively charged fields (Nekrasov)
  For hypers $J_i < 0$ residue is outside the unit circle for positively charged fields (Observed by Hollands, Keller, Song)
- These prescriptions agree with the the above derivations from ADHM QM
- For 5d $N = 1^* U(N)$ theory, the contribution from hypers are vanishing and only vectors contribute

Dependence on FI parameters; Wall Crossing?

- BPS states depend on the sign of FI parameters?
- BPS states have dependence on neither $SU(2)_R$ nor $SU(2)_I$
- no dependence on FI parameter
- related to the pole behavior at the infinities of cylinder (different from 2D)
- at the infinities of cylinders Coulomb branch can be developed and D0 can escape
- not a field theory degrees of freedom
- $Z = Z_{QFT} Z_{string}$
- all FI dependence comes from Z_{string}
- cf. (dimensional reduction of)(2,2) theory does have FI dependence -> Wall crossing (Hori, Kim, Yi ; Cordova, Shao)

Applications

- We apply the formalism to N=1 5d $Sp(N)$ theory with $N_f = 6, 7, 8$
- For $N_f = 6, 7$ we obtain the 5d superconformal index with E_7, E_8 symmetry
  (Improvement over Kim, Kim, Lee)
- For $N_f = 8$ we recover the E-string elliptic genus

Conclusions

- We derive the Nekrasov instanton partition in a systematic way using ADHM QM
- This holds for classical gauge groups with arbitrary matter representations
- With basic tools available, one can explore conformal zoo in 5d, 6d SCFTs
- For 6d (0,1) E_8 theory, we propose the tensionless string as IR limit of the related 2D gauge theory
- Obviously this method should work for many cases
- Good starting point for exploring details beyond the elliptic genus