Strings from a Higher Spin Perspective

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based mainly on

MRG, R. Gopakumar, 1501.07236 and work in progress
MRG, C. Peng, I.G. Zadeh, 1506.02045

Tensionless string theory

At tensionless point in moduli space, stringy excitations become massless: infinite number of massless higher spin fields, which generate a very large gauge symmetry

maximally unbroken phase of string theory

[Gross, ’88], [Witten, ’88]
[Moore, ’93], [Sagnotti, ’02..]

Tensionless string theory

A subsector of these massless higher spin fields forms a vector-like Vasiliev higher spin theory.

[Fradkin & Vasiliev, ’87], [Vasiliev, ’99…]
[Sundborg, ’01], [Witten, ’01], [Mikhailov, ’02],
[Klebanov & Polyakov, ’02], [Sezgin & Sundell, ’03..]

Understand String Theory using this Higher Spin Perspective!

Plan of talk

- Explicit realisation of idea
- Regge trajectories
- The Higher Spin Square
- String Theory as a HS theory

AdS3 example

[MRG, Gopakumar, '14]

Concrete realisation of this idea in context of AdS3:
CFT dual of string theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at tensionless point is

$$\mathrm{Sym}\big(\mathbb{T}^4\big) \equiv \big(\mathbb{T}^4\big)^{\otimes(N+1)} / S_{N+1}$$

$$\cup$$

$$\mathcal{W}_{\infty}^{(\mathcal{N}=4)}[0]$$

CFT dual of Vasiliev higher spin theory on AdS3

Symmetric orbifold

[MRG, Gopakumar, '14,'15]

Symmetric orbifold theory contains permutation symmetric combinations of the generators

$$\begin{array}{cc} \partial\phi^{ia} & \partial\bar{\phi}^{ia} \\ \psi^{ia} & \bar{\psi}^{ia} \end{array} \qquad i = 1, 2\ , \quad a = 1, \ldots, N+1$$

as well as the corresponding right-movers.

Single particle symmetry generators are then

$$\sum_a P_1^a \cdots P_m^a \qquad P_j \in \{\partial^\# \phi^i,\ \partial^\# \bar{\phi}^i,\ \partial^\# \psi^i,\ \partial^\# \bar{\psi}^i\}$$

Note: counted by partition function of $\mathbb{T}^4$.

Higher Spin Perspective

The generators of the $\mathcal{W}_\infty^{(\mathcal{N}=4)}[0]$ algebra that are dual to the higher spin fields of the Vasiliev theory consist of the subset of U(N) invariants

$$\sum_a P^a \ , \quad \sum_a \bar{P}^a$$

$$\sum\nolimits_a' P^a \bar{P}^a$$

$$P \in \{\partial^\# \phi^i, \partial^\# \psi^i\} \ , \quad \bar{P} \in \{\partial^\# \bar{\phi}^i, \partial^\# \bar{\psi}^i\}$$

Thus can organise the symmetry generators of the symmetric orbifold in terms of representations of the

$\mathcal{W}_\infty^{(\mathcal{N}=4)}[0]$ algebra.

Stringy algebra

The relevant decomposition is [MRG, Gopakumar, '15]

$$\mathcal{A}_{\rm sym.orb.} = \mathcal{W}_\infty^{(\mathcal{N}=4)}[0] \;\oplus\; \sideset{}{'}\bigoplus_{n,\bar{n}} \left(0;[n,0,\ldots,0,\bar{n}]\right)$$

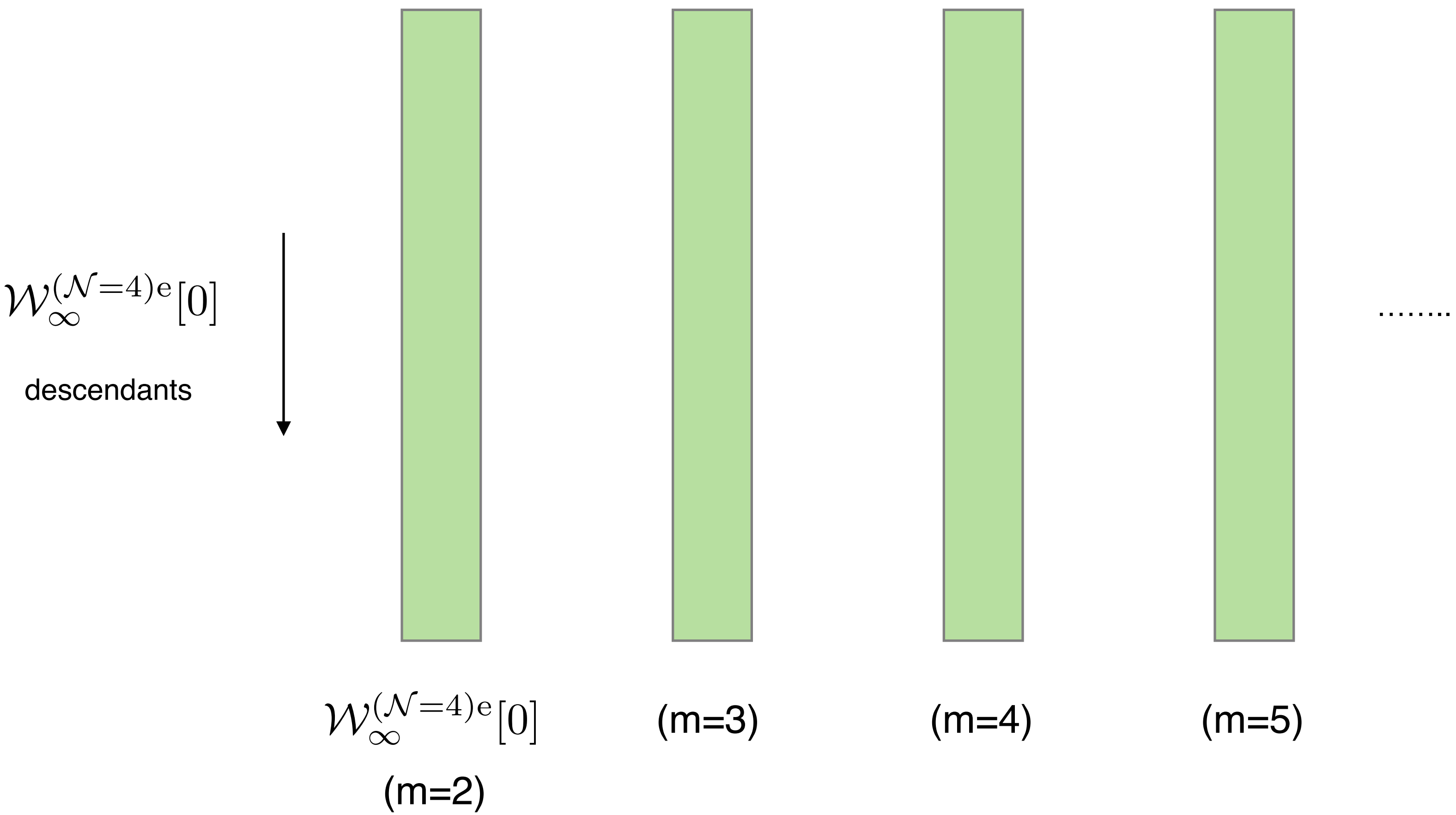
symmetric orbifold generators with

$$\#(P)=n\ , \quad \#(\bar{P})=\bar{n}$$

Convenient to collect together the generators according to total number of free fields $(m=n+\bar{n})$

$$\mathcal{A}_{\rm sym.orb.} = \mathcal{W}_\infty^{(\mathcal{N}=4){\rm e}}[0] \;\oplus\; \bigoplus_{m\geq 3} (0;m)$$

Column decomposition



Regge trajectories

This seems to correspond to the decomposition in terms of the different `Regge trajectories':

(m=2) quadratic terms = original higher spin fields

= leading Regge trajectory

(m=3) cubic terms = first subleading Regge trajectory

(m=4) quartic terms = 2nd subleading Regge trajectory

etc…

Tension perturbation

[MRG, Peng, Zadeh, '15]

Study perturbation by exactly marginal operator that corresponds to switching on string tension — SO(4) inv. combination of moduli from 2-cycle twisted sector.

Anomalous dimension from diagonalisation of mixing matrix

$$\boxed{\gamma^{ij} = \langle \mathcal{N}(W^i)\,,\,\mathcal{N}(W^j)\rangle}$$

where

$$\mathcal{N}(W^{(s)}) = \sum_{l=0}^{\lfloor s+h_\Phi \rfloor -1} \frac{(-1)^l}{l!}\,(L_{-1})^l W^{(s)}_{-s+1+l}\,\Phi \longleftarrow \text{perturbing field}$$

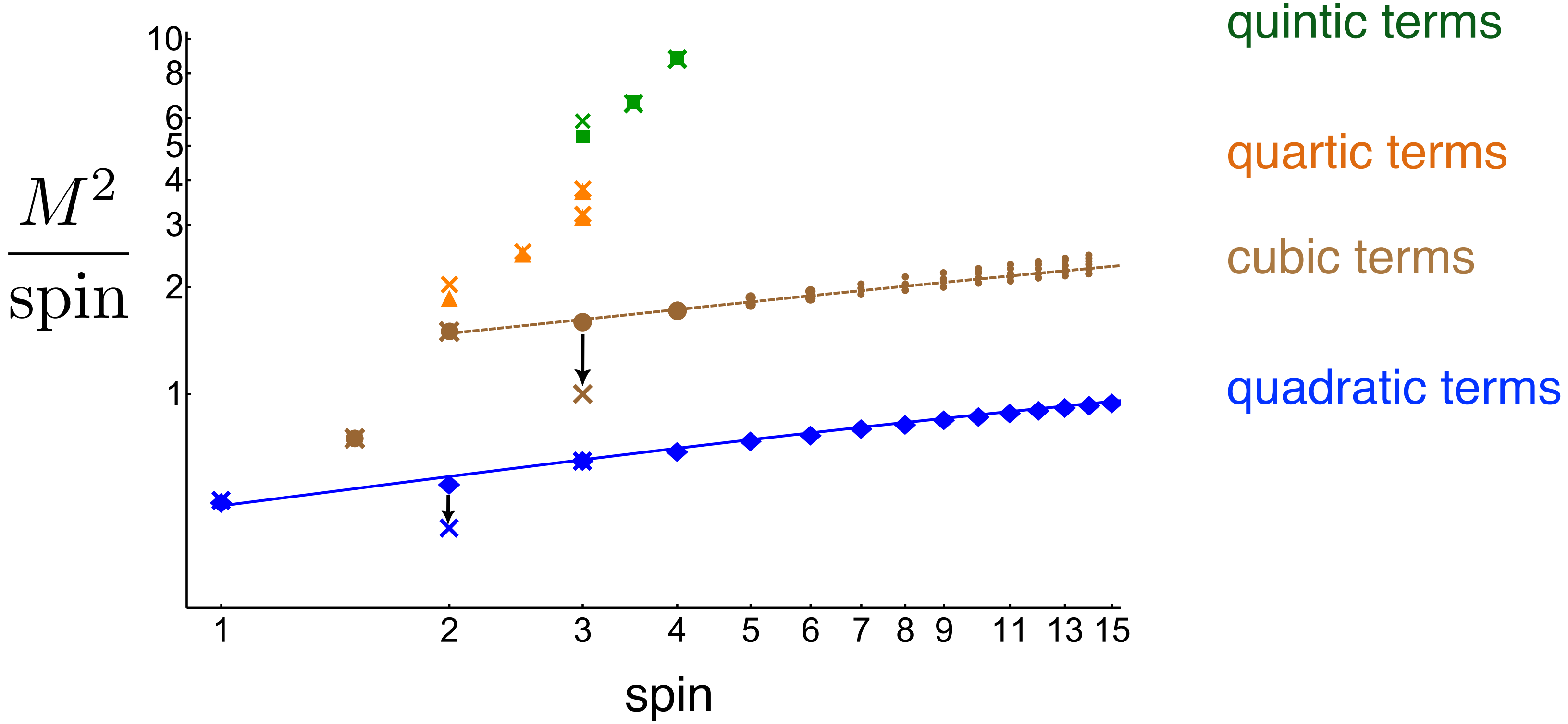
[Note: $\partial_{\bar{z}} W^{(s)} = g\,\pi\,\mathcal{N}(W^{(s)})$.]

cf. [Aharony, Clark, Karch, '06]

Explicit results

[MRG, Peng, Zadeh, ‘15]

Explicitly we find that



Mixing and log behaviour

[MRG, Peng, Zadeh, '15]

Generically, fields of the same unperturbed spin (incl. those from different columns) will mix, and the full diagonalisation problem is complicated.

However, have good estimate for diagonal entries for the original higher spin fields,

$$\gamma^{(s)} \simeq a \log s$$

and hence for associated dispersion relation

$$E(s) \simeq s + a \log s$$

RR background

Since this is of the form

$$E(s) \simeq s + a \log s + 0 \cdot (\log s)^2$$

suggests that AdS3 background has pure RR flux.

[Loewy, Oz, '03]
[David, Sadhukhan, '14]

This is also in line with situation for AdS5….

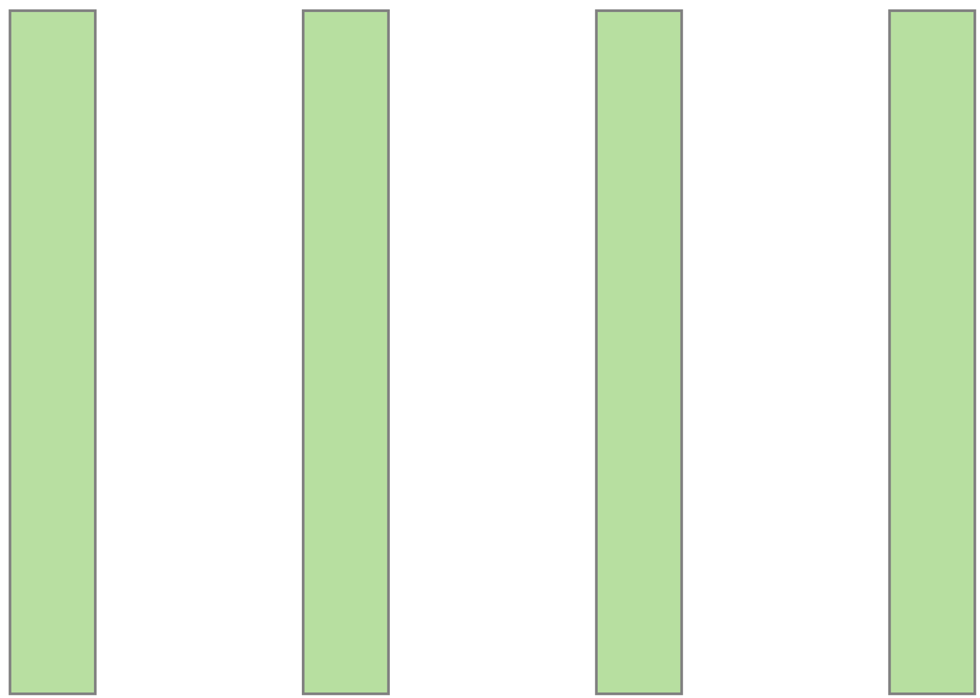
[Gubser, Klebanov, Polyakov '02], [Frolov, Tseytlin, '02]
[Roiban, Tirziu, Tseytlin, '07], [Roiban, Tseytlin, '07],…

Symmetric orbifold symmetry

Recall:

$\mathcal{W}_{\infty}^{(\mathcal{N}=4)\mathrm{e}}[0]$

descendants



....

$\mathcal{W}_{\infty}^{(\mathcal{N}=4)\mathrm{e}}[0]$ (m=2)

(m=3)

(m=4)

(m=5)

Horizontal product

[MRG, Gopakumar, '15]

In order to understand the full product structure, consider bosonic toy model, where

$$P \in \{\partial^{\#}\phi\}$$

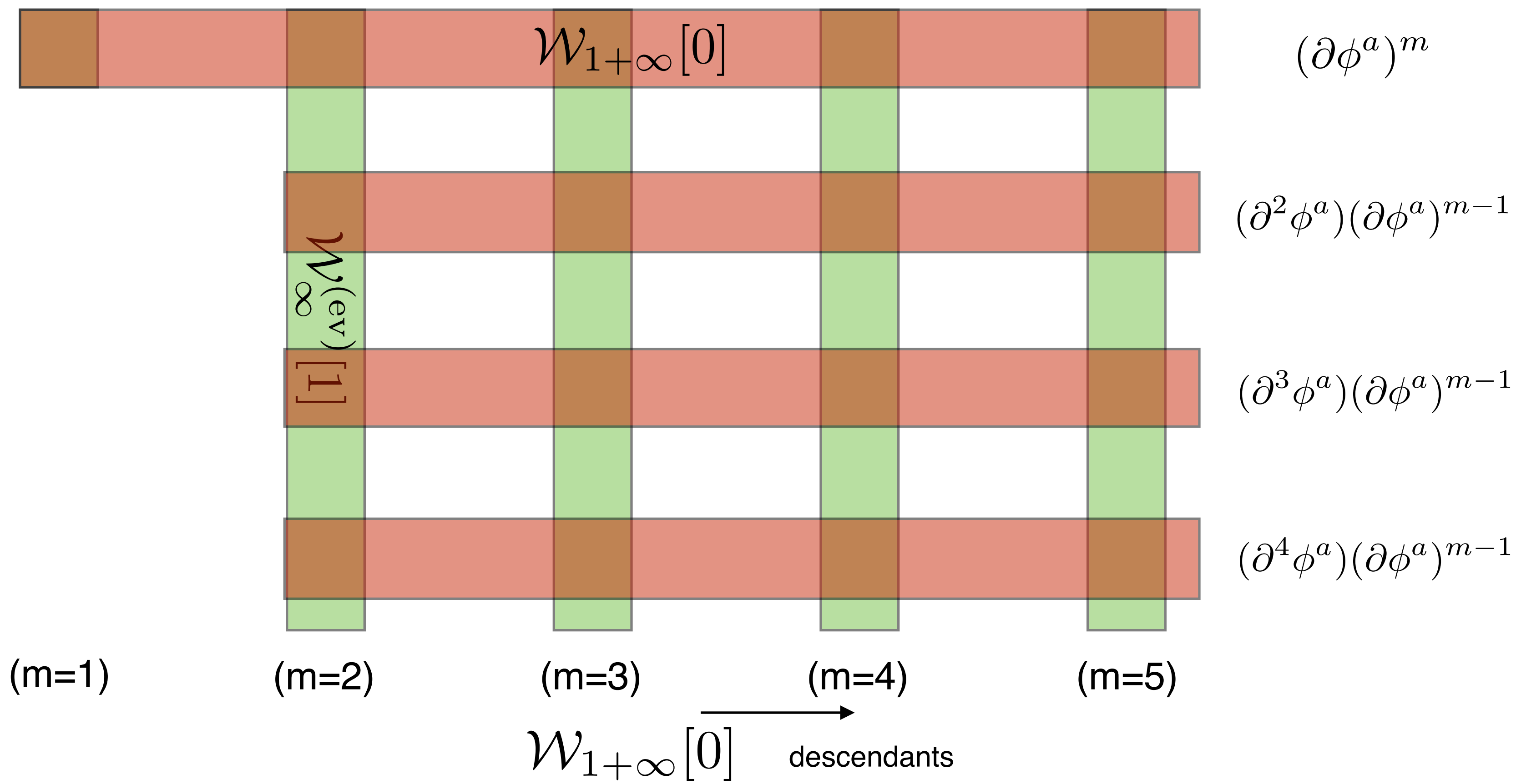
Then `vertical' W-algebra is $\mathcal{W}_{\infty}^{(\mathrm{ev})}[1]$.

Because of bosonisation/fermionisation, the sym. orb. algebra contains in this case a second higher spin algebra

$$\mathcal{W}_{1+\infty}[0] \qquad \text{generated by} \qquad \sum_a (\partial\phi^a)^m + \cdots$$

Higher Spin Square

[MRG, Gopakumar, ‘15]



The Higher Spin Square

The full stringy algebra is then generated by successive commutators of the two algebras

$$\mathcal{W}_{\infty}^{(\mathrm{ev})}[1] \qquad \text{and} \qquad \mathcal{W}_{1+\infty}[0]$$

Thus the full Lie algebra structure is fixed by this

Higher Spin Square

Clifford algebra analogy

An analogous finite-dimensional toy model is the gamma matrix algebra

$$\{\gamma^i, \gamma^j\} = 2\delta^{ij} \ , \quad (i, j = 1, \ldots, 2N)$$

We can in particular form the combinations

$$H^{(k)} : \quad \gamma^{[i_1}\gamma^{i_2} \ldots \gamma^{i_k]}$$

that transform in k'th antisymmetric tensor rep of

$$\mathfrak{so}(2N) \ \cong \ H^{(2)} \quad \Rightarrow$$

vertical decomposition

Horizontal product

Horizontal product is given by Clifford commutators that have the schematic structure

$$[H^{(k)}, H^{(m)}] = \sum_{r=\max(0,\frac{k+m}{2}-N)}^{\min(k,m)} \left(1-(-1)^{r+km}\right) H^{(k+m-2r)}$$

Resulting full Lie algebra is $\mathfrak{su}(2^N)$ — since the gamma matrices generate the full matrix algebra.

→ exhibits exponential growth of `stringy' algebra

Global Lie algebra

[MRG, Gopakumar, in progress]

For the bosonic case, the underlying global symmetry algebra (=wedge algebra) is explicitly generated by

$$S^{(s;m)}_{|n|} = \sum f^{(s-m)}(n_1, \ldots, n_m) \; : \alpha_{n_1} \cdots \alpha_{n_m} :$$

$$|n| = n_1 + \cdots + n_m$$

$$|n| \leq s-1$$

totally symmetric polynomial of degree s-m, such that non-zero terms have

$$n_1 \leq 0 \;, \quad n_m \geq 0$$

Differential algebra

Can think of the generators as

$$\begin{aligned} \alpha_n &\quad\longleftrightarrow\quad \partial_{z_n} \\ \alpha_{-n} &\quad\longleftrightarrow\quad n \cdot z_n \end{aligned} \qquad (n > 0)$$

Thus the stringy algebra is generated by the differential operators

$$z_{n_1} \cdots z_{n_p} \, \partial_{z_{n_1}} \cdots \partial_{z_{n_q}}$$

Geometric interpretation?

The Higher Spin Square

A more elaborate version of this construction works also for the $\mathcal{N}=4$ case.

The resulting structure is also somewhat reminiscent of a Yangian symmetry….

[Dolan, Nappi, Witten, ’03, ’04]
[Drummond, Henn, Plefka, ’09]

Perturbative HS

Perturbative part of Vasiliev HS theory is dual to 2d CFT partition function

$$Z_{\text{pert}} = \sum_{\Gamma} |Z_{\Gamma}|^2$$

=

$$Z_{\text{hs}} \cdot \prod_{r,\bar{r}=0}^{\infty} \frac{1}{(1 - q^{h+r}\bar{q}^{h+\bar{r}})}$$

sum over Young diagrams

(anti-)symmetrised powers of `minimal' CFT rep

=

corresponds to scalar field

[MRG, Gopakumar, Hartman, Raju,'11]

String Theory as HS

[MRG, Gopakumar, in progress]

Untwisted sector of symmetric orbifold has a similar structure

$$Z_{\mathrm{U}} = \sum_{\Gamma} \left| Z_{\Gamma}^{\mathrm{ext}} \right|^2$$

sum over
Young diagrams

(anti-)symmetrised powers
of `minimal' irrep of
stringy algebra

Thus untwisted sector has structure of perturbative `higher spin theory' for stringy higher spin algebra.

Twisted Sector

We are currently trying to understand structure of twisted sector from this viewpoint, i.e., as representation of the global stringy algebra.

In any case suggests that higher spin perspective is useful way to organise symmetric orbifold, i.e., CFT dual of string theory on AdS3 at tensionless point.

[MRG, Gopakumar, in progress]

Summary

- Stringy symmetry at tensionless point in AdS3 consists of HS algebra together with subleading Regge trajectories

- Lie algebra structure fixed by Higher Spin Square; described by differential algebra

- String Theory has structure of a HS theory w.r.t to global stringy algebra — together with twisted sectors…

Open problems & future directions

HS viewpoint: new perspective on stringy CFT

- Find Lie algebra structure of stringy symmetry
- Understand representation of twisted sector
- higher dimensional analogue?

cf. [Chang, Minwalla, Sharma, Yin ’12]
cf. [Beisert,Bianchi,Morales,Samtleben ’04]

Open problems:

- Interpretation from D1-D5 viewpoint
- Relation to spin chain picture

[Babichenko, Stefanski, Zarembo ‘09]
[Borsato, Ohlsson Sax, Sfondrini, Stefanski ‘14]

Thank You