

Bootstrapping the $(2, 0)$ theories in six dimensions

Balt van Rees

CERN / Durham

25 June 2015

together with C. Beem, M. Lemos, P. Liendo, W. Peelaers, L. Rastelli

The $(2, 0)$ theories in six dimensions

The $(2, 0)$ theories are six-dimensional conformal field theories with the maximal amount of supersymmetry (16 Q 's and 16 S 's). They are conventionally described using string- or M-theory.

[Witten (1995)]

We do not know of a standard field-theoretic definition, which severely limits our understanding of these theories.

Nevertheless, these theories are the “mother” of many lower dimensional field theories and of tremendous interest for the general study of supersymmetric field theories in $d < 6$.

[Witten (1995), Gaiotto (2009), ...]

The $(2, 0)$ theories in six dimensions

Some common lore:

- They are local and unitary quantum field theories with $\mathfrak{osp}(8^*|4)$ superconformal invariance. [Seiberg, Witten (1995)]
- The known interacting theories are classified by a simply-laced Lie algebra $\mathfrak{g} \in \{A_n, D_n, E_n\}$. [Witten (1995)]
- They are isolated: there are no marginal deformations that preserve $\mathfrak{osp}(8^*|4)$.
- The large n theories can be described through AdS/CFT.

The $(2, 0)$ theories in six dimensions

- They are local and unitary quantum field theories with $\mathfrak{osp}(8^*|4)$ superconformal invariance.

In this sense, they are like any other conformal field theory: we have an infinite set of local operators \mathcal{O}_i , transforming in unitary irreducible representations of the (super)conformal algebra. Its correlation functions satisfy the (super)conformal Ward identities, for example

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(y) \rangle = \frac{\delta_{ij}}{|x - y|^{2\Delta_i}},$$

and there exists an *operator product expansion* or OPE

$$\mathcal{O}_i(x) \mathcal{O}_j(y) \simeq \sum_k \lambda_{ij}^k C[x - y, \partial_y] \mathcal{O}_k(y)$$

which has finite radius of convergence.

Until last year, we did not know of any nontrivial λ_{ij}^k for finite n .

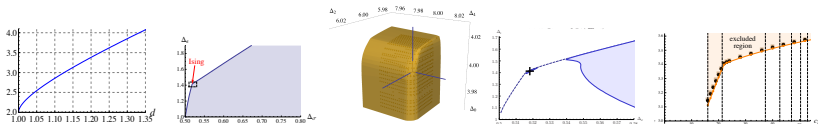
The (2, 0) theories in six dimensions

The (quantum numbers of the) \mathcal{O}_i and the OPE coefficients λ_{ij}^k are subject to the constraints of *crossing symmetry*.

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle$$

$$= \sum_k \lambda_{12}^k \lambda_{34}^k \text{ (diagram with internal line } k \text{)} = \sum_p \lambda_{13}^p \lambda_{24}^p \text{ (diagram with internal line } p \text{)}$$

The idea of the bootstrap is to exploit these constraints with the aspiration that they might completely determine the theory. We have made much progress in putting these constraints to use in higher-dimensional field theories.



[Rattazzi, Rychkov, Tonni, Vichi (2008); many others]

The $(2, 0)$ theories in six dimensions

It is therefore natural to ask

*What does crossing symmetry tell us about
six-dimensional $(2, 0)$ theories?*

Would it be possible to:

- Constrain the space of all theories?
- Find the spectrum for the allowed theories?
- Find the OPE coefficients?

The remainder of this talk discusses some initial results in trying to answer these questions.

Outline

- 1 Introduction
- 2 Analytical results
- 3 Numerical results
- 4 Conclusions

Outline

- 1 Introduction
- 2 Analytical results
- 3 Numerical results
- 4 Conclusions

Classification of operators

The local operators \mathcal{O}_i transform in unitary irreducible highest-weight representations of $\mathfrak{osp}(8^*|4)$ with maximal bosonic subalgebra $\mathfrak{so}(6, 2) \times \mathfrak{so}(5)_R$.

They are therefore labelled by $\Delta, [d_1, d_2, d_3], [b_1, b_2]$.

Examples:

- The free tensor multiplet with conformal primaries

| | | | |
|---------------------------------------|----------------|-------------|----------|
| 5 scalars: $\Phi^{\mathbf{a}}$ | $\Delta = 2$ | $[0, 0, 0]$ | $[1, 0]$ |
| 4 fermions: $\Psi_{\alpha\mathbf{A}}$ | $\Delta = 5/2$ | $[1, 0, 0]$ | $[0, 1]$ |
| 1 s.d. three-form: $H_{\mu\nu\rho}$ | $\Delta = 3$ | $[2, 0, 0]$ | $[0, 0]$ |

- The half-BPS operators

$$\mathcal{O}_k^{\{\mathbf{a}_1 \dots \mathbf{a}_k\}}(x) \qquad \Delta = 2k \qquad [0, 0, 0] \qquad [k, 0]$$

They form a ring with generators that are in one-to-one correspondence with the Casimirs of \mathfrak{g} . For example, for the A_n theories we have generators with $k \in \{2, 3, \dots, n+1\}$.

Chiral correlation functions

Consider now a correlation function

$$\langle \mathcal{O}^{I_1}(x_1) \dots \mathcal{O}^{I_n}(x_n) \rangle$$

with the following properties.

- 1 Consider \mathbb{Q} -chiral operators satisfying

$$\Delta = d_1/2 + d_2 + 2b_1 \quad b_2 = d_3 = 0 \quad b_1 \neq 0$$

Such operators are the highest weights of a nontrivial $\mathfrak{su}(2) \subset \mathfrak{so}(5)_R$. We add the index I running over the entire $\mathfrak{su}(2)$ multiplet.

- 2 Take all n points to lie in a plane $\mathbb{R}^2 \subset \mathbb{R}^6$.
- 3 Contract the $\mathfrak{su}(2)$ indices with *position-dependent* $v_I(\bar{z})$.
For example, for a doublet $v(\bar{z}) = (1, \bar{z})$.

Claim: the resulting correlation function is *meromorphic*.

The six-dimensional $(2, 0)$ theories have a chiral algebra!

Chiral correlation functions

Consider now a correlation function

$$\langle \mathcal{O}^{I_1}(z_1, \bar{z}_1) \dots \mathcal{O}^{I_n}(z_n, \bar{z}_n) \rangle$$

with the following properties.

- 1 Consider \mathbb{Q} -chiral operators satisfying

$$\Delta = d_1/2 + d_2 + 2b_1 \quad b_2 = d_3 = 0 \quad b_1 \neq 0$$

Such operators are the highest weights of a nontrivial $\mathfrak{su}(2) \subset \mathfrak{so}(5)_R$. We add the index I running over the entire $\mathfrak{su}(2)$ multiplet.

- 2 Take all n points to lie in a plane $\mathbb{R}^2 \subset \mathbb{R}^6$.
- 3 Contract the $\mathfrak{su}(2)$ indices with *position-dependent* $v_I(\bar{z})$.
For example, for a doublet $v(\bar{z}) = (1, \bar{z})$.

Claim: the resulting correlation function is *meromorphic*.

The six-dimensional $(2, 0)$ theories have a chiral algebra!

Chiral correlation functions

Consider now a correlation function

$$v_{I_1}(\bar{z}_1) \dots v_{I_n}(\bar{z}_n) \langle \mathcal{O}^{I_1}(z_1, \bar{z}_1) \dots \mathcal{O}^{I_n}(z_n, \bar{z}_n) \rangle$$

with the following properties.

- 1 Consider \mathbb{Q} -chiral operators satisfying

$$\Delta = d_1/2 + d_2 + 2b_1 \quad b_2 = d_3 = 0 \quad b_1 \neq 0$$

Such operators are the highest weights of a nontrivial $\mathfrak{su}(2) \subset \mathfrak{so}(5)_R$. We add the index I running over the entire $\mathfrak{su}(2)$ multiplet.

- 2 Take all n points to lie in a plane $\mathbb{R}^2 \subset \mathbb{R}^6$.
- 3 Contract the $\mathfrak{su}(2)$ indices with *position-dependent* $v_I(\bar{z})$.
For example, for a doublet $v(\bar{z}) = (1, \bar{z})$.

Claim: the resulting correlation function is *meromorphic*.

The six-dimensional $(2, 0)$ theories have a chiral algebra!

Chiral correlation functions

Consider now a correlation function

$$\frac{\partial}{\partial \bar{z}_k} (v_{I_1}(\bar{z}_1) \dots v_{I_n}(\bar{z}_n) \langle \mathcal{O}^{I_1}(z_1, \bar{z}_1) \dots \mathcal{O}^{I_n}(z_n, \bar{z}_n) \rangle) = 0$$

with the following properties.

- 1 Consider \mathbb{Q} -chiral operators satisfying

$$\Delta = d_1/2 + d_2 + 2b_1 \quad b_2 = d_3 = 0 \quad b_1 \neq 0$$

Such operators are the highest weights of a nontrivial $\mathfrak{su}(2) \subset \mathfrak{so}(5)_R$. We add the index I running over the entire $\mathfrak{su}(2)$ multiplet.

- 2 Take all n points to lie in a plane $\mathbb{R}^2 \subset \mathbb{R}^6$.
- 3 Contract the $\mathfrak{su}(2)$ indices with *position-dependent* $v_I(\bar{z})$.
For example, for a doublet $v(\bar{z}) = (1, \bar{z})$.

Claim: the resulting correlation function is *meromorphic*.

The six-dimensional $(2, 0)$ theories have a chiral algebra!

Chiral correlation functions

Claim:

$$\frac{\partial}{\partial \bar{z}_k} \langle v_{I_1}(\bar{z}_1) \mathcal{O}^{I_1}(z_1, \bar{z}_1) \dots v_{I_n}(\bar{z}_n) \mathcal{O}^{I_n}(z_n, \bar{z}_n) \rangle = 0$$

Proof:

- There exists a particular nilpotent supercharge \mathbb{Q} such that

$$[\mathbb{Q}, \mathcal{O}^1(0)] = 0.$$

for \mathbb{Q} -chiral operators. Roughly speaking $\mathbb{Q} = \mathcal{Q} - \mathcal{S}$.

- Holomorphic translations are \mathbb{Q} closed

$$[\mathbb{Q}, P_z] = 0$$

- In the antiholomorphic direction we find that

$$\partial_{\bar{z}} (v_I(\bar{z}) \mathcal{O}^I(z, \bar{z})) = v_I(\bar{z}) [P_{\bar{z}} + \mathcal{R}^-, \mathcal{O}^I(\bar{z})]$$

and such *twisted* antiholomorphic translations are \mathbb{Q} exact

$$P_{\bar{z}} + \mathcal{R}^- = \{\mathbb{Q}, \dots\}$$

and are trivial in the cohomology of \mathbb{Q} .

Example

Consider the free tensor multiplet where the scalar Φ^+ is \mathbb{Q} -chiral.

In this case I is a triplet index, and $v_I(\bar{z}) = (1 + \bar{z}^2, 2\bar{z}, i(1 - \bar{z}^2))/\sqrt{2}$.

The OPE is

$$\Phi^I(z, \bar{z})\Phi^J(0) \sim \frac{\delta^{IJ}}{(z\bar{z})^2}$$

so

$$v_I(\bar{z})v_J(0)\Phi^I(z, \bar{z})\Phi^J(0) \sim \dots = \frac{1}{z^2}$$

which is the OPE of a dimension one current. We write

$$[v_I(\bar{z})\Phi^I(z, \bar{z})]_{\mathbb{Q}} \rightsquigarrow j(z)$$

The other \mathbb{Q} -chiral operators are normal ordered products and holomorphic derivatives of this basic field.

→ The complete *chiral algebra* of a free tensor multiplet is the $u(1)$ AKM algebra generated from

$$j(z)j(0) \sim \frac{1}{z^2}$$

Chiral correlation functions

For the interacting theories:

- The $\frac{1}{2}$ -BPS operators are part of the chiral algebra;
- Generators of the $\frac{1}{2}$ -BPS chiral ring are also generators of the chiral algebra;
- The character of the chiral algebra is equal to a partially refined superconformal index computed by [Kim³, Lee (2009-2013)]. It indicates that there is no need for further generators.

We claim that the generators of the chiral algebra are in one-to-one correspondence with the Casimir invariants of \mathfrak{g} .

Conjecture: the chiral algebra for the $(2, 0)$ theories of type \mathfrak{g} is $W_{\mathfrak{g}}$.

From the twisted OPE of the stress tensor multiplet we find that

$$c_{2d} = c_{6d}$$

in conventions where $c_{6d} = 1$ for a free tensor multiplet.

Corollary: three-point functions of \mathbb{Q} -chiral operators are computable.

[Beem, Rastelli, BvR (2014)]

Chiral correlation functions

For the interacting theories:

- The $\frac{1}{2}$ -BPS operators are part of the chiral algebra;
- Generators of the $\frac{1}{2}$ -BPS chiral ring are also generators of the chiral algebra;
- The character of the chiral algebra is equal to a partially refined superconformal index computed by [Kim³, Lee (2009-2013)]. It indicates that there is no need for further generators.

We claim that the generators of the chiral algebra are in one-to-one correspondence with the Casimir invariants of \mathfrak{g} .

Conjecture: the chiral algebra for the $(2, 0)$ theories of type \mathfrak{g} is $W_{\mathfrak{g}}$.

From the twisted OPE of the stress tensor multiplet we find that

$$c_{2d} = c_{6d}$$

in conventions where $c_{6d} = 1$ for a free tensor multiplet.

Corollary: three-point functions of \mathbb{Q} -chiral operators are computable.

[Beem, Rastelli, BvR (2014)]

Chiral correlation functions

For the interacting theories:

- The $\frac{1}{2}$ -BPS operators are part of the chiral algebra;
- Generators of the $\frac{1}{2}$ -BPS chiral ring are also generators of the chiral algebra;
- The character of the chiral algebra is equal to a partially refined superconformal index computed by [Kim³, Lee (2009-2013)]. It indicates that there is no need for further generators.

We claim that the generators of the chiral algebra are in one-to-one correspondence with the Casimir invariants of \mathfrak{g} .

Conjecture: the chiral algebra for the $(2, 0)$ theories of type \mathfrak{g} is $W_{\mathfrak{g}}$.

From the twisted OPE of the stress tensor multiplet we find that

$$c_{2d} = c_{6d}$$

in conventions where $c_{6d} = 1$ for a free tensor multiplet.

Corollary: three-point functions of \mathbb{Q} -chiral operators are computable.

[Beem, Rastelli, BvR (2014)]

Three-point functions

OPE coefficients in the chiral algebra determine certain OPE coefficients in six dimensions. Consider for example the three-point functions of the half-BPS operators:

$$\langle \mathcal{O}_{k_1}^{\{\mathbf{a}_1 \dots \mathbf{a}_{k_1}\}}(x_1) \mathcal{O}_{k_2}^{\{\mathbf{b}_1 \dots \mathbf{b}_{k_2}\}}(x_2) \mathcal{O}_{k_3}^{\{\mathbf{c}_1 \dots \mathbf{c}_{k_3}\}}(x_3) \rangle = \frac{\lambda_g(k_1, k_2, k_3) \mathfrak{C}^{\mathbf{a}_1 \dots \mathbf{c}_{k_3}}}{x_{12}^{2k_{12}} x_{13}^{2k_{13}} x_{23}^{2k_{23}}}$$

The $\lambda_g(k_1, k_2, k_3)$ are completely determined by the chiral algebra.

Corollary: $\lambda_g(k_1, k_2, k_3)$ are three-point functions of the W_g currents.

This works wonderfully at large n where (for the A_n type theories) we know from AdS/CFT that

$$\lambda_{n \rightarrow \infty}(k_1, k_2, k_3) = \frac{2^{2 \sum k_i - 2}}{\pi n^{3/2}} \Gamma\left(\frac{\sum k_i}{2}\right) \frac{\Gamma(\frac{k_{12}+1}{2}) \Gamma(\frac{k_{13}+1}{2}) \Gamma(\frac{k_{23}+1}{2})}{\sqrt{\Gamma(2k_1-1) \Gamma(2k_2-1) \Gamma(2k_3-1)}}$$

with $k_{12} = k_1 + k_2 - k_3$ etc. [Corrado et al; Bastianelli et al (1999)]

This agrees with the large n limit of W_n ! (note: $c_{2d} \sim 4n^3$)

[Gaberdiel et al; Campoleoni et al (2011)]

Three-point functions

OPE coefficients in the chiral algebra determine certain OPE coefficients in six dimensions. Consider for example the three-point functions of the half-BPS operators:

$$\langle \mathcal{O}_{k_1}^{\{\mathbf{a}_1 \dots \mathbf{a}_{k_1}\}}(x_1) \mathcal{O}_{k_2}^{\{\mathbf{b}_1 \dots \mathbf{b}_{k_2}\}}(x_2) \mathcal{O}_{k_3}^{\{\mathbf{c}_1 \dots \mathbf{c}_{k_3}\}}(x_3) \rangle = \frac{\lambda_g(k_1, k_2, k_3) \mathfrak{C}^{\mathbf{a}_1 \dots \mathbf{c}_{k_3}}}{x_{12}^{2k_{12}} x_{13}^{2k_{13}} x_{23}^{2k_{23}}}$$

The $\lambda_g(k_1, k_2, k_3)$ are completely determined by the chiral algebra.

Corollary: $\lambda_g(k_1, k_2, k_3)$ are three-point functions of the W_g currents.

This works wonderfully at large n where (for the A_n type theories) we know from AdS/CFT that

$$\lambda_{n \rightarrow \infty}(k_1, k_2, k_3) = \frac{2^{2 \sum k_i - 2}}{\pi n^{3/2}} \Gamma\left(\frac{\sum k_i}{2}\right) \frac{\Gamma(\frac{k_{12}+1}{2}) \Gamma(\frac{k_{13}+1}{2}) \Gamma(\frac{k_{23}+1}{2})}{\sqrt{\Gamma(2k_1-1) \Gamma(2k_2-1) \Gamma(2k_3-1)}}$$

with $k_{12} = k_1 + k_2 - k_3$ etc. [Corrado et al; Bastianelli et al (1999)]

This agrees with the large n limit of W_n ! (note: $c_{2d} \sim 4n^3$)

[Gaberdiel et al; Campoleoni et al (2011)]

Three-point functions

OPE coefficients in the chiral algebra determine certain OPE coefficients in six dimensions. Consider for example the three-point functions of the half-BPS operators:

$$\langle \mathcal{O}_{k_1}^{\{\mathbf{a}_1 \dots \mathbf{a}_{k_1}\}}(x_1) \mathcal{O}_{k_2}^{\{\mathbf{b}_1 \dots \mathbf{b}_{k_2}\}}(x_2) \mathcal{O}_{k_3}^{\{\mathbf{c}_1 \dots \mathbf{c}_{k_3}\}}(x_3) \rangle = \frac{\lambda_g(k_1, k_2, k_3) \mathfrak{C}^{\mathbf{a}_1 \dots \mathbf{c}_{k_3}}}{x_{12}^{2k_{12}} x_{13}^{2k_{13}} x_{23}^{2k_{23}}}$$

The $\lambda_g(k_1, k_2, k_3)$ are completely determined by the chiral algebra.

Corollary: $\lambda_g(k_1, k_2, k_3)$ are three-point functions of the W_g currents.

This works wonderfully at large n where (for the A_n type theories) we know from AdS/CFT that

$$\lambda_{n \rightarrow \infty}(k_1, k_2, k_3) = \frac{2^{2 \sum k_i - 2}}{\pi n^{3/2}} \Gamma\left(\frac{\sum k_i}{2}\right) \frac{\Gamma(\frac{k_{12}+1}{2}) \Gamma(\frac{k_{13}+1}{2}) \Gamma(\frac{k_{23}+1}{2})}{\sqrt{\Gamma(2k_1-1) \Gamma(2k_2-1) \Gamma(2k_3-1)}}$$

with $k_{12} = k_1 + k_2 - k_3$ etc. [Corrado et al; Bastianelli et al (1999)]

This agrees with the large n limit of $W_n!$ (note: $c_{2d} \sim 4n^3$)

[Gaberdiel et al; Campoleoni et al (2011)]

Outline

- 1 Introduction
- 2 Analytical results
- 3 Numerical results
- 4 Conclusions

Outline

- 1 Introduction
- 2 Analytical results
- 3 Numerical results**
- 4 Conclusions

Numerical results

We have learned a great deal about OPE coefficients of a subset of protected operators. What about other operators? What about the *unprotected operators*? Can we for example constrain their spectrum?

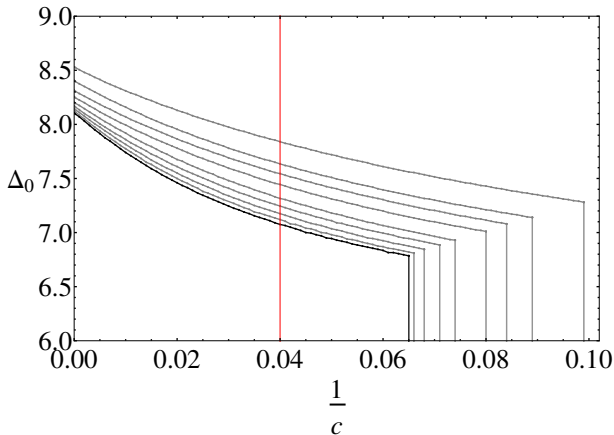
We will resort to numerical methods. I will give a snapshot of initial results, based on the crossing symmetry constraints and the methods of [Rattazzi, Rychkov, Tonni, Vichi (2008)].

We investigated the four point function of stress tensor multiplets. The superconformal Ward identities and superconformal block decomposition were essentially already solved in [Arutyunov, Sokatchev (2002); Dolan, Gallot, Sokatchev (2004); Heslop (2004)].

[Beem, Lemos, Rastelli, BvR (to appear)]

Numerical results

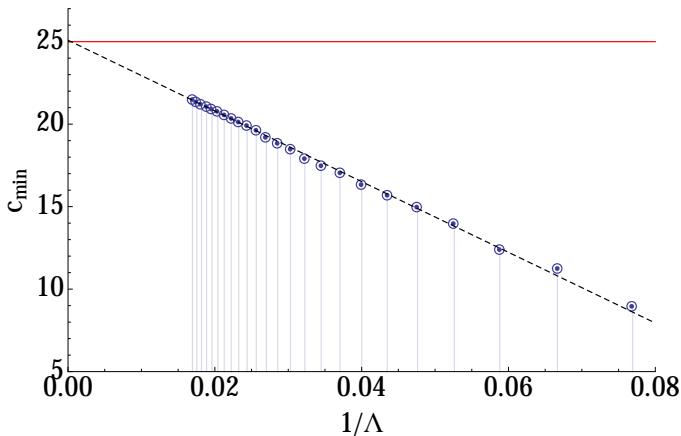
Upper bound Δ_0 for the dimension of the lowest-lying unprotected $\mathfrak{so}(5)_R$ singlet scalar operator.



- The bound is likely to improve with better numerics
- We recover the minimal value of c
- For very large c we find $\Delta_0 \lesssim 8.1$, in agreement with AdS/CFT.

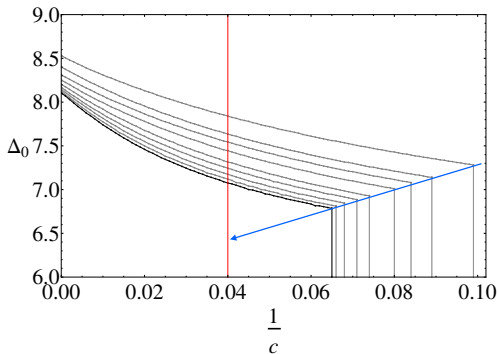
Numerical results

Lower bound on c (without higher spin currents).



The bound appears to converge to $c \simeq 25$!

Numerical Results

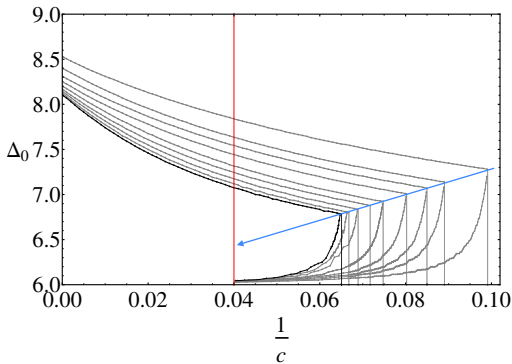


If the lower bound on c converges to 25, then

- the A_1 theory has an unprotected scalar of dimension $\Delta_0 \sim 6.4$
- this correlator is very likely *uniquely fixed* by crossing symmetry

→ quite possibly the A_1 theory can be *defined* as the only solution to the crossing symmetry equations at $c = 25$.

Numerical Results

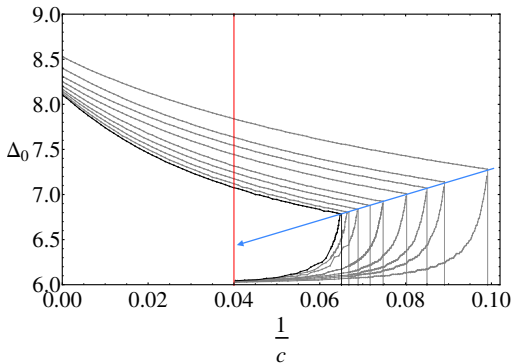


If the lower bound on c converges to 25, then

- the A_1 theory has an unprotected scalar of dimension $\Delta_0 \sim 6.4$
- this correlator is very likely *uniquely fixed* by crossing symmetry

→ quite possibly the A_1 theory can be *defined* as the only solution to the crossing symmetry equations at $c = 25$.

Numerical Results



If the lower bound on c converges to 25, then

- the A_1 theory has an unprotected scalar of dimension $\Delta_0 \sim 6.4$
- this correlator is very likely *uniquely fixed* by crossing symmetry

→ quite possibly the A_1 theory can be *defined* as the only solution to the crossing symmetry equations at $c = 25$.

Conclusions

We gave a snapshot of analytic and numerical results that follow from a study of crossing symmetry for the six-dimensional $(2, 0)$ theories.

- We find exact OPE coefficients through a chiral algebra
- We numerically constrain unprotected data
- Numerical evidence for uniqueness of the A_1 theory

Can we really bootstrap the $(2, 0)$ theories? Let's try!